

INTUITIVE LOOK:

Fourier Spectral Analysis *

by William T. Taylor

Introduction

Because many commodity and stock prices appear to exhibit some cyclical movement, it is important for the trader who relies on this characteristic feature to recognize these cycles and adjust the parameters of the particular trading system in use, accordingly. For example, the trader may wish to filter out or accentuate cycles of particular lengths using moving averages, Commodity Channel Index, or any number of popular trading systems.

Some cycles can be seen, more or less, on the price charts from commercial charting services or computer-generated charts from some of the popular microcomputer programs available to the trader. I say "more or less" because these charts show only a relatively short time period on each page, particularly if the data is on a daily basis.

Wouldn't it be nice if there were some technique for identifying and categorizing cyclical components by various frequencies and amplitudes, given actual price data over relatively long time periods? Well, there is such a technique. It is called Spectral Analysis.

Derived from the word spectrum, meaning a continuous sequence or range, Spectral Analysis is a statistical procedure used to evaluate a time series of data that produces an evaluation of the various cyclical components of that data. It is based on a mathematical concept called Fourier Analysis, named after the French mathematician Jean Baptiste Joseph Fourier. The fundamental theory of Fourier Analysis is that any time series is made up of components of sine and cosine wave forms of various frequencies and amplitudes. These components are summed for each point on the time series to produce a frequency/amplitude series of data, called a Fourier Series.

Consider Figure 1. Wave form A is a continuous cyclical function of time with a given frequency and amplitude.

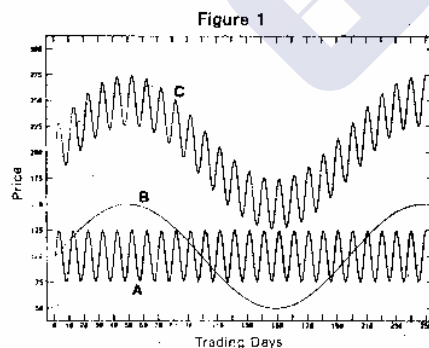
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Frequency is measured by the number of cycles over a given period of time, in this case, 25 over the period displayed on the graph. Amplitude is the height of the cycle, measured peak-to-peak. These properties will be explained further in the next section.

Wave form B is also a cyclical function of time, but with a different frequency and amplitude than that of A. In fact it goes through only one cycle in 200 units of time. A simple Fourier series can be constructed in the following way: If at every point of time, measured on the x-axis (horizontal) the value of wave form A is added to the value of wave form B, the resultant wave form is shown as C. The more components there are that make up a time series, the more convoluted that series appears.

Actual data may be quite convoluted, indicating that many components of sine and cosine functions are included in the composition of the data. Spectral Analysis can find these cycles and indicate their frequencies and relative amplitudes so that you, the trader, can determine the major, intermediate, and minor cycles of a particular stock or commodity.

The computation and statistical concepts that are used in this process require a very sophisticated mathematics background, which is why this technique is not widely used



CYCLES NOVEMBER 1985

by the average investor. This article attempts to remove as much of the mathematics as possible and concentrates on the interpretation of the results and the intuitive, common sense aspects of this useful tool.

The Nature of Cycles

Cycles have three basic properties, Frequency, Amplitude and Phase. Frequency is a measure of the number of cycles completed in a given time period. Alternatively, a cycle can be examined by its length, that is the number of units of time it takes to complete one full cycle. Frequency and length of the cycle measure basically the same things. They are merely the inverse of each other, that is:

$$\text{Frequency} = \frac{1}{\text{Length}}$$

$$\text{or Length} = \frac{1}{\text{Frequency}}$$

For example, if a cycle has a frequency of 12.5 cycles per year (250 days) the length is:

$$\text{Length} = \frac{1 \text{ Year}}{12.5 \text{ Cycles per Year}}$$

or 0.08 years. Note that 250 days is used to denote a year. This is because there are roughly 250 trading days in a year. Converting that to days is accomplished by multiplying 0.08 by 250, resulting in a cycle length of 20 trading days. The frequency of a cycle is measured in "cycles per time period." This period can be years, months, weeks or whatever time period is appropriate and meaningful for the particular analysis. As an alternative to frequency, the period or length of the cycle can be used. The units of measure can be days, weeks, months, or years, depending on the form of the input data. That is, if input is daily prices, then the cycle period or length is also measured in days; if input is weekly prices, then the cycle period is measured in weeks, and so on.

The second basic property of cycles is that of Amplitude. Figure 2 shows two cycles, each with the same frequency, but cycle B has an amplitude of twice that of cycle A. Amplitude is measured from the top of the cycle (the maximum value) to the bottom of the cycle (the minimum value). This measurement was referred to earlier as the "peak-to-peak" amplitude. In this case cycle B has an amplitude of 100 measured peak-to-peak whereas cycle A is only 50.

The third property of cycles is that of Phase. Phase is the positional relationship between one cyclical wave form and another. Cycles may either lead, lag or be coincident with each other. Figure 3 shows two cyclical wave forms of

Figure 2

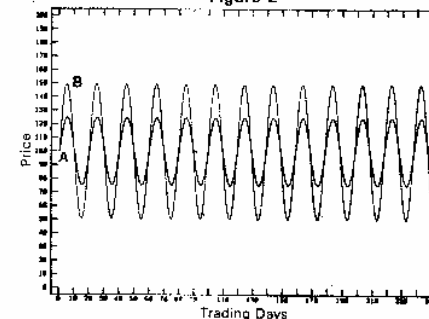
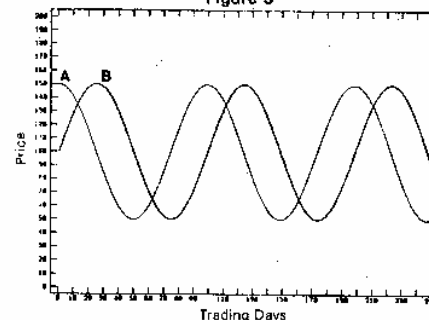


Figure 3



equal amplitude and frequency but cycle A leads cycle B. That is, A reaches its peak earlier in time than cycle B, alternatively cycle B is said to lag cycle A.

In order to develop a more intuitive feel for Spectral Analysis, cyclical time series (wave forms) with known properties, i.e., amplitude and frequency, will be run through a Spectral Analysis procedure. This allows the demonstration of changes in single properties while holding all the others constant. For example, frequency can be varied while holding amplitude constant.

A popular measure of the relative strength of cyclical components of a time period is the Periodogram. This measure is calculated from the coefficients of sine and cosine components of the time series measured at various frequencies. A more general measure of the relative strength in the cyclical components of a time series is the Spectral Density. This measure is a weighted version of the Periodogram, which provides, in many cases, a clearer indication of the cyclical components. Unless otherwise

CYCLES NOVEMBER 1985

stated, the Spectral Density will be used in the examples that follow. To the trader the most important thing to understand is how to interpret the results.

Simple Wave Forms

The following examples show the results of spectral analysis on input data with known properties. The first example, illustrated in Figure 2 curve A, shows a cyclical time series of fixed amplitude and cycle length, in this case, a cycle length of 20 days, or alternatively stated, a frequency of 12.5 cycles per year. Figure 4 shows the resultant spectral density graph. The period or length of the cyclical component of the time series is plotted on the x-axis (horizontal). The Spectral Density is plotted on the y-axis (vertical). As expected, the spectral density has its peak at a cycle length of 20 days.

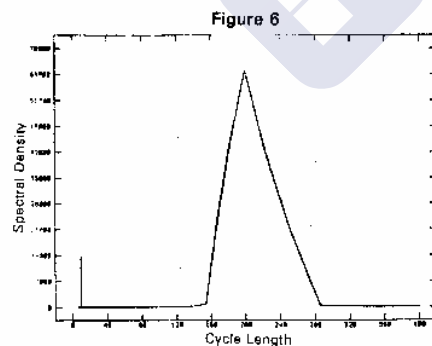
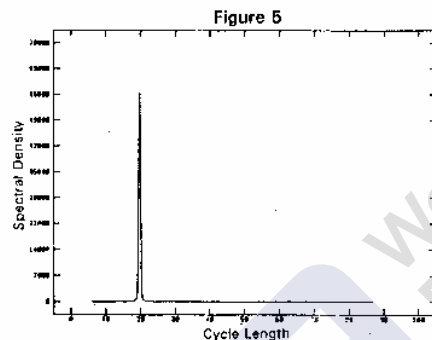
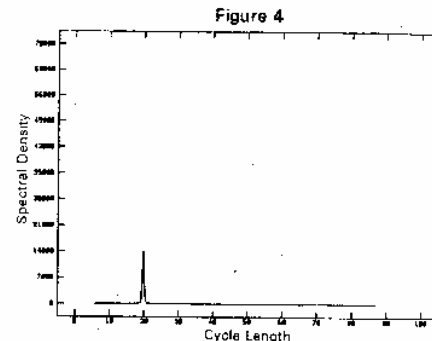
Consider now the cyclical time series whose amplitude is twice that in the previous example. Figure 2, curve B shows that the peaks occur at the same point in time, since they have the same frequency and phase. The amplitude is twice that of the original data.

When this data is run through the spectral analysis procedure, the results, shown in Figure 5, confirm that the only change is that in the magnitude of the spectral density. The peak occurs at the same cycle length, but the magnitude of the spectral density is four times that of Figure 4. The magnitude of both Spectral Density and Periodogram vary with the amplitude squared of the input, rather than proportionally.

What now can be generalized from the examples shown thus far? First of all we see that Spectral Analysis can identify cyclical components of a time series of data. Secondly, Spectral Density is highest, and "peaks" at the precise frequency or cycle length of the cyclical component. And thirdly, the magnitude of Spectral Density varies with the amplitude squared of the cyclical component.

Complex Wave Forms

The previous section demonstrated how Spectral Analysis could find the frequency of cyclical components in a time series of data. This section will extend the analysis by explaining time series that have more than one cyclical component. As was mentioned in the introduction, a Fourier series is comprised of several components of sine and cosine waves. Take, for example, cycles A and B in Figure 1. If at every point in time the amplitude of cycle A is added to the amplitude of cycle B, the result would be the Fourier series shown as C. This series C is run through the Spectral Analysis program resulting in the Spectral Density graph in Figure 6. Notice there are now two peaks, one for each cyclical component at 10 days and 200 days. (The apparent widthness of the 200 day component is an artifact of the display system used.)



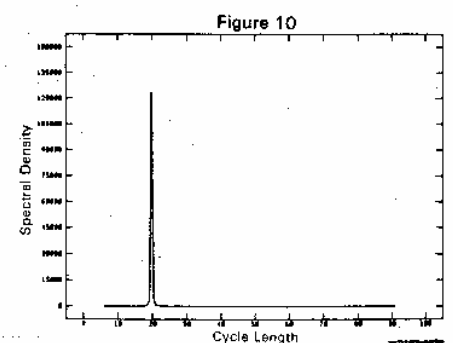
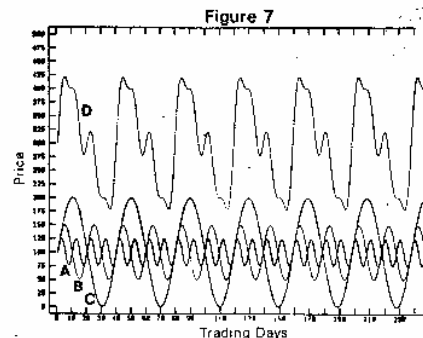
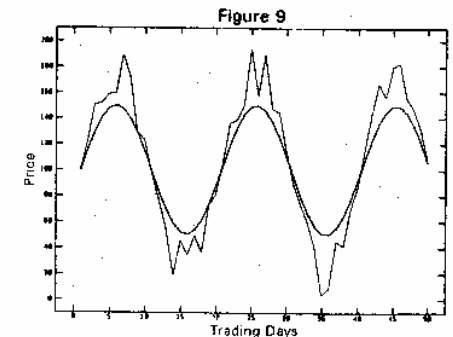
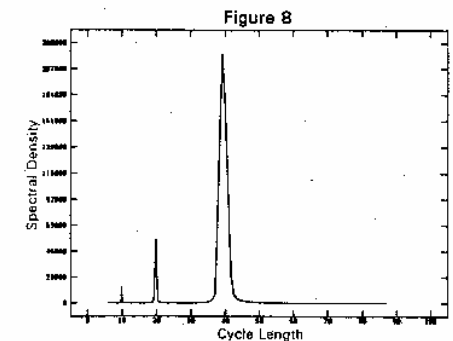
The general observation made above regarding the relative magnitudes of the "peaks" in the Spectral Density graph still holds. That is, they vary quadratically with the relative amplitudes of the cyclical components. This can be extended to a series with three components. Figure 7 shows three cyclical wave forms, each with different amplitudes and frequencies (or cycle lengths), represented by A, B, and C respectively. Wave form D, a Fourier series, is constructed by summing each of the amplitudes of the other three at each point in time. When this series D is run through Spectral Analysis, there are three peaks that result, one for each cyclical component. These are shown in Figure 8.

By now you should be getting the "feel" of what Spectral Analysis can do. In addition to the conclusions from the previous section, it has now been demonstrated that many cyclical components can be found by analysing a complex time series input. One can imagine a series consisting of hundreds of components, not all of which may be significant.

Effects of Random Components in the Data

Real data rarely behaves in a perfectly predictable manner and often contains random effects caused by unforeseen events or influences. The first type of random effect to be demonstrated is that of amplitude. Figure 9 shows one of the now familiar wave patterns with a cycle length of 20 days. Superimposed on this wave form is another, this one exhibiting random effects in amplitude. Note that the peaks and troughs are not always the same. The cycle length, or frequency, however, is precisely the same.

A quick look at the Spectral Density graph in Figure 10 shows the results of running the data with the random components of amplitude through the Spectral Analysis procedure. Because the cycle length is 20 days long in every



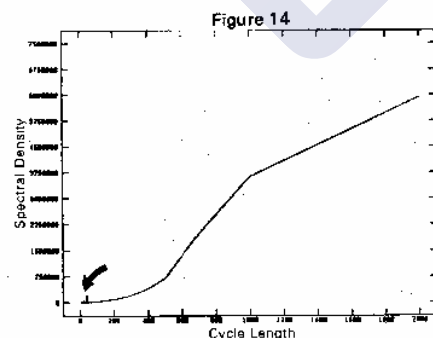
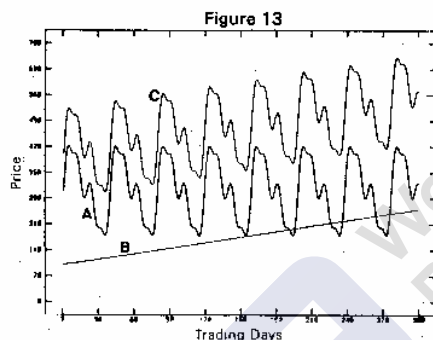
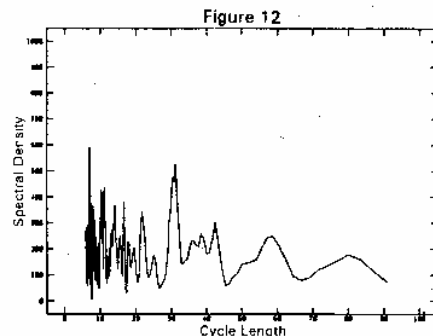
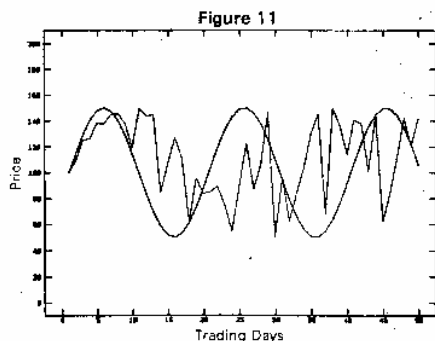
case, the indicated peak in Spectral Density occurs at precisely 20 days. We conclude that the random effects on amplitude will not have an effect on the position of the peak in Spectral Density. The magnitude of Spectral Density will vary depending on how much randomness is contained in the transform relative to the absolute amplitude of the cyclical component.

Random components affecting the frequency of the cyclical component, however, can be devastating to Spectral Analysis if the random component is large relative to the length of the cyclical component. Figure 11 shows a cyclical wave with a length of 20 days. Superimposed is another wave with random components that affect the length of the cyclical wave form. The results of the Spectral Analysis procedure on this data are shown in Figure 12, which indicate a multitude of "peaks" at virtually all wave lengths! If a spectrum exhibits many cyclical components of approximately the same relative magnitude then there really are no dominant cyclical components.

Effects of Trend on Spectral Analysis Results

While Spectral Analysis does find cyclical components and their relative strength within a time series, many times in real data there may exist an underlying trend up or down. Data trends can really mess up the performance of Spectral Analysis computer routines that do not adequately compensate for them.

To demonstrate what happens to the spectral density graph when trend is introduced to the Fourier series, one of the time series previously constructed is given a trend as shown in Figure 13. The original data, labeled A, is added to the trend line B. A new time series C is created that includes this mild trend. This trend is constructed such that the price rises 50 points in 100 days. The results of the Spectral Analysis program with 2000 observations of input data is shown in Figure 14. Note that the "known" cyclical components are so minor relative to that which the trend



CYCLES NOVEMBER 1985

causes, that they are barely visible.

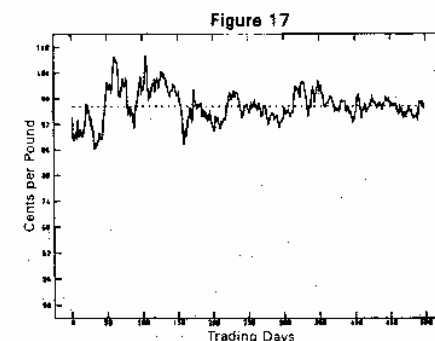
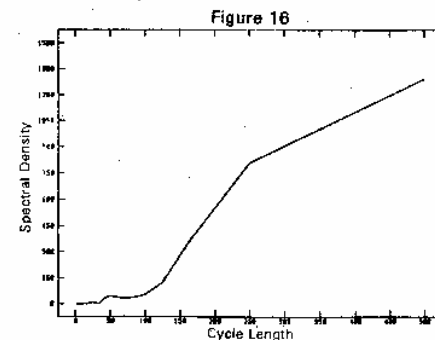
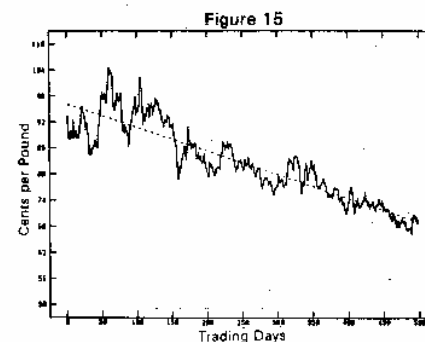
So what can be done? Most "real" data that an analyst is likely to encounter has some trend in it. The answer is that the data must be detrended. One approach involves the use of another statistical technique known as Linear Least Squares Regression Analysis. The actual procedure is found in the next section dealing with "live" data. Some computer packages may automatically do this for you. The one I am using for this article, unfortunately, does not.

Using Spectral Analysis on "Real" Data

This section shows how Spectral Analysis may be used and interpreted when applied to actual price data. The data used here is Comex Copper closing price on a near month basis, excluding spot month. The particular plotting program used for this paper does not have the facility to produce the time axis as calendar dates. It shows each day by observation number starting from 4/25/80 through 2/21/82.

The copper prices are shown in Figure 15. Notice the slight downward trend in the prices over this time period. When this data is run through the Spectral Analysis program, the Spectral Density chart shown in Figure 16 results. Over approximately 500 trading days, the long cyclical components have the greatest Spectral Density value. This is expected since the data shows some trend. The Spectral Density values of the other cyclical components are dwarfed by comparison.

The time series trend can be eliminated by means of the statistical technique called Linear Least Squares Regression Analysis. Regression analysis basically "fits" a straight line to the actual data so that the square of the errors (where the observed data misses the line) are minimized. The actual mathematics will not be discussed here. The results of



regression analysis are the estimates of the parameters of a straight line, namely the slope of the line and the y-intercept.

Recall the formula for the straight line:

$$y = a + bx$$

where: y = the dependent variable (price)
 a = y-intercept (the value at $x=0$)
 b = slope of the line
 x = independent variable (time)

For this data, y is the closing price; x is the "time" variable: 1 for the first observation, 2 for the second and so on; b represents the change in price over time for the data observed; and a is the y-intercept, the value from which price changes by the value of b for each passing day.

In the case of the Copper data on hand the estimated values for a and b are 96.28 and -0.054 respectively. This means that the formula for the price of copper, for the time

CYCLES NOVEMBER 1985

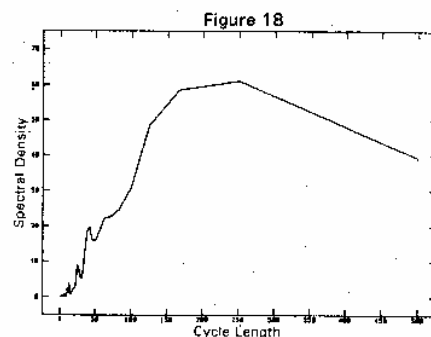
period analyzed is as follows:

$$P = 96.28 - 0.054 * \text{"time variable"} (1,2,3, \text{etc})$$

Remember, this is only an estimate. Figure 15 show this estimated trend line superimposed on the price data. A quick glance at the actual price data shows that the observed data misses the line quite a bit.

To remove the trend from the actual data, a simple transformation of the data is made where the estimated value of the price is subtracted from the observed value and the result added to the y-intercept, 96.28. Figure 17 shows the copper data with the trend "flattened out" or removed by this procedure.

This new time series is run through the Spectral Analysis program. The resulting Spectral Density graph is shown in Figure 18. Your attention is first directed to the left side of the graph. Notice that there are at least four cyclical components at 10, 13, 25, and 41 days. What happens at the right side of the graph, where the cyclical components are of greater length, is also interesting. There appears to be a relatively large cyclical component centered around 250 days. This peak, however, is very wide relative to its height. Recall from the earlier example in Figure 6, the longer cycle's peak was wider than the shorter cycle's peak. In general, if the Spectral Analysis program indicates a strong cyclical component whose length is greater than half of the number of observations in the input data, one



should question the validity of the results.

CONCLUSIONS

Spectral Analysis is as much an art as a science, much the same as price chart interpretation. The best instructor is experience. Because of its mathematical complexity, many traders and analysts have avoided using this powerful and revealing tool. As with any "statistical" tool, its use must be tempered with prudence and experience.

UNITED STATES BUSINESS CYCLE SUMMARY: Subjects on which additional material appears in this issue are marked by an asterisk. "Same Month, Previous Year" is the value for the month to match the most recent for which information is available.

	1985 JUL	AUG	SEP	OCT	NOV	Change from Previous Month	Change from Same Month Last Year
PRODUCTION							
FRB Index, Industrial Production (1977 = 100)	(a) 124.1	125.2	125.0	124.6	125.1	+0.40%	+1.38%
Manufacturing Total Index	(a) 126.9	128.2	127.9	127.5	128.1	+0.47	+1.67
Durable Goods	(a) 127.9	129.4	128.5	127.9	128.9	+0.78	+1.10
Nondurable Goods	(a) 125.6	126.6	127.0	127.0	127.1	+0.08	+2.67
Mining	(a) 108.7	108.3	107.7	106.0	105.6	-0.38	-2.94
Utilities	(a) 110.7	110.3	113.2	113.4	114.0	+0.53	+1.69
Overall Factory Operating Rates (%)	80.4	80.3	80.0	79.8	80.0	0	0
Steel Production (Raw, Mil Tons)	7.01	7.13	6.92	7.35		+6.21	+9.70
Aluminum Production (Mil Tons, Annual Rate)	3.79	3.75	3.76	3.70	3.56	-3.78	-18.72
CONSTRUCTION							
Total Construction (Bil \$, Annual Rate)	(a) 340.2	344.0	346.1	347.8		+0.49	+9.30
Private, Residential (Bil \$, Annual Rate)	(a) 153.2	147.3	149.0	151.4		+1.61	+5.14
Housing Starts (Private, Annual Rate, Thous)	(a) 1663	1746	1589	1761	1547	-12.15	-3.31
Mobile Home Shipments (Annual Rate, Thous)(a)	287(Apr)	287(May)	270(Jun)	286(Jul)	290(Aug)	+1.40	-3.97
TRADE							
Retail Sales (Bil \$)	(a) 114.42	117.04	119.41	114.55	115.85	+1.13	+5.07
Car Sales, Retail, Domestic (Annual Rate, Mil)	(a) 8.4(May)	7.6(Jun)	7.4(Jul)	9.7(Aug)	11.3(Sep)	+16.49	+44.87
Manufacturers' Shipments (Bil \$)	(a) 193.87(Jun)	193.79(Jul)	196.59(Aug)	194.23(Sep)	197.49(Oct)	+1.68	+3.25
Imports, General (Bil \$)	(a) 23.42(Jun)	26.63(Jul)	26.08(Aug)			-2.07	-2.95
Exports (Incl Reexports, Excl DOD, Bil \$)	(a) 17.44(Jun)	17.41(Jul)	17.42(Aug)	17.30(Sep)		-0.69	-5.00
AGRICULTURE							
Corn Production (Mil Bu, Crop Est, Grain Only)	(1985 Crop = 8603.0 VS 1984 Crop = 7656.2; UP 12.37%)						
Wheat Production (Mil Bu, Crop Estimate)	(1985 Crop = 2419 VS 1984 Crop = 2596; DOWN 6.82%)						
Cattle Slaughtered (Thousands, By Qtr-Yr.)	(1985 2nd QTR = 8670 VS 1984 2nd QTR = 8959; DOWN 3.23%)						
Hogs Slaughtered (Thousands, By Qtr-Yr.)	(1985 2nd QTR = 20,745 VS 1984 2nd QTR = 20,498; UP 1.20%)						
EMPLOYMENT							
Per Cent Unemployed (civilian)	7.3	7.0	7.1	7.1	7.0	-1.41%	-1.41%
Employees, Nonagricultural Payrolls (Mil)	(a) 97.71	97.99	98.12	98.6	98.80	+0.20	+3.05
Average Workweek (All Nonfarm)	(a) 33.0	33.1	33.1	33.1	33.0	-0.28	-0.57
PRICE INDICES							
Producer Prices (1967 = 100)	309.0	307.2	305.8	308.0		+0.72	-0.45
Consumer Prices (Urban, 1967 = 100)	322.8	323.5	324.5	323.5		+0.31	+3.24
Dow-Jones Spot Commodity Index	116.31	114.66	112.50	115.81	120.12	+3.72	-4.69
PRICES—COMMODITIES							
Beef, Omaha Choice (\$/cwt)	53.80	52.09	51.98	58.69	63.71	+8.55%	-1.91%
Coffee, Brazilian, N.Y. (\$/lb)	1.33	1.33	1.33	1.37	1.55	+13.14	+12.32
Copper, Electrolytic, Wirebars, Delivered (¢/lb)	69.17	68.72	67.99	68.83	68.65	-0.26	+0.94
Corn, #2 Yellow, Chicago (\$/bu)	2.76	2.50	2.31	2.26	2.48	+9.73	-10.79
Cotton, 1 1/16", str. lw-mid, Memphis (¢/lb)	58.89	56.97	56.64	56.50	56.36	-0.25	-6.70
* Eggs, Lrg White, Chicago (¢/doz)	58.6	66.4	70.5	70.7	74.6	+5.52	+5.97
Gold, Base Price, H & H, Troy oz., N.Y.	317.802	330.234	322.624	326.023	325.466	-0.17	+2.41
Hogs Sioux City (\$/cwt)	47.09	43.91	46.42	44.51	44.57	+0.13	-8.20