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How Neural Nets Work *

Alan Lapedes
Robert Farber
Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM 87545
January, 1988

Abstract:

There is presently great interest in the abilities of neural networks to mimic "qualitative reasoning" by manipulating neural incodings of symbols. Less work has been performed on using neural networks to process floating point numbers and it is sometimes stated that neural networks are somehow inherently inaccurate and therefore best suited for "fuzzy" qualitative reasoning. Nevertheless, the potential speed of massively parallel operations make neural net "number crunching" an interesting topic to explore. In this paper we discuss some of our work in which we demonstrate that for certain applications neural networks can achieve significantly higher numerical accuracy than more conventional techniques. In particular, prediction of future values of a chaotic time series can be performed with exceptionally high accuracy. We analyze how a neural net is able to predict well , and in the process show that a large class of functions from $R^n \rightarrow R^m$ may be accurately approximated by a backpropagation neural net with just two "hidden" layers. The network uses this functional approximation to perform either interpolation (signal processing applications) or extrapolation (symbol processing applications). Neural nets therefore use quite familiar methods to perform their tasks. The geometrical viewpoint advocated here seems to be a useful approach to analyzing neural network operation and relates neural networks to well studied topics in functional approximation.

1. Introduction

Although a great deal of interest has been displayed in neural network's capabilities to perform a kind of qualitative reasoning, relatively little work has been done on the ability of neural networks to process floating point numbers in a massively parallel fashion. Clearly, this is an important ability. In this paper we discuss some of our work in this area and show the relation between numerical, and symbolic processing. We will concentrate on the the subject of accurate prediction in a time series. Accurate prediction has applications in many areas of signal processing. It is also a useful, and fascinating ability, when dealing with natural, physical systems. Given some data from the past history of a system, can one accurately predict what it will do in the future?

Many conventional signal processing tests, such as correlation function analysis, cannot distinguish deterministic chaotic behavior from from stochastic noise. Particularly difficult systems to predict are those that are nonlinear and chaotic. Chaos has a technical definition based on nonlinear, dynamical systems theory, but intuitivly means that the system is deterministic but "random," in a rather similar manner to deterministic, pseudo random number generators used on conventional computers. Examples of chaotic systems in nature include turbulence in fluids (D. Ruelle, 1971; H. Swinney, 1978), chemical reactions (K. Tomita, 1979), lasers (H. Haken, 1975), plasma physics (D. Russel, 1980) to name but a few. Typically, chaotic systems also display the full range of nonlinear behavior (fixed points, limit cycles etc.) when parameters are varied, and therefore provide a good testbed in which to investigate techniques of nonlinear signal processing. Clearly, if one can uncover the underlying, deterministic algorithm from a chaotic time series, then one may be able to predict the future

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