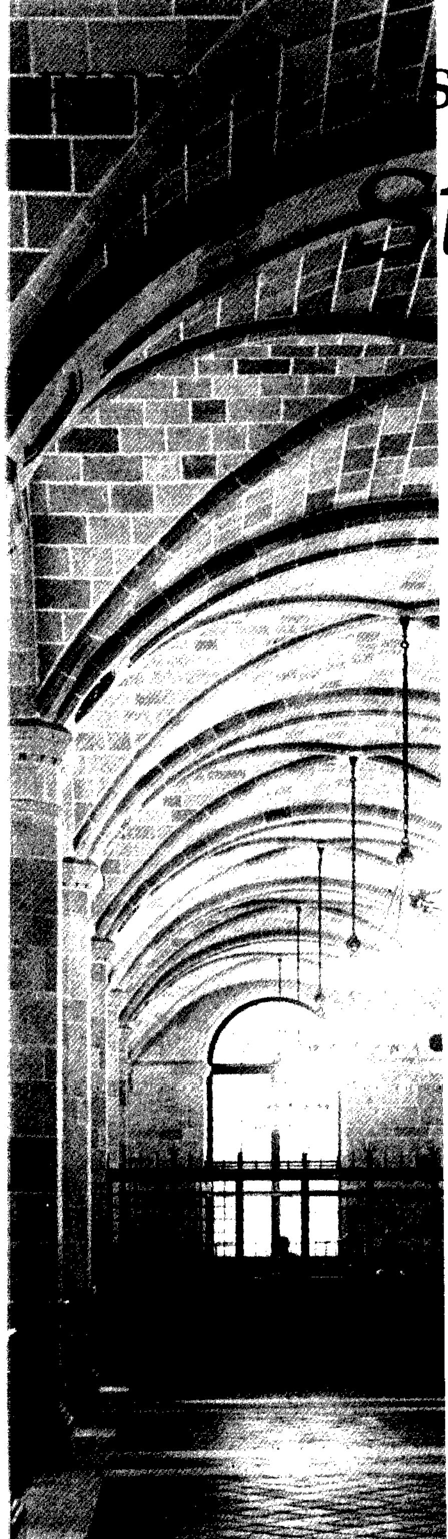

SERVE BANK OF NEW YORK

Staff Reports

Head and Shoulders: Not Just a Flaky Pattern

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Number 4
August 1995

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ABSTRACT

This paper evaluates rigorously the predictive power of the head-and-shoulders pattern as applied to daily exchange rates. Though such visual, nonlinear chart patterns are applied frequently by technical analysts, our paper is one of the first to evaluate the predictive power of such patterns. We apply a trading rule based on the head-and-shoulders pattern to daily exchange rates of major currencies versus the dollar during the floating rate period (from March 1973 to June 1994). We identify head-and-shoulders patterns using an objective, computer-implemented algorithm based on criteria in published technical analysis manuals. The resulting profits, replicable in real-time, are then compared with the distribution of profits for 10,000 simulated series generated with the bootstrap technique under the null hypothesis of a random walk.

Results: The head-and-shoulders trading rule appears to have some predictive power for the German mark and yen but not for the Canadian dollar, Swiss franc, French franc, or pound. Nonetheless, if one had speculated in all six currencies simultaneously, profits would have been both statistically and economically significant. Taken individually, profits in the markets for yen and marks are also substantial when adjusted for transactions costs, interest differentials, or risk. These results are robust to changes in the parameters of the head-and-shoulders identification algorithm, changes in the sample period, and the assumption that exchange rates follow a GARCH process rather than a random walk. These results are inconsistent with virtually all standard exchange rate models, and could indicate the presence of market inefficiencies. (JEL #F31, G12, G14)

I. Introduction[1]

Technical analysis, the prediction of price movements based on past price movements, has been shown to generate statistically significant profits despite its incompatibility with most economists' notions of "efficient markets." In the stock market, excess profits based on technical trading rules are documented by Brock, Lakonishok, and LeBaron (1992), and in the foreign exchange market such excess profits are found by Dooley and Shafer (1984); Logue, Sweeney, and Willett (1978); Sweeney (1986); and Levich and Thomas (1993).

Tests of technical analysis have largely limited their attention to techniques that are easily expressed algebraically, namely filter rules and moving averages. Practitioners, however, rely heavily on many other techniques, including a broad category of exclusively visual patterns. Typically known by fanciful names, this category includes "head and shoulders," "rounded tops" and "bottoms," "flags," "pennants," and "wedges." Highly nonlinear and complex, trading rules based on these patterns normally cannot be expressed algebraically.

The purpose of this paper is to begin evaluating this large set of visual

[1] We are greatly indebted to Peter Tordo for help with implementing our algorithms on the computer. Tariq Haider, Denny Wong, and Keith Yanowitz provided excellent research assistance. We acknowledge helpful comments from seminar participants at the Federal Reserve Bank of New York, Wharton, New York University, and London Business School.

trading rules by focusing on one of the best-known patterns, **head and shoulders**. Technical analysts claim that this pattern, identified when the second of a series of three peaks is higher than the first and the third, presages a trend reversal. Our computer-based identification algorithm locates such patterns using local maxima and minima.

Since the head-and-shoulders pattern is considered by practitioners to be the one of the most, if not *the* most, reliable of all chart patterns, it represents a natural point of departure for empirical research. If trading based on this pattern generates excess profits, investigating other patterns may prove interesting. Conversely, if profits are insignificant, then this entire branch of visually based technical analysis may be called into question.

We test the head-and-shoulders rule on daily spot rates for six currencies against the dollar: the yen, German mark, Canadian dollar, Swiss franc, French franc, and pound. Our data cover the entire floating rate period (from March 19, 1973, to June 13, 1994), a twenty-one year span that provided us with more than 5,500 daily observations. Currency markets seem especially appropriate for testing technical signals, as they are characterized by very high liquidity, low bid-ask spreads, and round-the-clock decentralized trading. Furthermore, because of their size, these markets are relatively immune to insider trading. In any event, technical analysts claim that "the principles that underlie analysis of currencies from a technical aspect are basically the same as those used in any other financial market or for individual stocks" (Pring 1985, p. 466).[2] We are aware of only two studies that evaluate—for any market—the visual, nonlinear patterns that are the focus of this paper. The two studies come to different conclusions: Levy (1971) tests the predictive power of all thirty-two possible five-point chart patterns, including the head and shoulders. He finds no evidence of profitable forecasting ability. As we will discuss later (see Section IVA), there are reasons to question the validity of these results with regard to the head-and-shoulders pattern. Brock et al. (1992), on the other hand, finds that breakouts from observed trading ranges are meaningful predictors of short-term returns in the Dow Jones Index during 1897-1986, a result corroborating technicians' claims regarding "support" and "resistance" levels. In short, research on these visual trading patterns is both scarce and inconclusive; thus, as Neftci (1991) notes, these visually based strategies are currently "a broad class of prediction rules with unknown statistical properties."

Our study can be viewed as contributing to a growing body of research testing for nonlinear dependence in financial prices. Early tests for the presence of nonlinearities, testing the null hypothesis of i.i.d. behavior, indicate that nonlinearities are indeed present in stock markets (Hsieh 1991) and in floating exchange rates (Hsieh 1989). The form of these nonlinearities remains unclear. Modeling financial prices as a GARCH process seems to

2 Other authors echo this same principle, as in the following statement: "In general, all of the chart patterns ... can be applied to virtually any market. One oft-stated claim of technical analysts is that it is not even necessary to know the name of the stock or commodity being analyzed in order to apply the principles of chart analysis. The statement that, 'a chart is a chart' is often heard" (Murphy 1986, p. 172).

capture some of the nonlinearities indicated by more general tests; more specifically, it is helpful for predicting volatilities (Hsieh 1989). Other sources of nonlinearity are also consistent with the data. Another potential source of nonlinearity is chaos, although the few available tests fail to confirm its existence in exchange rate data. [3] Our tests may be helpful in identifying another specific form of exchange rate nonlinearity that is consistent with data on floating exchange rates.

There are three parts to the methodology by which we calculate and interpret the profits earned by taking foreign exchange positions once the pattern is recognized (see Section III for a complete discussion of methodology). The first part is an objective, computerized identification of the head-and-shoulders pattern itself. The second part is a strategy, replicable in real time without knowledge of the future, for entering and exiting speculative positions after recognizing such patterns. The third part concerns

[3] For more information on chaos, see Brock et al. (1991).

Aczel and Josephy (1991) calculate correlation dimensions and associated standard errors for four European currencies and the Singapore dollar against the U.S. dollar. (The correlation dimensions are intended to capture the fact that the phase diagrams of chaotic series occupy a lower share of the phase space than do the phase diagrams of purely random processes). Although the authors find that the correlation dimensions of the European currencies are all statistically indistinguishable from each other and statistically different from the Singapore dollar, their evidence does not rule out the possibility that nonlinearities in exchange rates are due to other, nonchaotic generating processes. Further, correlation dimensions are considered unreliable in small samples, and for this study the authors have only 115 observations (Hsieh 1991). Gilmore (1991) considers an alternative approach to testing for chaos in exchange rates, the method of close returns, which he asserts is more robust in small samples. On a sample of 935 daily observations for four exchange rates against the U.S. dollar, Gilmore finds no evidence to indicate the presence of chaos.

evaluating whether the profits obtained from this trading rule imply that there were predictable profit opportunities in the data.

Our approach is to evaluate whether these profits are statistically greater than those that would have been found had there been no intertemporal dependence in exchange rate changes. We identify reliable confidence intervals via the bootstrap methodology, implemented by constructing 10,000 new exchange rate series. In each simulated series, daily changes are determined by drawing randomly, with replacement, from the original series of exchange rate changes and applying these changes consecutively to the exchange rate's actual starting value. When we apply our trading rules on these constructed series, we obtain a distribution for profits under the null hypothesis that there are no predictable profit opportunities. From this distribution we calculate confidence intervals.

The results indicate that during the floating rate period, the head-and-shoulders pattern had significant predictive power for the mark and yen, but not for the Canadian dollar, Swiss franc, French franc, or pound, all of which were quoted against the dollar. Overall, if one had speculated in all six currencies simultaneously, profits would have been both statistically and economically significant. Taken individually, profits in the markets for yen and marks are also substantial when adjusted for transactions costs, interest differentials, or risk. These results are robust to changes in the parameters of the head-and-shoulders identification algorithm, changes in the sample period, and the assumption that exchange rates follow a GARCH process rather than a random walk.

In Section II, we discuss technical analysis in general and the head-and-shoulders pattern in particular. In Section III, we describe our methodology. In Section IV, we present our results. Section V discusses possible explanations for our results, and Section VI presents conclusions.

II. Technical Analysis and the Head-and-Shoulders Pattern

This section provides a careful delineation of the features of head-and-shoulders patterns. Since we are neither specialists in this area, nor advocates, our material is primarily drawn from textbooks; numerous conversations with practitioners indicate that actual practice conforms closely with the published recommendations. We begin with a few paragraphs introducing technical analysis to place the head-and-shoulders pattern in context.

A. Technical Analysis

According to one of the classic references in technical analysis, "The whole purpose of charting ... is to identify trends in the early stages of their development for the purpose of trading in the direction of those trends" (Murphy 1986, p. 3). This activity became popular after John Magee and R.D. Edwards published the first of many editions of *Technical Analysis of Stock Trends* in 1948. We surveyed seven technical analysis manuals and found substantial consistency across authors and across time with regard to the visual patterns that we discuss here. The chart patterns highlighted are largely similar from one source to the next, and their significance is consistently interpreted.[4]

There is little in these books to please an academically trained economist. For example, profits are measured in isolation, without regard for opportunity cost or risk. The analysis is also imprecise: authors do not hesitate to characterize a pattern as frequent or reliable, but they virtually never attempt to quantify those assessments. Few empirical studies are cited despite the fact that the indicators are frequently described as "scientifically researched" (Pring 1985, p. x).

Despite the failure of technical analysts to document the validity of their claims, those claims are clearly testable, since their directions for taking positions are quite explicit. This testability is addressed below.

B. The Head-and-Shoulders Chart Pattern

A textual analysis of the seven manuals indicates near unanimity in the list of primary requirements for a head-and-shoulders pattern as well as strong agreement on minor requirements. The primary defining characteristic of a head-and-shoulders pattern is a sequence of three peaks with the highest in the middle (Chart 1). The left and right peaks are referred to as "shoulders," the center peak as the "head." The straight line connecting the troughs separating the head from right and left shoulders is called "the neckline." The head-and-shoulders pattern is complete when the price path

4 These seven books are Arnold and Rahfeldt (1986), Edwards and Magee (1966), Hardy (1978), Kaufman (1978), Murphy (1986), Pring (1985), and Sklarew (1980). These seven were chosen because they were available to us through the Federal Reserve Bank of New York, New York University, and Wharton libraries. We understand that they are among the better known references on this subject.

crosses the neckline after forming the right shoulder. In addition to these basic features, all seven of the manuals list the following criteria:

1. The head and shoulders is a reversal pattern, that is, the head-and-shoulders pattern is supposed to indicate that an earlier upward trend is about to be reversed. States one manual: "an important reversal of trend is not conclusively signalized until the neckline has been penetrated" (Edwards and Magee 1966, p. 55).
2. The head and shoulders is one of the most reliable of all chart patterns. "This is one of the most common and, by all odds, the most reliable of the major reversal patterns" (Edwards and Magee, 1966, p. 50).
3. No position should be entered into until the pattern is confirmed by the price line crossing the neckline. "It is essential to wait for a decisive break below the neckline" (Pring 1985, p. 64).
4. After crossing the neckline, it is not uncommon for the price line to "pull back" to the neckline briefly before continuing on its new trend. "After the signal has been given by the penetration of the neckline, there is sometimes a return move back to the neckline before the main move gets under way in the direction signaled" (Sklarew 1980, p. 23).
5. Head-and-shoulders patterns can occur both at peaks, where they are "tops," and at troughs, where they are "bottoms." All the price factors that apply to head-and-shoulders tops apply equally, with a sign change, to head-and-shoulders bottoms. "In a head-and-shoulders bottom the formation is inverted and all the above principles apply in the reverse direction" (Sklarew 1980, p. 23).

Four of the books explicitly discuss the extent to which head-and-shoulders patterns exhibit diversity.[5] All of them emphasize that the individual realizations of the patterns can indeed vary: "Chart patterns are like music. There are scores of variations on a theme" (Hardy 1978, p. 36).

III. Methodology

Evaluating the predictive powers of the head-and-shoulders pattern involved three challenges: to create a replicable strategy for recognizing such patterns in the data; to establish a strategy for entering and exiting positions on the basis of such recognition; and to evaluate the statistical properties of profits derived from following these identification and profit-taking strategies.

A. Identifying a Head-and-Shoulders Pattern

We identify head-and-shoulders patterns with a parameterization intended to capture the features described in Section II as closely as possible in an objective manner. For both head-and-shoulders tops and bottoms, we begin by tracing out a zigzag pattern in the data. Essentially, a zigzag pattern is a smoothed version of the original data consisting of peaks and troughs connected with upward and downward diagonal trend lines.

The concept of beginning with a zigzag comes directly from the technical analysis manuals themselves: "*This zigzag pattern is the foundation of all chart formations, and is the key to their forecasting value. ... All chart formations are some variation of either the minor or major zigzag trend*

[5] The four books are Arnold and Rahfeldt (1986), Edwards and Magee (1966), Pring (1985), and Hardy (1978).

pattern" (Sklarew 1980, pp. 15-16, italics in original). Academics have also noted the relationship between visual chart patterns and a zigzag pattern: Neftci (1991) asserts that "most patterns used by technical analysts need to be characterized by appropriate sequences of local minima and/or maxima" (p. 550).

We define a peak as a local maximum at least χ percent higher than the preceding trough and a trough as a local minimum at least χ percent lower than the preceding peak, where χ is referred to as the "cutoff." Examples of the zigzag applied to the pound during the early 1990s are shown in Charts 2a through 2d, where cutoff is set at 0 percent (that is, the original data), 1 percent, 3 percent, and 5 percent, respectively. The reader can observe that the number of peaks and troughs identified in the data is inversely related to the value of the cutoff. An increase or a decrease in the cutoff generates a different series of peaks and troughs, which will result in a different set of head-and-shoulders patterns.[6]

To capture most of the head-and-shoulders patterns occurring for these currencies we scan the data ten times, each time with a different value of cutoff. The ten different levels of cutoff are chosen with reference to each currency's volatility. Specifically, we set one standard deviation of daily exchange rate changes as a lower bound for cutoff to distinguish local peaks and troughs from a single day's upward or downward movement, and consider the following higher multiples of that standard deviation:

[6] In our analysis, we exclude duplicates when examining the aggregate of *all* head-and-shoulders positions found for a range of cutoffs.

1.25, 1.50, 1.75, 2.00, 2.50, 3.00, 3.50, 4.00, and 4.50.[7] Each time we scan the data at a new cutoff we eliminate duplicate patterns.[8] Using these ten indices typically leads us to take positions in each currency one to two times a year on average, a frequency consistent with anecdotal descriptions in the practitioners' literature.

For each cutoff, the computer searches for a head-and-shoulders top following each peak and a head-and-shoulders bottom following each trough. In order to qualify as a head-and-shoulders top, a given set of four consecutive peaks—the last three of which represent the left shoulder, head, and right shoulder of the head-and-shoulders pattern itself—is required to satisfy a number of criteria, which are defined below and are best illustrated with the help of Chart 1. (The defining criteria for head-and-shoulders bottoms are the reverse.)

1. The height of the head must exceed the height of the left and right shoulders.
2. To ensure that the pattern could *presage a trend reversal,* we require that the pattern occur following an uptrend. Specifically, we require

7 In this sample, standard deviations of daily percent changes were as follows: deutsche mark 0.697 percent, Japanese yen 0.615 percent, Canadian dollar 0.274 percent, Swiss franc 0.794 percent, British pound 0.657 percent, and French franc 0.698 percent.

Increasing the cutoff beyond 4.50 times the standard deviation of daily exchange rate changes resulted in very few additional positions taken.

8 If a head-and-shoulders pattern using one cutoff suggested entering a position within two days of a previously identified entry date, we did not include the new position in our analysis. Our results are unaffected by whether we identify such positions with successively higher cutoffs or successively lower cutoffs.

that (a) the price at the left shoulder exceed the price at the previous peak (point P), and (b) the price at the first trough within the head-and-shoulders (point T_L) exceed the price at the previous trough (point T_{LL}).

3. To ensure that the patterns do not display extreme *vertical asymmetries,* the pattern should only be moderately sloped. We require that the right shoulder exceed, and the right trough not exceed, the mid-point between the left shoulder and left trough (T_L). Likewise, we require that the left shoulder exceed, and the left trough not exceed, the midpoint between the right shoulder and right trough (T_R).
4. To ensure that the patterns do not display extreme *horizontal asymmetries,* we require that the time between the left shoulder and the head be no more than 2.5 times greater than that between the head and the right shoulder; likewise the time between the head and the right shoulder should be no more than 2.5 times greater than the time between the left shoulder and the head.
5. Since the head-and-shoulders pattern is meant to indicate an *imminent trend reversal,* we impose a time limit by which the series must cross the neckline in order to constitute a bona fide confirmation of the head-and-shoulders signal. In particular, the time required for this confirmation to occur must be no longer than the time interval between right and left shoulders.

The set of signals by which chartists identify head-and-shoulders patterns, summarized above, were originally determined to be important in pre-1948 equity data (Edwards and Magee 1966, pp. 50-72). However, since we test the significance of this set of signals on post-1972 exchange rate data (that is, data from a later time period on a different set of financial prices), the reader can be sure that our test data are independent from the data that originally suggested the pattern's significance. For this reason, any significance we might find for this pattern would be unconnected to the observations that originally motivated the attention to the pattern. Our tests are analogous to out-of-sample tests of other predictive rules.

B. Measuring Profits

Once a head-and-shoulders pattern is identified, we evaluate the profits that a market participant would receive if he or she were to act upon this information.

Entry: The technical analysis manuals are very explicit about exactly where to enter. Following these manuals, we enter a position once the price line breaks the neckline.[9] We generally take as our entry price the closing price on the day of the neckline's penetration.[10] Since peaks and troughs are identified only after they occur, the pattern could cross the neckline before the price has declined by the cutoff percent required to identify the

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9 Entry always entails going long or short one unit in the priced currency (the denominator in a conventional quote), with profits measured in terms of the numeraire currency (the numerator in the quote). Thus, for all currencies but the pound, positions involve a long or short position in one dollar. For the pound, positions involve one pound. This has no effect, however, on the interpretations of our results, which are always expressed in terms of percent return.

In practice, we find that 25 to 40 percent of all head-and-shoulders patterns are confirmed by penetration of the neckline, depending on the currency.

10 In our analysis, the closing price was used rather than the price at which the head-and-shoulders patterns actually crossed the neckline. Manuals giving detailed instructions on this point stress that the penetration must be decisive, and furthermore, we do not observe prices intraday.

right shoulder as a peak. When this occurs, we permit the analyst to enter the position only when cutoff has been reached. In this way, we ensure that our positions do not benefit from future information.[11]

The technical analysis manuals provide very little guidance about when to exit. For this reason, we examine the implications of two different approaches to choosing an exit point. One approach is endogenous, the other is exogenous.

An Endogenous Exit Rule: Our first approach to calculating profits endogenously chooses an exit point, in a manner aiming to mimic the way an actual market participant might act on the basis of head-and-shoulders technical signals. The manuals make clear that participants would choose different holding periods each time a position is taken to try to maximize returns each time. In this approach, speculators maintain their positions until prices seem to have conclusively stopped moving in the predicted direction. On the basis of the upward and downward trends that constitute the zigzag, a price conclusively stops moving when a new peak or trough has occurred. Thus, for a head-and-shoulders top, investors exit their short positions as soon as a new trough has been identified, that is, when the price has risen the cutoff percent above that local minimum. Two caveats apply to this basic endogenous exit rule.

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[11] Neftci (1991) asserts that acting on the head-and-shoulders pattern necessarily requires future information. This assertion could be correct only in the context of his particular representation of the head-and-shoulders patterns, which does not correspond to representations in the technical analysis manuals we studied. The assertion also relies on the requirement that the right and left trough be very nearly identical, a condition that is not required in the manuals, and that we do not require in our alogrithm.

Caveat 1—Stoploss: To limit losses when the theory proves wrong, we close all positions whenever the exchange rate moves sufficiently far in the direction opposite to that predicted by chartist theory. In our empirical analysis, we set this stoploss at 1 percent of the original price at entry, although the actual loss could exceed this because we exit at the day's closing price.

Caveat 2—Bounce: The technical analysis manuals consistently stress that in a trend reversal (following a confirmed head-and-shoulders top), prices may in certain instances exhibit a temporary upward movement back toward the neckline. To exit one's position at this point would, according to these guidelines, be premature, as the reversal would not have completed its course and reached its true minimum. For this reason, we build a bounce possibility into this exit algorithm to account for a brief interruption in the reversal pattern. Specifically, if the downward movement of prices following a confirmed head-and-shoulders top turns upward before falling by at least 25 percent of the vertical distance from the head to the neckline (a distance generally referred to as the measuring objective or price objective in the manuals), then investors hold their positions unchanged until one of two conditions is met: (1) prices cross the neckline by at least 1 percent, triggering the stoploss discussed above; or (2) a second trough (of any size) is reached in the zigzag. The 25 percent bounce parameter was arrived at through discussions with practicing technical analysts.[12]

An Exogenous Exit Rule: In our second approach to determining an exit point, we simply unwind a position after an exogenously specified

[12] In practice, we found that for most currencies the bounce possibility was invoked about 15 percent of the time.

number of days (one, three, five, ten, twenty, thirty, and sixty days). Profits are calculated as the gain or loss between entry and exit. In order to compare statistically independent returns, we focus on the *marginal* (incremental) returns accruing from holding the position from one of those exit points to the next (for example, after holding the position for ten days, the returns accruing in days eleven to twenty). These results enable us to evaluate technical rules for *entering* positions, though they do not permit us to evaluate technical rules on when to exit.

Data: We apply the head-and-shoulders recognition and profit-taking programs to daily exchange rate data on the dollar against the yen, mark, Canadian dollar, Swiss franc, French franc, and pound. The data cover the entire floating rate period (March 19, 1973, to June 13, 1994), a twenty-one year span that provided us with more than 5,500 daily observations.[13]

The technical analysis community is not unanimous about the appropriate price data to use for charting. Some chartists prefer daily bar charts, while others "believe that because the closing price is the most critical price of the trading day, a ... close-only ... chart is a more valid measure of price activity" (Murphy 1986, p. 36). The use of daily closing prices seems especially appropriate for currencies, since the daily highs and lows are not recorded in foreign exchange spot markets. Since technical analysis manu-

[13] Japanese data were sampled as the Tokyo market closing middle rate. Mark data were taken at the official fixing, 1 p.m. Frankfurt time. The pound and Swiss franc rates represent current market rates at 2:15 p.m. Swiss time. French franc rates are rates at 2:15 p.m. Canadian dollar rates represent London market middle rates at around noon Swiss time (Bank for International Settlements Data Bank, courtesy of the Federal Reserve Bank of New York). Canadian data were available only beginning August 1, 1976.

als generally share the view that their techniques apply equally to all financial price series, the absence of daily highs and lows should not compromise the accuracy of our analysis.[14]

Comparison with Filter Rules: Since the trading rules based on the head-and-shoulders pattern may superficially resemble filter rules, which have been the subject of earlier studies (Logue and Sweeney 1978; Sweeney 1986), it is worthwhile to clarify why these two approaches are in fact very different. Under a standard filter rule, one enters a short position when the spot rate has fallen by X percent (where X is the size of the filter) from the previous high, and exits from a position when the rate has risen by X percent. Comparison with the algorithm based on head-and-shoulders patterns indicates that both the entry and exit criteria for these two strategies differ sharply.

Under a filter rule, entry occurs when the price has fallen a fixed X percent from the price's local maximum. When following a head-and-shoulders strategy, however, the entry point depends on the characteristics of price behavior preceding the peak in the ways shown in Chart 1 (three successive peaks, the highest in the center, and so forth). Furthermore, the head-and-shoulder rules specify that the price cross the neckline, a requirement not imposed by filter rules. A final distinction between these two strategies is the fact that, under a filter rule, one takes a position following every identified peak; for the head-and-shoulders-based strategy one takes a position following only a small subset of identified peaks. (The distinc-

[14] The technical traders with whom we spoke were fully confident that head-and-shoulders signals appearing in these daily exchange rate closes were fully adequate for predicting future price trends.

tion between the two trading strategies is analogous with long positions following price increases or head-and-shoulders bottoms.)

With regard to exit, the filter rule strategy requires that positions be liquidated following the first large upward price movement after entry, while the head-and-shoulders strategy indicates that traders should ride out large upward price movements—specifically those that occur when the price has not fallen near to its price objective.

C. Evaluating the Statistical Significance of Profits

Once we have run the head-and-shoulders identification and profit-taking programs on actual exchange rate data, we need to evaluate the resulting profit values. Our chosen benchmark is the distribution of profits in series for which it is known with certainty that intertemporal patterns are meaningless. This provides confidence intervals consistent with a null hypothesis of "no meaning" to head-and-shoulders patterns.[15]

We construct confidence intervals using the bootstrap method, a technique described in Efron (1979, 1982). We first create 10,000 artificial exchange rate series, each of which is the same length as the original series for that currency. These new series begin at the same starting point as the original series and change each period according to percent changes drawn at random, with replacement, from the original series of percent exchange

[15] An alternative approach to calculating confidence intervals would be to combine an assumed distribution of exchange rate changes and our profit-taking program. There is no consensus in the profession, however, regarding the closest approximation to exchange rate changes (Westerfield 1977; Boothe and Glassman 1987; Hsieh 1988). For this reason, we choose instead a strategy for finding confidence intervals that seems the most agnostic; the random walk.

rate changes. This specification is based on the null hypothesis that the exchange rate follows a random walk. Since each constructed sequence of prices changes is drawn from the same population as the original exchange rate series, the mean, variance, skewness, and kurtosis of each series should be representative of the same population that underlies the actual exchange rate series. The new series can differ from the original exchange rate series in terms of the intertemporal dependence of prices; however, each of these new series is *by construction* not intertemporally dependent, whereas the original exchange rate series may or may not be intertemporally dependent. By contrast, if technical analysts are correct and head-and-shoulders patterns predict future trends, then the original exchange rate series *would* exhibit intertemporal dependence.

On each of these new series, we run the head-and-shoulders identification and profit-taking programs described above. The distribution of the profits from these 10,000 series provides us with confidence intervals against which to measure the corresponding values found when the same programs were run on *actual* exchange rates. The distributions of other variables associated with each program, such as the number of positions taken, are also calculated. As shown in Table 1, profit measures for the artificial series are extremely small, as one would expect. Further information concerning these profits can be found in Appendix A.

IV. Empirical Profitability of Head-and-Shoulders Signals

We now apply the head-and-shoulders recognition and profit-taking programs to actual data, and compare these results with the distribution of results in the simulated series. A brief characterization of our basic results indicates that the head-and-shoulders pattern has predictive power for the mark and yen but not for the other four currencies of our sample. These results are robust to many modifications in our baseline methodology. Further results indicate that other claims made by technical analysts, such as the predictive power of the measuring objective, are not supported by the data. We finish this section with an extensive analysis of our basic results. We show that the profits from our head-and-shoulders based strategy in the mark and yen are economically as well as statistically meaningful, a finding that holds true when we speculate in all six currencies simultaneously. Finally, we show that the statistical significance of profits is not affected by adjustments for interest differentials, transactions costs, or risk.

Our data consist of the average of daily bid and ask quotes at the close of the New York business day, compiled by the Bank for International Settlements. Data are expressed according to the market convention of foreign currency units per U.S. dollar, except for the pound and the Canadian dollar, which are expressed in dollars per unit of foreign currency. To provide some perspective on the frequency, magnitude, and variability of the profits from these positions, Charts 3a through 3f depict, for each currency, the percent profit for *each* position taken over the 1973-94 period. Since the reader may wish to inspect the actual patterns detected by our parameterization, we show those for the mark and yen in Appendix B, together with a sketch of their necklines (patterns for other currencies are not shown for reasons of space). For each currency, our algorithm leads us to take one to two positions per year that last on average a few weeks, leaving us with open positions (either short or long) about 10 percent of the time.

A. Basic Results

Profits for the mark and the yen average about 1 percent on positions typically held for a few weeks. These values are significantly above profits calculated in the simulated series based on a null hypothesis of a random walk, thus supporting technical analysts' claims. The significance of these profits is shown in Table 2a, which summarizes profits from each currency from the endogenous exit rule strategy and presents corresponding *p*-values. Average percent returns for the other four currencies are consistently insignificant.[16] This suggests that the head-and-shoulders pattern has predictive power for exchange rate trends, as claimed by technical analysts, but only for the mark and the yen. (The magnitude of profits is discussed at greater length in Part D of this Section.)

Profits measured according to the exogenous exit rules (after a fixed number of days), shown in Table 2b, confirm the poor predictive power of the head-and-shoulders signals for the Canadian dollar, Swiss franc, French franc, and the pound. For the mark and the yen most exogenously specified exit dates also led to insignificant profits, with two exceptions. Speculative positions in the yen tended to yield substantial profits within three days, while positions in the mark proved most profitable in the interval between twenty and thirty days after entry. The contrast between these results and our results based on the endogenous exit rule lends support to the claims of technical analysts that determine the timing of exit.

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[16] The contrast between the mark and the French franc, pound, and Swiss franc may seem surprising, since these currencies tend to move in line against the dollar. During this time period, the correlation of daily changes were 0.583 French franc-mark, 0.307 pound-mark, and 0.468 Swiss franc-mark.

It may interest the reader to compare these results with those of Levy (1971), who analyzed the predictive power for stock prices of thirty-two distinct five-point chart patterns, four of which represent variations of the head-and-shoulders pattern (Levy's patterns eleven and nineteen correspond to head-and-shoulders tops; fourteen and twenty-two to head-and-shoulders bottoms). Levy found that "neither the best nor the worst of these thirty-two rules performed very differently from the market. As a result, *after taking trading costs into account, none of the thirty-two patterns showed any evidence of profitable forecasting ability in either (bullish or bearish) direction*" (p. 318, italics in original).

Although Levy's patterns eleven, fourteen, nineteen, and twenty-two correspond to head-and-shoulders patterns, his profit results are not representative of profits one would obtain by carefully following technical trading rules. In identifying the patterns, Levy does not impose any symmetry requirements. In entering positions, he does not incorporate the requirement, consistently stressed by technical analysts, that one wait for the price series to cross the neckline before taking a position. In closing positions, Levy's approach fails to incorporate the endogenous exit rules central to technical analysts' claims. Instead, he calculates profit after fixed intervals (one, four, fourteen, or twenty-six weeks), disregarding the evolution of prices after entry. The insignificance of Levy's results is consistent with our finding (albeit on a different data set and over a different time period), that it is advantageous to time one's exit according to actual price behavior following entry. Overall, the patterns Levy identifies correspond to head-and-shoulders, but his trading strategy differs substantially from that advocated by technical analysts.

B. How Robust Are These Results?

The results from our study are strikingly robust. We examined many alternative versions of our basic methodology, including changes in the parameter configuration defining a head-and-shoulders pattern, changes in the sample period, changes in the allowed size of head-and-shoulders patterns, and changes in the assumed underlying behavior of exchange rates. None of these produced results significantly different from those reported above.[17]

Robustness of Results to Parameter Variation: To determine how our results might be influenced by our particular specification of the head-and-shoulders pattern, we recalculate our results under various alternative parameter configurations. First, we relax the horizontal symmetry requirement (number four in Section IIIA). In particular, we try two alternative values for the maximum ratio of number of days between the left shoulder and the head and number of days between the head and right shoulder. The base value for this ratio is 2.5, and our alternatives are 2.0 and 3.0.

Second, we relax the vertical symmetry requirement regarding the height of the left and right shoulders relative to other local peaks and troughs (number three in Section IIIA). Specifically, in the head-and-shoulders top pattern, we simply require that the right shoulder exceed the left trough, rather than exceeding the midpoint between the left shoulder and left trough, and so forth.[18]

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[17] Full details of these sensitivity analyses are available from the authors upon request.

[18] For all these recalculations, the conditions are relaxed symmetrically for the head-and-shoulders tops and bottoms.

Third, we reduce by half stoploss the parameter determining the time of exit when exchange rates fail to reverse as predicted by the head-and-shoulders signal (see page 15). Fourth, we double the bounce parameter, from 0.25 to 0.5 (see page 15). None of these parameter adjustments has any noticeable effect on the significance of our profit results.

Robustness of Results to Changes in Sample Period: To determine whether opportunities for chart-based profits had increased or decreased in the past decade, we split the sample in two, the first half covering March 19, 1973, through December 31, 1983; the second half covering January 1, 1984, through June 13, 1994. This enables us to evaluate the widespread conviction among professional analysts that a dramatic rise in professional attention to these patterns since the early 1980s has partially undermined their usefulness as trend predictors. (The source of this conviction is unclear to the authors, since this trading strategy is a positive feedback strategy that would intensify rather than mitigate the predicted exchange rate changes.) The results, shown in Table 3, do not show a consistent increase or decrease in significance of profits, thereby refuting the traders' belief that profits had been dissipated as a result of increased market participation.

Robustness of Results to Differences in the Sizes of Head-and-Shoulders Patterns: To determine whether smaller or larger cutoffs tend to lead to higher profits, we also perform these tests using only a single cutoff at a time. Though the number of positions declines as cutoff increases, the overall profitability results appear to be largely unchanged by this breakdown: the yen and the mark exhibit unusually high profits (by a number of measures) for multiple cutoffs, while the majority of profit measures for the other currencies have insignificant p-values. In general, we find no systematic correlation between profitability and the size of cutoff.

Robustness of Results to the Assumption of Random-Walk Exchange Rate Behavior: To investigate whether the random-walk assumption about exchange rate behavior could have affected our results, we also generate simulated exchange rate series in which the data follow a GARCH (1,1) process. (The standard deviation was used, in place of the more familiar variance, for computational reasons.)[19] This seemed appropriate given the extensive literature documenting the serial correlation of exchange rate volatility.[20]

To create GARCH-based exchange rate series, we began by estimating the parameters for the following exchange rate process:

$$S_t = \alpha_0 + \alpha_1 s_{t-1} + \varepsilon_t ,$$

where

$$e_t \sim N(0,\sigma_t^2)$$

$$\sigma_t = \beta_0 + \beta_1 \sigma_{t-1} + \beta_2 \sqrt{\varepsilon_{t-1}{}^2} + \eta_t$$

and

$$\eta_t \sim N(0,\sigma_\eta^2) .$$

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[19] The choice was motivated by computational ease in our simulations; randomly drawn shocks to variance often resulted in negative variances.

[20] For example, see Domowitz and Hakkio (1985), Bollersley (1987), and Diebold and Pauly (1988).

From this we were able to reconstruct the series of volatility residuals, η_t, and a series of standardized exchange rate residuals, $e_t = \varepsilon_t/\sigma_t^2$. Then, drawing randomly with replacement from these 2 residual series and combining these draws according to the process specified above, we were able to construct our 10,000 simulated exchange rate (and volatility) series. The results from these GARCH-based simulations, presented in Table 4, give essentially the same conclusions as those from the random-walk-based simulations.

C. Testing Technical Analysts' Other Claims

So far this paper has provided mixed support for the claims of technical analysts regarding head-and-shoulders patterns: the pattern seems to have some predictive power for two major currencies, but no predictive power for four other major currencies. Our framework also provides the means for validity of other claims.

Price Objectives: Technical analysis manuals consistently emphasize that the vertical distance from the neckline to the head represents the minimum price objective that will characterize the trend reversal.[21] Therefore, the head height in a head-and-shoulders top represents the minimum distance by which the price is expected to fall after it has penetrated the neckline. An identical condition is associated with head-and-shoulders bottoms. (The rest of this section will refer to all head-and-shoulders patterns as if they were tops.) The term "mirror" represents the ratio of maximum post-head-and-shoulders downward vertical movement to the original head

[21] Our conversations with practicing technical analysts indicate that these claims are supported by practitioners as well as by the written manuals.

height. According to technical analysts, mirror should normally be greater than or equal to one.

In order to test this claim empirically, we measure the maximum fall to the next trough after every head-and-shoulders top confirmed by a penetration of the neckline. We choose our measure of mirror in every case to give the benefit of the doubt to technicians' claims. For example, note that the maximum fall will be greater in magnitude than actual profits earned from following our endogenous exit rule (since that exit rule requires one to wait until a trough has been identified by an upward price movement). In the case of a bounce (when the vertical distance from entry to the first trough is less than 25 percent of the original head height), we calculate the difference between the entry price and the first and second troughs and calculate mirror using the larger of the two. Finally, when the head-and-shoulders signal fails utterly and we exit at a stoploss, we assign mirror a value of zero.

Our results find no support for technical analysts' claims that this minimum price objective characterizes the trend reversal, if any, following a confirmed head-and-shoulders pattern. As shown in Table 5, 73 percent of the mirrors calculated fall below the proclaimed minimum value of one. In fact, for all currencies but the yen, even the *average* value of mirror is well below one.

Reliability: Another almost universal claim of the technical analysis manuals we surveyed is that the head-and-shoulders pattern is reliable. Since no benchmark is provided by disciples with which to measure this concept, it is hard to test whether the claim is valid. By one measure, the consistency with which profits are positive, our results suggest that head-and-shoulders patterns are certainly not reliable. For all six currencies, a speculator would earn negative or zero profits more than 40 percent of the time.

D. How Profitable Is the Head-and-Shoulders Strategy?

From 1973-94, head-and-shoulders signals for the mark resulted in thirty-two speculative positions, held on average seventeen business days. On any given position, returns reached a maximum of 9 percent and averaged 0.78 percent. For the yen, head-and-shoulders signals resulted in twenty speculative positions, held on average twenty-two business days. Total returns per position ranged as high as 12.8 percent and averaged 1.52 percent. For the other currencies, the number of positions and the average holding periods do not differ markedly from those of the mark and yen. Average profits for the French franc, while statistically insignificant, are nonetheless sizable, averaging over more than 0.50 percent earned on average holding periods slightly longer than three weeks. For the remaining three currencies, average profits were actually negative.

Annualized Profits: To get a sense of the magnitude of the profits on the mark and yen positions, we can express them in annual terms. If the average 0.78 percent profits in the mark could be sustained for a full year, rather than seventeen days, the compound return would be roughly 13 percent. This far exceeds the average annual buy-and-hold yield on the mark, which would have been 2.48 percent if one had been so fortunate as to hold marks rather than dollars. The head-and-shoulders-based yields also exceed bond yields during the period, as well as average yields on the New York Stock Exchange (as measured by the Standard and Poor's (S&P) 500, these average 6.78 percent over March 1973 to June 1994). If one could sustain the 1.52 percent average return on yen positions for a full year, the compound return would be roughly 19 percent. In addition to exceeding U.S. equity yields, this yield also exceeds the 4.35 percent average annual buy-and-hold return to holding yen (rather than dollars).

Profits from Multicurrency Strategies: So far our results are mixed, in that the head-and-shoulders pattern produced statistically and economically meaningful profits for two currencies but not for the other four. Is there any way to summarize the predictive power of the head-and-shoulders-based strategy? To answer this question, we consider the profits that would be earned by a speculator trading in all six currencies. Such a strategy would have produced total profits of 69.9 percent over the period, which differ significantly from profits generated in our simulated data (*p*-value of .0155).

This *p*-value could potentially be biased downward for the following reason. The historical data from which we derive our actual profit series include some positive correlation across currencies (that is, when the dollar strengthens against the mark, it has some tendency to strengthen against other currencies as well). If simulated profits from each currency were constructed from uncorrelated exchange rate series, then profits from each currency would likewise be uncorrelated. In turn, this would cause simulated aggregate profit (over all six currencies) to be relatively concentrated around the mean, resulting in more extreme *p*-values for actual profit. In actuality, we largely avoid this potential bias because, when we construct our simulated data, each random draw corresponds to the same historical day's change for five of our six currencies (the exception is the Canadian dollar, because data for this currency do not begin until 1976). The aggregate simulation based on just the five currencies with identical coverage is scarcely different from the results already presented: total profit from 1976 to 1994 is 74.5 percent, with a p-value of 0.0117. Thus, p-values from the multicurrency strategies remain significant.

V. Interpretation of Significant Profits

What could explain the profitability of the head-and-shoulders trading strategy? The first place to turn in answering this question would naturally be technical analysis manuals themselves. Unfortunately, the limited explanations in these manuals are not based on rigorous analysis and generally proceed by using metaphors. For example: "One does not bring instantly to a stop a heavy car moving at seventy miles an hour and, all within the same split second, turn it around and get it moving back down the road in the opposite direction at seventy miles an hour" (Edwards and Magee 1966, p. 48).

These explanations would not be regarded by most economists as very helpful. As economists, we would choose to examine whether the profits simply compensate investors for international interest differentials or risk, the two issues we examine next.

Profits Adjusted for Interest Differentials: Our reported profits on mark and yen positions do not incorporate the interest costs and earnings associated with using one currency to invest in another. The claims of technical analysis make no reference to interest costs, but it would be interesting to know, nonetheless, whether they make any difference to our profitability measures. To adjust for interest differentials, we calculate daily differentials in overnight Eurocurrency rates between the foreign currency in question and the U.S. dollar and apply them to the holding periods associated with our head-and-shoulders positions.[22] Adjusting for interest differentials raises average profits for both the mark and the yen, but by so little that the change gets lost in the rounding. The interest adjustment raises mark profits more substantially, from 0.78 percent to 0.85 percent. In both cases, the fact that profits *increase* suggests that the speculative positions were more often long the higher interest rate currency and short the lower interest rate currency.

Profits and Risk: While profitable for the mark and the yen, the head-and-shoulders strategy is extremely risky. Ideally, we could evaluate the profits on a risk-adjusted basis. There is no consensus, however, on how to risk-adjust excess returns to foreign exchange speculation. One approach is to calculate the Sharpe ratio of annualized excess returns to their standard deviation. These are shown below, along with the comparable statistic for the S&P 500.[23]

	H&S Yen	**H&S Mark**	**S&P 500**
Sharpe Ratio	1.47	1.00	0.32

On this basis, it would seem reasonable to conclude that speculating on the head-and-shoulders pattern in yen and marks is more profitable on a risk-adjusted basis than speculating on the S&P 500, though this approach does not provide confidence intervals from which to conclude statistical significance.

Since investors are presumably more concerned with systematic risk

[22] In performing this adjustment, we accounted for the cost of funds over weekends and, of course, for the direction of our position.

[23] The Sharpe ratio for the S&P 500 is calculated using annual total returns with data from March 1973 through March 1994.

than absolute risk, it would be desirable to adjust our profit measures for systematic risk. The universe of assets against which to measure the systematic portion of exchange rate risk is difficult to define. One creative solution to this problem was originally presented by Sweeney (1986). Appendix C describes how we modified his original methodology for the present context. The centerpiece of the methodology is a statistic, *X,* that represents the difference between returns to the head-and-shoulders strategy on the one hand and buy-and-hold on the other, each appropriately adjusted for holding periods. The *t*-statistics for *X* for the mark and yen are 1.357 and 1.762, respectively, indicating in each case that risk-adjusted excess profits are not significantly different from zero when risk is measured according to this approach.

There are many reasons to question the validity of these results based on Sweeney's methodology. First, the methodology assumes that the risk premium on excess returns is constant, an assumption called into question by relatively recent research showing that conditional exchange rate volatilities are time-varying (see, for example, Cumby and Obstfeld 1984). Second, the assumption that short-term interest rates are nonstochastic is also a potential source of concern. Finally, during our sample period it appears that very little of exchange rate risk can be characterized as systematic: the correlations between monthly exchange rate changes and corresponding monthly total returns to the S&P 500 are -.03 for the mark and -.071 for the yen.

To verify that the profits we obtained for the mark and yen were unlikely to be explained as a reward for bearing systematic risk, we calculated the beta relating excess returns from our head-and-shoulders positions to excess U.S. and foreign stock market returns. We regressed daily excess returns to our speculative positions in the mark on daily excess returns to the S&P 500 and, separately, the Deutsche Ahtien Index. Similar regressions were performed for the yen, using the S&P 500 and the Nikkei 225 index. None of our point estimates of beta was statistically significant, and the largest was .03.[24]

In sum, there is little evidence that the profits we obtained can be attributed to systematic risk and, in the absence of such risk, it is reasonable to focus on total risk. While speculation based on head-and-shoulders patterns is clearly risky, the Sharpe ratios indicate that the magnitude of the returns more than amply compensates for the accompanying risk.

Profits Adjusted for Transactions Costs: Our observed profits apparently cannot be dismissed as compensation for interest rate differentials or risk. It is natural to question, however, whether these profits would vanish if we accounted for transactions costs. In currency markets, transactions costs take the form of bid-ask spreads, which differ by customer. We take as our benchmark a mid-sized spread, one that would be paid by good corporate customers of a major foreign exchange house. Other foreign exchange banks would generally do better; smaller corporations would generally do worse. Per dollar traded, this spread would have cost at most 0.0010 marks or 0.1 yen and would have shaved about 0.05 percent off

[24] We also regressed total excess returns for a given position on stock market excess returns for corresponding time intervals. Again, point estimates for the betas were statistically insignificant and numerically tiny.

returns for each round-trip transaction in either currency.[25] After adjustment for both interest differentials and transactions costs, mark returns would have averaged 0.86 percent per position, and yen returns would have averaged 1.48 percent, still remarkably high given the brevity of our holding periods.

Other Explanations for Profits: The first three specific interpretations of our results have failed to explain the profits found from following a head-and-shoulders-based trading strategy. We turn next to three other possible interpretations, all of which are less readily tested.

1. Uncertainty and Imperfect Information: Brown and Jennings (1990) suggest that, when markets are characterized by uncertainty and imperfect information, technical analysis may be useful because the current price is not a sufficient statistic for the market's full information set. Though their model incorporates only two periods, for which reason it could not be applied directly to the case of the head-and-shoulders pattern, its basic insight might generalize to the many-period setting of that pattern.

2. Self-Fulfilling Predictions: Likewise, relatively few theories explain why we find significant profits for only the mark and yen. One possible explanation could be self-fulfilling expectations: since technical traders who follow head-and-shoulders trading rules sell as soon as they perceive downtrends and buy when they perceive uptrends, they could potentially

[25] For both currencies, the average percent bid-ask cost was calculated by taking a given point spread and dividing it by the average exchange rate over the sample period. The given point spreads, provided for foreign exchange salespersons at Natwest Markets, New York City, actually represent a very large estimate of the typical spreads for good corporate customers.

create persistence in what might otherwise be purely transitory price movements.[26] In practice, such speculative activity tends to be heavily concentrated on the mark and the yen, so the self-fulfilling tendency would be most apparent in those currencies.

If market activity and the significance of technical signals were positively associated, as suggested by the hypothesis that these predictions are self-fulfilling, then the steeply increasing market attention received by the yen since 1973 would generate increasing significance of the head-and-shoulders pattern over time. As was shown in Table 3, however, the opposite has been true: the significance of the head-and-shoulders pattern for the yen seems to be slightly lower from 1983 on. This suggests that a currency's prominence in global markets, at least taken in isolation, is unable to explain the observed significance of technical patterns for only the mark and the yen.

3. Central Bank Intervention: Another potential explanation for the profitability of our technical trading rule is central bank intervention, since introduction of a major player with non-profit-maximizing objectives can introduce departures from random-walk behavior. The hypothesis that intervention is the source of these profits receives some indirect support in Silber (1994), who examines profits from moving-average trading rules in futures contracts for currencies, short-term interest rates, the S&P 500, and commodities. He finds abnormal profits in the markets characterized by government intervention to smooth prices, such as currencies and interest rates, but not in the other markets.

[26] This speculative activity provides an example of positive feedback trading of the sort examined by De Long et al. (1990) and Frankel and Foot (1990).

Within our sample of currencies, the hypothesis that central bank intervention is the source of the extra profitability of head-and-shoulders-based speculation in the mark and yen is not more directly testable at present: intervention data are available for only a few of the seven central banks associated with our six exchange rates, and the available intervention series do not cover our full sample period. Nonetheless, for those few currencies and time periods where data are available, it could be fruitful to conduct a careful analysis of this hypothesis. This is left for future research.

Casual empiricism suggests that our results are not fully consistent with the hypothesis that profits can be attributed to central bank intervention. As is widely known, the Bank of Canada intervenes almost daily in the market for Canadian dollars, while intervention is reported only infrequently in the market for marks and yen. If intervention was the source of our trading profits, then the profits would likely be statistically significant for the Canadian dollar as well as the mark and the yen. However, our results indicate that profits are actually negative for the Canadian dollar.

VI. Concluding Comments

This paper assesses the predictive power of the nonlinear, head-and-shoulders visual chart pattern, extending the existing literature on technical analysis, which has focused on linear, easy-to-calculate trading rules. These patterns make up an important class of trading rules that have not heretofore been evaluated rigorously. We choose the head-and-shoulders pattern because it is cited by technical analysts as particularly frequent and reliable.

The paper first describes a head-and-shoulders pattern. It then devises algorithms to identify these signals and to measure profits derived from technical analysts' rules in daily foreign exchange data. It compares those profits with the distribution of similarly derived profits based on data in which head-and-shoulders patterns are meaningless by design.

For the mark and the yen, head-and-shoulders-based profits derived from actual foreign exchange data are significantly greater than those derived from artificial data. For the Canadian dollar, the Swiss franc, the French franc, and the pound, head-and-shoulders-based profits derived from actual foreign exchange data are not significantly different from those derived from artificial data. To summarize the predictive power of our trading strategy, we investigate profits from speculating in all six currencies simultaneously over the same time horizon. Our findings show that these aggregate profits would have been both statistically and economically meaningful regardless of transactions costs, interest differentials, or risk. On this basis we conclude that head-and-shoulders signals have some predictive power for the mark and yen during the twenty years since the advent of floating exchange rates.

The technical strategy examined here can be viewed as one of a large class of nonlinear prediction rules potentially deriving from nonlinear versions of structural models such as the monetary model (Meese and Rose 1991), target-zone models (Krugman 1991), chaos models (Gilmore 1991), ARCH-in-Mean and other econometrically based models with structural content (Diebold and Pauly 1988), or the Self-Exciting Threshold Autoregressive model (Krager and Kugler 1993). Many of these models have been shown to fit the data with some acceptable level of explanatory power within sample, and some appear to be helpful in forecasting conditional exchange rate variances. Nonetheless, out-of-sample tests of these models indicate that they generally forecast short-term exchange rate changes with little or no greater success than the random-walk model (Hsieh 1988; Diebold and Nason 1990; Meese and Rose 1991). In contrast, we find that significant profits could be obtained by following a nonlinear prediction rule, although the trading strategy has no known theoretical foundation.

Much more research is needed before there will be any consensus regarding the predictive value of visual chart patterns in general, or the head-and-shoulders pattern, in particular. The research presented here has some very clear limitations that deserve to be addressed in future research: it applies only to foreign exchange markets, it considers only one market signal (price) even though technical analysis manuals frequently assert the importance of checking volume signals as well as price movements, and it covers a period of only twenty years in which typically only thirty head-and-shoulders patterns are observed in each currency. Though we began our inquiry with foreign exchange markets since they are the most liquid in the world, some of these data problems could potentially be overcome by applying our methodology to equity or commodity price data.

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Table 1. Profit Statistics on 10,000 Simulated Exchange Rate Series (Percent)

	JAPANESE YEN	DEUTSCHE MARK	CANADIAN DOLLAR	SWISS FRANC	FRENCH FRANC	BRITISH POUND
Endogenous exit rule						
Average percent profit	-0.0267	0.0132	-0.0085	-0.0344	-0.0304	-0.0261
Exogenous returns						
1-day return	-0.0147	-0.0162	-0.0079	-0.0234	-0.0189	-0.0200
3-day marginal return	-0.0093	-0.0133	-0.0038	-0.0129	-0.0073	-0.0071
5-day marginal return	-0.0005	0.0015	-0.0005	0.0014	0.0012	0.0001
10-day marginal return	-0.0064	-0.0025	-0.0021	0.0015	-0.0033	-0.0031
20-day marginal return	-0.0141	-0.0037	-0.0057	-0.0012	-0.0005	0.0031
30-day marginal return	-0.0164	-0.0036	-0.0041	-0.0043	-0.0030	-0.0035
60-day marginal return	-0.0175	0.0005	-0.0150	-0.0209	0.0051	0.0006
10-day **total** return	0.0561	0.0094	-0.0142	-0.0335	-0.0282	-0.0301
30-day **total** return	-0.0615	-0.0378	-0.0240	-0.0389	-0.0318	-0.0305

NOTES: Head-and-shoulders identification and profit-taking algorithms are applied to simulated exchange rates. These exchange rates are constructed by sampling with replacement from actual exchange rate changes.

Table 2a. Profitability of Positions Following Head-and-Shoulders Patterns, 1973-94
Endogenous Exit Rule

	JAPANESE YEN	**DEUTSCHE MARK**	**CANADIAN DOLLAR**	**SWISS FRANC**	**FRENCH FRANC**	**BRITISH POUND**
Average profit (percent)	1.5158% **0.0035**	0.7784% **0.0438**	-0.1874% **0.7655**	0.0959% **0.3638**	0.5653% **0.1109**	-0.0699% **0.5022**
Average holding period	22.2	16.6	18.6	11.5	16.2	20.0
Number of positions	20	32	25	27	33	27

NOTES: Head-and-shoulders identification and profit-taking algorithms are applied to actual exchange rate data, assuming exit takes place when trough (or peak) has been identified following entry. *P*-values, shown in bold type, for the hypothesis of no predictive power for the head-and-shoulders patterns are calculated by comparing profits from the actual data with those from data simulated by sampling with replacement from original exchange rate changes.

Table 3. Profitability of Positions Following Head-and-Shoulders Patterns, 1973-83 and 1984-94
Random-Walk-Based Estimates (*P*-values)

	JAPANESE YEN		DEUTSCHE MARK		CANADIAN DOLLAR		SWISS FRANC		FRENCH FRANC		BRITISH POUND	
	1h	2h	1h	2h	1h	2h	1h	2h	1h	2h	1h	2h
Average profit (percent)	0.002	0.090	0.118	0.095	0.649	0.870	0.352	0.396	0.050	0.350	0.255	0.634
1-day return	0.355	0.400	0.720	0.223	0.823	0.406	0.169	0.666	0.393	0.821	0.480	0.763
3-day marginal return	0.003	0.106	0.170	0.722	0.176	0.893	0.097	0.547	0.730	0.866	0.644	0.214
5-day marginal return	0.681	0.182	0.970	0.512	0.206	0.351	0.947	0.893	0.749	0.507	0.324	0.135
10-day marginal return	0.321	0.885	0.304	0.272	0.961	0.549	0.958	0.333	0.331	0.170	0.097	0.629
20-day marginal return	0.732	0.003	0.384	0.434	0.018	0.419	0.927	0.393	0.107	0.470	0.064	0.807
30-day marginal return	0.889	0.759	0.067	0.102	0.219	0.592	0.379	0.284	0.084	0.199	0.799	0.709
60-day marginal return	0.047	0.475	0.254	0.856	0.590	0.424	0.760	0.432	0.362	0.876	0.011	0.772

NOTES: Head-and-shoulders identification and profit-taking algorithms are applied to actual exchange rate data, assuming exit takes place when trough (or peak) has been identified following entry. *P*-values for the hypothesis of no predictive power for the head-and-shoulders patterns are calculated by comparing profits from the actual data with those from data simulated by sampling with replacement from original exchange rate changes.

Table 4. Profitability of Positions Following Head-and-Shoulders Patterns, 1973-94
GARCH-based Estimates

	JAPANESE YEN	DEUTSCHE MARK	CANADIAN DOLLAR	SWISS FRANC	FRENCH FRANC	BRITISH POUND
Endogenous exit rule						
Average profit	1.5158%	0.7784%	-0.1874%	0.0959%	0.5653%	0.0699%
p-value	**0.0024**	**0.0505**	**0.7620**	**0.3646**	**0.1109**	**0.4957**
Exogenous exit rules						
1-day return	0.0274%	0.0403%	-0.0233%	-0.0119%	-0.1117%	-0.0995%
p-value	**0.3441**	**0.3439**	**0.6025**	**0.4770**	**0.7693**	**0.7386**
3-day marginal return	0.4793%	-0.0299%	-0.0698%	0.0973%	-0.2581%	0.0908%
p-value	**0.0032**	**0.5411**	**0.8016**	**0.3095**	**0.9121**	**0.2887**
5-day marginal return	0.0939%	-0.1673%	0.0595%	-0.4059%	0.0025%	0.2214%
p-value	**0.2746**	**0.7951**	**0.2243**	**0.9697**	**0.6468**	**0.1045**
10-day marginal return	-0.2481%	0.2350%	-0.1173%	-0.1647%	0.0058%	0.0781%
p-value	**0.8322**	**0.2452**	**0.8142**	**0.6881**	**0.1272**	**0.3843**
20-day marginal return	0.9179%	0.1103%	0.2801%	-0.2308%	0.2748%	0.0280%
p-value	**0.0073**	**0.4107**	**0.0491**	**0.6955**	**0.2501**	**0.5207**
30-day marginal return	-0.4967%	0.7919%	0.0098%	0.3223%	0.6324%	-0.3735%
p-value	**0.9092**	**0.0506**	**0.4586**	**0.2394**	**0.0611**	**0.8397**
60-day marginal return	-0.5688%	-0.5323%	0.1401%	-0.1751%	-0.7319%	0.2996%
p-value	**0.1711**	**0.7507**	**0.2933**	**0.5899**	**0.8563**	**0.3157**

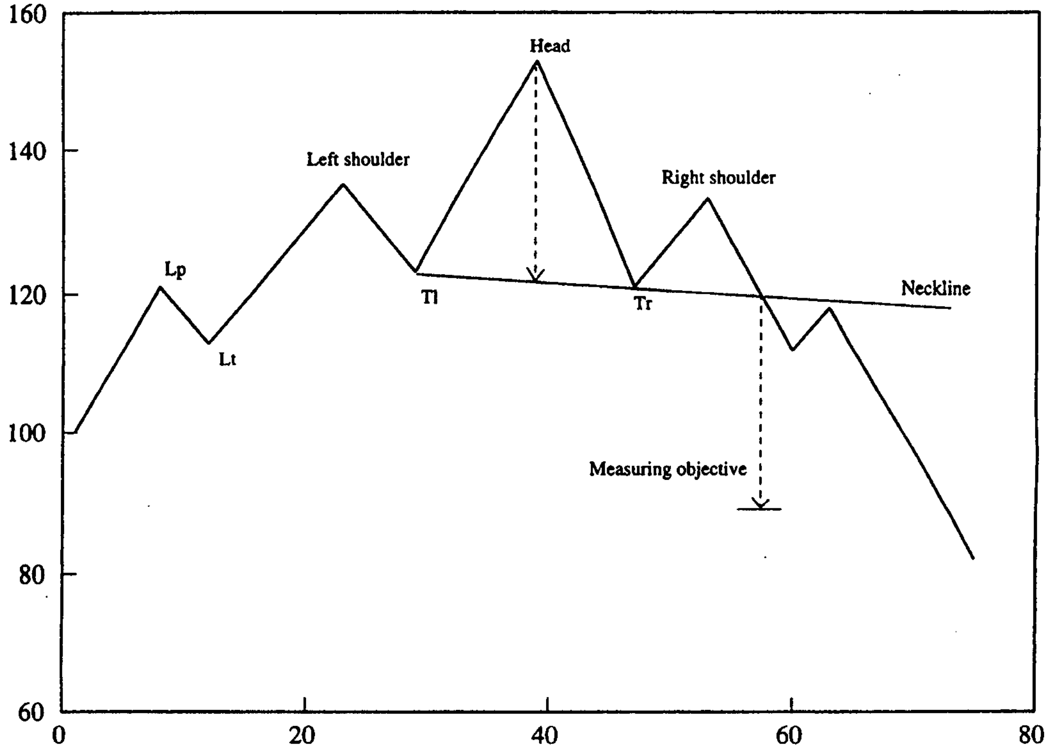
NOTES: Head-and-shoulders identification and profit-taking algorithms are applied to actual exchange rate data, assuming exit takes place when trough (or peak) has been identified following entry. *P*-values, shown in bold type, for the hypothesis of no predictive power for the head-and-shoulders patterns are calculated by comparing profits from the actual data with those from data simulated by sampling with replacement from original exchange rate changes and original series of standard deviations. Original standard deviations estimated using GARCH methodology; simulated standard deviations are estimated by applying that same methodology.

Table 5. Distribution of Values for Mirror

	JAPANESE YEN	DEUTSCHE MARK	CANADIAN DOLLAR	SWISS FRANC	FRENCH FRANC	BRITISH POUND
Average mirror	1.21	0.87	0.58	0.51	0.69	0.55
Range:	No. (%)	No. (%)	No. (%)	No. (%)	No. (%)	No. (%)
0.0	5 (23)	11 (34)	11 (44)	11 (41)	14 (42)	11 (41)
0.0-0.5	5 (23)	5 (16)	3 (12)	7 (26)	4 (12)	8 (30)
0.5-1.0	3 (14)	9 (28)	4 (16)	4 (15)	3 (9)	2 (7)
1.0 or higher	7 (32)	8 (25)	7 (28)	5 (19)	12 (36)	6 (22)
Total positions	22%	32%	25%	27%	33%	27%
P-value, total-positions	0.90%	0.49%	0.51%	0.73%	0.20%	0.63%

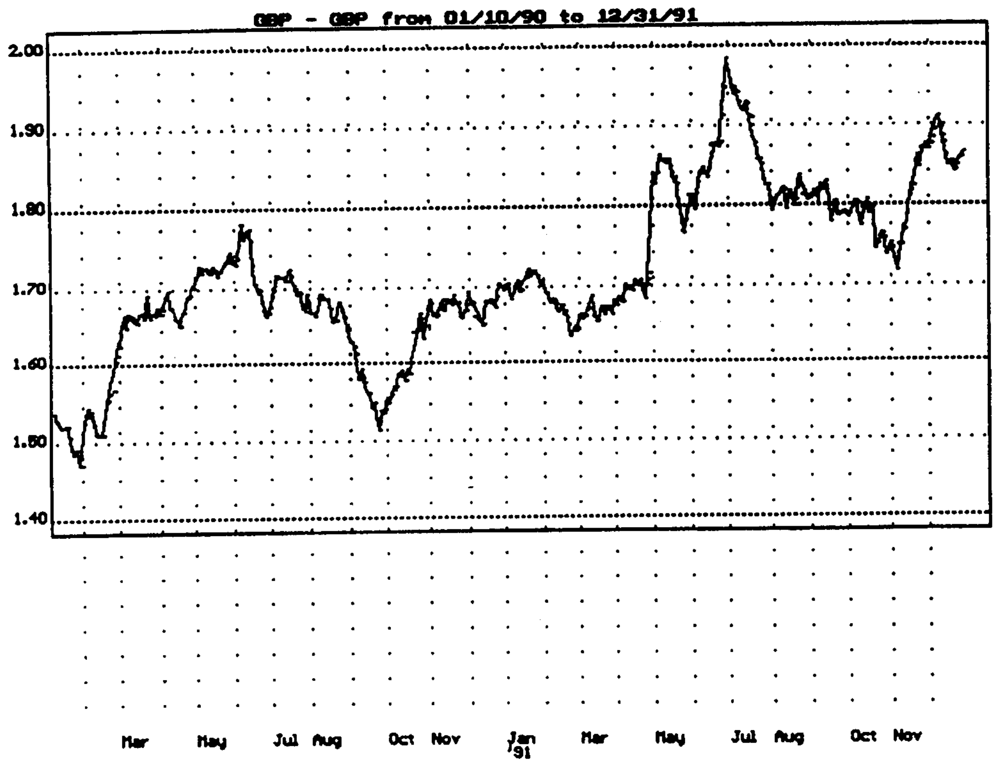
NOTES: Mirror represents the ratio of (a) the difference between the entry price and the final trough before exit to (b) the vertical distance from the head-and-shoulder's peak to its neckline. Technical analysts claim this should at least be unity.

Chart 1
Schematic Head-and-Shoulders Pattern



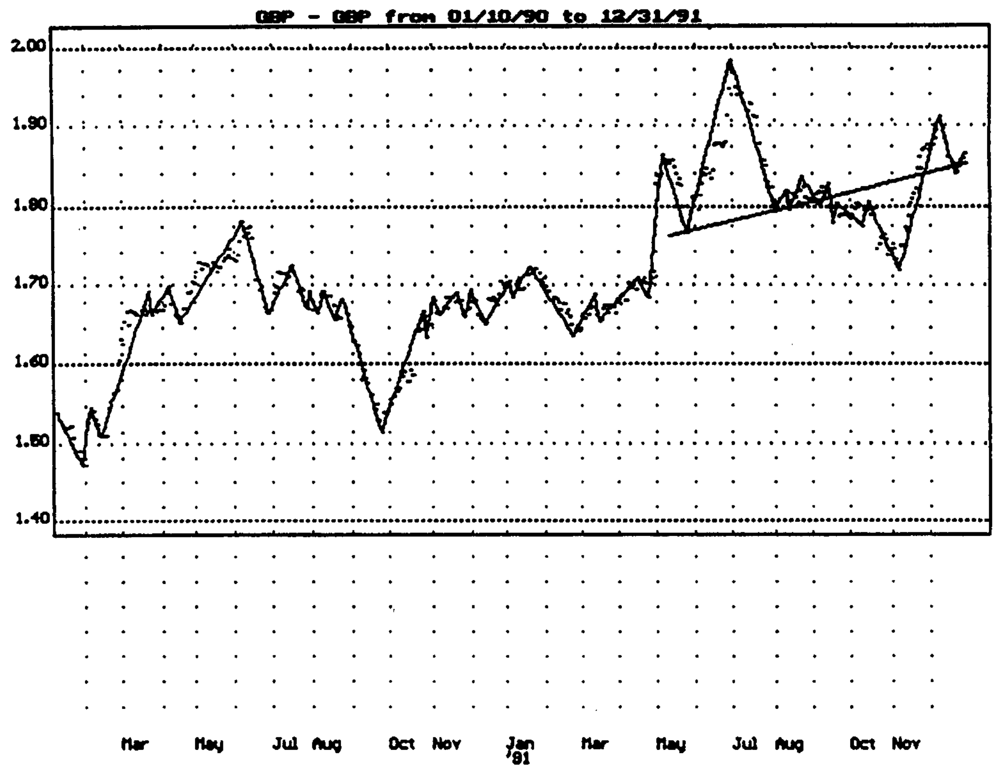
Original Data

Chart 2a



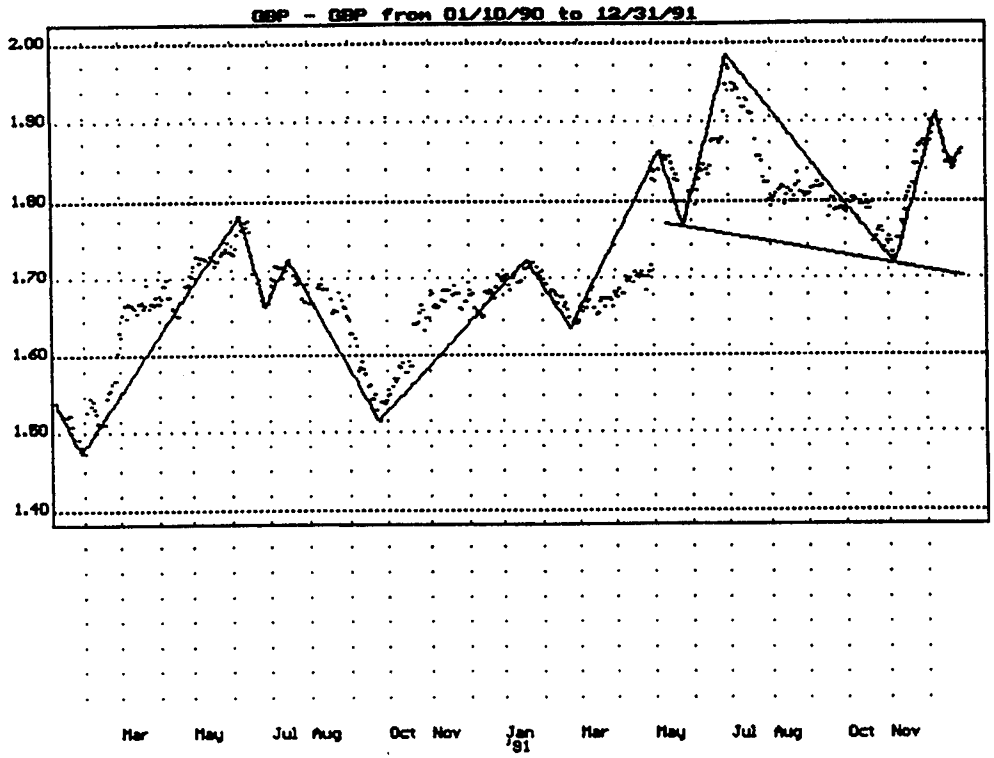
Cutoff = 1 %

Chart 2b



Cutoff = 3 %

Chart 2c



Cutoff = 5 %

Chart 2d

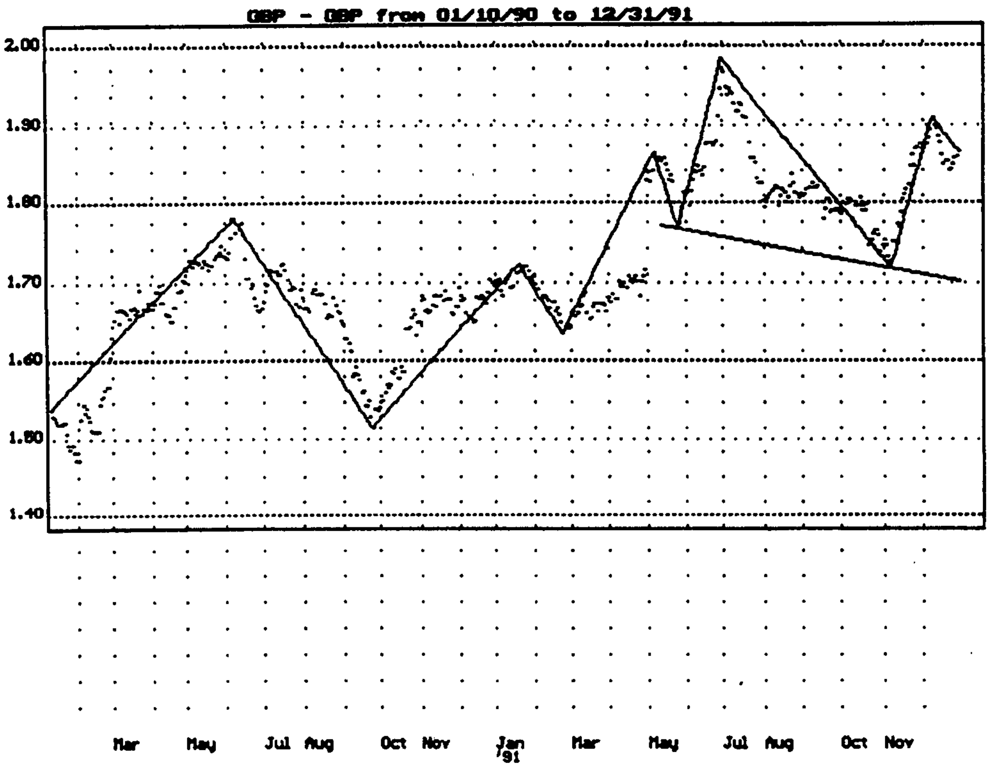


Chart 3a
Profits from Yen Positions

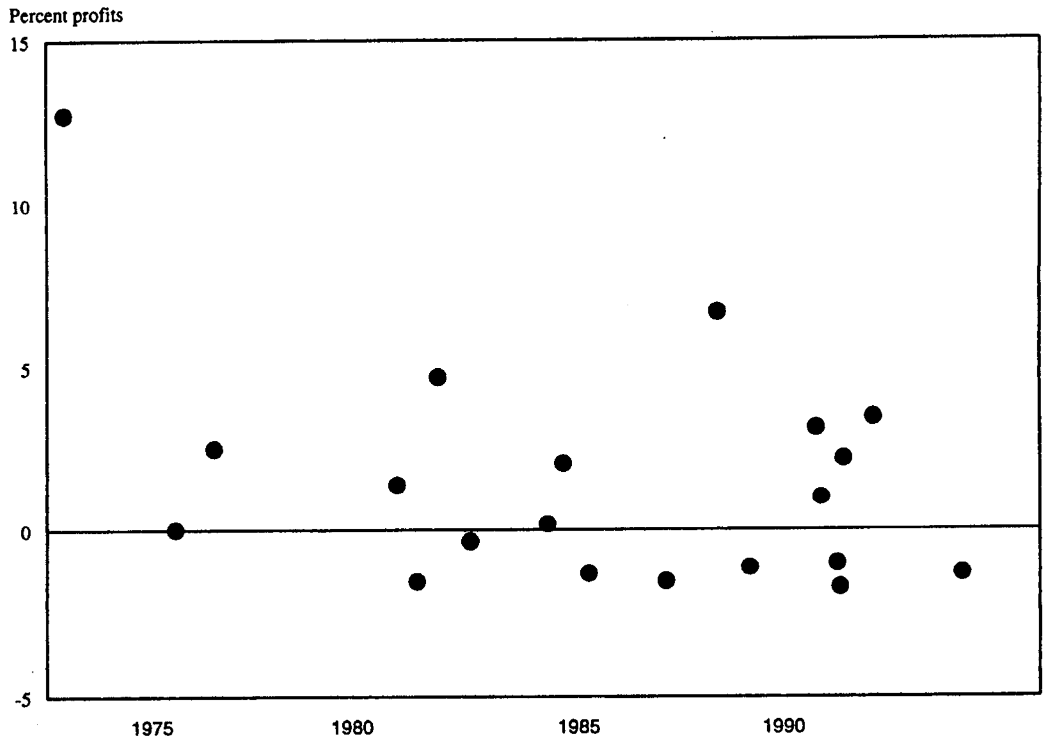


Chart 3b
Profits from Deutsche Mark Positions

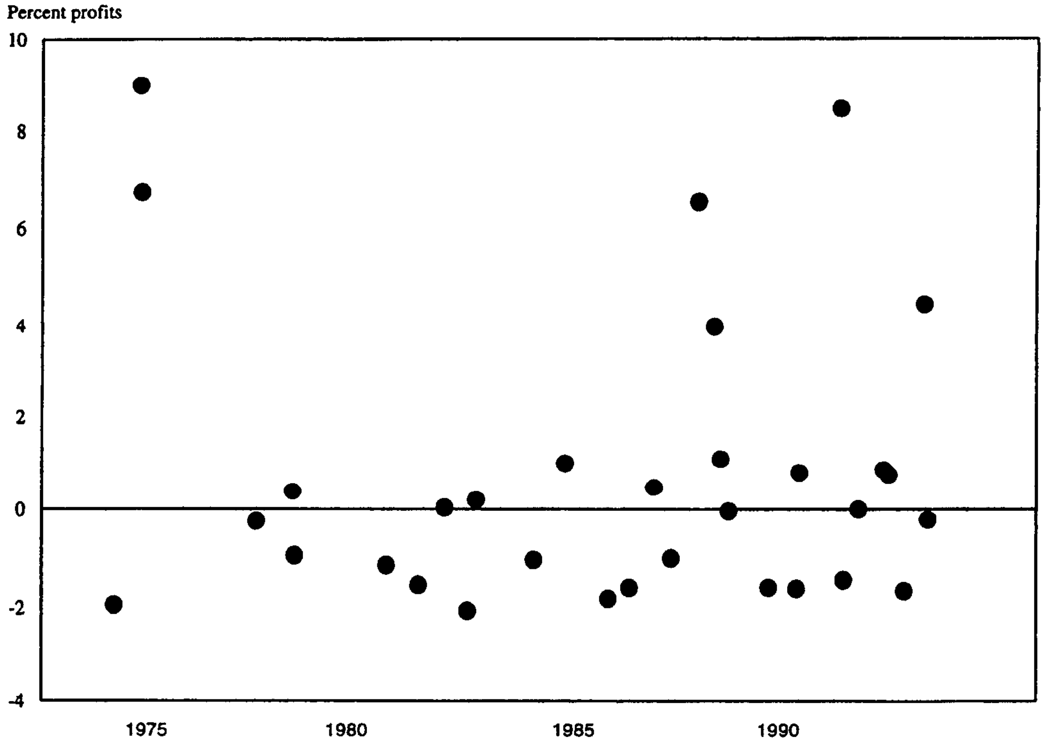


Chart 3c
Profits from Canadian Dollar Positions

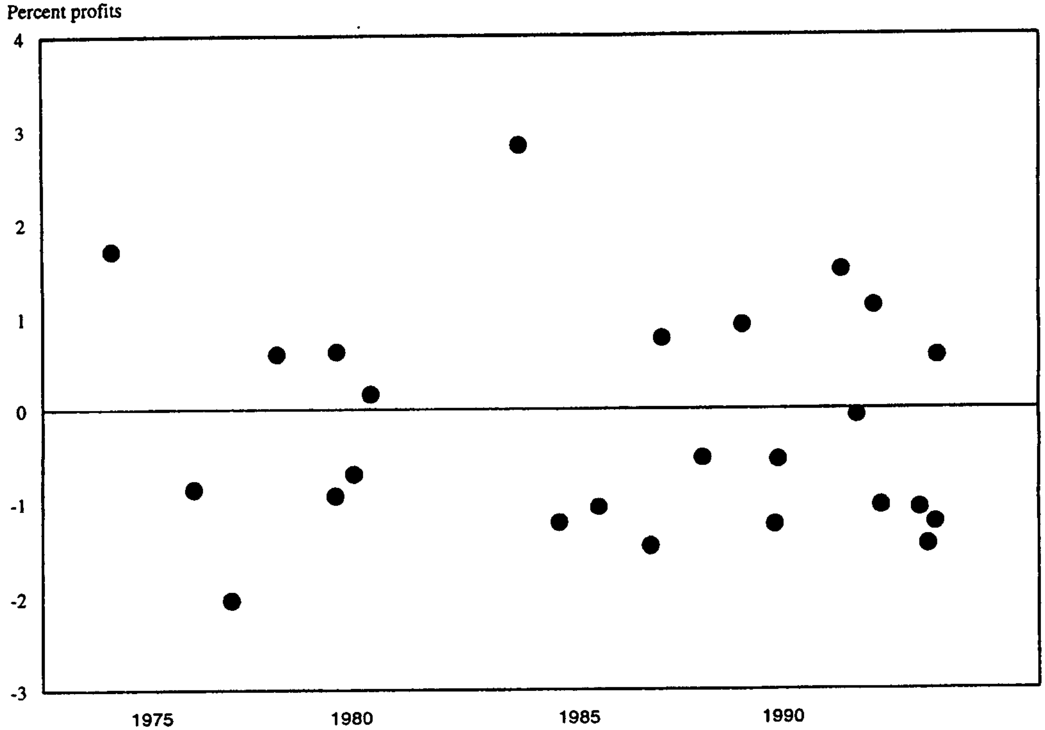


Chart 3d
Profits from Swiss Franc Positions

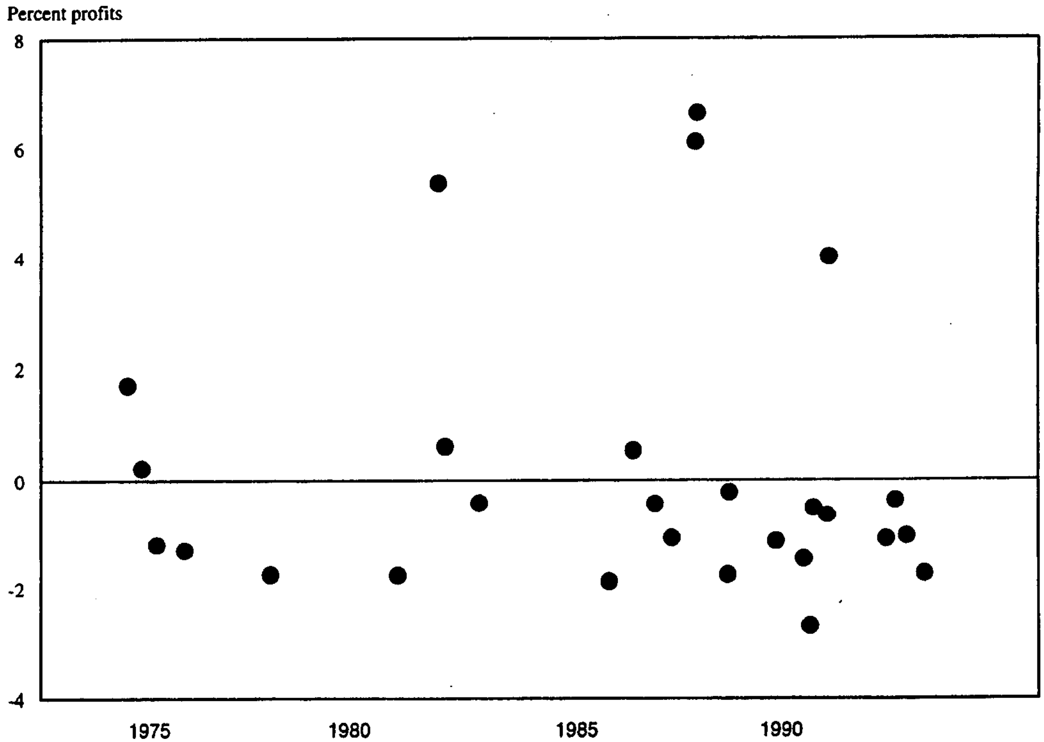


Chart 3e
Profits from French Franc Positions

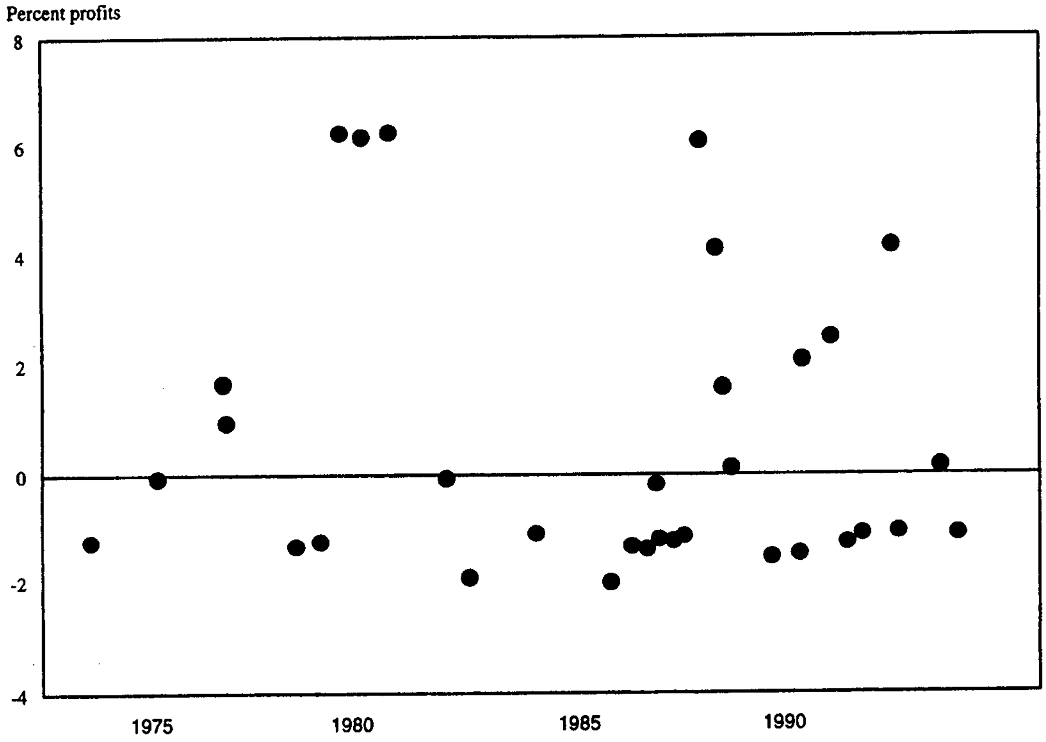
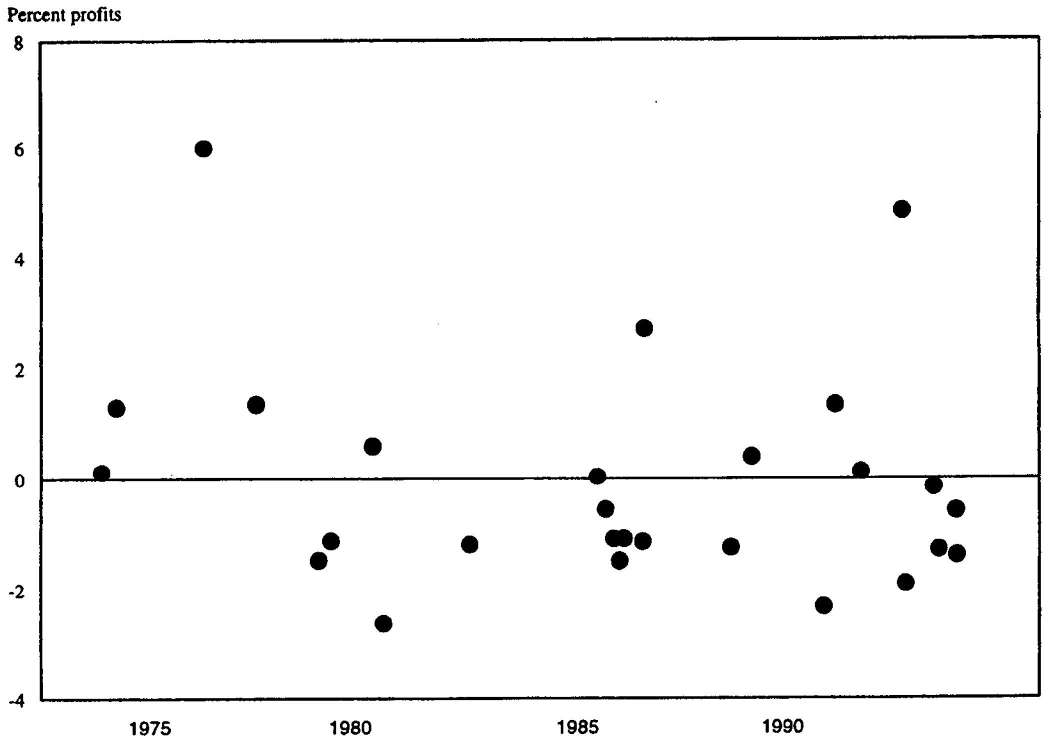


Chart 3f
Profits from U.K. Pound Positions



APPENDIX A: EXPLAINING NEGATIVE AVERAGE PROFITS IN SIMULATED SERIES

In the 10,000 simulated series, the average percent profit is significantly negative for all currencies. Our bootstrapping results are unaffected by this, since our tests of significance are based on *p*-values. Since floating exchange rates have closely approximated a random walk, the finding of significantly negative average profits was initially perplexing. It can be explained, however, by exchange rate trends over our sample. For example, yen per dollar declined from 264 on March 19, 1973, to 103 on June 13, 1994. Because of the overall downward trend in the exchange rate (expressed as yen/U.S. dollar), head-and-shoulders bottoms (both confirmed and unconfirmed), which occur after downward trends, appear more frequently than head-and-shoulders tops. For this reason, we take long positions more frequently than short positions. Since first differences are drawn at random from a distribution with a negative unconditional mean, these long positions on average produce losses.

The same holds in reverse for exchange rates that trended upwards during this period, that is, rates whose first difference is drawn from a distribution with a positive unconditional mean. The higher frequency of head-and-shoulders tops results in more short positions, which on average generate a loss, given the positive unconditional mean. These effects are summarized in the table below.

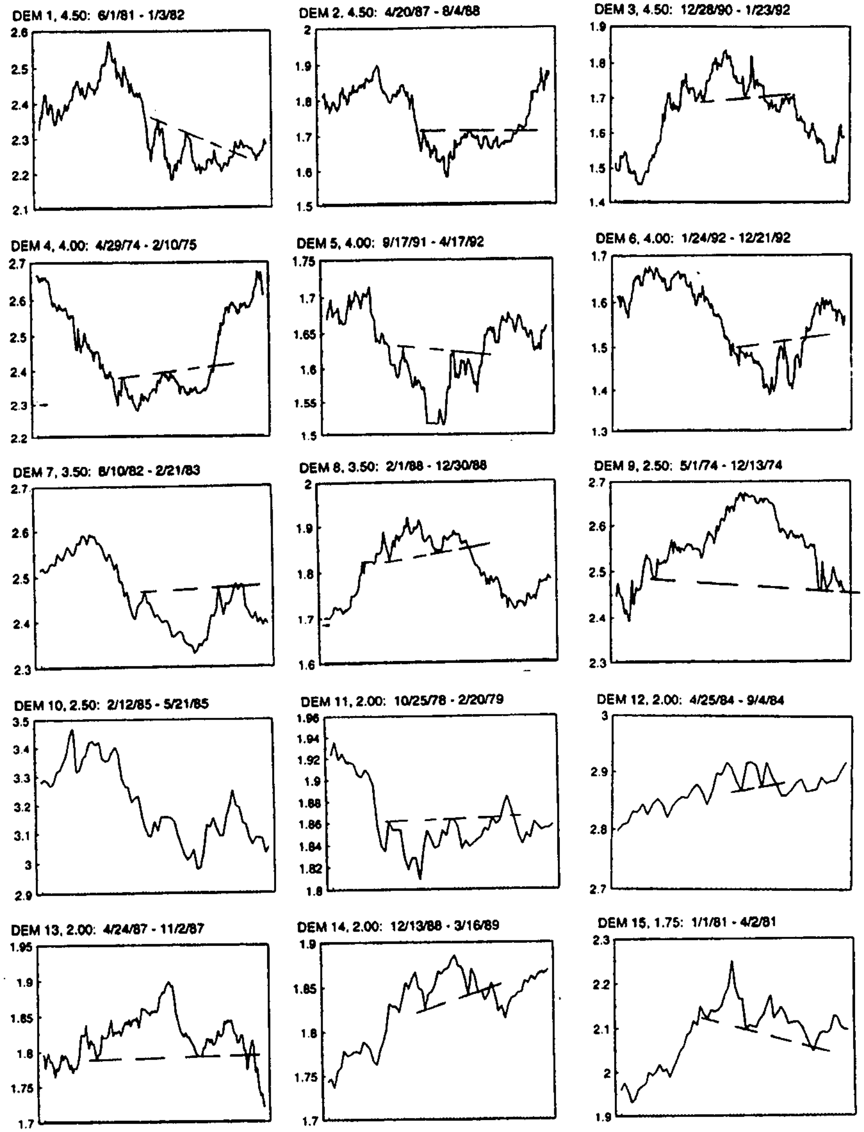
Average Simulated Profits, Exchange Rate Trends, and Direction of Head-and-Shoulders Patterns

	Yen (JPY/USD)	**Mark** (DEM/USD)	**Can. Dollar** (USD/CAD)	**Swiss Fr.** (CHF/USD)	**French Fr.** (FRF/USD)	**Pound** (USD/GBP)
Average profits (percent) (Standard error)	-0.0194 (0.0045)	-0.0376 (0.0044)	-0.0085 (0.0025)	-0.0344 (0.0049)	-0.0304 (0.0048)	-0.0261 (0.0045)
Exchange rate trends						
March 19, 1973*	264	2.82	0.977	3.21	4.55	2.46
June 13, 1994	103	1.65	1.383	1.40	5.63	1.51
Percent change	-61	-41	42	-56%	24%	-39%
Direction of head-and-shoulders pattern						
Mean percent of tops (Standard error)	43.96 (0.09)	47.31 (0.08)	55.63 (0.09)	43.09 (0.09)	50.99 (0.09)	47.43 (0.09)
Percent short positions (Standard error)	46.31 (0.10)	48.44 (0.09)	53.63 (0.10)	47.84 (0.09)	51.18 (0.09)	48.50 (0.09)

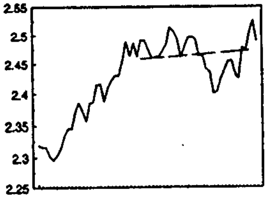
* The Canadian exchange rate series begins August 1, 1976.

Appendix B: Head-and-shoulders Patterns in the Mark and Yen

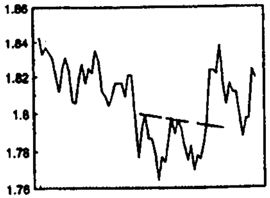
The charts below show confirmed patterns detected under the baseline parameterization for the Mark.



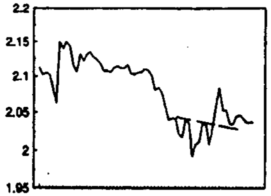
DEM 16, 1.75: 5/18/82 - 8/10/82



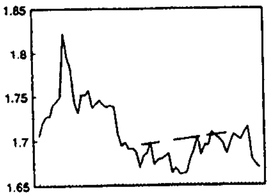
DEM 17, 1.75: 3/16/87 - 6/16/87



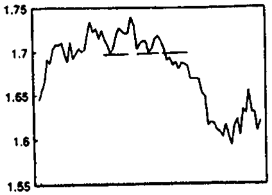
DEM 18, 1.50: 12/28/77 - 3/27/78



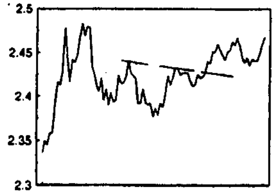
DEM 19, 1.50: 8/8/91 - 11/1/91



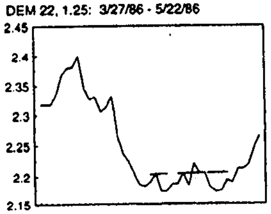
DEM 20, 1.50: 6/16/93 - 9/30/93



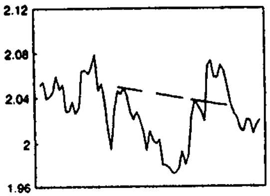
DEM 21, 1.25: 1/11/83 - 5/17/83



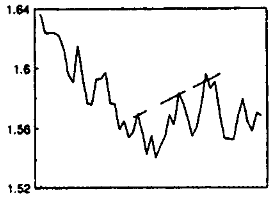
DEM 22, 1.25: 3/27/86 - 5/22/86



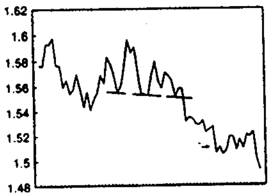
DEM 23, 1.25: 8/20/86 - 11/24/86



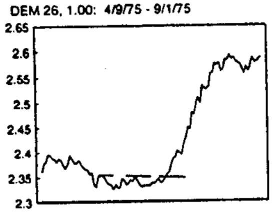
DEM 24, 1.25: 7/23/90 - 9/27/90



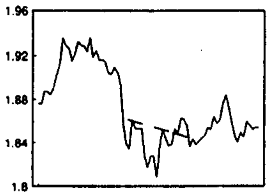
DEM 25, 1.25: 8/6/90 - 11/5/90



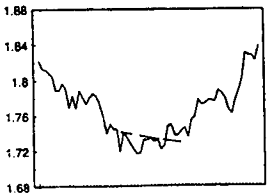
DEM 26, 1.00: 4/9/75 - 9/1/75



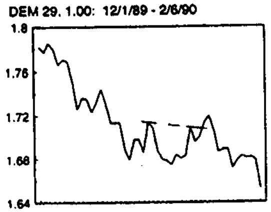
DEM 27, 1.00: 11/8/78 - 2/19/79



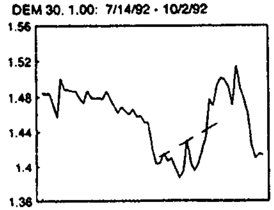
DEM 28, 1.00: 10/14/88 - 1/13/89



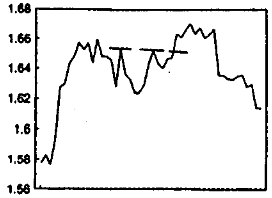
DEM 29, 1.00: 12/1/89 - 2/6/90



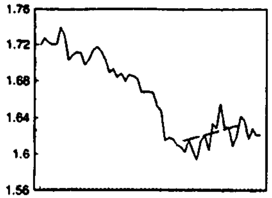
DEM 30, 1.00: 7/14/92 - 10/2/92



DEM 31, 1.00: 1/26/93 - 4/1/93

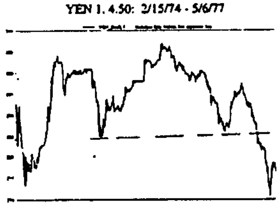


DEM 32, 1.00: 7/23/93 - 10/8/93

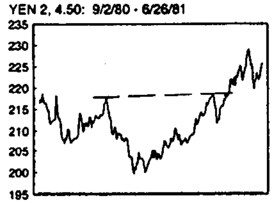


. The charts below show confirmed patterns detected under the baseline parameterization for the Yen.

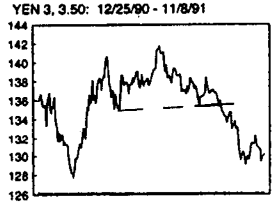
YEN 1, 4.50: 2/15/74 - 5/6/77



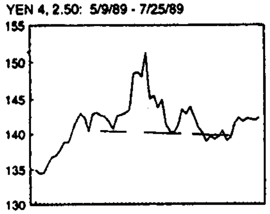
YEN 2, 4.50: 9/2/80 - 6/26/81



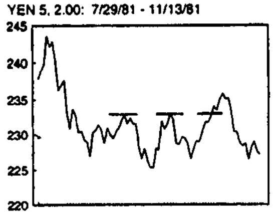
YEN 3, 3.50: 12/25/90 - 11/8/91



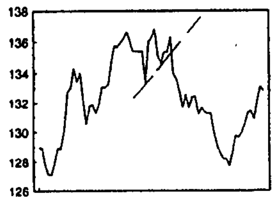
YEN 4, 2.50: 5/9/89 - 7/25/89



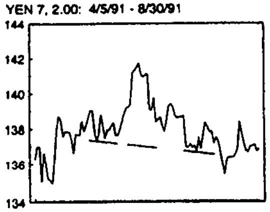
YEN 5, 2.00: 7/29/81 - 11/13/81



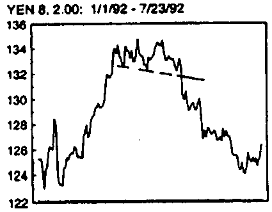
YEN 6, 2.00: 11/19/90 - 2/27/91



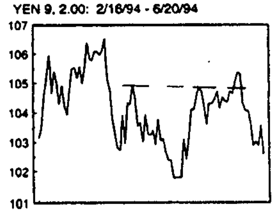
YEN 7, 2.00: 4/5/91 - 8/30/91



YEN 8, 2.00: 1/1/92 - 7/23/92

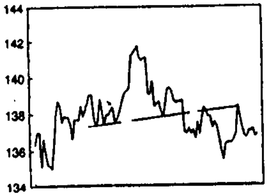


YEN 9, 2.00: 2/16/94 - 6/20/94



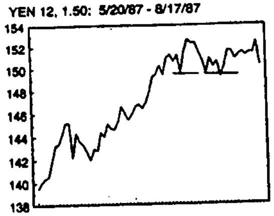
Appendix B (Continued)

YEN 10, 1.75: 4/5/91 - 8/30/91

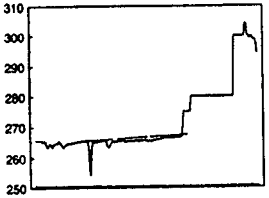


YEN 11, 1.50: 7/29/81 - 11/13/81

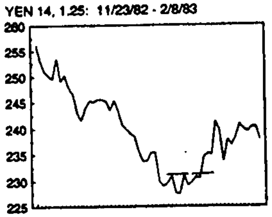
YEN 12, 1.50: 5/20/87 - 8/17/87



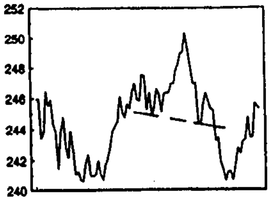
YEN 13, 1.25: 5/1/73 - 2/5/74



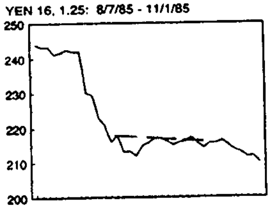
YEN 14, 1.25: 11/23/82 - 2/8/83



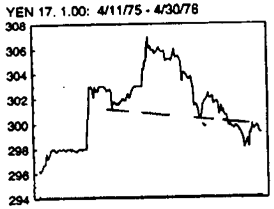
YEN 15, 1.25: 7/24/84 - 11/28/84



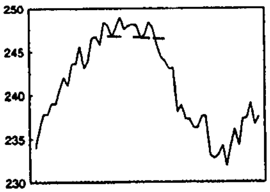
YEN 16, 1.25: 8/7/85 - 11/1/85



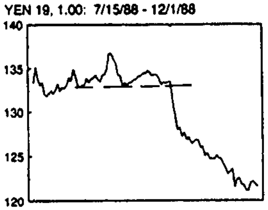
YEN 17, 1.00: 4/11/75 - 4/30/76



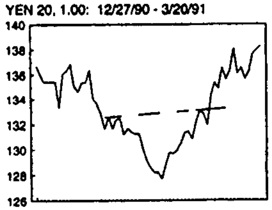
YEN 18, 1.00: 3/8/82 - 5/24/82



YEN 19, 1.00: 7/15/88 - 12/1/88



YEN 20, 1.00: 12/27/90 - 3/20/91



APPENDIX C: APPLYING THE SWEENEY RISK-ADJUSTMENT METHODOLOGY TO THE HEAD-AND-SHOULDERS CONTEXT

Sweeney's (1986) methodology for risk-adjusting returns to foreign exchange trading strategy involves calculating the difference, X, between excess returns to the particular trading strategy (in this case the head-and-shoulders-based strategy) and excess returns to a buy-and-hold strategy for the whole period, adjusting the latter for the difference between total holding periods to the two strategies. In our case, this is expressed as follows:

$$X = R_{HS} - (L-S)R_{BH}.$$

Here L and S represent the share of total days in which the head-and-shoulders strategy indicated taking a long and short position in foreign currency, respectively. In addition,

$$R_{HS} = \frac{\sum_{N_{long}} (\Delta s_t + r_{ft} - r_{dt})}{N} - \frac{\sum_{N_{short}} (\Delta s_t + r_{ft} - r_{dt})}{N},$$

and

$$R_{BH} = \frac{\sum_{N} (\Delta s_t + r_{ft} - r_{dt})}{N},$$

where Δs represents the daily percent exchange rate changes, r_f represents the foreign overnight euromarket interest rate, r_d represents the domestic currency overnight euromarket rate (both rates are calculated on a continuously compounded basis), N represents the total days in the sample period (including weekends), N_{long} represents the total days in which the head-and-shoulders strategy indicated taking a long position in foreign currency, and N_{short} represents the total days in which the head-and-shoulders strategy indicated taking a short position in foreign currency.

Sweeney shows that X will be distributed approximately normally, assuming the standard deviation of exchange rate changes exists, with mean zero. Applying his logic, which assumes that interest rates are not stochastic, we find that X will have the following variance:

$$\sigma_X^2 = \frac{\sigma_{\Delta s}^2}{N} [L(1-L) + S(1-S)+2LS].$$