A New Micro Model of Exchange Rate Dynamics

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Abstract

We show how aggregation of information in general equilibrium can resolve the exchange rate determination puzzle. Unlike macro DGE models, which enrich tastes or technology, we enrich the information structure. Unlike micro-based models, our model connects dispersed information to the macroeconomic variables that anchor traditional analysis. Results relevant to the determination puzzle include: (1) persistent gaps between exchange rates and fundamentals, (2) excess volatility relative to fundamentals, (3) exchange rate movements without macro news, (4) little or no exchange rate movement when macro news occurs, and (5) a structural-economic resolution of the "order flow puzzle"—that macro variables cannot account for monthly exchange rate changes, whereas transaction flows can. Calibration results match the empirical findings of Meese and Rogoff (1983).

Keywords: Exchange Rates, Dispersed Information, General Equilibrium, Microstructure.

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Introduction

Two new approaches to exchange rates emerged in the 1990s, both micro-founded. This paper addresses whether connecting them can resolve the most researched puzzle in international macroeconomics—that macro fundamentals do not explain monthly exchange rate changes (the determination puzzle; see Meese and Rogoff 1983). The two new approaches are the dynamic general equilibrium (DGE) approach and the microstructure approach.[2] DGE modeling introduces rich preference structures and production structures, but has not yet ventured beyond common-knowledge information structures (e.g., toward information that originates in a dispersed form). The microstructure approach focuses explicitly on richer information structures, at the cost of relying on stylized, partial equilibrium analysis (e.g., informative signals are introduced without links to deep economic fundamentals, and without considering that the fundamentals themselves are determined by the signals received, through real allocation choices). The "new micro" approach we propose here connects the two by embedding a micro process of information aggregation into a macro DGE setting.

The driving force behind the exchange rate in our model is productivity. Though not necessary, anchoring exchange rates with a real variable shows that information dynamics are not special to financial transactions and the associated nominal variables. (The information approach produces broadly similar results when instead focused on shocks to nominal variables like money demand, or to other real variables.) The essential ingredient is that individuals' currency trades are more correlated with unobserved shocks to home-country productivity than with shocks to foreign productivity. Consider an economy in which bits of information about realized productivity are initially present at the micro level, i.e., at the level of individual firms. None of these firms considers itself to have superior information. But if the currency trades of individual firms are correlated with their own micro-level productivities (e.g., due to increased export revenues), then *aggregated* trades initiated by home agents convey incremental information about the home shock. This information structure differentiates our model from the DGE macro literature. Beyond this, the macro features are quite standard, in fact rather streamlined.

The micro features of the model relate closely to micro models of asset trade. In these models, financial intermediaries act as marketmakers who provide two-way prices. We introduce liquidity provision of this type by assuming that all agents engage in both consumption and marketmaking.[3] This consolidates the activities of households with that of financial institutions in a way similar to the "yeoman farmer" consolidation of consumption and production decisions in the new-macro branch of DGE models (i.e., individual agents, or households, engage in both consumption and production in those models). The consolidation greatly facilitates integration of the microstructure and DGE approaches.[4] In particular, it ensures that the

[2] DGE examples include Backus, Kehoe, and Kydland (1994), Obstfeld and Rogoff (1995,1998), and Chari, Kehoe, and McGrattan (2002), among many others. Microstructure analysis, particularly its hallmark use of signed transaction quantities, includes Lyons (1995), Rime (2001), Evans (2002), Evans and Lyons (2002a), Hau, Killeen, and Moore (2002), Bjonnes and Rime (2003), and Payne (2003), among many others.

[3] Note the emphasis here on liquidity provision that is private, in contrast to the public provision of liquidity (in the form of central banks) at the center of the monetary approach to exchange rates.

[4] To non-macro readers, this type of consolidation is surely unfamiliar. The assumption facilitates general equilibrium analysis because the agent population remains defined over a single continuum, and differences along that continuum arise as

objectives of financial-market participants are aligned with those of consumers. All trading is therefore consistent with expected utility maximization; noise traders, behavioral traders, and other non-rational agent types are absent. The resulting financial markets are semi-strong form efficient (reflect all public information), but not strong form efficient: the endogenous pace of aggregation cannot keep up with the evolution of fundamentals.

The model shows that richer, more realistic information structures produce an exchange rate that aligns closely with empirical facts. For the determination puzzle in particular, relevant results include: (1) persistent gaps between exchange rates and fundamentals, (2) excess volatility relative to fundamentals, (3) exchange rate movements without macro news, and (4) little or no exchange rate movement when macro news occurs. Intuition for these results is as follows. Persistent gaps between exchange rates and fundamentals arise because the underlying state of fundamentals—which corresponds to the union of all information sets—is revealed only gradually. Excess volatility occurs because real allocations are distorted by rational exchange rate errors—an "embedding effect"; these distorted real allocations induce additional volatility in exchange rates.[5] (Note that past micro models cannot produce excess volatility from this source since they do not permit feedback from information and exchange rates back to real fundamentals.) Exchange rates move without macro news because microeconomic actions—in particular, trades—convey information, even when public macro news is not present. There may be no impact on exchange rates from macro news if prior microeconomic aggregation of information renders that news redundant.

We explore the model's empirical implications with numerical simulations. These simulations reveal two important features: First, exchange rates are disconnected from fundamentals over monthly horizons. We show, for example, that Meese-Rogoff style regressions would have almost no explanatory power. Not only can this explain the Meese-Rogoff results, it does so in a way consistent with the empirical literature linking order flow and exchange rates (see footnote 2). Second, the presence of the "embedding effect" makes the empirical link between exchange rates and fundamentals appear only at horizons that are far longer than the horizon at which past states of the economy are publicly known. For example, fundamentals account for only 50% of the variance in exchange rates at the two year horizon, and 75% at the five year horizon (consistent with empirical work, such as that in Mark 1995).

This paper belongs to a recent literature that addresses why exchange rates are well explained by signed transaction flows (e.g., 40 to 80 percent of daily changes explained, for a host of major currencies; see Evans and Lyons 2002a,b). Our model shows why signed transaction flows *should* have more explanatory power than macro variables: in a setting of dispersed information, aggregated transaction flows provide a stronger signal of current and expected future changes in macro fundamentals than lagged macro variables do. The

parsimoniously as possible to capture the model's essential features.

[5] For further intuition on embedding, recognize that the exchange rate, as an asset price, is free to jump, whereas real variables (like total output) are not. Suppose home agents over-estimate real ouput and consume too much today (resulting in part from an overvalued real exchange rate). The following period the exchange rate must depreciate from its over-valued level, not only enough to reduce consumption to reflect lower-than-expected output, but also to compensate for the distorted consumption decision last period.

model of Hau and Rey (2002) goes a different route in addressing the empirical significance of transaction flows. Their focus for understanding currency movements is on shocks to cross-country equity returns and the resulting flows into less-than-perfectly-elastic currency markets. No information aggregation takes place in their model. A more relevant paper along this theoretical line is Bacchetta and van Wincoop (2003), which does explicitly address how transaction flows relate to information aggregation. Unlike our DGE setting, theirs is a model based on rational expectations equilibrium (in the spirit of Grossman and Stiglitz 1980). An important finding is that greater dispersion of information can lead to greater price impact from non-fundamental trades (from rational confusion between fundamental and non-fundamental trades). They too generate simulation results that broadly match the empirical findings of Meese and Rogoff (1983).

Though our model includes private information, it should not be interpreted as "inside" information in the sense that one or a few insiders have large information advantages (and know it). The dispersed information we have in mind in fact characterizes most variables at the center of exchange rate modeling, such as output, money demand, inflation, consumption preferences, and risk preferences. These variables are not realized at the macro level, but rather first as dispersed micro realizations, and only later aggregated by markets and/or governments.[6] For some of these measures, such as risk preferences and money demands, government aggregations of the underlying micro-level shocks do not exist, leaving the full task of aggregation to markets. For other variables, government aggregations exist, but publication lags underlying realizations by 1-4 months, leaving room for market-based aggregation in advance of publication.

Methodologically, the DGE environment we study has the following novel features. First, financial markets in our model are incomplete, which, among other things, makes room for the exchange rate to be determined from more than just the marginal rate of substitution between home and foreign consumption goods (see also Duarte and Stockman 2001). In particular, the exchange rate is pinned down by expectations via a present-value relation in a manner familiar to the asset approach. Second, the model embeds social learning: agents learn from the equilibrium actions of others. Third, the presence of social learning means that we need to solve each agent's decision problem and inference problem jointly. More concretely, the solution begins with a conjecture about each agent's information set, and concludes with verification that these conjectured information sets line up with information provided by market outcomes. Fourth, our solution accounts for agent risk aversion. Risks associated with incomplete knowledge about the economy's state influence consumption and trading decisions (which, in turn, affect inferences agents draw from market outcomes). To our knowledge, this is the first paper to solve a DGE model with this combination of risk-averse decision-making, heterogeneous information, and social learning.

Section 1 presents the model's basic characteristics. Section 2 provides model details. Section 3 specifies the steps involved in solving for equilibrium. Sections 4 and 5 study the equilibrium, with focus on pricing dynamics at both high frequencies and low. Section 6 concludes. An appendix presents analytical detail.

[6] Our information specification abstracts from strategic behavior. Strategic interaction is important for understanding collapsing fixed exchange rates (see, e.g., Corsetti et al. 2001), but not the everyday functioning of major floating-rate currencies.

1 Theoretical Overview

Our genre of information model identifies primitive shocks and their propagation in ways that partial-equilibrium models cannot. There are three essential ingredients: (1) specification of an endowment process (or production technology) for dispersed information, (2) specification of the information available for financial pricing, and (3) a solution methodology that maps individual information sets into equilibrium actions that, once observed, support those individual information sets. The model below is one set of choices in these three dimensions. Other options are indicated so that those with different modeling preferences have a sense of the wider possibilities.

The first of these essential ingredients—specification of dispersed information—is a qualitative departure from existing DGE work in macro. The focus here is on price effects from information that persist, not on "microstructure effects" (by the latter we mean price effects that are transitory, e.g., from marketmaker risk management or from bouncing between bid and ask prices); from a macro perspective, microstructure effects of this kind are second order. Though the dispersed information that drives the exchange rate in our model is productivity, as noted, this need not be the case. Other key features of open-macro modeling like sticky goods prices and imperfectly competitive firms can be introduced in the usual way. Our macro structure is streamlined to highlight the information dimension.

The second essential ingredient is that modeling liquidity provision needs to take a stand on information sets: what information do agents have when setting transactable prices? The genre of models we work with here relaxes the common assumption of "strong-form informational interdependence"—where actions at any given time are conditioned on information aggregated from all other actions occurring at that same time (see, e.g., Grossman and Stiglitz 1980). When simultaneous actions are informationally interdependent in this way, resulting transaction quantities convey no information that is not already embedded in the transaction price; there is no learning from order flow ex post, and indeed, there is no information content in transaction flows whatsoever (both counterfactual).[7] In the model below, we choose instead a "simultaneous trade" design (see, e.g., Lyons 1997). Under this design, trades at any point in time occur simultaneously throughout the economy, but realizations cannot be conditioned on one another (a standard assumption within the class of simultaneous-move games in game theory). One cannot condition on information revealed by the trading intentions of every other agent at the time one chooses to trade, save doing one's best to forecast them. We find this an inherently realistic assumption. Though a convenient way to relax the assumption of trades being strong-form information interdependent, it is not the only possibility. For example, an intermediate road would be to assume that financial transactions at any "point" in time are executed sequentially (à la Glosten and Milgrom 1985). In this case, the earlier the trade in the sequence, the more limited the

[7] Another unfortunate feature of Walrasian mechanisms is that agents generally do not take positions that they intend in the future to liquidate (because all trades are conditioned on all concurrent trading information). Among other things, this produces counterfactual predictions about how liquidity is provided in financial markets: transitory position-taking is a deep property of liquidity provision, and is important for understanding how trade quantities (i.e., realized order flow) map into price changes.

conditioning information (because early trades cannot condition on information conveyed by later trades). This alternative produces a constraint on information sets that is qualitatively similar to the one we employ here.

The third essential ingredient of this modeling genre is its solution methodology, which needs to map individual information sets into equilibrium actions, and then back to information sets. Here we adopt a guess-and-verify method with the following 5 steps, the first and last of which sharply distinguish our information-theoretic approach from past DGE modeling. In step 1, we make a conjecture about the information available to agents at each point in time. This involves specifying agents' information endowments as well as what they learn by observing trading outcomes. Based on this information structure, in step 2 we guess the form of the equilibrium pricing rules for spot rates. In step 3, we solve for optimal consumption and portfolio allocations (based on analytic approximation methods in Campbell and Viceira 2002). Step 4 verifies that agent choices for consumption, investment, and currency holdings clear markets. In step 5, we verify that the conjectured information structure from step 1 can be supported by an inference problem based on endowment information and information from trading (the latter includes both prices and order flows).

A fourth ingredient of our model below is that consumers and financial liquidity providers are consolidated. This is not an essential ingredient. Whereas new-macro DGE models focus on richer micro-foundations on the economy's supply side, hence their consolidation of consumers with producers, our focus is instead on richer micro-foundations in financial price determination. This consolidation serves three main purposes. First, it consolidates budget constraints across the two sets of activities, which simplifies the analytics. Second, it ensures that messy incentive misalignments do not arise (e.g., there are no agency problems). Third, it ensures that the preferences of liquidity providers are in no sense special, as is often the case in partial-equilibrium microstructure modeling. We recognize that for some questions it will be necessary to drop this fourth, non-essential ingredient.

Finally, we note that currency trades in the model are quite general, in the sense that they include all three of the fundamental motivations addressed in the literature: a transactions motive (e.g., for purchasing imported goods), a speculative motive (based on information differentials), and a hedging motive (for risk management).

2 The Model

2.1 Environment

2.1.1 Preferences

The world is populated by a continuum of infinitely-lived agents indexed by $z \in [0, 1]$ who are evenly split between the home country (i.e., for $z \in [0, 1/2)$) and foreign country ($z \in [1/2, 1]$). For concreteness, we refer to the home country as the US and the foreign country as the UK. Preferences for the z'th agent are given by:

$$\mathbb{U}_{t,z} = \mathbb{E}_{t,z} \sum_{i=0}^{\infty} \beta^i U(C_{t+i,z}, \hat{C}_{t+i,z}) \quad (1)$$

where $0 < \beta < 1$ is the discount factor, and $U(.)$ is a concave sub-utility function, which we specialize to log (which exhibits constant relative risk aversion, CRRA):

$$U(\hat{C}_{t,z}, C_{t,z}) = \tfrac{1}{2} \ln \hat{C}_{t,z} + \tfrac{1}{2} \ln C_{t,z}.$$

All agents have identical preferences over the consumption of US goods $C_{t,z}$ and UK goods $\hat{C}_{t,z}$. $\mathbb{E}_{t,z}$ denotes expectations conditioned on agent z's information set at time t, $\Omega_{t,z}$. $\mathbb{E}_t$ denotes expectations conditioned on common time-t information (i.e., $\Omega_t \equiv \cap_{z \in [0,1]} \Omega_{t,z}$).

2.1.2 Timing

Decision-making in the model takes place at two frequencies. Consumption-savings decisions take place at a lower frequency than financial decision-making (where the latter includes determination of asset prices and reallocation of portfolios via trading). To implement this idea, we split each "month" t into four periods (see Figure 1). Consumption-savings decisions are made "monthly," while financial decisions are made periodically within the month. As will become clear, the use of the term "month" is nothing more than a convenient label: the economic intuition developed by the model is exactly the same if we replaced "month" t by some other consumption-relevant period. That said, let us now describe the structure of the model by considering the "monthly" sequence of four events.

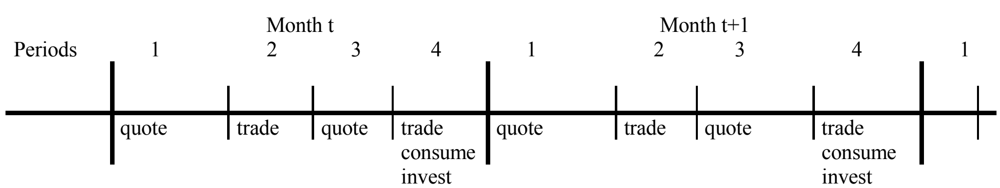

Figure 1: Model Timing

Period 1 (Quoting): Agents begin the month with holdings in three assets: dollar deposits $B^1_{t,z}$, pound sterling deposits $\hat{B}^1_{t,z}$, and domestic capital ($K_{t,z}$ for US agents and $\hat{K}_{t,z}$ for UK agents). Each agent then quotes a spot price $S^1_{t,z}$ (\$/£) at which he is willing to buy or sell pound deposits in exchange for dollar deposits. (Exchanging pound deposits for dollar deposits are the model's currency trades. Note too that the absence of money in this economy means that these deposits are best viewed as real deposits.) Quotes are observable to all agents.[8]

[8] It will be clear below that consumers in this model have both speculative and non-speculative motives for trading (the

Period 2 (Trading): Each agent z chooses the amount of pounds (i.e., deposits) $T^2_{t,z}$ that he wishes to purchase (negative values for sales) by initiating a trade with other agents. The sum of these signed trade quantities across all agents z is what we shall refer to as the period's *order flow*. Trading is simultaneous, trading with multiple partners is feasible, and trades are divided equally among agents offering the same quote. (That trades are divided equally is important: in equilibrium it will imply that all agents receive the same incoming order-flow realization.) Once these transactions have taken place, agent z's deposits at the start of period 3 are given by:

$$\begin{aligned} B^3_{t,z} &= B^1_{t,z} + S^1_t T^2_{t,z*} - S^1_t T^2_{t,z}, \\ \hat{B}^3_{t,z} &= \hat{B}^1_{t,z} + T^2_{t,z} - T^2_{t,z*}, \end{aligned}$$

where $T^2_{t,z*}$ denotes the *incoming* foreign currency orders, in total, from other agents trading at z's quoted price. S^1_t is the period-1 spot rate quote at which z purchases pounds (so $S^1_t T^2_{t,z*}$ is the dollar deposits received for having sold $T^2_{t,z*}$ pound deposits in response to the incoming order). In equilibrium, this will be the spot rate quoted by all agents (i.e., $S^1_t = S^1_{t,z}$) for reasons we explain below. Notice that period-3 currency holdings depend not only on the transactions initiated by z, (i.e., $T^2_{t,z}$) but also on the transactions initiated by other agents $T^2_{t,z*}$. An important assumption of our model is that the choice of $T^2_{t,z}$ by agent z cannot be conditioned on the incoming orders $T^2_{t,z*}$ because period-2 trading takes place simultaneously (save for doing one's best to forecast $T^2_{t,z*}$). Consequently, though agents target their desired allocation across dollar and pound assets, resulting allocations include a stochastic component from the arrival of unexpected orders from others.

Period 3 (Quoting): All agents again quote a spot price and also a pair of one-month interest rates for dollar and pound deposits.[9] The spot quote, $S^3_{t,z}$, is good for a purchase or sale of pounds, while the interest rates, $R_{t,z}$ and $\hat{R}_{t,z}$ indicate the rates at which the agent is willing to borrow or lend one-month in dollars and pounds, respectively. (Later, we use R^k_{t+1} and $\hat{R}^k_{t+1}$ to denote the one-month returns on US and UK real capital, respectively.) As in period 1, all quotes are publicly observable.

Period 4 (Trading and Real Decisions): In period 4, agents choose a second round of foreign currency purchases (if there remain motives for further intra-month trade).[10] They also choose their real allocations: consumption of US and UK goods and real investment expenditures. After US agents z have chosen their

non-speculative motive arising from the need to facilitate periodic consumption and investment). That the speculative motive is not the only motive obviates concern about so-called "no trade" results (i.e., the theorem proposed by Milgrom and Stokey 1982, that if I know that your only motive for trade with me is superior information, then I would never want to trade with you at any price at which you want to trade).

[9] Deposit rates are only quoted in period 3 to keep the structure of the model as simple as possible. Allowing interest to accrue on intra-month deposits, based on rates quoted period 1, would not materially affect our results. In particular, the existence of period-1 deposit rate quotes would not affect the trading decisions and inference problems facing agents in period 2, which lie at the heart of our analysis.

[10] That motives for further currency trade within the month will indeed remain is one of the model's important properties (established below in Section 4). It addresses the question of why agents would want to trade at such high frequencies.

consumption of US and UK goods, $C_{t,z}$ and $\hat{C}_{t,z}$, their foreign currency purchases $T^4_{t,z}$, and their real investment $I_{t,z}$, the resulting deposit holdings in period 1 of month $t+1$ are:

$$B^1_{t+1,z} = R_t(B^3_{t,z} - S^3_t T^4_{t,z} - I_{t,z} + S^3_t T^4_{t,z*} + C_{t,z*}), \tag{2}$$

$$\hat{B}^1_{t+1,z} = \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - \hat{C}_{t,z} - T^4_{t,z*}) \tag{3}$$

where R_t and $\hat{R}_t$ are the dollar and pound interest rates (gross) that are quoted by all agents in period 3 of month t (in equilibrium, $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$ for all z, as shown below). As in period 2 trading, actual deposit holdings following period-4 trading also depend on the actions of other agents. In particular, total incoming orders for foreign currency $T^4_{t,z*}$ and total incoming orders for US goods $C_{t,z*}$ affect the deposit levels in the first period of the following month. Notice, for example, that $B^1_{t+1,z}$ is augmented by $C_{t,z*}$: these are deposits received in exchange for exports of US goods. We assume that C_{t,z^*} includes an aggregate component, $\frac{1}{2}\int_{1/2}^{1} C_{t,z}dz$, (common to all US agents; $z < 1/2$), and an idiosyncratic component, $\nu_{t,z}$, with $\int_{1/2}^{1} \nu_{t,z}dz = 0$.[11] For UK agents, the dynamics of deposit holdings are similarly determined by:

$$B^1_{t+1,z} = R_t(B^3_{t,z} - S^3_t T^4_{t,z} - C_{t,z} + S^3_t T^4_{t,z*}), \tag{4}$$

$$\hat{B}^1_{t+1,z} = \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - \hat{I}_{t,z} - T^4_{t,z*} + \hat{C}_{t,z*}). \tag{5}$$

The export of UK goods by agent $z > 1/2$ also is composed of an aggregate component, $\frac{1}{2}\int_{0}^{1/2} \hat{C}_{t,z}dz$, and an idiosyncratic component, $\hat{\nu}_{t,z}$, with $\int_{0}^{1/2} \hat{\nu}_{t,z}dz = 0$.

Finally, we turn to the dynamics of the capital stocks, which are central in our model for determining equilibrium exchange rates. The production of US and UK goods at the start of month $t+1$, $Y_{t+1,z}$ and $\hat{Y}_{t+1,z}$, is given by:

$$Y_{t+1,z} = A_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right),$$

$$\hat{Y}_{t+1,z} = \hat{A}_{t+1}(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}),$$

where A_{t+1} and $\hat{A}_{t+1}$ capture US and UK productivity. Note our convention: $K_{t,z}$ is the real capital stock at the *beginning* of month t, so that the term in parenthesis is the capital stock at the end of month t, i.e., after period 4 trading and consumption. These production functions lead to the following capital accumulation

[11] We have to make an assumption about the distribution of the incoming export orders C_{t,z^*} in the same sense that we had to make an assumption about the distribution of incoming foreign exchange trades $T^2_{t,z*}$. For $T^2_{t,z*}$, we assumed that trades are divided equally among agents offering the same price. (This is conservative, informationally, since it maximizes the trading information available to each.) For exports, a more decentralized market, it is perhaps more natural to allow for idiosyncratic noise. This noise will not, however, slow down the pace of information revelation: as we shall see, in equilibrium agents will precisely infer the state of world productivity after period-4 trading without needing an additional signal from the real product markets.

equations:

$$K_{t+1,z} = R^k_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right), \tag{6}$$

$$\hat{K}_{t+1,z} = \hat{R}^k_{t+1}(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}), \tag{7}$$

where $R^k_{t+1} \equiv 1 + A_{t+1}$, and $\hat{R}^k_{t+1} \equiv 1 + \hat{A}_{t+1}$ denote the one-month returns on US and UK capital. (Depreciation is zero in both countries.) Equation (6) describes how US consumers' holdings of capital evolve; equation (7) describes the dynamics of UK consumers' holdings.

2.1.3 Productivity and the Information Structure

Our model becomes explicitly "international" with the specification of relative productivity, the driving force behind the exchange rate. The key feature that differentiates US from UK agents is that each agent type is better informed about the productivity of home firms than foreign firms. (This could result, for example, through direct observation of the productivity realization for one's own firm.) As a result, agents in different countries do not share the same expectation about current and future returns to real capital. Below we examine how this dispersed information is impounded in exchange rates and interest rates via trading. Our focus is thus on the process of information transmission, not so much on the specific type of underlying information. The analysis can be extended to include dispersed information about alternative underlying information types.

The exogenous productivity processes are expressed here in terms of log returns on real capital. Though we specify these separately for the US and UK, as we shall see, only relative productivity will matter for exchange rate determination:

$$\ln R^k_t \equiv r^k_t = r + u_t + e_t + \theta(e_{t-1} - \hat{e}_{t-1}), \tag{8a}$$

$$\ln \hat{R}^k_t \equiv \hat{r}^k_t = r + \hat{u}_t + \hat{e}_t + \theta(\hat{e}_{t-1} - e_{t-1}). \tag{8b}$$

We assume that the u_t, $\hat{u}_t$, e_t, and $\hat{e}_t$ are normally distributed mean-zero shocks. The u_t and $\hat{u}_t$ shocks have a common variance σ^2_u and the e_t and $\hat{e}_t$ shocks have a common variance σ^2_e. We allow for the possibility of non-zero covariance between the u_t and $\hat{u}_t$ shocks:

$$Cov[u_t, \hat{u}_t] = \rho\sigma^2_u$$

For tractability, we assume that the e_t and $\hat{e}_t$ shocks are independently distributed.

Our specification for log capital returns includes two random components beyond the constant r: a transitory component u_t ($\hat{u}_t$) and a persistent component e_t ($\hat{e}_t$). The transitory component u_t ($\hat{u}_t$) is a one-month effect on US (UK) returns with cross-country correlation ρ. Unlike u_t ($\hat{u}_t$), the random variable e_t ($\hat{e}_t$) is contemporaneously independent across countries, but gives rise to an intertemporal impact that depends on this component's cross-country differential from the previous period. It should be clear from these two productivity processes that their differential, i.e., $r_t^k - \hat{r}_t^k$, follows a simple MA(1) process. This greatly facilitates analysis of the differential as a driving force (richer processes for this differential get technically difficult quickly). Though not intended as precise empirical representations, we consider it uncontroversial that capital returns should include both transitory and persistent components.

In most of the analysis below we examine information structures in which for each month t, all US agents observe in period 1 their home shocks $\{u_t, e_t\}$, whereas all UK agents observe their home shocks $\{\hat{u}_t, \hat{e}_t\}$.[12] Dispersed information thus exists inter-nationally, but not intra-nationally. This specification highlights the theoretical consequences of dispersed information in the simplest possible way. (In the version of the model that we calibrate, we include information that is dispersed both internationally and intra-nationally.) The timing structure depicted in Figure 1 is also motivated by the desire for analytical clarity. At the heart of our analysis is the following question: Can dispersed information about the month t state of productivity be completely aggregated via trading and hence reflected in exchange rates and interest rates before real allocation decisions are made? In our timing structure, this boils down to the question of whether information aggregation is complete by the start of period 3. Periods 1 and 2 are in this sense a metaphor for the many rounds of quoting and trading that facilitate information aggregation in actual markets before real allocation decisions are made.[13] We offset the dampening effect that this structure has on revelation by making actions (trades) observable, which is much more transparency than is present in actual markets. Allowing for fewer trading periods with greater transparency enables us to examine the process of information aggregation in a clear yet meaningful way.

2.2 Decision-Making

Agents make two types of decisions: consumption-savings decisions and financial decisions (quoting and trading). The former are familiar from standard macro models, but the latter are new. By quoting spot prices and interest rates at which they stand ready to trade, agents are taking on the liquidity-providing role of financial intermediaries. Specifically, the quote problem facing agents in periods 1 and 3 is identical to that facing a marketmaker in a simultaneous trading model (see, for example, Lyons 1997, Rime 2001, Evans and Lyons 2002a). We therefore draw on this literature to determine how quotes are set.

Equilibrium quotes are derived as a Nash equilibrium with the following two properties: (i) they are consistent with market clearing, and (ii) they are a function of public information only. Though the latter property is not necessary for the information transmission role of transaction flows, it is still important for this role, so let us address it more fully. With this property, the information in unanticipated flow can only be impounded into price after it is realized and publicly observed. This lies at the opposite pole

[12] This is not the same as assuming two representative consumers: two consumers would interact strategically (a rather implausible notion here), whereas in our continuum consumers are perfectly competitive.

[13] For an example of a model with timing structure along these lines, see Evans and Lyons (2004).

of the information assumptions underlying Walrasian mechanisms (Grossman and Stiglitz 1980 being an example) in which the market price at a given time impounds information in *every* trade occurring at that time. The Walrasian mechanism is akin to assuming that all trades are conditioned on one another. This is obviously counter-factual in most markets, and certainly so in FX.[14] As noted in the previous section, what is really necessary for the transmission role of transaction flow is that market prices do not impound all of the information in concurrent executed transactions. This is insured in our model by an assumption that marketmakers cannot condition their price quotes on the concurrent trading intentions of all other agents. This aspect of the model can be viewed as taking seriously the information constraints that price-setters actually face.

We should stress, though, that quotes being conditioned only on public information in our model is a result, not an assumption. Put differently, we make other assumptions that are sufficient for this outcome (drawing from the simultaneous-trade references above). Those assumptions are (1) that actions within any given quoting or trading period are simultaneous, (2) that quotes are a single price good for any size, and (3) that trading with multiple marketmakers is feasible.[15] The resulting solution to the quote problem facing agent z in periods $j = \{1, 3\}$ will be a quote $S^j_{t,z} = S^j_t$, where S^j_t is a function of public information Ω^j_t (determined below). Similarly, the period-3 interest rate quotes are given by $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$, where R_t and $\hat{R}_t$ are functions of Ω^3_t (recall that interest rates are set by liquidity-providing marketmakers here, not by a central bank). To understand why these quotes represent a Nash equilibrium, consider a marketmaker who is pondering whether to depart from this public-information price by quoting a weighted average of public information and his own individual information. Any price that deviates from other prices would attract pure arbitrage trade flows, and therefore could not possibly represent an equilibrium. Instead, it is optimal for marketmakers to quote the same price as others (which means the price is necessarily conditioned on public information), and then exploit their individual information by initiating trades at other marketmakers' prices. (In some models, marketmakers can only establish desired positions by setting price to attract incoming trades, which is not the case here since they always have the option of initiating outgoing trades.)

Next we turn to the consumption and portfolio choices made in periods 2 and 4. Let $W^j_{t,z}$ denote the wealth of individual z at the beginning of period j in month t. This comprises the value of home and foreign

[14] Even if the FX market were organized as a centralized auction with full transparency, this would not be sufficient for Walrasian-type aggregation: it would also have to be true that in equilibrium all agents would actually choose to trade simultaneously (so that each could condition on the price effects of others' trades). In any case, actual FX markets are not centralized auctions, but rather decentralized dealer markets with trade transparency that is is relatively low.

[15] The assumption of no spreads is not necessary, though it greatly facilitates the analytics. Specifically, the economics of the model would not be significantly changed if each trader-consumer's quote were a schedule of prices, one for each incoming order quantity from minus infinity to plus infinity, so long as that schedule is conditioned only on the incoming order, as opposed to the realization of all other orders in the market (i.e., the quoting trader would in this way be able to protect against adverse selection in the single incoming trade). To go this route, we would have to relax the assumption that trades are split equally across marketmakers, since, as noted, equal splits means that everybody is receiving their share of the marketwide aggregate order. Alternatively, marketmakers could charge one another an unconditional, fixed commission on incoming orders. The existence of this commission would create an incentive to quote beyond the reciprocity requirement we impose, but would not affect the equilibrium we analyze since all agents are marketmakers (and the marketmaker problems are symmetric, so the redistribution is nil).

deposit holdings and domestic capital:

$$\begin{aligned} W^2_{t,z} &\equiv B^1_{t,z} + S^1_t \hat{B}^1_{t,z} + K_{t,z} + S^1_t \hat{K}_{t,z} \\ W^4_{t,z} &\equiv B^3_{t,z} + S^3_t \hat{B}^3_{t,z} + K_{t,z} + S^3_t \hat{K}_{t,z} \end{aligned}$$

Notice that wealth is valued in dollars using the equilibrium spot rate quoted in the period before trading takes place.[16]

In period 2 agents initiate transactions (i.e., choose $T^2_{t,z}$) to allocate wealth optimally between dollar and pound assets. Because trading takes place simultaneously, however, the choice of $T^2_{t,z}$ cannot be conditioned on the orders they simultaneously receive from others, $T^2_{t,z*}$. Of course, in choosing $T^2_{t,z}$ agents do their best to forecast $T^2_{t,z*}$, but they cannot condition on its realization. We denote this forecast of the incoming order as $\mathbb{E}^2_{t,z}T^2_{t,z*}$. (Hereafter we use $\mathbb{E}^j_{t,z}$ to denote expectations conditioned on information available to individual z at the *beginning* of period j in month t.)

Let $J^2_z(W^2_{t,z})$ and $J^4_z(W^4_{t,z})$ denote the value functions for agent z at the beginning of periods 2 and 4. $T^2_{t,z}$ is determined as the solution to the following dynamic programming problem:

$$J^2_z(W^2_{t,z}) = \max_{\lambda_{t,z}} \mathbb{E}^2_{t,z}\left[J^4_z(W^4_{t,z})\right], \tag{9}$$

s.t.

$$W^4_{t,z} = H^3_{t,z} W^2_{t,z}, \tag{10}$$

where

$$\begin{aligned} H^3_{t,z} &\equiv \left(1 + \left(\tfrac{S^3_t}{S^1_t} - 1\right)(\lambda_{t,z} - \xi_t)\right), \\ \lambda_{t,z} &\equiv S^1_t \left(\hat{B}^1_{t,z} + \hat{K}_{t,z} + T^2_{t,z} - \mathbb{E}^2_{t,z}T^2_{t,z*}\right)/W^2_{t,z}, \\ \xi_{t,z} &\equiv S^1_t(T^2_{t,z*} - \mathbb{E}^2_{t,z}T^2_{t,z*})/W^2_{t,z}. \end{aligned}$$

The choice variable $\lambda_{t,z}$ is key. It identifies the target fraction of wealth agents wish to hold within the month in pounds, given their expectations about incoming orders they will receive during trading, $\mathbb{E}^2_{t,z}T^2_{t,z*}$. (Outgoing orders $T^2_{t,z}$ are determined from the optimal choice of $\lambda_{t,z}$ given $\mathbb{E}^2_{t,z}T^2_{t,z*}$, $\hat{B}^1_{t,z} + \hat{K}_{t,z}$, and $W^2_{t,z}$.) $H^3_{t,z}$ identifies the within-month return on wealth (i.e., between periods 1 and 3). This depends on the rate of appreciation in the pound and the actual faction of wealth held in foreign deposits at the end of period-2 trading. The latter term is $\lambda_{t,z} - \xi_{t,z}$, where $\xi_{t,z}$ represents the position-effect of unexpected incoming pound orders from other agents (a shock). This means that the return on wealth, $H^3_{t,z}$, is subject to two sources of uncertainty: uncertainty about the future spot rate S^3_t, and uncertainty about order flow in the form of trades initiated by other agents $T^2_{t,z*}$.

In period 4, agents choose consumption of US and UK goods, foreign currency orders, and investment expenditures. Let $\alpha_{t,z}$ and $\gamma_{t,z}$ denote the desired, cross-month fractions of wealth (weights) held in pounds

[16] No single agent can hold both $K_{t,z}$ and $\hat{K}_{t,z}$ since agents hold domestic real capital only; thus, depending on whether z is above or below 1/2, one of these two terms in each equation will equal zero.

and domestic capital respectively:

$$\alpha_{t,z} \equiv \left\{S_t^3\hat{K}_{t,z} + S_t^3\hat{B}_{t,z}^3 + S_t^3\left(T_{t,z}^4 - \mathbb{E}_{t,z}^4 T_{t,z*}^4\right) - S_t^3\hat{C}_{t,z}\right\}/W_{t,z}^4,$$

$$\gamma_{t,z} \equiv \begin{cases} \left(K_{t,z} + I_{t,z} - C_{t,z} - \mathbb{E}_{t,z}^4 C_{t,z*}\right)/W_{t,z}^4 & z < 1/2, \\ \\ \left(\hat{K}_{t,z} + \hat{I}_{t,z} - \hat{C}_{t,z} - \mathbb{E}_{t,z}^4 \hat{C}_{t,z*}\right)/W_{t,z}^4 & z \geq 1/2. \end{cases}$$

The period-4 problem can now be written as:

$$J_z^4(W_{t,z}^4) = \max_{\{C_{t,z},\hat{C}_{t,z},\alpha_{t,z},\gamma_{t,z}\}} \left\{U(\hat{C}_{t,z}, C_{t,z}) + \beta\mathbb{E}_{t,z}^4\left[J_z^2(W_{t+1,z}^2)\right]\right\}, \tag{11}$$

s.t.

$$W_{t+1,z}^2 = R_t H_{t+1,z}^1 W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3\hat{C}_{t,z}\right), \tag{12}$$

where:

$$H_{t+1,z}^1 = \begin{cases} 1 + \left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t} - 1\right)(\alpha_{t,z} - \varsigma_{t,z}) + \left(\frac{R_{t+1}^k}{R_t} - 1\right)(\gamma_{t,z} - \zeta_{t,z}) & z < 1/2 \\ \\ 1 + \left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t} - 1\right)(\alpha_{t,z} - \varsigma_{t,z}) + \left(\frac{S_{t+1}^1\hat{R}_{t+1}^k}{S_t^3 R_t} - \frac{S_{t+1}^1\hat{R}_t}{S_t^3 R_t}\right)\left(\gamma_{t,z} - \hat{\zeta}_{t,z}\right) & z \geq 1/2 \end{cases}.$$

with $R_{t+1}^k \equiv 1 + A_{t+1}$ and $\hat{R}_{t+1}^k \equiv 1 + \hat{A}_{t+1}$.

$H_{t+1,z}^1$ is the excess return on wealth (measured relative to the dollar one-month interest rate R_t). As above, realized returns depend on the actual faction of wealth held in pounds $\alpha_{t,z} - \varsigma_{t,z}$, where $\varsigma_{t,z} \equiv S_t^3(T_{t,z*}^4 - \mathbb{E}_{t,z}^4 T_{t,z*}^4)/W_{t,z}^4$ represents the effects of unexpected currency orders that arise from period-4 trading. Monthly returns also depend on the fraction of wealth held in real capital. For the US case this is given by $\gamma_{t,z} - \zeta_{t,z}$, where $\zeta_{t,z} \equiv \left(C_{t,z*} - \mathbb{E}_{t,z}^4 C_{t,z*}\right)/W_{t,z}^4$ identifies the effects of unexpected demand for US goods (i.e. US exports).[17] In the UK case, the fraction is $\gamma_{t,z} - \hat{\zeta}_{t,z}$, where $\hat{\zeta}_{t,z} \equiv (\hat{C}_{t,z*} - \mathbb{E}_{t,z}^4\hat{C}_{t,z*})/W_{t,z}^4$. Monthly returns are therefore subject to four sources of uncertainty: uncertainty about future spot rates (i.e., S_{t+1}^1, which affects deposit returns); uncertainty about future productivity (which affects real capital returns); uncertainty about incoming currency orders; and uncertainty about export demand.

The first-order conditions governing consumption and portfolio choice (i.e., $C_{t,z}, \hat{C}_{t,z}, \lambda_{t,z}, \alpha_{t,z}$) take the

[17] When superior information about home-country income is not symmetrized by month's end, one manifestation of the residual uncertainty is a shock to export demand.

same form for both US and UK agents:

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_{t,z}, C_{t,z}) = \beta R_t S_t^3 \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \right], \tag{13}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_{t,z}, C_{t,z}) = \beta R_t \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \right], \tag{14}$$

$$\lambda_{t,z} \quad : \quad 0 = \mathbb{E}_{t,z}^2 \left[V_{t,z} \left(\frac{S_t^3}{S_t^1} - 1 \right) \right], \tag{15}$$

$$\alpha_{t,z} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \left(\frac{S_{t+1}^1 R}{S_t^3 R_t} - 1 \right) \right], \tag{16}$$

where $V_{t,z} \equiv dJ_z^4(W_{t,z}^4)/dW_{t,z}^4$ is the marginal utility of wealth. The first-order conditions governing real investment (i.e. $\gamma_{t,z}$) differ between US and UK agents and are given by:

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \left(\frac{R_{t+1}^k}{R_t} - 1 \right) \right], \tag{17}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \left(\frac{S_{t+1}^1 \hat{R}_{t+1}^k}{S_t^3 R_t} - 1 \right) \right]. \tag{18}$$

To further characterize the form of optimal consumption, portfolio and investment decisions, we need to identify the marginal utility of wealth. This is implicitly defined by the recursion:

$$V_{t,z} = \beta R_t \mathbb{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 H_{t+1,z}^1 \right]. \tag{19}$$

In a standard macro model where agents provide no liquidity provision, equations (15) through (19) together imply that $V_{t,z} = U_c(\hat{C}_{t,z}, C_{t,z})$. The first-order conditions can then be rewritten in familiar form using the marginal rate of substitution. This is not generally the case in our model. As we shall show, the marginal utility of wealth $V_{t,z}$ can diverge from the marginal utility of consumption because unexpected currency and export orders affect portfolio returns.

2.3 Market Clearing

Market clearing in the currency market requires that the dollar value of pound orders initiated equals the dollar value of pound orders received:

$$\int T_{t,z}^j dz = \int T_{t,z^*}^j dz,$$

for $j = \{2, 4\}$.

We assume that dollar and pound deposits are in zero net supply, so that aggregate deposit holdings at the start of periods 1 and 3 are given by:

$$\int B_{t,z}^1 dz = 0, \qquad \int \hat{B}_{t,z}^1 dz = 0, \tag{20}$$

$$\int B_{t,z}^3 dz = 0, \qquad \int \hat{B}_{t,z}^3 dz = 0. \tag{21}$$

Combining these conditions with the budget constraints for dollar and pound deposits implies that $\int I_{t,z}dz = \int \hat{I}_{t,z}dz = 0$, so that aggregate real investment expenditures must equal zero if the deposit and goods markets are to clear. This does *not* imply that the investment expenditures of individual agents are zero. The appendix shows that each agent chooses real investment expenditures to offset the effects of the idiosyncratic component of last month's export shock. This is achieved via trading in domestic deposits. (For example, recipients of positive idiosyncratic export shocks last month will undertake more real investment this month by selling domestic deposits to the recipients of negative idiosyncratic export shocks—the latter being happy with a lower level of real investment.) Of course, this form of trading has no impact on the aggregate level of investment expenditure; it simply allows expenditures to be redistributed among agents. Aggregate expenditure could only vary if there were a change in aggregate deposit holdings, an implication that is inconsistent with market clearing. As a consequence, the aggregate capital stock available for production after period-4 trading is complete is $\int (K_{t,z} - C_{t,z})\,dz$ in the US and $\int \left(\hat{K}_{t,z} - \hat{C}_{t,z}\right) dz$ in the UK. Each capital stock is augmented by production that takes place between month t and $t+1$, so that the stock of US and UK capital in period-1 of month $t+1$ are (from equations 6 and 7):

$$K_{t+1} = R^k_{t+1}\left(K_t - \int C_{t,z}dz\right), \tag{22}$$

$$\hat{K}_{t+1} = \hat{R}^k_{t+1}\left(\hat{K}_t - \int \hat{C}_{t,z}dz\right). \tag{23}$$

where $K_t \equiv \frac{1}{2}\int K_{t,z}dz$ and $\hat{K}_t \equiv \frac{1}{2}\int \hat{K}_{t,z}dz$. These equations summarize the implications of market clearing for the dynamics of the aggregate capital stocks.

3 Solving for Equilibrium

An equilibrium in this model is described by: (i) a set of quote functions that clear markets given the consumption, investment, and portfolio choices of agents; and (ii) a set of consumption, investment, and portfolio rules that maximize expected utility given spot rates, interest rates, and exogenous productivity. In this section we describe how the equilibrium is constructed.

We solve for equilibrium using a guess-and-verify method. This includes the following five steps, the first and last of which distinguish our information approach quite sharply from other DGE macro modeling:

1. Information Conjecture: We make a conjecture about information available to agents at each point in time. This involves specifying what information agents receive directly and what they learn by observing trading.

2. Quote Decisions: Based on this information structure, we then guess the form of equilibrium quote functions for spot rates and interest rates (periods 1 and 3).

3. Allocation Decisions: We use log linearized first-order conditions and the budget constraint to approximate agents' optimal consumption, investment, and currency choices given the spot and interest rates from step 2.

4. Market Clearing: We check that agent choices for consumption, investment, and currency holdings clear markets.

5. Information Conjecture Verified: We verify that the conjectured information structure (from step 1) can be supported by an inference problem based on exogenous information available to each agent, and their observations of quotes and trading activity.

As suggested in the previous section, capital stock dynamics are at the center of the model's equilibrium. Capital dynamics are approximated from the market-clearing conditions in (22) and (23):

$$k_{t+1} - k_t \cong r^k_{t+1} + \ln(1-\mu) - \tfrac{\mu}{2(1-\mu)} \left(s^3_t + \hat{k}_t - k_t + \int \delta_{t,z} dz \right), \tag{24}$$

$$\hat{k}_{t+1} - \hat{k}_t \cong \hat{r}^k_{t+1} + \ln(1-\mu) - \tfrac{\mu}{2(1-\mu)} \left(k_t - s^3_t - \hat{k}_t + \int \delta_{t,z} dz \right), \tag{25}$$

where $\delta_{t,z}$ is the log consumption/wealth ratio: $\delta_{t,z} \equiv c_{t,z} - w^4_{t,z} - \ln(\mu/2)$ and μ is twice the unlogged consumption/wealth ratio in steady state (overbar denotes steady-state value): $\mu \equiv 2\bar{C}_{t,z}/\bar{W}^4_{t,z}$. In deriving these equations for capital dynamics, we have assumed that deposit holdings always represent a small fraction of agent wealth. This condition is met trivially in the steady state because both US and UK agents hold all their wealth in the form of domestic capital. The accuracy of these approximations deteriorates when away from the steady state if agents accumulate substantial financial assets/liabilities relative to their capital holdings.

4 Exchange Rate Dynamics

Given that productivity is the forcing variable, exchange rate dynamics will depend on how dispersed productivity information is embedded in spot rates. Recall that the processes for log capital returns in the US and UK, respectively, follow:

$$r^k_t = r + u_t + e_t + \theta(e_{t-1} - \hat{e}_{t-1}),$$

$$\hat{r}^k_t = r + \hat{u}_t + \hat{e}_t + \theta(\hat{e}_{t-1} - e_{t-1}),$$

where we allow the transitory components u_t and $\hat{u}_t$ have correlation ρ, but the persistent components e_t and $\hat{e}_t$ are independent across countries (for tractability). Recall also that we assumed in section 2 that information about the return on capital arrives as follows:

1. US Shocks: US agents all observe the realization of their home shocks $\{u_t, e_t\}$ at t:1,

2. UK Shocks: UK agents all observe the realization of their home shocks $\{\hat{u}_t, \hat{e}_t\}$ at t:1,

where the shorthand t:j denotes period j in month t.

The following propositions characterize the exchange rate process implied by this information structure. They clarify the model's essential features, including the central role of endogenous information revelation.

Proposition 1 (Spot Rates) *The log nominal exchange rate implied by spot quotes in periods 1 and 3 are given by:*

$$s_t^1 = \mathbb{E}_t^1 \nabla k_t, \tag{27}$$

$$s_t^3 = \mathbb{E}_t^3 \nabla k_t, \tag{28}$$

where the operator ∇ denotes the difference between US and UK values (e.g., $\nabla k_t = k_t - \hat{k}_t$).

Thus, spot rates in each of the two price-setting periods are pinned down by the log capital-stock differential, where expectations are conditioned on common information, $\Omega_t^j = \{1, 3\}$. To develop intuition for this result, first note that markets here are incomplete, so that unlike complete-markets models, the spot rate is not determined by the ratio of marginal utilities of consumption across US and UK goods.[18] Rather, the spot rate is pinned down by the international distribution of wealth, which here means the international distribution of capital. This can be seen by combining the definitions of the realized capital shares $\gamma_{t,z} - \zeta_{t,z}$ (see the definition of $H_{t+1,z}^1$ in equation 12) with the dynamics of US and UK capital:

$$\frac{W_{t,\mathrm{US}}^4}{W_{t,\mathrm{UK}}^4} = \left(\frac{\gamma_{t,\mathrm{UK}} - \zeta_{t,\mathrm{UK}}}{\gamma_{t,\mathrm{US}} - \zeta_{t,\mathrm{US}}}\right)\left(\frac{K_t}{S_t^3 \hat{K}_t}\right)^{\frac{1}{1-\mu}}.$$

The ratio of US to UK wealth is proportional to the ratio of US to UK capital, with the proportionality factor that depends on the ratio of realized capital shares. In equilibrium, changes in the wealth ratio are highly correlated with changes in the capital ratio because allocation choices (i.e., $\gamma_{t,z}$) are determined by expected excess returns that are comparatively stable. This means that any equilibrium restrictions on the distribution of wealth will have their counterpart on the distribution of capital. One such restriction is that the wealth of each consumer remains positive (i.e., $W_{t,z}^i > 0$ for $i = \{2, 4\}$), or equivalently, that log wealth remains bounded. In equilibrium, order flows aggregate dispersed information about productivity because consumers have an incentive to trade based on their individual information. This process of social learning is crucial to the equilibrium (see Propositions 3 and 4 below), but it breaks down if the wealth of either US or UK consumers falls to zero. (For example, if $W_{t,\mathrm{US}}^2 = 0$, then there is no period-2 order flow that can

[18] The nature of market incompleteness is somewhat novel in this model, so we discuss it in some detail in the working paper version (Evans and Lyons 2004a). Even if agents could hold foreign real capital, financial markets here would still be incomplete: the deeper source of incompleteness is that dispersed information precludes a full set of state-contingent claims.

convey dispersed information about US productivity shocks, u_t and e_t.) This bound on log wealth ties down the spot rate. In particular, the period-3 spot rate must satisfy:

$$s_t^3 = \mathbb{E}_t^3 \nabla k_t + \mathbb{E}_t^3 \sum_{i=1}^{\infty} (1-\mu)^i \left\{ r_{t+i}^k - \left(\hat{r}_{t+i}^k + \Delta s_{t+i}^3 \right) \right\}. \tag{29}$$

Equation (29) identifies the unique value for the spot rate that places $K_t/S_t^3 \hat{K}_t$ on an expected future path consistent with the equilibrium bound on log wealth. To see why, consider what would happen if the expected $t+1$ return on US capital rose relative to the return on UK capital, with no change in current or future spot rates. This change in returns would raise the expected ratio of US to UK capital in $t+1$. It would also lower $W_{t+1,\mathbb{UK}}/K_{t+1}$ and raise $W_{t+1,\mathbb{US}}/S_{t+1}^3 \hat{K}_{t+1}$, thereby reducing US exports and raising UK exports (relative to domestic capital). These wealth effects induce a self-perpetuating cycle of higher growth in US capital and lower growth in UK capital from $t+1$ onwards (see equations 22 and 23). And, as a result, $K_t/S_t^3 \hat{K}_t$ would rise without bound and $W_{t,\mathbb{UK}}^4$ would be driven to zero. This outcome can be avoided only if the current spot rate is raised to offset the effects of higher returns on the distribution of capital in $t+1$. The present value term in equation (29) shows the extent to which the current spot rate must be raised to offset the effects of future return differentials, such that the international distribution of log capital and wealth remain bounded.

The quote equations of Proposition 1 follow in a straightforward manner from (29). The equilibrium dynamics of spot rates insure that expected future returns on US and UK capital are equal (when expressed in terms of a common currency). Under these circumstances, the present value term disappears from (29), leaving $s_t^3 = \mathbb{E}_t^3 \nabla k_t$ as shown in equation (28). Period-1 spot rate quotes are set so that expected intra-month returns are equal.[19] Since no intra-month interest is paid on US or UK deposits, this requirement implies that $s_t^1 = \mathbb{E}_t^1 s_t^3 = \mathbb{E}_t^1 \nabla k_t$ as in equation (27).

Proposition 1 identifies the different factors that contribute to the dynamics of spot rates. In particular, combining (27) and (28) with the dynamics of US and UK capital in (24) and (25) we find:

$$s_t^1 - s_{t-1}^3 \quad = \quad \mathbb{E}_t^1 \nabla r_t^k + \tfrac{1}{1-\mu} \mathbb{E}_t^1 (\nabla k_{t-1} - \mathbb{E}_{t-1}^3 \nabla k_{t-1}), \tag{30}$$

$$s_t^3 - s_t^1 \quad = \quad \left(\mathbb{E}_t^3 - \mathbb{E}_t^1 \right) \nabla r_t^k + \tfrac{1}{1-\mu} \left(\mathbb{E}_t^3 - \mathbb{E}_t^1 \right) (\nabla k_{t-1} - \mathbb{E}_{t-1}^3 \nabla k_{t-1}). \tag{31}$$

These equations show how changing expectations about the distribution of capital and the return on capital contribute to spot rate dynamics. Specifically, equation (30) shows that across months the revision in spot rate quotes has two components. The first is the common knowledge expectation of the difference in capital returns, $\mathbb{E}_t^1 \nabla r_t^k \equiv \mathbb{E}_t^1 [r_t^k - \hat{r}_t^k]$. The second component is proportional to the current estimate (conditional on

[19] This property of the equilibrium arises from the absence of hedging terms in the period-2 portfolio choices (see appendix for further details).

Ω_t^1) of the last month's error in estimating the distribution of capital, $\nabla k_{t-1} - \mathbb{E}_{t-1}^3 \nabla k_{t-1}$. The within-month spot rate change shown in (31) also has two components. The first term $\left(\mathbb{E}_t^3 - \mathbb{E}_t^1\right) \nabla r_t^k$ conveys what agents learn about capital returns during the current month. The second term identifies what they learned during the current month about last month's error in estimating the distribution of capital.

Equations (30) and (31) clarify how exchange rates are driven by the arrival of information. In particular, common-knowledge information and its evolution through time are key to understanding the contribution of the various components of spot-rate adjustment. We now study this evolution in detail.

Proposition 2 (Revelation Special Case) *Immediate revelation of new information about the month-t state of the economy occurs only when $\rho = -1$.*

Recall that US (UK) agents learn the values of e_t and u_t ($\hat{e}_t$ and $\hat{u}_t$) at the start of period 1 in month t, with $Cov[u_t, \hat{u}_t] = \rho\sigma_u^2$. Although all four shocks contribute to the current difference in capital returns, ∇r_t^k, they cannot affect the spot rate until they become common knowledge. In the special case where $\rho = -1$, both u_t and $\hat{u}_t$ are immediately common knowledge, leading to their full impounding in quotes immediately in period 1 via $\mathbb{E}_t^1 \nabla r_t^k$ in (30). When $\rho > -1$, however, none of the dispersed information about current returns is immediately common knowledge, so none of the new information about the state of the economy is reflected in the period-1 spot rate (despite the information existing in dispersed form).[20]

Proposition 3 addresses the general case, clarifying the degree to which period-2 trading contributes to learning.

Proposition 3 (Revelation General Case) *Let $T_t^2 \equiv \int T_{t,z}^2 dz$ denote aggregate order flow toward pounds in period-2 trading. In equilibrium, aggregate order flow augments the common-knowledge information set between the start of periods 2 and 3: $\Omega_t^3 = \left\{T_t^2 \cup \Omega_t^2\right\}$. In the special case where $\rho = -1, \nabla e_t \in \Omega_t^3$. For the general case where $\rho > -1$ ($\neq 0$), $\{\nabla e_t, \nabla u_t\} \notin \Omega_t^3$, and*

$$\mathbb{E}_t^3 \left[\nabla e_t + \nabla u_t\right] = \psi \xi_t, \tag{32}$$

where $\xi_t \equiv S_t^1 \left(T_t^2 - \mathbb{E}_t^2 T_t^2\right) / \beta R W_{t-1}^2$ is the scaled innovation to period-2 order flow (relative to Ω_t^2) that depends on all four return shocks:

$$\xi_t \cong \xi_e \nabla e_t + \xi_u \nabla u_t. \tag{33}$$

At the start of period 3, residual uncertainty about the true distribution of capital is:

$$\begin{aligned} \nabla k_t - \mathbb{E}_t^3 \nabla k_t &= \nabla e_t + \nabla u_t - \mathbb{E}_t^3 \left[\nabla e_t + \nabla u_t\right] \\ &= \pi_e \nabla e_t + \pi_u \nabla u_t, \end{aligned} \tag{34}$$

[20] Of course, when ρ=1 the same is true, but in this case there is not even a change in the relative value of u_t and $\hat{u}_t$, i.e., no change in this fundamental to be revealed.

where $\pi_e = (1 - \psi\xi_e) \neq 0$ *and* $\pi_u = (1 - \psi\xi_u) \neq 0$.

This proposition shows the pace at which period-2 trading aggregates dispersed information. (Coefficient values are in the appendix.) Period-2 order flow is informative because $s_t^3 - s_t^1$ is forecastable based on agents' individual information, $\Omega_{t,z}^2$. Hence, each agent has an incentive to trade, and in so doing, some of their individual information is revealed to others via order flow. When $\rho = -1$, the innovation in order flow is a function of e_t and $\hat{e}_t$. This means that each agent can infer the value of ∇e_t from incoming order flow and their individual information. Under these circumstances, dispersed information concerning e_t and $\hat{e}_t$ becomes common knowledge after a single trading period. The key to this result is that with $\rho = -1$, the values of e_t and $\hat{e}_t$ represent the sole source of individual information that motivates trade. In particular, u_t and $\hat{u}_t$ play no role because they are common knowledge at the beginning of the month, so their implications are fully reflected in the period-1 spot rate, s_t^1. When $\rho > -1$, by contrast, the values of e_t and u_t ($\hat{e}_t$ and $\hat{u}_t$) are both sources of superior information to US (UK) agents because the values of u_t and $\hat{u}_t$ are not reflected in s_t^1. This means that order flow innovations contain information on all four shocks, as approximated by (33).[21] As a consequence, it is not generally possible for any agent to infer the exact values of $\nabla e_t + \nabla u_t$ by combining their individual information with their observation of period-2 order flow.[22] Consequently, aggregation of dispersed information at the end of period-2 trading is incomplete.

We can gain further perspective from the composition of period-2 order flow:

$$T_t^2 = \int \lambda_{t,z} \left(W_{t,z}/S_t^1 \right) dz - \hat{K}_t + \int \mathbb{E}_{t,z}^2 T_{t,z^*}^2 dz.$$

This shows that order flow aggregates information from: (i) the portfolio allocation decisions of US and UK consumers $\lambda_{z,t}$, (ii) the distribution of wealth $W_{t,z}$, (iii) the outstanding UK capital stock $\hat{K}_t$, and (iv) expectations of incoming order flow $\mathbb{E}_{t,z}^2 T_{t,z^*}^2$. This means that order flow reflects both individual information about the current state, as well as other variables that affect the distribution of wealth, capital stock, and so on. In general, these additional variables are not common knowledge. Rather, they represent a source of noise that makes precise inferences about the current state from observations of order flow impossible. This source of informational inefficiency is likely to occur in any model that combines dispersed information with CRRA utility: Since CRRA asset demands depend on wealth, less-than-full information about the distribution of wealth creates noise, more difficult signal extraction, and informational inefficiency.

Order flow ξ_t is the model's key variable in terms of information transmission. Per the definition in

[21] Importantly, it is the unexpected component of aggregate order flow that conveys news — a component that is uniquely determined. The expected component of order flow is not uniquely determined, however: if agents could all coordinate in adding some constant $\varkappa$ to $T_{t,z}^2$, then because they are trading with one another, the resulting prices and allocations of foreign exchange following trading would remain unchanged. (We are grateful to Eric van Wincoop for pointing out this feature of expected order flow.) As a theoretical matter, adding even infinitesimal trading costs would eliminate this source of indeterminacy from the expected component of equilibrium order flow.

[22] An artifactual exception occurs when $\rho = 0$. In this case, the trades of US (UK) consumers happen to be a function of the unweighted sum $e_t + u_t$ ($\hat{e}_t + \hat{u}_t$), so observation of ξ_t combined with private information could fully reveal the value of $\nabla e_t + \nabla u_t$ to all consumers. We ignore this artifactual case in the propositions that follow.

proposition 3, it is a reflection of the gap between trade initiations that agents are expecting (based on public information) and those that actually occur, i.e., $T_t^2 - \mathbb{E}_t^2 T_t^2$. From (33) it is clear that in equilibrium this one-dimensional signal is a weighted average of the underlying productivity realizations. Naturally, the weights play a direct role in governing the degree to which uncertainty about the state remains once trading is concluded and the order flow outcome is observed.

Next we turn to period-4 trading.

Proposition 4 (Revelation Month End) *After period-4 trading, information aggregation is complete. In particular, the components of returns $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ are all common knowledge:*

$$\{u_t, \hat{u}_t, e_t, \hat{e}_t\} \in \left\{T_t^4 \cup \Omega_t^4\right\}.$$

where $T_t^4 \equiv \int T_{t,z}^4 dz$ denotes aggregate order flow for pounds in period-4 trading.

When $\rho > -1$, period-3 spot rates cannot fully reflect all information relevant to the state of the economy. This means that agents still have individual information that is relevant for forecasting returns between t:4 and t+1:1, and hence have an incentive to trade in period 4. Order flow in period 4 will therefore constitute a second signal on the underlying distribution of individual information. This signal contains incremental information sufficient to reveal fully the values of $\hat{u}_t$, and $\hat{e}_t$ to US consumers, and the values of u_t and e_t to UK consumers. As a result, the values of $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ become common knowledge by the end of period-4 trading.

Two features of our model lie behind the speed of information aggregation. First, each consumer has only to learn about a limited amount of information, namely, the values of two foreign shocks. Second, our model makes trading very transparent because in equilibrium, incoming orders are equally divided among all consumers. This means that the order flow received by each consumer is completely representative of the market as a whole. This high level of transparency insures that incoming orders are only a function of $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ in each period. Consequently, consumers can precisely infer the values of the two foreign shocks from incoming orders in periods 2 and 4. We shall see in the next section that information does not aggregate so quickly with less transparency.

Note, however, that proposition 4 does not imply that the exchange rate is fully revealing. Indeed, the exchange rate in this model is never fully revealing. We have already shown that when $\rho > -1$, period-3 spot rates cannot reflect all information. And while agents do learn the values of $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ by the end of period-4 trading, their quotes in period 1 of month t+1 will not fully reflect intervening changes in the macro state variables $u_{t+1}, \hat{u}_{t+1}, e_{t+1}$, and $\hat{e}_{t+1}$. Put differently, the macro-economy is evolving at a pace that never allows asset prices to catch up fully.

We may summarize the implications of Propositions 2 through 4 as follows. For the special case where $\rho = -1$, common information evolves according to:

$$\Omega_t^1 = \left\{u_t, \hat{u}_t \cup \Omega_{t-1}^4\right\}, \quad \Omega_t^2 = \Omega_t^1,$$
$$\Omega_t^3 = \left\{e_t, e_t \cup \Omega_t^2\right\}, \qquad \Omega_t^4 = \Omega_t^3.$$

This information structure implies that $\nabla k_{t-1} = \mathbb{E}_{t-1}^3 \nabla k_{t-1}$, $\mathbb{E}_t^1 \nabla r_t^k = 2\theta \nabla e_{t-1} + \nabla u_t$ and $\left(\mathbb{E}_t^3 - \mathbb{E}_t^1\right) \nabla r_t^k = \nabla e_t$, so equations (30) and (31) become:

$$s_t^1 - s_{t-1}^3 = 2\theta \nabla e_{t-1} + \nabla u_t, \tag{35a}$$
$$s_t^3 - s_t^1 = \nabla e_t. \tag{35b}$$

The exchange rate dynamics described by these equations reflect the rapid pace of information aggregation. With perfectly correlated productivity shocks u_t and $\hat{u}_t$, seeing one means seeing the other, so both are immediately in the common-knowledge information set (i.e., at t:1). Consequently, u_t and $\hat{u}_t$ have an immediate, one-to-one effect on the period-1 spot rate. Given this, all consumers can make precise inferences about the remaining uncertainty (the values of e_t and $\hat{e}_t$) from their observation of period-2 order flow. The period-3 price is perfectly revealing.

In the general case where $\rho > -1$ ($\neq 0$), common information evolves according to:

$$\Omega_t^1 = \left\{u_{t-1}, \hat{u}_{t-1}, e_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1}^4\right\}, \quad \Omega_t^2 = \Omega_t^1,$$
$$\Omega_t^3 = \left\{\xi_t \cup \Omega_t^2\right\}, \qquad \Omega_t^4 = \Omega_t^3.$$

where the aggregate order flow ξ_t now plays the central information communication role. This information structure implies that $\nabla k_{t-1} = \mathbb{E}_{t-1}^3 \nabla k_{t-1} + \pi_e \nabla e_{t-1} + \pi_u \nabla u_{t-1}$, $\mathbb{E}_t^1 \nabla r_t^k = 2\theta \nabla e_{t-1}$ and $\left(\mathbb{E}_t^3 - \mathbb{E}_t^1\right) \nabla r_t^k = \psi \xi_t$, so equations (30) and (31) become:

$$s_t^1 - s_{t-1}^3 = 2\theta \nabla e_{t-1} + \tfrac{1}{1-\mu}\left(\pi_e \nabla e_{t-1} + \pi_u \nabla u_{t-1}\right), \tag{36}$$
$$s_t^3 - s_t^1 = \psi \xi_t. \tag{37}$$

The exchange rate dynamics described here reflect the slower speed of information aggregation. Equation (36) shows that u_t and $\hat{u}_t$ have no immediate impact on the spot rate because they are not common knowledge at the time of their realization. Instead, dispersed information on ∇u_t and ∇e_t becomes gradually impounded in spot rates via the order flows generated in periods 2 and 4. Impounding via period-2 order flow is shown in (37). The second term in (36) shows the effect of period-4 order flow.

5 Exchange Rates and Fundamentals

We now examine the implications of our model for the relationship between exchange rates and fundamentals. For this purpose, we first examine how real allocation decisions are affected by the speed of information aggregation. We then show how the pace of learning affects the volatility of spot prices over short horizons. Finally, we extend the model to allow for lower transparency in foreign exchange trading. This allows us to examine the relationship between exchange rates and fundamentals over a broad range of horizons.

5.1 Embedding, Volatility, and Macro Announcements

The speed of information aggregation affects real allocation decisions. In the general case with $\rho > -1$, consumers make real consumption and investment decisions at the start of period 4 before the complete state of the economy is known. This means that real allocations will be distorted by (rational) expectation errors. In Propositions 5 and 6 below we examine the implications of these distortions for the dynamics of fundamentals and the volatility of exchange rates.

Proposition 5 (Embedding in Fundamentals) *Expectational errors are embedded in fundamentals via the relation:*

$$\nabla k_{t+1} - \nabla k_t = \nabla r_{t+1}^k + \tfrac{\mu}{1-\mu}\left(\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right).$$

Proposition 5 shows that the monthly change in the realized distribution of capital includes two components: the difference in capital returns ∇r_{t+1}^k, and residual uncertainty after period-2 trading concerning the distribution of capital, $\nabla k_t - \mathbb{E}_t^3 \nabla k_t$. When $\rho = -1$, there is common knowledge about the full state of the economy by period 3 and $s_t^3 = \nabla k_t$. Accordingly, we refer to ∇k_t as identifying common-knowledge fundamentals. In this special case, $\nabla k_t \in \Omega_t^3$, so changes in fundamentals are driven solely by the difference in capital returns. In the general case with $\rho > -1$, both components contribute to the dynamics of fundamentals. In particular, Proposition 3 shows that $\nabla k_t - \mathbb{E}_t^3 \nabla k_t = \pi_e \nabla e_t + \pi_u \nabla u_t$, so:

$$\nabla k_{t+1} = \nabla k_t + \nabla r_{t+1}^k + \tfrac{\mu}{1-\mu}\left(\pi_e \nabla e_t + \pi_u \nabla u_t\right).$$

Thus, residual uncertainty about the distribution of capital becomes embedded in the dynamics of fundamentals via the π_e and π_u terms. The economic intuition behind this result is straightforward. Recall that $s_t^3 = \mathbb{E}_t^3 \nabla k_t$, so residual uncertainty creates a gap between the month t spot rate, s_t^3 and its fundamental level, ∇k_t, that affects the international distribution of wealth. This, in turn, affects exports in both the US and UK, thereby influencing the rate of capital accumulation in both countries between month t and $t+1$. Thus, past exchange rates affect the current level of fundamentals. Notice, too, that the effects of residual uncertainty are not transitory. Even though the value of past fundamentals becomes common knowledge with just a one-month lag, effects on the level of fundamentals persist indefinitely: Although consumers learn about their "consumption mistakes" once information aggregation is complete, their optimal response does not involve immediate reversal of those mistakes.[23]

Proposition 6 (Excess Volatility) *When $\rho > -1$ and $\pi_u > \bar{\pi}_u \equiv 2\left(\frac{1-\mu}{2-\mu}\right)^2$, the monthly depreciation rate displays volatility in excess of that implied by fundamentals:*

$$\mathbb{V}\left[\Delta s_{t+1}^3\right] > \mathbb{V}\left[\nabla r_{t+1}^k\right].$$

When $\rho = -1$, the volatility of the monthly depreciation rate is determined by the volatility of full-information fundamentals:

$$\mathbb{V}\left[\Delta s_{t+1}^3\right] = \mathbb{V}\left[\nabla k_{t+1} - \Delta k_t\right] = \mathbb{V}\left[\nabla r_{t+1}^k\right].$$

This proposition links the speed of information aggregation to excess volatility. Recall that when $\rho > -1$, consumers make real consumption and investment decisions at the start of period 4 before the complete state of the economy is known. Proposition 5 shows how this affects the dynamics of fundamentals via expectational errors. These errors can also be a source of excess volatility. Consider the monthly rate of depreciation implied by equations (30) and (31):

$$\Delta s_{t+1}^3 = \mathbb{E}_{t+1}^3 \nabla r_{t+1}^k + \tfrac{1}{1-\mu}\mathbb{E}_{t+1}^3\left[\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right]$$

Here we see that monthly changes in the exchange rate depend on current shocks, via $\mathbb{E}_{t+1}^3 \nabla r_{t+1}^k$, and on corrections for past-month expectational errors, via $\mathbb{E}_{t+1}^3\left[\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right]$. Squaring both sides of this equation and taking expectations gives:

$$\begin{aligned}\mathbb{V}\left[\Delta s_{t+1}^3\right] - \mathbb{V}\left[\nabla r_{t+1}^k\right] \quad &= \quad \left(\mathbb{V}\left[\mathbb{E}_{t+1}^3 \nabla r_{t+1}^k\right] - \mathbb{V}\left[\nabla r_{t+1}^k\right]\right) + \tfrac{1}{(1-\mu)^2}\mathbb{V}\left[\mathbb{E}_{t+1}^3[\nabla k_t - E_t^3 \nabla k_t]\right] \\ &\quad + \tfrac{2}{1-\mu}\mathbb{CV}\left[\mathbb{E}_{t+1}^3 \nabla r_{t+1}^k, \mathbb{E}_{t+1}^3\left[\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right]\right].\end{aligned}$$

We shall refer to $\mathbb{V}\left[\nabla r_{t+1}^k\right]$ as the "fast-aggregation benchmark," since this would pin down exchange rate volatility if aggregation were instantaneous. Now, $\mathbb{V}\left[\mathbb{E}_{t+1}^3 \nabla r_{t+1}^k\right] - \mathbb{V}\left[\nabla r_{t+1}^k\right] < 0$ (from the definition of a variance), so the first term on the right suggests that the lack of common knowledge should *reduce* volatility. (This corresponds to the intuition—mistaken here—that less information can only translate into less price adjustment, and therefore less volatility.) But, as the equation shows, this argument overlooks the effects of agents' learning about past states of the economy. In our model, $\mathbb{E}_{t+1}^3 \nabla k_t = \nabla k_t$, so the second and third terms become:

$$\tfrac{1}{(1-\mu)^2}\mathbb{V}\left[\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right] + \tfrac{2}{1-\mu}\mathbb{CV}\left[\mathbb{E}_{t+1}^3 \nabla r_{t+1}^k, \nabla k_t - \mathbb{E}_t^3 \nabla k_t\right].$$

[23] This embedding effect on consumption and real capital provides a natural link to the current account dynamics at the center of new macro modeling.

Clearly the first term is positive because it is proportional to the variance of forecast errors for fundamentals. The second term will also be positive when agents use information learned about past fundamentals to estimate capital's current return. The size of these terms depends on how much is learnt from period-2 trading. When period-2 order flow is relatively uninformative, the π_u and π_e coefficients will be larger and the effects of subsequent learning will contribute more to the volatility of spot rates. (Recall from Proposition 3 that the coefficients π_u and π_e measure the uncertainty that still remains after period-2 order flow is observed.) Proposition 6 identifies a sufficient condition for excess volatility (i.e., $\pi_u > \bar{\pi}_u$), where the learning effects dominate.

Though enormous, the literature on macro announcements has been unsuccessful in resolving the puzzle that lies at its center, namely, that even the most careful of event studies using the most comprehensive announcement samples are unable to account for 5 percent of total exchange rate variation (see, e.g., Andersen et al. 2003, Evans and Lyons 2003). Our model provides a sensible resolution: macro news will have little impact on exchange rates if prior microeconomic aggregation is doing most of the work.

Proposition 7 (Announcements) *When $\rho > -1$, public announcements concerning the values of r_t^k and $\hat{r}_t^k$ will only affect exchange rates if the announcements are made before period 4 in month t.*

When the transitory capital-return shocks u_t and $\hat{u}_t$ have correlation greater than -1, the state of fundamentals is not fully revealed until agents observe actions from period 4. Any announcement of realized capital returns prior to that time would itself convey new information to the market, and the amount of information it would convey would depend on how early in the month it occurs.

If information aggregation is slower, there will of course be greater scope for announcements to impact exchange rates. We turn now to calibrations of the model in which aggregation is slowed in a patently realistic way: noise is introduced to the signal of marketwide order flow that agents observe.

5.2 Calibration with Lowered Transparency

Though the model specification above clarifies how information is impounded in analytic detail, it is not designed to capture the full potential of learning for addressing macro issues. The extension we consider now slows aggregation, at the expense of considerably greater analytic complexity. We solve this version of the model numerically.

We make three modifications. Under the trading rules in our basic model, all agents received the same flow of FX orders in equilibrium, so that following trading, each could make a noiseless inference about aggregate order flow. Our first modification is to assume that the flow of FX orders is distributed randomly among agents quoting the same spot rate. As a result, in equilibrium each agent observes aggregate order flow at the end of each trading period with noise. Second, we allow for idiosyncratic noise in the information each agent receives in period 1 about domestic productivity shocks. This modification allows for dispersed information to exist both intra- and inter-nationally. The third modification is to add a public announcement that fully reveals r_t^k and $\hat{r}_t^k$ at $t+4{:}1$; i.e., there is a delay of one quarter after trading in month t has ended before the true state of the month-t economy is fully revealed. This latter assumption is consistent with, for example, the fact that U.S. final GDP announcements occur on average 4.5 months after the underlying real activity being measured.

The reduced form of our modified model is described by:

$$\nabla k_t = \nabla k_{t-1} + \nabla e_t + \nabla u_t + 2\theta \nabla e_{t-1} - \tfrac{\mu}{1-\mu}(s_{t-1}^3 - \nabla k_{t-1}), \tag{38}$$

$$s_t^3 = \mathbb{E}\left[\nabla k_t | \tilde{\Omega}_t^3\right], \qquad \tilde{\Omega}_t^3 \equiv \{\nabla \tilde{e}_{t-i}, \nabla \tilde{u}_{t-i}, \nabla \tilde{r}_{t-i}^k\}_{i \geq 0}, \tag{39}$$

$$\nabla \tilde{r}_t^k = \nabla r_{t-4}^k, \tag{40}$$

$$\nabla \tilde{e}_t = \nabla e_t + \varepsilon_t^e, \tag{41}$$

$$\nabla \tilde{u}_t = \nabla u_t + \varepsilon_t^u. \tag{42}$$

Equation (38) combines the capital accumulation equations in (24) and (25), with the returns processes in (8). As in Proposition 1, equation (39) shows that the period-3 spot rate is determined by the expected capital differential conditioned on information $\tilde{\Omega}_t^3$. Notice that $\tilde{\Omega}_t^3$ differs from the period-3 information set in the basic model. We assume $\tilde{\Omega}_t^3$ comprises the history of public announcements on past returns $\nabla \tilde{r}_t^k$, and the history of signals on ∇e_t and ∇u_t, denoted by $\nabla \tilde{e}_t$ and $\nabla \tilde{u}_t$ respectively. The four-month reporting lag for public announcements is shown in (40). The relation between the shock signals and actual shocks are summarized by (41) and (42). In our basic model we constructed the counterparts to these equations by solving the optimal portfolio and inference problem facing each agent (see Propositions 3 and 4). Here we model the relation in reduced form. ε_t^e and ε_t^u represent signaling noise that arises (endogenously) from the presence of intra-nationally dispersed information on domestic productivity and the lower level of market transparency. We assume that ε_t^e and ε_t^u are independent normally distributed mean zero random variables with variances $\tilde{\sigma}_e^2$ and $\tilde{\sigma}_u^2$.

We solve the modified model by guessing and verifying the form of the equilibrium estimation error for fundamentals, $\nabla k_t - \mathbb{E}[\nabla k_t | \tilde{\Omega}_t^3]$. To implement this numerical procedure (described in the appendix), we parametrize the capital returns processes so the quarterly change in spot rates has a standard deviation that matches the historical behavior for G-3 currencies over the floating-rate period. For this purpose we set $\sigma_u = \sigma_e = 1.3$, $\theta = 0.1$ and $\rho = 0$. These values imply that the standard deviation of quarterly spot rate changes is approximately 6%. Our log specification for utility implies that $\mu = 1 - \beta$, so we set μ equal to $1 - 0.97^{1/12}$. The remaining parameters are $\tilde{\sigma}_e^2$ and $\tilde{\sigma}_u^2$. We set $\tilde{\sigma}_e^2$ equal to $(1-\varkappa)2\sigma_e^2/\varkappa$ and $\tilde{\sigma}_u^2 = (1-\varkappa)2(1-\rho)\sigma_u^2/\varkappa$, where the parameter $0 \leq \varkappa \leq 1$ identifies the signal to noise variance ratio in both (41) and (42) (i.e., $\mathbb{V}[\nabla e_t]/\mathbb{V}[\nabla \tilde{e}_t] = \mathbb{V}[\nabla u_t]/\mathbb{V}[\nabla \tilde{u}_t] = \varkappa$). Low (high) values for $\varkappa$ imply that agents have relatively imprecise (precise) estimates of the productively shocks based on period-2 trading.

Table 1 reports summary statistics from calibration experiments using two values for $\varkappa$. In each case, the model was solved and then simulated over 100,000 months. The table reports statistics calculated from these simulated samples. Columns (i) and (iii) report the R^2 statistic from the regression of the $h-$month change in spot rates, $\Delta^h s_{t+h}$, on the $h-$month change in fundamentals $\Delta^h \nabla k_{t+h} \equiv \nabla k_{t+h} - \nabla k_t$.[24] Columns (ii) and (iv) report the variance ratio for $\Delta^h s_{t+h}$ relative to $\Delta^h \nabla k_{t+h}$.

Table 1: Calibration Results

	Experiment I $\varkappa = 0.5$		Experiment II $\varkappa = 0.99$	
Horizon h (months)	R^2	$\frac{\mathbb{V}\left(\Delta^h s_{t+h}\right)}{\mathbb{V}\left(\Delta^h \nabla k_{t+h}\right)}$	R^2	$\frac{\mathbb{V}\left(\Delta^h s_{t+h}\right)}{\mathbb{V}\left(\Delta^h \nabla k_{t+h}\right)}$
	(i)	(ii)	(iii)	(iv)
1	0.001	4.516	0.003	2.791
2	0.080	2.598	0.116	1.818
3	0.041	2.034	0.057	1.527
4	0.020	2.170	0.026	1.789
5	0.011	1.953	0.014	1.658
6	0.001	1.787	0.001	1.546
12	0.200	1.387	0.214	1.270
24	0.492	1.192	0.508	1.135
36	0.632	1.127	0.649	1.090
48	0.713	1.094	0.727	1.067
60	0.763	1.076	0.776	1.054
120	0.876	1.037	0.883	1.027

The results in Table 1 display two important features. First, the slower pace of learning almost completely masks the link between spot rate changes and changing fundamentals over horizons of one year or less (consistent with the well known results of Meese and Rogoff 1983). In particular, the R^2 statistics columns (i) and (iii) are less than 20% for $h < 12$. Intuitively, spot rates are being driven by order flow that changes estimated fundamentals $\Delta^h \mathbb{E}[\nabla k_t|\tilde{\Omega}_t^3]$, but the process by which information is aggregated makes actual $\Delta^h \nabla k_t$ a very poor proxy for $\Delta^h \mathbb{E}[\nabla k_t|\tilde{\Omega}_t^3]$ at high frequencies. At low frequencies, where the change in fundamentals is measured over many years, $\Delta^h \nabla k_t$ is much more closely correlated with $\Delta^h \mathbb{E}[\nabla k_t|\tilde{\Omega}_t^3]$, so the link between exchange rates and fundamentals appears as higher R^2 statistics.

The second noteworthy feature of Table 1 concerns volatility. Even though the complete state of fundamentals is known with a four-month reporting lag, the effects of incomplete information have significant impact on volatility at much lower frequencies: the variance ratios (reported in columns ii and iv) are approximately 1.3 at the $h = 12$ month horizon. In sum, the effects of learning on volatility extend well beyond the horizon at which learning about fundamentals is complete. Notice, also, that there is little difference

[24] The results in columns (i) and (iii) represent an upper bound on the R^2 statistics we could expect to find in empirical applications because they are computed under the assumption that data on actual fundamentals is available. In reality, a researcher may only have access to a subset of the variables that comprise fundamentals.

between the results based on $\varkappa = 0.5$ where market transparency comparatively low, and those based on $\varkappa = 0.99$ where transparency is quite high. A small degree of residual uncertainty appears sufficient to de-couple changes in spot rates and fundamentals at high frequency.[25]

To summarize, our calibrated model not only delivers a quantitative account for the determination puzzle, it does so in a way consistent with another important stylized fact—that order flows *can* account for monthly exchange rate changes. Our results, both analytic and simulated, show that the pace of aggregation is central to exchange rate dynamics. Importantly, our results do not depend on information aggregation being slow in any absolute sense: to have long-lasting effects on both fundamentals and exchange rates, the pace of aggregation need only be slow enough to affect real decisions. Moreover, to get slow aggregation in the model, it is not the case that conditions have to be special; rather, slow aggregation is the general case, and fast aggregation the special case. The learning that is occurring here is different from the symmetric learning that occurs elsewhere in international macroeconomics. In symmetric learning models, the time when (all) agents learn something is exogenous, e.g., the arrival of a macroeconomic announcement. Here, the timing of learning is endogenous—it depends on the actions of private agents.

6 Conclusion

Our new micro model of exchange rates connects the DGE and microstructure approaches. A challenge for past DGE models is finding more traction in the data. Our results suggest that enriching their information structure (as opposed to their preference or production structures) may provide that traction. The shortcomings of microstructure modeling are more on the theoretical side: these models warrant a richer placement within the underlying real economy if they are to realize their potential for addressing macro phenomena. This joint need is what motivates our paper.

DGE analysis highlights several implications of dispersed information that are not evident in partial equilibrium analysis. First, though the timing of information receipt is exogenous, the timing of impounding in price is endogenous. This is because the market signals that lead to that impounding are themselves endogenous (e.g., the signals in agents' decision to trade). Second, DGE modeling of price discovery shows that real decisions are affected, with the degree depending on the pace of endogenous revelation. Accordingly, in a DGE setting such as this, one can address questions such as, What is the welfare-optimal pace of revelation? (It is well known that fast revelation may not be optimal because, for example, it can impede risk sharing.) Third, the information structure of the DGE model provides needed clarity on why transaction effects on exchange rates should persist and, importantly, whether that persistence applies to real exchange

[25] For perspective on this result, consider the problem of estimating z_t from noisy observations $\tilde{\eta}_{t,} = \eta_t + \varpi_t$, where $\eta_t \sim N(0, \sigma^2_\eta)$, $\varpi_t \sim N(0, \sigma^2_\varpi)$, and $\Delta z_t = \eta_t$. It is straightforward to show that the estimation errors $z^{er}_t \equiv z_t - \mathbb{E}[z_t | \{\tilde{\eta}_{t-i}\}_{i \geq 0}]$, follow a random walk $\Delta z^{er}_t = \kappa \tilde{\eta}_t$ where $\kappa > 0$ for any $\sigma^2_\varpi > 0$. Thus, the sample standard deviation of z^{er}_t increases with the sample size whenever there is *any* noise in the η_t signal. Our model has a similar knife-edge property: Although the fundamentals' estimation errors $\nabla k_t - \mathbb{E}[\nabla k_t | \tilde{\Omega}^3_t]$ are stationary because agents learn the true value for ∇k_t after four months, the presence of any noise in the signaling equations (41) and (42) induces a large amount of volatility in $\nabla k_t - \mathbb{E}[\nabla k_t | \tilde{\Omega}^3_t]$.

rates or only to nominal rates. Persistence will apply to real exchange rates if, for example, signed transaction flow is conveying information about underlying shocks that are themselves both real and persistent.

We use the new framework to address the determination puzzle—that monthly exchange rates are not well explained by fundamentals empirically. Four analytical results include: (1) persistent gaps between exchange rates and fundamentals, (2) excess volatility relative to fundamentals, (3) exchange rate movements without macro news, and (4) little or no exchange rate movement when macro news occurs. Important for understanding all of these is the fact that the underlying state of fundamentals—the union of all information sets—is revealed only gradually. Meantime, real allocations are distorted by (rational) expectation errors, which induce additional exchange rate volatility. Calibration shows that this mechanism induces the propagation of micro-based learning effects to exchange rate dynamics at macro-relevant frequencies.

Our calibrated model not only delivers a quantitative account for the determination puzzle, it also accounts for why monthly exchange rate changes can be explained by order flow. When dispersed information is present, aggregate order flows provide a stronger signal of current and expected future changes macro fundamentals than lagged macro variables do. But is dispersed information actually present? Dispersed information characterizes most variables at the center of exchange rate modeling, including output, money demand, inflation, consumption preferences, and risk preferences. These variables are not realized at the macro level, but rather at the micro level, with macro aggregations provided by official institutions only for a subset of these, and even then only with considerable lags. Some of this information is clearly aggregated by markets. These ideas are borne out in recent empirical findings (Evans and Lyons 2004b,2005): order flows do indeed help to forecast future changes in macro fundamentals, and in fact do a much better job than the spot rate itself.

This paper is a first venture in an unexplored direction. We view it as the natural direction for synthesizing the microstructure and DGE-macro approaches to exchange rates. To microstructure-based work, the synthesis brings discipline and real-economy insights. To macro DGE modeling, it brings information-structure realism and an ability to account for exchange rates empirically.

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A Appendix

This appendix includes three sections. The first is the longest—it describes the model's solution and presents proofs of Propositions 1-7. The second addresses market clearing conditions. The third describes the numerical solution to the extended model used for calibration.

A.1 Solving the Model

A.1.1 Conjectured Equilibrium

There are three parts to the conjectured equilibrium: (1) the evolution of the exchange rate and interest rate, (2) the evolution of information sets, and (3) the form of the decision rules.

Equilibrium exchange rates and interest rates are conjectured to follow ($\rho > -1$ and $\rho \neq 0$):

$$s_{t+1}^1 - s_t^3 = 2\theta\nabla e_t + \tfrac{1}{1-\mu}\left(\pi_e\nabla e_t + \pi_u\nabla u_t\right), \tag{A1}$$

$$s_t^3 - s_t^1 = \psi\xi_t, \tag{A2}$$

$$r_t = r + \eta\xi_t, \tag{A3}$$

$$\hat{r}_t = r - \eta\xi_t, \tag{A4}$$

where $\xi_t \equiv S_t^1\left(T_t^2 - \mathbb{E}_t^2 T_t^2\right)/\beta RW_{t-1}^2$ is the scaled innovation in period-2 order flow (relative to Ω_t^2) that depends on all four return shocks:

$$\xi_t \cong \xi_e\nabla e_t + \xi_u\nabla u_t. \tag{A5}$$

The π_i and ξ_i coefficients are related by $\pi_i = (1 - \psi\xi_i) \neq 0$.

Country-level information sets are conjectured to evolve according to:

$$\begin{array}{ll} \Omega_{t,\mathbb{US}}^1 = \left\{u_t, e_t, \hat{u}_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1,\mathbb{US}}^4\right\}, & \Omega_{t,\mathbb{UK}}^1 = \left\{\hat{u}_t, \hat{e}_t, u_{t-1}, e_{t-1} \cup \Omega_{t-1,\mathbb{US}}^4\right\}, \\ \Omega_{t,\mathbb{US}}^2 = \Omega_{t,\mathbb{US}}^1, & \Omega_{t,\mathbb{UK}}^2 = \Omega_{t,\mathbb{UK}}^1, \\ \Omega_{t,\mathbb{US}}^3 = \left\{\xi_{t,\mathbb{US}} \cup \Omega_{t,\mathbb{US}}^2\right\}, & \Omega_{t,\mathbb{UK}}^3 = \left\{\xi_{t,\mathbb{UK}}, \cup\Omega_{t,\mathbb{UK}}^2\right\}, \\ \Omega_{t,\mathbb{US}}^4 = \Omega_{t,\mathbb{US}}^3, & \Omega_{t,\mathbb{UK}}^4 = \Omega_{t,\mathbb{UK}}^3, \end{array} \tag{A6}$$

where $\xi_{t,z} \equiv S_t^1\left(T_{t,z^*}^2 - \mathbb{E}_{t,z}^2 T_{t,z^*}^2\right)/W_{t,z}^2$ is the order flow innovation received by consumer z in period-2 trading ($z = \{\mathbb{US}, \mathbb{UK}\}$). (Hereafter, we use $X_{t,\mathbb{US}}$ to denote $X_{t,z}$ for $z < 1/2$, and $X_{t,\mathbb{UK}} = X_{t,z}$ for $z \geq 1/2$ for any variable X.)

Public information is conjectured to evolve according to:

$$\begin{aligned}
&\Omega_t^1 = \left\{u_{t-1}, \hat{u}_{t-1}, e_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1}^4\right\}, \\
&\Omega_t^2 = \Omega_t^1, \\
&\Omega_t^3 = \left\{\xi_t \cup \Omega_t^2\right\}, \\
&\Omega_t^4 = \Omega_t^3.
\end{aligned} \tag{A7}$$

Based on this information structure, individual and public expectations regarding productivity shocks can be represented by:

$$\mathbb{E}_{t,z}^i[\varepsilon_t] = b_z^i \varepsilon_t, \tag{A8}$$

$$\mathbb{E}_t^i[\varepsilon_t] = b^i \varepsilon_t, \tag{A9}$$

where b_z^i and b^i are 4×4 matrices and $\varepsilon_t \equiv [\ e_t \quad \hat{e}_t \quad u_t \quad \hat{u}_t\ ]'$. Equations (A8) and (A9) imply that $\mathbb{V}_{t,z}^i[\varepsilon_t] = \left(I - b_z^i\right) \Sigma_\varepsilon \left(I - b_z^i\right)$ and $\mathbb{V}_t^i[\varepsilon_t] = \left(I - b^i\right) \Sigma_\varepsilon \left(I - b^i\right)$ where Σ_ε is the (exogenous) unconditional covariance of ε_t.

Decision rules for portfolio shares and log consumption-wealth ratios are conjectured to take the form:

$$\lambda_{t,z} = \lambda_z + \underline{\lambda}_z' \varepsilon_t \tag{A10}$$

$$\omega_{t,z} = \omega_z + \underline{\omega}_z' \varepsilon_t \tag{A11}$$

$$\delta_{t,z} = \delta_z + \underline{\delta}_z' \varepsilon_t \tag{A12}$$

where $\omega_{t,z}$ is introduced here to summarize the two period-4 portfolio decisions, i.e., $\omega_{t,\mathbb{US}}' \equiv [\ \alpha_{t,\mathbb{US}} \quad \gamma_{t,\mathbb{US}}\ ]$ and $\omega_{t,\mathbb{UK}}' \equiv [\ \alpha_{t,\mathbb{UK}} - \gamma_{t,\mathbb{UK}} \quad \gamma_{t,\mathbb{UK}}\ ]$. $\underline{\lambda}_z$, $\underline{\omega}_z$ and $\underline{\delta}_z$ are 4×1 vectors of coefficients, while λ_z, ω_z and δ_z are constants.

A.1.2 Verification

Decision Rule Verification: We start by verifying the third part of the conjectured equilibrium—the form of the decision rules in (A10)–(A12). First we derive log approximations for returns and the budget constraint. For the within-month returns, we use the definition of the period-3 return $H_{t,z}^3$, to yield

$$h_{t,z}^3 \cong \lambda_{t,z}\left(s_t^3 - s_t^1\right) + \tfrac{1}{2}\lambda_{t,z}\left(1 - \lambda_{t,z}\right) \mathbb{V}_{t,z}^2\left[s_t^3\right] - \mathbb{CV}_{t,z}^2\left[s_t^3, \xi_{t,z}\right], \tag{A13}$$

where lowercase letters denote natural logs. $\mathbb{V}_{t,z}^j[.]$ and $\mathbb{CV}_{t,z}^j[.]$ denote the variance and covariance conditioned on agent $z's$ information at the start of period j in month t. This approximation is similar to those adopted by Campbell and Viceira (2002) and is based on a second-order approximation that holds exactly in continuous time when the change in spot rates and unexpected order flow follow Wiener processes. Monthly returns are approximated in a similar fashion. Specifically, for US agents (i.e. $z < 1/2$) we use:

$$\begin{aligned} h^1_{t+1,z} \quad \cong \quad & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(r^k_{t+1} - r_t\right) + \tfrac{1}{2}\alpha_{t,z}\left(1-\alpha_{t,z}\right)\mathbb{V}^4_{t,z}\left[s^1_{t+1}\right] \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathbb{V}^4_{t,z}\left[r^k_{t+1}\right] - \alpha_{t,z}\gamma_{t,z}\mathbb{CV}^4_{t,z}\left[s^1_{t+1}, r^k_{t+1}\right] \\ & - \mathbb{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] - \mathbb{CV}^4_{t,z}\left[r^k_{t+1}, \zeta_{t,z}\right], \end{aligned} \tag{A14}$$

and for UK agents (i.e. $z \geq 1/2$):

$$\begin{aligned} h^1_{t+1,z} \quad \cong \quad & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(\hat{r}^k_{t+1} - \hat{r}_t\right) + \tfrac{1}{2}\left(\alpha_{t,z} - \gamma_{t,z}\right)\left(1 - \left(\alpha_{t,z} - \gamma_{t,z}\right)\right)\mathbb{V}^4_{t,z}\left[s^1_{t+1}\right] \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathbb{V}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1}\right] - \left(\alpha_{t,z} - \gamma_{t,z}\right)\gamma_{t,z}\mathbb{CV}^4_{t,z}\left[s^1_{t+1}, \hat{r}^k_{t+1} + s^1_{t+1}\right] \\ & - \mathbb{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] - \mathbb{CV}^4_{t,z}\left[r^k_{t+1}, \hat{\zeta}_{t,z}\right]. \end{aligned} \tag{A15}$$

The monthly budget constraint is approximated by combining the two periodic budget constraints in (10) and (12):

$$\Delta w^4_{t+1,z} \cong r_t + h^3_{t+1,z} + \ln\left(1-\mu\right) + \left(\frac{1}{1-\mu}\right) h^1_{t+1,z} - \left(\frac{\mu}{1-\mu}\right)\delta_{t,z}. \tag{A16}$$

Next we turn to the first order conditions. We combine the log linearized versions of equations (13) - (19) and our assumption of log utility to obtain the log marginal utility of wealth $v_{t,z}$ as:

$$v_{t,z} = -c_{t,z} - \phi_{t,z}, \tag{A17}$$

where the wedge, $\phi_{t,z}$, is equal to $\mathbb{CV}^4_{t,z}[s^1_{t+1}, \varsigma_{t,z}] + \mathbb{CV}^4_{t,z}[r^k_{t+1}, \zeta_{t,z}]$ for $z < 1/2$ (US agents) and $\mathbb{CV}^4_{t,z}[s^1_{t+1}, \varsigma_{t,z}] + \mathbb{CV}^4_{t,z}[\hat{r}^k_{t+1}, \hat{\zeta}_{t,z}]$ for $z \geq 1/2$ (UK agents). Approximations to the model's first-order conditions are derived by substituting for $v_{t,z}$ $(=\ln V_{t,z})$ in the log linearized versions of (13) - (18):

$$\begin{aligned} \lambda_{t,z} \quad &: \quad \mathbb{E}^2_{t,z}s^3_t - s^1_t + \tfrac{1}{2}\mathbb{V}^2_{t,z}\left[s^3_t\right] = \mathbb{CV}^2_{t,z}\left[c_{t,z} + \phi_{t,z}, s^3_t\right], & \text{(A18)} \\ \alpha_{t,z} \quad &: \quad \mathbb{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] + \tfrac{1}{2}\mathbb{V}^4_{t,z}\left[s^1_{t+1}\right] = \mathbb{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, s^1_{t+1}\right], & \text{(A19)} \\ c_{t,z} \quad &: \quad \ln\beta + r_t = \mathbb{E}^4_{t,z}\left[\Delta c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right] - \tfrac{1}{2}\mathbb{V}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right], & \text{(A20)} \\ \hat{c}_{t,z} \quad &: \quad c_{t,z} = s^3_t + \hat{c}_{t,z}, & \text{(A21)} \end{aligned}$$

for both US and UK agents. The linearized versions of (17) and (18) are:

$$\begin{aligned} \gamma_{t,z<1/2} \quad &: \quad \mathbb{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] + \tfrac{1}{2}\mathbb{V}^4_{t,z}\left[r^k_{t+1}\right] = \mathbb{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, r^k_{t+1}\right], & \text{(A22)} \\ \gamma_{t,z\geq 1/2} \quad &: \quad \mathbb{E}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] + \tfrac{1}{2}\mathbb{V}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1}\right] = \\ & \qquad \mathbb{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, \hat{r}^k_{t+1} + s^1_{t+1}\right]. & \text{(A23)} \end{aligned}$$

It is straightforward to derive the decision rules in (A10)–(A12) from these log approximations. The derivation is somewhat tedious, however, and conveys little insight by itself. Accordingly, we refer readers to the derivation in our working paper, Evans and Lyons (2004a), pages A2-A5.

Two results do warrant attention before we proceed. The first relates to the wedge in the decision problems that captures the risk in liquidity provision, $\phi_{t,z}$. In particular, it is constant over time and equal across agents, i.e., $\phi_{t,z} = \phi$. Constancy is a reflection of the ergodic structure of the model at the monthly frequency: the risk inherent in liquidity provision is not varying from month to month. Equality across agents reflects the symmetry in conditional second moments that agents are facing when solving their liquidity-provision problem. The second noteworthy result is an explicit expression for $\lambda_{t,z}$ that arises from the linearized first-order condition for $\lambda_{t,z}$:

$$\lambda_{t,z} = \tfrac{1}{2} + \left(\tfrac{1}{\mathbb{V}^2_{t,z}[s^3_t]}\right)\mathbb{E}^2_{t,z}\left[s^3_t - s^1_t\right]. \tag{A24}$$

This will be useful below.

A.1.3 Information Structure Verification:

Now we verify the second part of the conjectured equilibrium—the evolution of information sets. To set the stage, note that at the start of period 1 consumers observe home productivity shocks so that $\{e_t, u_t\} \in \Omega^2_{t,\mathbb{US}}$ and $\{\hat{e}_t, \hat{u}_t\} \in \Omega^2_{t,\mathbb{UK}}$. Expectations of the productivity shocks can be calculated from the (Kalman Filter) updating equation:

$$\mathbb{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = \mathbb{E}\left[\varepsilon_t|\Omega^4_{t-1,z}\right] + \mathcal{K}^1_z\varepsilon_{t,z},$$

with $\varepsilon_{t,\mathbb{US}} \equiv \iota_{\mathbb{US}}\varepsilon_t$ and $\varepsilon_{t,\mathbb{UK}} \equiv \iota_{\mathbb{UK}}\varepsilon_t$ where $\varepsilon_{t,z}$ denotes the vector of shocks directly observed by consumer z at the start of period 1 and $\mathcal{K}^1_z \equiv \mathbb{V}\left[\varepsilon_{t,z}\right]^{-1}\mathbb{CV}\left[\varepsilon_t, \varepsilon'_{t,z}\right]$. Note that $\iota'_{\mathbb{US}} \equiv [\iota'_1, \iota'_3]$ and $\iota'_{\mathbb{UK}} \equiv [\iota'_2, \iota'_4]$ where ι_i is the 1× 4 vector that selects the i'th. element from ε_t.The updating equation can therefore be rewritten as:

$$\mathbb{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = \left(\iota_z\Sigma_\varepsilon\iota'_z\right)^{-1}\Sigma_\varepsilon\iota'_z\iota_z\varepsilon_t = b^1_z\varepsilon_t.$$

as shown in equation (A8) where the superscript i in (A8) is set to 1.

We proceed by characterizing this expectation for successive values of b^i_z , $i = \{1, 2, 3, 4\}$. Since no new information arrives during period 1, $\Omega^2_{t,z} = \Omega^1_{t,z}$ and hence $b^1_z = b^2_z$.Specifically, (A8) implies that $\mathbb{E}^2_{t,\mathbb{US}}\hat{e}_t = \mathbb{E}^2_{t,\mathbb{UK}}e_t = 0$, $\mathbb{E}^2_{t,\mathbb{US}}\hat{u}_t = \rho u_t$ and $\mathbb{E}^2_{t,\mathbb{UK}}u_t = \rho\hat{u}_t$. For the public information described by (A9), since the elements of ε_t are not common knowledge by the start of period 2, $\mathbb{E}\left[\varepsilon_t|\Omega^2_t\right] = \mathbb{E}\left[\varepsilon_t|\Omega^1_t\right] = \mathbb{E}\left[\varepsilon_t|\Omega^4_{t-1}\right] = 0$. This is the form of (A9) with $b^i = 0$ for $i = \{1, 2\}$.

Next, we consider the information that accrues between the start of periods 2 and 3, i.e., the $i = 3$ case. Under the rules of trading, all consumers receive the same incoming orders in equilibrium, so aggregate order flow, $T^2_t \equiv \int T^2_{t,z^*}dz^*$ is observed by all consumers by the end of period-2 trading. Hence $\Omega^3_t = \left\{T^2_t, \Omega^1_t\right\}$.

Combining the market clearing condition, $\int T^2_{t,z^*}dz^* = \int T^2_{t,z}dz$, with the definitions of T^2_t and the target fraction of wealth in pounds, $\lambda_{t,z}$, we obtain:

$$\begin{aligned} S^1_t T^2_t &= \int S^1_t T^2_{t,z} dz \\ &= \int \left\{ \lambda_{t,z} W^2_{t,z} - S^1_t \left(\hat{B}_{t,z} + \hat{K}_t \right) + S^1_t \mathbb{E}^2_{t,z} T^2_{t,z^*} \right\} dz \\ &= \int \lambda_{t,z} W^2_{t,z} dz - S^1_t \hat{K}_t + S^1_t \int \mathbb{E}^2_{t,z} T^2_{t,z^*} dz. \end{aligned}$$

The scaled innovation in order flow is defined as $\xi_t \equiv S^1_t \left(T^2_t - \mathbb{E}^2_t T^2_t \right) / \beta R W^2_{t-1}$, where $W^2_{t-1} = \int W^2_{t-1,z} dz$ is world-wide wealth. Bond-market clearing implies that $W^2_{t-1} = S^1_{t-1} \hat{K}_{t-1} + K_{t-1}$, which according to the conjectured information structure in (A7) is common-knowledge at t:1 (i.e., $W^2_{t-1} \in \Omega^1_t$). We may therefore represent common-knowledge information at t:3 as $\Omega^3_t = \left\{ \xi_t, \Omega^1_t \right\}$. Substituting for T^2_t in the definition of ξ_t we get:

$$\xi_t = \lambda_{t,\mathbb{US}} \left(\frac{W_{t,\mathbb{US}}}{\beta R W^2_{t-1}} \right) + \lambda_{t,\mathbb{UK}} \left(\frac{W_{t,\mathbb{UK}}}{\beta R W^2_{t-1}} \right) - \left(\frac{S^1_t \hat{K}_t}{\beta R W^2_{t-1}} \right) + \left(\frac{\left(\mathbb{E}^2_{t,\mathbb{US}} - \mathbb{E}^2_t \right) S^1_t T^2_t}{\beta R W^2_{t-1}} \right) + \frac{1}{2} \left(\frac{\left(\mathbb{E}^2_{t,\mathbb{UK}} - \mathbb{E}^2_t \right) S^1_t T^2_t dz}{\beta R W^2_{t-1}} \right). \quad \text{(A25)}$$

This expression can be written as:

$$\begin{aligned} \xi_t \quad = \quad & \left(\frac{\lambda_{t,\mathbb{US}} \exp\left(w^2_{t,\mathbb{US}} - k_t \right) \exp(\Delta k_t - \Delta k)}{\left(\exp\left(s^3_{t-1} - \nabla k_{t-1} \right) + 1 \right)} \right) + \left(\frac{\lambda_{t,\mathbb{UK}} \exp(w^2_{t,\mathbb{UK}} - s^1_t - \hat{k}_t) \exp\left(s^1_t - s^3_{t-1} + \Delta \hat{k}_t - \Delta k \right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1} \right) + 1 \right)} \right) \\ & - \left(\frac{\exp\left(s^1_t - s^3_{t-1} + \Delta \hat{k}_t - \Delta k \right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1} \right) + 1 \right)} \right) + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{US}} \xi_t + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{UK}} \xi_t. \end{aligned} \quad \text{(A26)}$$

Recall that bond-market clearing implies that $W^i_{t,\mathbb{US}} + W^i_{t,\mathbb{UK}} = K_t + S^{i-1}_t \hat{K}_t$, or:

$$w^i_{t,\mathbb{US}} - k_t = \ln \left((1 + \exp\left(s^{i-1}_t - \nabla k_t \right) - \exp\left(w^i_{t,\mathbb{UK}} - k_t \right) \right)$$

for $i = \{1, 3\}$. Approximating the right hand side around the steady state gives,

$$w^i_{t,\mathbb{US}} - k_t \cong s^{i-1}_t + \hat{k}_t - w^i_{t,\mathbb{UK}}, \quad \text{(A27)}$$

for $i = \{1, 3\}$. Linearizing (A26) around the steady state and combining the result with (A27) for $i = 2$, we find:

$$\xi_t \cong \tfrac{1}{2}(\lambda_{t,\mathbb{US}} - \tfrac{1}{2}) + \tfrac{1}{2}(\lambda_{t,\mathbb{UK}} - \tfrac{1}{2}) - \tfrac{1}{4} \left(s^1_t - \nabla k_t \right) + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{US}} \xi_t + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{UK}} \xi_t. \quad \text{(A28)}$$

Substituting for $\lambda_{t,\mathbb{US}}$ and $\lambda_{t,\mathbb{UK}}$ with expressions derived in Evans and Lyons (2004b) for establishing the decision rule (A10), we have:

$$\xi_t \cong \tfrac{1}{2} \lambda_e \nabla e_t + \tfrac{1}{2} \lambda_u \nabla u_t - \tfrac{1}{4} \left(s^1_t - \nabla k_t \right) + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{US}} \xi_t + \tfrac{1}{2} \mathbb{E}^2_{t,\mathbb{UK}} \xi_t, \quad \text{(A29)}$$

where $\lambda_e = \psi\xi_e/\sigma_s^2$ and $\lambda_u = \psi\xi_u(1-\rho)/\sigma_s^2$. Substituting for $s_t^1 - \nabla k_t$ with (A49) (derived below) gives:

$$\xi_t \cong \tfrac{1}{2}\left(\lambda_e + \tfrac{1}{2}\right)\nabla e_t + \tfrac{1}{2}\mathbb{E}^2_{t,\mathbb{US}}\xi_t + \tfrac{1}{2}\mathbb{E}^2_{t,\mathbb{UK}}\xi_t + \left(\tfrac{1}{2}\lambda_u + \tfrac{1}{2}\right)\nabla u_t. \tag{A30}$$

To determine the expectations terms, $\mathbb{E}^2_{t,\mathbb{US}}\xi_t$ and $\mathbb{E}^2_{t,\mathbb{UK}}\xi_t$, we guess and verify that $\xi_t = \varpi_e\nabla e_t + \varpi_e\nabla u_t$ for some coefficients ϖ_i. Under our information structure, this guess implies that $\mathbb{E}^2_{t,\mathbb{US}}\xi_t = \varpi_e e_t + \varpi_u(1-\rho)u_t$ and $\mathbb{E}^2_{t,\mathbb{UK}}\xi_t = -\varpi_e\hat{e}_t - \varpi_u(1-\rho)\hat{u}_t$. Substituting these expressions into our guess for ξ_t and equating coefficients gives:

$$\begin{aligned}\xi_t &\cong \left(\lambda_e + \tfrac{1}{2}\right)\nabla e_t + \tfrac{1}{1+\rho}\left(\lambda_u + \tfrac{1}{2}\right)\nabla u_t \\ &\cong \xi_e\nabla e_t + \xi_u\nabla u_t = \underline{\xi}'\varepsilon_t,\end{aligned} \tag{A31}$$

as shown in (A5).

Inferences about the vector of productivity shocks based on Ω_t^3 are derived from the Kalman filter updating equation:

$$\mathbb{E}\left[\varepsilon_t|\Omega_t^3\right] = \mathbb{E}\left[\varepsilon_t|\Omega_t^1\right] + \mathcal{K}^3\left(\xi_t - \mathbb{E}\left[\xi_t|\Omega_t^1\right]\right), \tag{A32}$$

where $\mathcal{K}^3 \equiv \mathbb{V}_t^1\left[\xi_t\right]^{-1}\mathbb{CV}_t^1\left[\varepsilon_t,\xi_t\right]$. Now (A5) and (A7) imply that $\mathbb{E}\left[\varepsilon_t|\Omega_t^1\right] = 0$, so

$$\mathbb{E}\left[\varepsilon_t|\Omega_t^3\right] = \left(\underline{\xi}'\Sigma_\varepsilon\underline{\xi}\right)^{-1}\Sigma_\varepsilon\underline{\xi}\underline{\xi}'\xi\varepsilon_t = b^3\varepsilon_t.$$

We thus have our expression for (A9) with $i = 3$.

Turning to the $i = 4$ case, inferences about the productivity shocks based on $\Omega_{t,z}^4$ are calculated as follows. Let $\tilde{\xi}_{t,z} \equiv \left(W_{t,z}^2/\beta RW_{t-1}^2\right)\xi_{t,z}$ denote the re-scaled unexpected order flow consumer z received during period-2 trading. Since $W_{t,z}^2/\beta RW_{t-1}^2 \in \Omega_{t,z}^1$, we can use $\tilde{\xi}_{t,z}$ to represent individual information accruing to consumer z between the start of periods 2 and 4. (Since period-3 spot rates are a function of Ω_t^3, no new individual information accrues between the start of periods 3 and 4.) Combining the definitions of $\tilde{\xi}_{t,z}$ and ξ_t with (A5) gives

$$\tilde{\xi}_{t,z} \equiv \left(\xi_t - \mathbb{E}^2_{t,z}\xi_t\right) \cong \xi_e\left(\nabla e_t - \mathbb{E}^2_{t,z}\nabla e\right) + \xi_u\left(\nabla u_t - \mathbb{E}^2_{t,z}\nabla u_t\right).$$

Using (A7) to evaluate the expectations terms on the right, we find that

$$\tilde{\xi}_{t,\mathbb{US}} \cong -\xi_e\hat{e}_t + \xi_u\left(\rho u_t - \hat{u}_t\right) = \underline{\tilde{\xi}}'_{\mathbb{US}}\varepsilon_t, \tag{A33}$$

$$\tilde{\xi}_{t,\mathbb{UK}} \cong \xi_e e_t + \xi_u\left(u_t - \rho\hat{u}_t\right) = \underline{\tilde{\xi}}'_{\mathbb{UK}}\varepsilon_t. \tag{A34}$$

Inferences about the productive shocks can now be calculated using these expressions and the updating equation

$$\mathbb{E}\left[\varepsilon_t|\Omega^4_{t,z}\right] = \mathbb{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] + \mathcal{K}^4_z\left(\tilde{\xi}_{t,z} - \mathbb{E}\left[\tilde{\xi}_{t,z}|\Omega^1_{t,z}\right]\right),$$

where $\mathcal{K}^4_z \equiv \mathbb{V}^1_{t,z}\left(\tilde{\xi}_{t,z}\right)^{-1}\mathbb{CV}^1_{t,z}\left(\varepsilon_t, \tilde{\xi}_{t,z}\right)$. Now equations (A33) and (A34) imply that $\mathbb{V}^1_{t,z}\left(\tilde{\xi}_{t,z}\right) = \underline{\tilde{\xi}_z}'\mathbb{V}^1_{t,z}\left(\varepsilon_t\right)\underline{\tilde{\xi}_z}$ and $\mathbb{CV}^1_{t,z}\left[\varepsilon_t, \tilde{\xi}_{t,z}\right] = \mathbb{V}^1_{t,z}\left[\varepsilon_t\right]\underline{\tilde{\xi}_z}$. Further, recall that $\mathbb{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = b^1_z\varepsilon_t$ and $\mathbb{V}^1_{t,z}[\varepsilon_t] = \left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)$. Substituting these results into the updating equation above gives:

$$\begin{aligned}
\mathbb{E}\left[\varepsilon_t|\Omega^4_{t,z}\right] &= \left(b^1_z + \left(\underline{\tilde{\xi}_z}'\left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)\underline{\tilde{\xi}_z}\right)^{-1}\left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)\underline{\tilde{\xi}_z\tilde{\xi}_z'}\right)\varepsilon_t \\
&= b^4_z\varepsilon_t.
\end{aligned}$$

We thus have our expression for equation (A8) when $i = 4$.

We turn back now to $i = 1$, i.e., we examine the information revealed by order flow at the end of the month in period-4. As in period 2, all consumers receive the same incoming orders in equilibrium, so aggregate order flow, $T^4_t \equiv \int T^{42}_{t,z^*}dz^*$ is observed by all consumers by the end of period-4 trading. Hence $\Omega^1_{t+1} = \left\{T^4_t, \Omega^4_t\right\}$. Combining the market clearing condition, $\int T^4_{t,z^*}dz^* = \int T^4_{t,z}dz$, with the definitions of T^4_t, the target fraction of wealth in pounds $\alpha_{t,z}$, and the log consumption/wealth ratio $\delta_{t,z}$, we obtain:

$$S^3_t T^4_t = \int\left(\alpha_{t,z} + \tfrac{\mu}{2}\exp\left(\delta_{t,z}\right)\right)W^4_{t,z}dz - S^3_t\hat{K}_t + S^3_t\int\mathbb{E}^4_{t,z}T^4_{t,z^*}dz.$$

The scaled innovation in period-4 order flow is defined as $\varsigma_t \equiv S^3_t\left(T^4_t - \mathbb{E}^4_t T^4_t\right)/\beta R W^2_{t-1}$. Substituting for T^4_t in this definition, linearizing around the steady state (where $\alpha_{t,z} = \alpha_z = (1-\mu)/2$, $\delta_{t,z} = 0, W^2_{t,\mathbb{US}} = K_t$, $W^2_{t,\mathbb{UK}} = \hat{K}_t$, and $S^3_t = K_t/\hat{K}_t$), and combining this with (A27) for $i = 4$, we find that:

$$\begin{aligned}
\varsigma_t &= \tfrac{1}{2}\left(\alpha_{t,\mathbb{US}} - \alpha_{\mathbb{US}}\right) + \tfrac{1}{2}\left(\alpha_{t,\mathbb{UK}} - \alpha_{\mathbb{UK}}\right) \\
&\quad + \tfrac{\mu}{4}\left(\delta_{t,\mathbb{US}} + \delta_{t,\mathbb{UK}}\right) + \tfrac{1}{4}(\nabla k_t - s^3_t) + \tfrac{1}{2}\mathbb{E}^4_{t,\mathbb{US}}\varsigma_t + \tfrac{1}{2}\mathbb{E}^4_{t,\mathbb{UK}}\varsigma_t. \qquad \text{(A35)}
\end{aligned}$$

Substituting for $\nabla k_t - s^3_t$ with (A47) and the decision rules for $\delta_{t,z}$ and $\alpha_{t,z}$ in (A11) and (A12) gives

$$\varsigma_t = \tfrac{1}{2}\varrho_{\mathbb{US}}\underline{\omega}'_{\mathbb{US}}\varepsilon_t + \tfrac{1}{2}\varrho_{\mathbb{UK}}\underline{\omega}'_{\mathbb{UK}}\varepsilon_t + \tfrac{\mu}{4}\left(\underline{\delta}'_{\mathbb{US}} + \underline{\delta}'_{\mathbb{UK}}\right)\varepsilon_t + \tfrac{1}{4}\pi'\varepsilon_t + \tfrac{1}{2}\mathbb{E}^2_{t,\mathbb{US}}\varsigma_t + \tfrac{1}{2}\mathbb{E}^2_{t,\mathbb{UK}}\varsigma_t.$$

where $\alpha_{t,z} \equiv \varrho_z\omega_{t,z}$. As above, we solve this equation with the guess and verify method using (A8) to give:

$$\varsigma_t \cong \Phi_e e_t + \Phi_{\hat{e}}\hat{e}_t + \Phi_u u_t + \Phi_{\hat{u}}\hat{u}_t. \qquad \text{(A36)}$$

where the Φ_i coefficients are implicitly defined by:

$$\Phi_i = \iota_i\left(\frac{1}{2}\varrho_{\mathbb{US}}\underline{\omega}'_{\mathbb{US}} + \frac{1}{2}\varrho_{\mathbb{UK}}\underline{\omega}'_{\mathbb{UK}} + \frac{\mu}{4}\left(\underline{\delta}'_{\mathbb{US}} + \underline{\delta}'_{\mathbb{UK}}\right) + \frac{1}{4}\pi + \frac{1}{2}\Phi b^4_{\mathbb{US}} + \frac{1}{2}\Phi b^4_{\mathbb{UK}}\right),$$

with $\Phi \equiv [\ \Phi_e \quad \Phi_{\hat{e}} \quad \Phi_u \quad \Phi_{\hat{u}}\ ]$.

To solve for the information revealed by period-4 trading, we need to identify the unexpected order flows received by each consumer in period-4, as well as the unexpected export orders. Innovations in period-4 order flow can be written as $\varsigma_{t,z} \equiv \left(\beta R W^2_{t-1}/W^4_{t,z}\right)\left(\varsigma_t - \mathbb{E}^4_{t,z}\varsigma_t\right)$. Taking a log-linear approximation around the steady state values of $W^2_{t-1}, W^4_{t,z}$ and ς_t produces $\varsigma_{t,z} \cong \frac{1}{2}\left(\varsigma_t - \mathbb{E}^4_{t,z}\varsigma_t\right)$. Combining this approximation with (A36) and (A9) gives:

$$\varsigma_{t,z} \cong \tfrac{1}{2}\Phi\left(I - b^4_z\right)\varepsilon_t. \tag{A37}$$

We approximate unexpected export orders $\zeta_{t,\mathbb{US}}$ and $\hat{\zeta}_{t,\mathbb{UK}}$ in a similar manner. This produces:

$$\zeta_{t,\mathbb{US}} \cong \tfrac{\mu}{2}\left(\delta_{t,\mathbb{UK}} + s^3_t - \nabla k_t\right). \tag{A38}$$

$$\hat{\zeta}_{t,\mathbb{UK}} \cong \tfrac{\mu}{2}\left(\delta_{t,\mathbb{US}} - s^3_t + \nabla k_t\right). \tag{A39}$$

The final step in verifying the conjectured information sets is to show how (A31) and (A36) can be combined with elements of $\Omega^4_{t,z}$ so that $\{e_t, \hat{e}_t, u_t, \hat{u}_t\} \in \left\{\varsigma_t \cup \Omega^4_{t,z}\right\}$ for $z = \{\mathbb{US}, \mathbb{UK}\}$. For the case of US consumers, we rewrite (A31) and (A36) as:

$$\begin{aligned}
\chi^2_{t,\mathbb{US}} &\equiv \xi_t - \xi_e e_t - \xi_u u_t = -\xi_e \hat{e}_t - \xi_u \hat{u}_t, \\
\chi^4_{t,\mathbb{US}} &\equiv \varsigma_t - \Phi_e e_t - \Phi_u u_t = \Phi_{\hat{e}}\hat{e}_t + \Phi_{\hat{u}}\hat{u}_t.
\end{aligned}$$

$\chi^2_{t,\mathbb{US}}$ and $\chi^4_{t,\mathbb{US}}$ provide two signals of the values of $\hat{e}_t$ and $\hat{u}_t$ that can be constructed from information available to US consumers at the end of period-4 trading (i.e., $\left\{\chi^2_{t,\mathbb{US}}, \chi^4_{t,\mathbb{US}}\right\} \in \left\{\varsigma_t \cup \Omega^4_{t,\mathbb{US}}\right\}$). Combining these equations, we find that:

$$\begin{aligned}
\hat{e}_t &= \left(\frac{1}{\Phi_{\hat{e}}\xi_u - \Phi_{\hat{u}}\xi_e}\right)\left(\Phi_{\hat{u}}\chi^2_{t,\mathbb{US}} + \xi_u \chi^4_{t,\mathbb{US}}\right), \\
\hat{u}_t &= -\left(\frac{1}{\Phi_{\hat{e}}\xi_u - \Phi_{\hat{u}}\xi_e}\right)\left(\Phi_{\hat{e}}\chi^2_{t,\mathbb{US}} + \xi_e \chi^4_{t,\mathbb{US}}\right).
\end{aligned}$$

Similarly, UK consumers can combine their observations of order flow from periods 2 and 4 with their knowledge of $\hat{u}_t$ and $\hat{e}_t$ to infer the values of e_t and u_t precisely. Thus, $\{e_t, \hat{e}_t, u_t, \hat{u}_t\}$ are indeed common knowledge after period-4 trading. This completes the verification of the information structure shown in (A6)–(A9).

Exchange and Interest Rate Process Verification: The last step in verifying the equilibrium concerns the conjectured processes for the exchange rate and interest rate. First we verify that the processes for equilibrium quotes made in periods 1 and 3 follow (A1)–(A4). To derive the exchange rate process, we start by combining the capital accumulation equations, (24) and (25), to give:

$$\nabla k_{t+1} = \nabla k_t + \nabla r^k_{t+1} - \left(\tfrac{\mu}{1-\mu}\right)\left(s^3_t - \nabla k_t\right). \tag{A40}$$

Combining this equation with the identity $s^3_{t+1} - \nabla k_{t+1} \equiv \Delta\left(s^3_{t+1} - \nabla k_{t+1}\right) + s^3_t - \nabla k_t$ gives:

$$s^3_{t+1} - \nabla k_{t+1} = \left(\tfrac{1}{1-\mu}\right)\left(s^3_t - \nabla k_t\right) + \Delta s^3_{t+1} - \nabla r^k_{t+1}. \tag{A41}$$

Next, we take conditional expectations of both sides of this equation:

$$\mathbb{E}^3_t\left[s^3_{t+1} - \nabla k_{t+1}\right] = \left(\tfrac{1}{1-\mu}\right)\left(s^3_t - \mathbb{E}^3_t \nabla k_t\right) + \mathbb{E}^3_t\left[\Delta s^3_{t+1} - \nabla r^k_{t+1}\right].$$

By iterated expectations, the left-hand side is equal to $\mathbb{E}^3_t\left[s^3_{t+1} - \mathbb{E}^3_{t+1}\nabla k_{t+1}\right]$. Substituting this expression on the left and iterating forward gives:

$$s^3_t = \mathbb{E}^3_t \nabla k_t + \mathbb{E}^3_t \sum_{i=1}^{\infty} (1-\mu)^i \left\{r^k_{t+i} - \left(\hat{r}^k_{t+i} + \Delta s^3_{t+i}\right)\right\}. \tag{A42}$$

This establishes an important equation; it corresponds to (29) in the text.

Next we note from (A1), (A2), and (A5) that:

$$\begin{aligned}
\mathbb{E}^3_t[\Delta s^3_{t+1} - \nabla r^k_{t+1}] &= \mathbb{E}^3_t\left[\left(\tfrac{\pi_e}{1-\mu}\right)\nabla e_t + \left(\tfrac{\pi_u}{1-\mu}\right)\nabla u_t - \pi_e \nabla e_{t+1} - \pi_u \nabla u_{t+1}\right] \\
&= \mathbb{E}^3_t\left[\left(\tfrac{\pi_e}{1-\mu}\right)\nabla e_t + \left(\tfrac{\pi_u}{1-\mu}\right)\nabla u_t\right] = \left(\tfrac{1}{1-\mu}\right)\mathbb{E}^3_t\left[\nabla k_t - \mathbb{E}^3_t \nabla k_t\right] = 0.
\end{aligned}$$

Since $\mathbb{E}^3_t\left[\Delta s^3_{t+i} - \nabla r^k_{t+i}\right] = \mathbb{E}^3_t\left[\mathbb{E}^3_{t+i-1}\left[\Delta s^3_{t+i} - \nabla r^k_{t+i}\right]\right]$ for $i > 1$ by iterated expectations, the expression above implies that $\mathbb{E}^3_t\left[\Delta s^3_{t+i} - \nabla r^k_{t+i}\right] = 0$ for $i > 0$, so (A42) simplifies to:

$$s^3_t = \mathbb{E}^3_t \nabla k_t. \tag{A43}$$

Importantly, in equilibrium the expected return on holding pounds conditioned on public information Ω^1_t (i.e., $\mathbb{E}^1_t\left[s^3_t - s^1_t\right]$) must equal zero. To establish this, recall from (A25) that under market clearing:

$$\xi_t = \lambda_{t,\mathbb{US}}\left(\tfrac{W_{t,\mathbb{US}}}{\beta R W^2_{t-1}}\right) + \lambda_{t,\mathbb{UK}}\left(\tfrac{W_{t,\mathbb{UK}}}{\beta R W^2_{t-1}}\right) - \left(\tfrac{S^1_t \hat{K}_t}{\beta R W^2_{t-1}}\right) + \left(\tfrac{\left(\mathbb{E}^2_{t,\mathbb{US}} - \mathbb{E}^2_t\right) S^1_t T^2_t}{\beta R W^2_{t-1}}\right) + \tfrac{1}{2}\left(\tfrac{\left(\mathbb{E}^2_{t,\mathbb{UK}} - \mathbb{E}^2_t\right) S^1_t T^2_t dz}{\beta R W^2_{t-1}}\right).$$

Applying the conditional expectations operator $\mathbb{E}^1_t$ to both sides of this equation gives:

$$0 = \mathbb{E}\left[\lambda_{t,\mathbb{US}}\left(\tfrac{W_{t,\mathbb{US}}}{\beta R W^2_{t-1}}\right) + \lambda_{t,\mathbb{UK}}\left(\tfrac{W_{t,\mathbb{UK}}}{\beta R W^2_{t-1}}\right) - \left(\tfrac{S^1_t \hat{K}_t}{\beta R W^2_{t-1}}\right)\Big|\,\Omega^1_t\right],$$

which implies that:

$$0 = \mathbb{E}\left[\lambda_{t,\mathbb{US}} W_{t,\mathbb{US}} + \lambda_{t,\mathbb{UK}} W_{t,\mathbb{UK}} - S^1_t \hat{K}_t | \Omega^1_t\right], \tag{A44}$$

because $W_{t-1}^2 \in \Omega_t^1$. Notice that this restriction follows as an implication of market clearing and rational expectations (it does not rely on any approximations). As such, it must hold true for any equilibrium distribution of wealth, including the case where $W_{t,\mathbb{US}} = W_{t,\mathbb{UK}} = S_t^1 \hat{K}_t \in \Omega_t^1$. Under these circumstances, (A44) simplifies further to:

$$\mathbb{E}\left[\lambda_{t,\mathbb{US}} - \tfrac{1}{2}|\Omega_t^1\right] = -\mathbb{E}\left[\lambda_{t,\mathbb{UK}} - \tfrac{1}{2}|\Omega_t^1\right].$$

Substituting for $\lambda_{t,\mathbb{US}}$ and $\lambda_{t,\mathbb{UK}}$ with (A24) gives:

$$\mathbb{E}\left[\left(\tfrac{1}{\mathbb{V}_{t,\mathbb{US}}^2[s_t^3]}\right)\mathbb{E}_{t,\mathbb{US}}^2\left[s_t^3 - s_t^1\right]\Big|\,\Omega_t^1\right] = -\mathbb{E}\left[\left(\tfrac{1}{\mathbb{V}_{t,\mathbb{UK}}^2[s_t^3]}\right)\mathbb{E}_{t,\mathbb{UK}}^2\left[s_t^3 - s_t^1\right]\Big|\,\Omega_t^1\right].$$

When period-3 spot rates are set according to (A43) and the distribution of capital follows (A40), $\mathbb{V}_{t,\mathbb{US}}^2\left[s_t^3\right] = \mathbb{V}_{t,\mathbb{UK}}^2\left[s_t^3\right] = \sigma_s^2$, a constant. This means that the equation above further simplifies to:

$$\mathbb{E}\left[s_t^3 - s_t^1\big|\,\Omega_t^1\right] = -\mathbb{E}\left[s_t^3 - s_t^1\big|\,\Omega_t^1\right],$$

a condition that can only be met when $\mathbb{E}\left[s_t^3 - s_t^1\big|\,\Omega_t^1\right] = 0$. Notice that we would not be able to derive this simple implication of rational expectations and market clearing if hedging terms were present in the period-2 portfolio decisions. Combining $\mathbb{E}\left[s_t^3 - s_t^1\big|\,\Omega_t^1\right] = 0$ with (A43) gives us the equilibrium exchange rate quoted by all consumers in period 1:

$$s_t^1 = \mathbb{E}_t^1 \nabla k_t. \tag{A45}$$

Equilibrium exchange rate dynamics are derived by combining (A43) and (A45) with (A40). For this purpose, we take expectations conditioned on Ω_{t+1}^1 on both sides of (A40) to give:

$$s_{t+1}^1 = \mathbb{E}_{t+1}^1 \nabla k_t + \mathbb{E}_{t+1}^1 \nabla r_{t+1}^k + \left(\tfrac{\mu}{1-\mu}\right)\mathbb{E}_{t+1}^1\left[\nabla k_t - s_t^3\right].$$

Subtracting s_t^3 from both sides yields:

$$\begin{aligned} s_{t+1}^1 - s_t^3 &= \mathbb{E}_{t+1}^1 \nabla r_{t+1}^k + \left(\tfrac{1}{1-\mu}\right)\mathbb{E}_{t+1}^1\left[\nabla k_t - s_t^3\right] \\ &= \mathbb{E}_{t+1}^1 \nabla r_{t+1}^k + \left(\tfrac{1}{1-\mu}\right)\mathbb{E}_{t+1}^1\left[\nabla k_t - \mathbb{E}_t^3 \nabla k_t\right], \end{aligned} \tag{A46}$$

as shown in (30) in the text. Now (A40) implies that:

$$\nabla k_t - \mathbb{E}_t^3 \nabla k_t = \nabla r_t^k - \mathbb{E}_t^3 \nabla r_t^k - \left(\tfrac{\mu}{1-\mu}\right)\left((s_{t-1} - \nabla k_{t-1}) - \mathbb{E}_t^3\left[s_{t-1} - \nabla k_{t-1}\right]\right).$$

Under our information structure, $\{s_{t-1}, \nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega_t^3$, so this expression simplifies to:

$$\begin{aligned} \nabla k_t - \mathbb{E}_t^3 \nabla k_t &= \nabla e_t + \nabla u_t - \mathbb{E}\left[\nabla e_t + \nabla u_t|\Omega_t^3\right], \\ &= \pi_e \nabla e_t + \pi_u \nabla u_t = \pi' \varepsilon_t, \end{aligned} \tag{A47}$$

where $\pi_e = \left(1 - \imath\mathcal{K}^3\xi_e\right)$ and $\pi_u = \left(1 - \imath\mathcal{K}^3\xi_u\right)$ with $\imath \equiv [\ 1 \quad -1 \quad 1 \quad -1\ ]$. The form of the π_i coefficients follow from (A32) and (A5). Combining (A47), (A46) and the fact that $\mathbb{E}^1_{t+1}\nabla r^k_{t+1} = 2\theta\nabla e_t$ under our information structure, gives:

$$s^1_{t+1} - s^3_t = 2\theta\nabla e_t + \left(\tfrac{1}{1-\mu}\right)\left(\pi_e\nabla e_t + \pi_u\nabla u_t\right).$$

Thus, we have established the first of our conjectured exchange rate equations in (A1).

To derive the second of our two exchange rate equations in (A2), we take expectations conditioned on Ω^3_t on both sides of (A40) (lagged one month), to give:

$$s^3_t = \mathbb{E}^3_t\nabla k_{t-1} + \mathbb{E}^3_t\nabla r^k_t + \left(\tfrac{\mu}{1-\mu}\right)\mathbb{E}^3_t\left[\nabla k_{t-1} - s^3_{t-1}\right].$$

Subtracting s^1_t from both sides and combing the result with (A46) gives:

$$s^3_t - s^1_t = \left(\mathbb{E}^3_t - \mathbb{E}^1_t\right)\nabla r^k_t + \left(\tfrac{1}{1-\mu}\right)\left(\mathbb{E}^3_t - \mathbb{E}^1_t\right)\left[\nabla k_{t-1} - s^3_{t-1}\right]. \tag{A48}$$

Under the information structure, $\{\nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega^1_t$, so $\left(\mathbb{E}^3_t - \mathbb{E}^1_t\right)\left[\nabla k_{t-1} - s^3_{t-1}\right] = 0$ and $\left(\mathbb{E}^3_t - \mathbb{E}^1_t\right)\nabla r^k_t = \left(\mathbb{E}^3_t - \mathbb{E}^1_t\right)\left[\nabla e_t + \nabla u_t\right]$. Since $\mathbb{E}^1_t\varepsilon_t = 0$, the latter term simplifies to $\mathbb{E}^3_t\left[\nabla e_t + \nabla u_t\right]$. Now (A32) and (A5) imply that $\mathbb{E}^3_t\left[\nabla e_t + \nabla u_t\right] = \imath\mathcal{K}^3\xi_t$. Combining these results with the equation above gives:

$$s^3_t - s^1_t = \imath\mathcal{K}^3\xi_t = \psi\xi_t,$$

as shown in (A2).

As a side point, we can use (A46) and (A40) to calculate the value of $s^1_t - \nabla k_t$ used in the derivation of period-2 order flow above. Specifically, by combining (A46) and (A40) we can write:

$$s^1_t - \nabla k_t = \left(\mathbb{E}^1_t - 1\right)\nabla r^k_t + \left(\tfrac{1}{1-\mu}\right)\left(\mathbb{E}^1_t - 1\right)\left(\nabla k_{t-1} - \mathbb{E}^3_{t-1}\nabla k_{t-1}\right).$$

According to the conjectured information structure in (A7), ∇k_{t-1} and ∇e_{t-1} are common knowledge by t:1, i.e., $\{\nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega^1_t$. This means that the second term in the expression above equals zero. (A7) also implies that $\left(\mathbb{E}^1_t - 1\right)\nabla r^k_t = -\nabla e_t - \nabla u_t$. Substituting these results into the equation above gives the value used in the derivation:

$$s^1_t - \nabla k_t = -\nabla e_t - \nabla u_t. \tag{A49}$$

Finally, we turn to the interest rate quotes made in period 3. From (A37), (A38) and (A39) we see that innovations to period-4 order flow, $\varsigma_{t,z}$, and exports, $\zeta_{t,\mathbb{US}}$ and $\hat{\zeta}_{t,\mathbb{UK}}$, depend on the choices for $\omega_{t,z}$ and $\delta_{t,z}$ made at the start of the period. This means that $\omega_{t,z}$ and $\delta_{t,z}$ cannot be functions of Ω^3_t, otherwise $\varsigma_{t,z}$, $\zeta_{t,\mathbb{US}}$ and $\hat{\zeta}_{t,\mathbb{UK}}$ would not be orthogonal to Ω^3_t as rational expectations requires. For this to be the case, expected excess returns on capital cannot be correlated with elements of Ω^3_t. Thus, market clearing requires that the interest rates quoted in period 3 satisfy:

$$r_t = \mathbb{E}_t^3 r_{t+1}^k, \tag{A50}$$

$$\hat{r}_t = \mathbb{E}_t^3 \hat{r}_{t+1}^k. \tag{A51}$$

Given the process for capital returns, and the conjecture information structure, these equations become

$$\begin{aligned} r &= r + \theta \mathbb{E}_t^3 \nabla e_t \\ &= r + \theta (\iota_1 - \iota_2) \mathcal{K}^3 \xi_t, \quad \text{(A52)} \\ \hat{r} &= r - \theta \mathbb{E}_t^3 \nabla e_t \\ &= r - \theta (\iota_1 - \iota_2) \mathcal{K}^3 \xi_t, \quad \text{(A53)} \end{aligned}$$

where $\mathcal{K}^3$ is defined in (A32). Equations (A52) and (A53) take the same form as the last two of the conjectured process equations we set out to verify, namely (A3) and (A4), in this case with $\eta = \theta (\iota_1 - \iota_2) \mathcal{K}^3$.

Investment and Idiosyncratic Export Shocks: To show how investment expenditures depend on the idiosyncratic components of last month's export shock, we first note from (A11) and (A12) that the values of $\omega_{t,z}$ and $\delta_{t,z}$ are the same for all agents within each country. Next, notice from (A14), (A15) and (A16) that export shocks only affect wealth dynamics via the covariance terms $\mathbb{CV}_{t,z}^4 \left[r_{t+1}^k, \zeta_{t,z} \right]$ and $\mathbb{CV}_{t,z}^4 \left[\hat{r}_{t+1}^k, \hat{\zeta}_{t,z} \right]$. Since neither term depends on the idiosyncratic components of the export shocks, these shocks have a negligible (i.e. 3rd order) effect on the distribution of wealth within a country. Together, these observations imply that period-4 investment decisions are made so that $K_{t,z} + I_{t,z} = \mathfrak{K}_t$ and $\hat{K}_{t,z} + \hat{I}_{t,z} = \hat{\mathfrak{K}}_t$ where $\mathfrak{K}_t$ ($\hat{\mathfrak{K}}_t$) is the same across US (UK) agents. Aggregating each of these expressions, and imposing the market clearing condition, implies that $\mathfrak{K}_t = K_t$ and $\hat{\mathfrak{K}}_t = \hat{K}_t$. Combining these results with (6) and (7) and the aggregate capital dynamics in (22) and (23) gives:

$$\begin{aligned} I_{t,z} &= K_t - K_{t,z} = R_t^k \nu_{t-1,z}, , \\ \hat{I}_{t,z} &= \hat{K}_t - \hat{K}_{t,z} = \hat{R}_t^k \hat{\nu}_{t-1,z}. \end{aligned}$$

Thus, in equilibrium, each US (UK) agent chooses real investment to offset the effects of the idiosyncratic component of last month's export shock.

A.1.4 Equilibrium when $\rho = -1$

When $\rho = -1$, equilibrium interest rates and the exchange rate follow:

$$s^1_{t+1} - s^3_t = \nabla u_{t+1} + 2\theta \nabla e_t, \tag{A54}$$

$$s^3_t - s^1_t = \nabla e_t, \tag{A55}$$

$$r_t = r + \theta \nabla e_t, \tag{A56}$$

$$\hat{r}_t = r - \theta \nabla e_t, \tag{A57}$$

Individual information sets evolve according to:

$$\begin{array}{ll} \Omega^2_{t,\mathbb{US}} = \Omega^1_{t,\mathbb{US}} = \left\{ u_t, \hat{u}_t, e_t \cup \Omega^{43}_{t-1,\mathbb{US}} \right\}, & \Omega^2_{t,\mathbb{UK}} = \Omega^1_{t,\mathbb{UK}} = \left\{ u_t, \hat{u}_t, \hat{e}_t \cup \Omega^4_{t-1,\mathbb{US}} \right\}, \\ \Omega^4_{t,\mathbb{US}} = \Omega^3_{t,\mathbb{US}} = \left\{ \hat{e}_t \cup \Omega^2_{t,\mathbb{US}} \right\}, & \Omega^4_{t,\mathbb{UK}} = \Omega^3_{t,\mathbb{UK}} = \left\{ e_t \cup \Omega^2_{t,\mathbb{UK}} \right\}, \end{array} \tag{A58}$$

and the evolution of public information is given by:

$$\begin{array}{ll} \Omega^1_t = \left\{ u_t, \hat{u}_t \cup \Omega^4_{t-1} \right\}, & \Omega^2_t = \Omega^3_t, \\ \Omega^3_t = \left\{ e_t, \hat{e}_t \cup \Omega^2_t \right\}, & \Omega^4_t = \Omega^3_t. \end{array} \tag{A59}$$

Unexpected order flow in period-2 is perfectly correlated with ∇e_t, while order flows in period 4 are perfectly predictable. Period-2 portfolio choices are given by:

$$\lambda_{t,\mathbb{US}} = \frac{1}{2} + \left(\frac{1}{\sigma^2_e} \right) e_t, \qquad \text{and} \qquad \lambda_{t,\mathbb{UK}} = \frac{1}{2} - \left(\frac{1}{\sigma^2_e} \right) \hat{e}_t. \tag{A60}$$

The consumption-wealth ratio and period-4 portfolio shares are constant.

We can verify that these equations describe the equilibrium following the verification procedure in the general case. In this special case things are much simpler, so we only outline the argument. Start with the observation that $\{u_t, \hat{u}_t\} \in \Omega^1_t$ because the "u" shocks are perfectly (negatively) correlated. Thus, the "e" shocks are the only source of individual information at the start of period-2 trading. In equilibrium, consumers use this information in choosing their desired portfolio, as (A60) shows, with the result that the innovation in order flow, ξ_t, is a known function of ∇e_t. Thus, $\{e_t, \hat{e}_t\} \in \left\{ \xi_t \cup \Omega^1_{t,z} \right\}$ for all z, so the "e" shocks become common knowledge by the start of period 3. This means that $\mathbb{E}^3_t \nabla k_t = \nabla k_t$, $\mathbb{E}^1_t \nabla R^k_t = 2\theta \nabla e_{t-1} + \nabla u_t$ and $\left(\mathbb{E}^3_t - \mathbb{E}^1_t \right) \nabla r^k_t = \nabla e_t$. Substituting these results into (A46) and (A48) gives (A54) and (A55). The information structure also implies that $\mathbb{E}^3_t r^k_{t+1} = \theta \nabla e_t$ and $\mathbb{E}^3_t \hat{r}^k_{t+1} = -\theta \nabla e_t$, so (A56) and (A57) follow from (A50) and (A51). All that now remains is to verify the form of the decision rules. (A54) – (A58) imply that the vector of expected excess returns $\mathbb{E}^4_{t,z} x_{t+1,z}$ is zero. Under these circumstances, $\omega_{t,z}$ and $\delta_{t,z}$ are constant (details in Evans and Lyons 2004b, pages A3-A4). Equation (A60) follows from (A55), (A16), and the linearized first-order condition for $\lambda_{t,z}$.

A.1.5 Proofs of Propositions

Proposition 1 (Spot Rates): See the text following (29).

Proposition 2 (Revelation in a Special Case): See the text following the proposition.

Proposition 3 (Revelation in the General Case): The only part of the proposition not covered in section A.1.2 concerns the values of the updating coefficients π_e and π_u. We argue by contradiction to show that $\pi_e \neq 0$ and $\pi_u \neq 0$. If $\pi_e = (1 - \psi\xi_e) = 0$ and $\pi_u = (1 - \psi\xi_u) = 0$, then $\xi_e = \xi_u$, so (A31) implies that:

$$(1+\rho)\left(\lambda_e + \frac{1}{2}\right) = \lambda_u + \frac{1}{2}.$$

The assumption that $\pi_e = \pi_u = 0$ together with (A24) also implies that:

$$\begin{aligned}\lambda_e &= \left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right)^{-1}, \\ \lambda_u &= (1-\rho)\left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right)^{-1}.\end{aligned}$$

Combining these expressions with the equation above gives $\left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right) = -4$; a contradiction.

Proposition 4 (Revelation at Month End): This proposition is proved within the Verification subsection of Appendix section A.1.

Proposition 5 (Embedding): The equation in this proposition is a simple combination of the results in (A40) and (A43).

Proposition 6 (Excess Volatility): The first variance expression follows directly from the capital returns processes (8a) and (8b), and the exchange rate equations (A54) and (A55). To derive the second expression, combine (A1) and (A2) to give:

$$\Delta s_{t+1}^3 = \psi\xi_{t+1} + 2\theta\nabla e_t + \left(\tfrac{1}{1-\mu}\right)(\pi_e \nabla e_t + \pi_u \nabla u_t).$$

Substituting for ξ_{t+1} with (A5) yields:

$$\begin{aligned}\Delta s_{t+1}^3 &= \psi\xi_e\nabla e_{t+1} + \psi\xi_u\nabla u_{t+1} + 2\theta\nabla e_t + \left(\tfrac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t) \\ &= \psi\xi_e\nabla e_{t+1} + \psi\xi_u\nabla u_{t+1} + \nabla r_{t+1}^k - \nabla e_{t+1} - \nabla u_{t+1} + \left(\tfrac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t) \\ &= \nabla r_{t+1}^k - (\pi_e\nabla e_{t+1} + \pi_u\nabla u_{t+1}) + \left(\frac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t).\end{aligned}$$

Using the last line in this expression we compute:

$$\begin{aligned}\mathbb{V}\left[\Delta s_{t+1}^3\right]-\mathbb{V}\left[\nabla r_{t+1}^k\right] &= \left(\tfrac{\pi_e}{1-\mu}+\pi_e\right)^2 2\sigma_e^2+\left(\tfrac{\pi_u}{1-\mu}+\pi_u\right)^2 2(1-\rho)\sigma_u^2+4\pi_e\left(\tfrac{1+\mu}{1-\mu}\right)\sigma_e^2-4\pi_u(1-\rho)\sigma_u^2 \\ &= \left(2\pi_e^2\left(\tfrac{2-\mu}{1-\mu}\right)^2+4\pi_e\left(\tfrac{1+\mu}{1-\mu}\right)\right)\sigma_e^2+\left(2\pi_u^2\left(\tfrac{2-\mu}{1-\mu}\right)^2-4\pi_u\right)(1-\rho)\sigma_u^2.\end{aligned}$$

The first term in unambiguously positive because $\pi_e > 0$. The second term is positive if $\pi_u > \bar{\pi}_u \equiv 2\left(\frac{1-\mu}{2-\mu}\right)^2$. Note that $\bar{\pi}_u < 1$ because $1 > \mu > 0$, so $\bar{\pi}_u$ is the lower bound on π_u sufficient to generate excess volatility.

Proposition 7 (Announcements): We established in (A43) and (A45) that the equilibrium log exchange rate can be written as $s_t^i = \mathbb{E}\left[\nabla k_t|\Omega_t^i\right]$, for $i = \{1,3\}$, where Ω_t^i denotes public information at $t{:}i$ identified in (A7) without announcements. Thus, a public announcement about the values of r_t^k and $\hat{r}_t^k$ in $t{:}i$ will have no impact on the exchange rate if $\mathbb{E}\left[\nabla k_t|\Omega_t^i\right] = \mathbb{E}\left[\nabla k_t|\Omega_t^i, r_t^k, \hat{r}_t^k\right]$. Since $\nabla k_t \in \Omega_{t+1}^1$, announcements made after $t{:}4$ have no exchange rate effects because all the information they contain has been aggregated by consumers via trading.

Suppose the announcement is made in $t{:}3$. Equation (A47) implies that $\nabla k_t = \mathbb{E}_t^3 \nabla k_t + \pi_e \nabla e_t + \pi_u \nabla u_t$, and (A40) with (A7) imply that $\mathbb{E}\left[\nabla k_t|\Omega_t^3, r_t^k, \hat{r}_t^k\right] = \nabla k_t$, so:

$$\mathbb{E}\left[\nabla k_t|\Omega_t^1, r_t^k, \hat{r}_t^k\right] - \mathbb{E}\left[\nabla k_t|\Omega_t^1\right] = \pi_e \nabla e_t + \pi_u \nabla u_t.$$

Under these circumstances, the effect of the announcement on the exchange rate is captured by the second term in:

$$s_t^3 - s_t^1 = \psi \xi_t + \left(\pi_e \nabla e_t + \pi_u \nabla u_t\right).$$

Period-1 announcements will also affect the exchange rate because:

$$\begin{aligned}\mathbb{E}\left[\nabla k_t|\Omega_t^1, r_t^k, \hat{r}_t^k\right] - \mathbb{E}\left[\nabla k_t|\Omega_t^1\right] &= \nabla k_t - \mathbb{E}\left[\nabla r_t^k + \nabla k_{t-1} - \left(\tfrac{\mu}{1-\mu}\right)\left(s_{t-1}^3 - \nabla k_{t-1}\right)|\Omega_t^1\right] \\ &= \nabla e_t + \nabla u_t.\end{aligned}$$

A.2 Market Clearing Conditions

Market clearing in US deposits in period 1 of day $t+1$ implies that (see equations 2, 4, and 21):

$$(B_{t,\mathbb{US}}^3 + S_t^3 T_{t,z*}^4 - S_t^3 T_{t,\mathbb{US}}^4 + C_{t,\mathbb{UK}} - I_{t,\mathbb{US}}) + (B_{t,\mathbb{UK}}^3 + S_t^3 T_{t,z*}^4 - S_t^3 T_{t,\mathbb{UK}}^4 - C_{t,\mathbb{UK}}) = 0.$$

With deposit-market clearing in period 3, this condition further simplifies to:

$$S_t^3 T_{t,z*}^4 - S_t^3 T_{t,\mathbb{US}}^4 + S_t^3 T_{t,z*}^4 - S_t^3 T_{t,\mathbb{UK}}^4 - I_{t,\mathbb{US}} = 0.$$

Since market clearing in currency markets implies that $\int T^j_{t,z}dz = \int T^j_{t,z^*}dz^*$, this condition implies that $I_{t,\mathbb{US}} = 0$. Imposing this restriction on the overnight dynamics of US capital gives (22). Similarly, market clearing in the UK deposit markets implies that:

$$\begin{aligned} 0 &= (\hat{B}^1_{t,\mathbb{US}} + T^4_{t,\mathbb{US}} - T^4_{t,z*} - \hat{C}_{t,\mathbb{US}}) + (\hat{B}^1_{t,\mathbb{UK}} + T^4_{t,\mathbb{UK}} - T^4_{t,z*} + \hat{C}_{t,\mathbb{US}} - \hat{I}_{t,\mathbb{US}}) \\ &= T^4_{t,\mathbb{US}} - T^4_{t,z*} + T^4_{t,\mathbb{UK}} - T^4_{t,z*} - \hat{I}_{t,\mathbb{US}} \\ &= -\hat{I}_{t,\mathbb{US}}. \end{aligned}$$

Imposing $\hat{I}_{t,\mathbb{UK}} = 0$ on the overnight dynamics of UK capital gives (23).

A.3 Solving the Modified Model

To solve the modified model, we first combine (24) and (25) to write fundamentals as a function of past capital returns and spot rates. Assuming that capital is equally distributed between the US and UK in month $t = 0$, this gives:

$$\nabla k_t = \sum_{i=0}^{t-1} \left(\tfrac{1}{1-\mu}\right)^i \left(\nabla r^k_{t-i} - \tfrac{\mu}{1-\mu} s_{t-1-i}\right).$$

Lagging this equation by four periods, and taking expectations conditional on $\tilde{\Omega}^3_t$ establishes that $\nabla k_{t-4} = \mathbb{E}\left[\nabla k_{t-4}|\tilde{\Omega}^3_t\right]$. Thus, the announced value for ∇r^k_{t-4} at t:1 combined with the information in $\tilde{\Omega}^3_{t-1}$ reveals the true value of ∇k_{t-4}. We incorporate this informational implication of the modified model by substituting:

$$\nabla \tilde{k}_t = \nabla k_{t-4} \tag{A61}$$

for (40). $\nabla \tilde{k}_t$ is value for fundamentals in $t-4$ implied by the history of announcements at t:1 (i.e., based on information $\left\{\nabla \tilde{r}^k_t \cup \tilde{\Omega}^3_{t-1}\right\}$).

Next, we write the estimation error for fundamentals $\nabla k^{err}_t \equiv \nabla k_t - \mathbb{E}[\nabla k_t|\tilde{\Omega}^3_t]$ as a linear function of variables that characterize the true state at month t but are not elements in Ω^3_t. In particular, we posit that:

$$\begin{aligned} \nabla k^{err}_t &= \beta_0 \nabla k^{err}_{t-1} + \beta_1 \nabla e_t + \beta_2 \nabla u_t + \beta_3 \varepsilon^e_t + \beta_4 \varepsilon^u_t \\ &\quad + \beta_5 \nabla e_{t-1} + \beta_6 \nabla u_{t-1} + \beta_7 \varepsilon^e_{t-1} + \beta_8 \varepsilon^u_{t-1} \\ &\quad + \beta_9 \Delta \nabla k_{t-1} + \beta_{10} \Delta \nabla k_{t-2} + \beta_{11} \Delta \nabla k_{t-3}. \end{aligned} \tag{A62}$$

for some undetermined coefficients β_i. Note that (A62) imposes the restriction that fundamentals errors follow a stationary process.

We can now write the dynamics of the modified model (i.e. equations (38), (39), (41), (42), (A61) and

(A62)) in state space form:

$$X_{t+1} = AX_t + BU_t \tag{A63}$$
$$Y_t = CX_t \tag{A64}$$

where $\mathbb{V}(U_t) = \mathbb{W}$ and:

$$\begin{aligned} X'_t &\equiv \left[\begin{array}{cccccccccc} \nabla k^{err}_t & \nabla k_t & \nabla k_{t-1} & \nabla k_{t-2} & \nabla k_{t-3} & \nabla k_{t-4} & \nabla e_t & \nabla u_t & \varepsilon^e_t & \varepsilon^u_t \end{array} \right], \\ U'_t &\equiv \left[\begin{array}{cccc} \nabla e_t & \nabla u_t & \varepsilon^e_t & \varepsilon^u_t \end{array} \right], \\ Y'_t &= \left[\begin{array}{ccc} \nabla \tilde{e}_t & \nabla \tilde{u}_t & \nabla \tilde{k}_t \end{array} \right]. \end{aligned}$$

X_t is the (unobserved) state vector for model and Y_t identifies the vector of signals observed at t:1. Notice also that the first rows of the A and B matrices contain the unknown coefficients β_i. Hence (A63) and (A64) summarize the dynamics of the modified model given a conjecture about the estimation error for fundamentals.

To find the equilibrium values for the β_i coefficients, we note that $\mathbb{CV}\left[\nabla k^{err}_t, z_t|\tilde{\Omega}^3_t\right] = \mathbb{CV}\left[\nabla k_t, z_t|\tilde{\Omega}^3_t\right]$, for any variable z_t. We use this moment restriction to find the values for the β_i as follows: First, we apply the Kalman filtering algorithm to (A63) and (A64) for a given set of β_i coefficients. This yields the following recursion:

$$\mathbb{S}_{t+1} = A(I - \mathbb{K}_t C)\mathbb{S}_t A' + B\mathbb{W}B',$$

where $\mathbb{S}_{t+1} = \mathbb{V}\left[X_{t+1}|\tilde{\Omega}^3_t\right]$ and $\mathbb{K}_t = \mathbb{S}_t C'(C\mathbb{S}_t C')^{-1}$. Starting with $\mathbb{S}_{t=0} = I_{10}$, we iterate on this recursion until there is no change in the gain matrix, say at $t = \tau$. (The form of the C matrix in our model insures that $\mathbb{K}_t$ always converges to a constant matrix $\mathbb{K}$) At this point we compare the first two columns of $\mathbb{S}_{\tau+1} = A(I - \mathbb{K}C)\mathbb{S}_\tau A' + B\mathbb{W}B'$. If all the entries match, then $\mathbb{CV}\left[\nabla k^{err}_t, z_t|\tilde{\Omega}^3_t\right] = \mathbb{CV}\left[\nabla k_t, z_t|\tilde{\Omega}^3_t\right]$ for z_t equal to each element of X_t and we have a solution to the model. In practice we find the equilibrium values for the β_i by minimizing a quadratic form in the difference between the first two columns of $\mathbb{S}_{\tau+1}$. Once these values are found, we can identify spot rates as $s_t = \mathbb{E}[\nabla k_t|\tilde{\Omega}^3_t] \equiv \nabla k_t - \nabla k^{err}_t$.