# Crash-Neutral Currency Carry Trades

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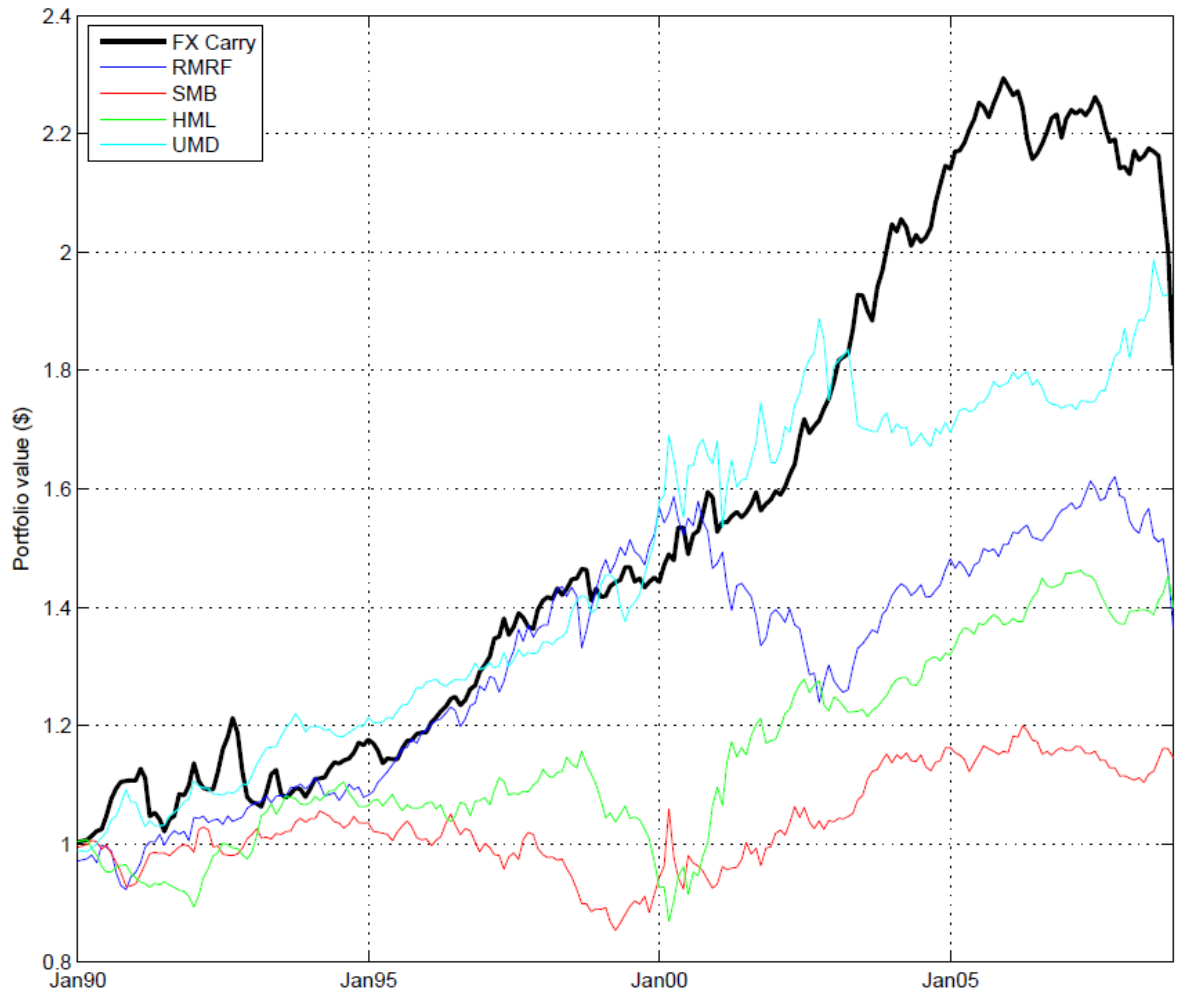
# Currency Carry Trade

Currency carry trades exploit violations of uncovered interest parity (UIP) by buying (selling) currencies with relatively high (low) interest rates.



# Currency Carry Trade

Historical returns:

- Before (USD/G10; monthly, 1990:1-2007:03)

| | RMRF | SMB | HML | UMD | FX Carry |
|---|---|---|---|---|---|
| Mean | 0.0730 | 0.0227 | 0.0477 | 0.0985 | 0.0478 |
| t-stat | 2.13 | 0.75 | 1.72 | 2.51 | 3.91 |
| St. dev. | 0.1422 | 0.1261 | 0.1153 | 0.1630 | 0.0507 |
| Skewness | -0.68 | 0.81 | 0.11 | -0.66 | -0.95 |
| SR | 0.51 | 0.18 | 0.41 | 0.60 | 0.94 |

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| St. dev. | 0.1422 | 0.1261 | 0.1153 | 0.1630 | 0.0507 |
| Skewness | -0.68 | 0.81 | 0.11 | -0.66 | -0.95 |
| SR | 0.51 | 0.18 | 0.41 | 0.60 | 0.94 |

- After (USD/G10; monthly, 1990:1-2008:10)

| | RMRF | SMB | HML | UMD | FX Carry |
|---|---|---|---|---|---|
| Mean | 0.0477 | 0.0191 | 0.0392 | 0.1060 | 0.0331 |
| t-stat | 1.39 | 0.68 | 1.50 | 2.83 | 2.55 |
| St. dev. | 0.1485 | 0.1223 | 0.1136 | 0.1628 | 0.0563 |
| Skewness | -0.84 | 0.83 | 0.11 | -0.60 | -1.63 |
| SR | 0.32 | 0.16 | 0.35 | 0.65 | 0.59 |

# Paper Summary

This paper tests the hypothesis that violations of UIP are attributable to crash risk premia by examining data on foreign exchange options:

1. Dynamics of risk-neutral skewness:
   - Negatively related to contemporaneous interest rate differential in the cross-section; weak time-series relation.
   - Does not forecast currency excess returns.
   - Moves opposite to future realized skewness in response to realized currency moves.

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   - Does not forecast currency excess returns.
   - Moves opposite to future realized skewness in response to realized currency moves.

2. Crash-neutral currency carry trades:
   - Returns smaller than for standard carry trades, but positive and statistically significant.
   - Results robust to inclusion of transaction costs.
   - Crash-based explanation would requires option implied volatilities that are 2-4x the actual values observed in the data.

# Uncovered Interest Parity

- *Uncovered interest parity (UIP)* predicts that high interest currencies should depreciate relative to low interest rate currencies, such that investors are indifferent between holding deposits in the two.
  - $S_t$ – spot exchange rate (price of foreign currency in USD)
  - $F_{t,\tau}$ – forward exchange rate

$$F_{t,\tau} = S_t \cdot \exp\left((r_{d,t} - r_{f,t}) \cdot \tau\right)$$

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$$F_{t,\tau} = S_t \cdot \exp\left((r_{d,t} - r_{f,t}) \cdot \tau\right)$$

- Following Hansen and Hodrick (1989), UIP is typically tested by running a regression of the log currency return on the log forward spread:

$$\begin{aligned} s_{t+1} - s_t &= a_0 + a_1 \cdot (f_t - s_t) + \varepsilon_{t+1} \\ &= a_0 + a_1 \cdot (r_{d,t} - r_{f,t}) \cdot \tau + \varepsilon_{t+1} \end{aligned}$$

  - Null hypothesis ($H_0$): $a_0 = 0$ and $a_1 = 1$.

# Uncovered Interest Parity

## Testing UIP in the panel of G10 currencies (Table I)

The intercept of the UIP regression is negative for 6 of 9 countries in the full sample (1990-2007) and in all countries during the 1999-2007 sample.

| | 1990-2007 | | | | 1999-2007 | | | |
|---|---|---|---|---|---|---|---|---|
| Currency | $\hat{a}_0$ | $\hat{a}_1$ | $R^2_{NFE}$ | $\chi^2$ test | $\hat{a}_0$ | $\hat{a}_1$ | $R^2_{NFE}$ | $\chi^2$ test |
| AUD | -0.0025 | -1.7483 | 0.0105 | 8.87 | -0.0028 | -3.9018 | 0.0310 | 9.34 |
| | (0.0023) | (1.0522) | | (0.01) | (0.0036) | (1.9520) | | (0.01) |
| CAD | -0.0001 | -0.5077 | 0.0019 | 9.13 | 0.0027 | -2.5012 | 0.0115 | 5.06 |
| | (0.0009) | (0.5104) | | (0.01) | (0.0015) | (2.1091) | | (0.08) |
| CHF | 0.0026 | -1.2815 | 0.0069 | 5.60 | 0.0096 | -4.5238 | 0.0350 | 9.03 |
| | (0.0024) | (1.0008) | | (0.06) | (0.0041) | (1.8485) | | (0.01) |
| EUR | 0.0002 | -0.0320 | -0.0002 | 1.34 | 0.0036 | -4.4836 | 0.0447 | 11.19 |
| | (0.0016) | (0.9072) | | (0.51) | (0.0024) | (1.6590) | | (0.00) |
| GBP | 0.0021 | 0.7061 | 0.0020 | 3.65 | 0.0001 | -1.7371 | 0.0061 | 4.55 |
| | (0.0019) | (1.2755) | | (0.16) | (0.0025) | (1.7738) | | (0.10) |
| JPY | 0.0058 | -2.0823 | 0.0165 | 12.33 | 0.0048 | -1.8183 | 0.0099 | 6.20 |
| | (0.0025) | (0.8787) | | (0.00) | (0.0045) | (1.4003) | | (0.05) |
| NOK | 0.0013 | 0.6255 | 0.0042 | 1.43 | 0.0007 | -1.4005 | 0.0090 | 5.46 |
| | (0.0017) | (0.6351) | | (0.49) | (0.0026) | (1.2175) | | (0.07) |
| NZD | -0.0047 | -2.4128 | 0.0147 | 15.46 | -0.0067 | -4.7728 | 0.0482 | 17.15 |
| | (0.0034) | (1.1975) | | (0.00) | (0.0045) | (1.6837) | | (0.00) |
| SEK | 0.0004 | 0.6081 | 0.0046 | 0.51 | 0.0026 | -3.5247 | 0.0405 | 11.09 |
| | (0.0017) | (0.6046) | | (0.77) | (0.0024) | (1.3764) | | (0.00) |
| Pooled | FE | -0.1795 | 0.0002 | 2.59 | FE | -3.0503 | 0.0248 | 24.17 |
| | | (0.6589) | | (0.99) | | (1.1190) | | (0.01) |
| XS | 0.0005 | -0.1883 | 0.1070 | - | 0.0012 | -0.5994 | 0.0966 | - |
| | (0.0003) | (0.0836) | | | (0.0005) | (0.1087) | | - |

# Uncovered Interest Parity

## Testing UIP in the panel of G10 currencies (Table I)

At the 10% significance level, UIP is rejected in 5 of 9 countries in the full sample (1990-2007) and in all countries during the 1999-2007 sample.

| | 1990-2007 | | | | 1999-2007 | | | |
|---|---|---|---|---|---|---|---|---|
| Currency | $\hat{a}_0$ | $\hat{a}_1$ | $R^2_{NFE}$ | $\chi^2$ test | $\hat{a}_0$ | $\hat{a}_1$ | $R^2_{NFE}$ | $\chi^2$ test |
| AUD | -0.0025 | -1.7483 | 0.0105 | 8.87 | -0.0028 | -3.9018 | 0.0310 | 9.34 |
| | (0.0023) | (1.0522) | | (0.01) | (0.0036) | (1.9520) | | (0.01) |
| CAD | -0.0001 | -0.5077 | 0.0019 | 9.13 | 0.0027 | -2.5012 | 0.0115 | 5.06 |
| | (0.0009) | (0.5104) | | (0.01) | (0.0015) | (2.1091) | | (0.08) |
| CHF | 0.0026 | -1.2815 | 0.0069 | 5.60 | 0.0096 | -4.5238 | 0.0350 | 9.03 |
| | (0.0024) | (1.0008) | | (0.06) | (0.0041) | (1.8485) | | (0.01) |
| EUR | 0.0002 | -0.0320 | -0.0002 | 1.34 | 0.0036 | -4.4836 | 0.0447 | 11.19 |
| | (0.0016) | (0.9072) | | (0.51) | (0.0024) | (1.6590) | | (0.00) |
| GBP | 0.0021 | 0.7061 | 0.0020 | 3.65 | 0.0001 | -1.7371 | 0.0061 | 4.55 |
| | (0.0019) | (1.2755) | | (0.16) | (0.0025) | (1.7738) | | (0.10) |
| JPY | 0.0058 | -2.0823 | 0.0165 | 12.33 | 0.0048 | -1.8183 | 0.0099 | 6.20 |
| | (0.0025) | (0.8787) | | (0.00) | (0.0045) | (1.4003) | | (0.05) |
| NOK | 0.0013 | 0.6255 | 0.0042 | 1.43 | 0.0007 | -1.4005 | 0.0090 | 5.46 |
| | (0.0017) | (0.6351) | | (0.49) | (0.0026) | (1.2175) | | (0.07) |
| NZD | -0.0047 | -2.4128 | 0.0147 | 15.46 | -0.0067 | -4.7728 | 0.0482 | 17.15 |
| | (0.0034) | (1.1975) | | (0.00) | (0.0045) | (1.6837) | | (0.00) |
| SEK | 0.0004 | 0.6081 | 0.0046 | 0.51 | 0.0026 | -3.5247 | 0.0405 | 11.09 |
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| | | (0.6589) | | (0.99) | | (1.1190) | | (0.01) |
| XS | 0.0005 | -0.1883 | 0.1070 | - | 0.0012 | -0.5994 | 0.0966 | - |
| | (0.0003) | (0.0836) | | | (0.0005) | (0.1087) | | - |

# Currency Carry Trade

The *currency carry trade* exploits deviations from UIP by borrowing funds in currencies with low interest rates and investing them in currencies with high interest rates.

- Investor constructs carry trades in X/USD currency pairs (X $\in$ G10)
- Funds are borrowed/invested at the relevant one-month LIBOR rates.
- Positions are held for one month.
- Payoffs:

$$\widetilde{CT}_{t+1} = \begin{cases} r_{f,t} > r_{d,t}: & \exp\left(r_{f,t} \cdot \tau\right) \cdot \tilde{S}_{t+\tau} - \exp\left(r_{d,t} \cdot \tau\right) \cdot S_t \\ r_{d,t} > r_{f,t}: & \exp\left(r_{d,t} \cdot \tau\right) \cdot S_t - \exp\left(r_{f,t} \cdot \tau\right) \cdot \tilde{S}_{t+\tau} \end{cases}$$

# Currency Carry Trade

## Portfolio strategies

Consider the following portfolio formation rules:

1. Equal-weighted (EQL)
2. Spread-weighted (SPR)
3. Equal-weighted dollar-neutral (EQL-$N)
4. Spread-weighted dollar-neutral (SPR-$N)

**Portfolio USD Exposure**

| Currency | $r_{f,t} - r_{d,t}$ | EQL | SPR |
|---|---|---|---|
| AUD | 1.88% | -0.11 | -0.14 |
| CAD | 0.05% | -0.11 | -0.00 |
| GBP | 1.28% | -0.11 | -0.09 |
| NOK | 1.29% | -0.11 | -0.09 |
| NZD | 2.78% | -0.11 | -0.20 |
| CHF | -2.11% | 0.11 | 0.15 |
| EUR | -0.42% | 0.11 | 0.03 |
| JPY | -3.45% | 0.11 | 0.25 |
| SEK | -0.32% | 0.11 | 0.02 |
| Net USD | - | **-0.11** | **-0.07** |

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| AUD | 1.88% | -0.11 | -0.14 | -0.20 | -0.26 |
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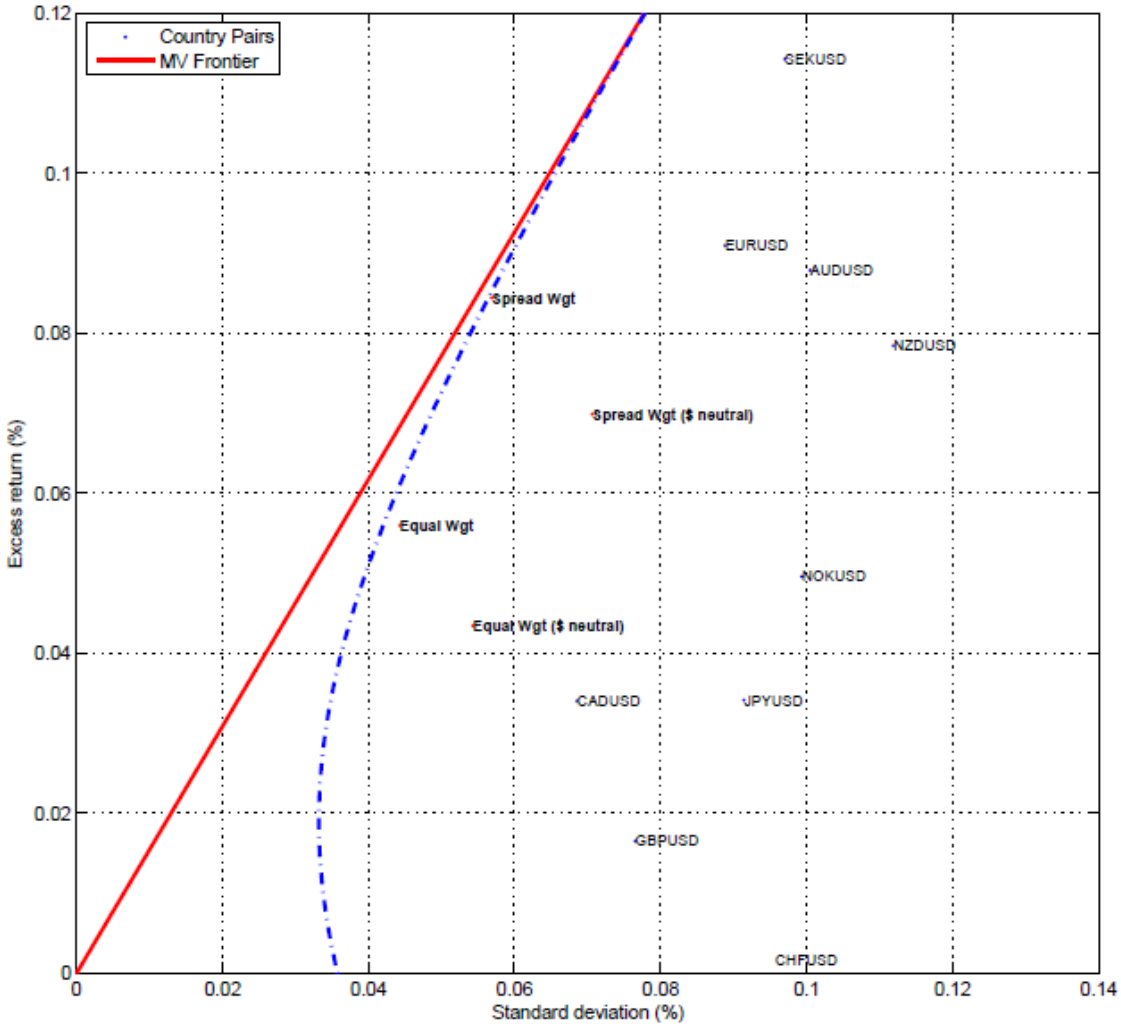
| GBP | 1.28% | -0.11 | -0.09 | -0.20 | -0.18 |
| NOK | 1.29% | -0.11 | -0.09 | -0.20 | -0.18 |
| NZD | 2.78% | -0.11 | -0.20 | -0.20 | -0.38 |
| CHF | -2.11% | 0.11 | 0.15 | 0.25 | 0.33 |
| EUR | -0.42% | 0.11 | 0.03 | 0.25 | 0.07 |
| JPY | -3.45% | 0.11 | 0.25 | 0.25 | 0.55 |
| SEK | -0.32% | 0.11 | 0.02 | 0.25 | 0.05 |
| Net USD | - | **-0.11** | **-0.07** | **0.00** | **0.00** |

# Currency Carry Trade

## Historical performance (Figure 1)

Simple portfolio construction rules (e.g. equal- and spread-weighting) were close to being *ex post* efficient.



# Currency Carry Trade

## Historical performance – portfolio strategies (Table II)

| 1999:1-2007:3 (N = 99) | | | | |
|---|---|---|---|---|
| | EQL | SPR | EQL-$N | SPR-$N |
| Mean | 0.0560 | 0.0844 | 0.0434 | 0.0699 |
| t-stat | 3.63 | 4.26 | 2.30 | 2.84 |
| Std. dev. | 0.0443 | 0.0569 | 0.0543 | 0.0707 |
| Skewness | -0.42 | -0.20 | -0.52 | -0.23 |
| Kurtosis | 3.73 | 2.97 | 4.31 | 4.33 |
| Min | -0.0368 | -0.0335 | -0.0569 | -0.0661 |
| Max | 0.0375 | 0.0455 | 0.0369 | 0.0651 |
| Carry | 0.0200 | 0.0307 | 0.0345 | 0.0478 |
| SR | 1.26 | 1.48 | 0.80 | 0.99 |

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| Max | 0.0375 | 0.0455 | 0.0369 | 0.0651 |
| Carry | 0.0200 | 0.0307 | 0.0345 | 0.0478 |
| SR | 1.26 | 1.48 | 0.80 | 0.99 |

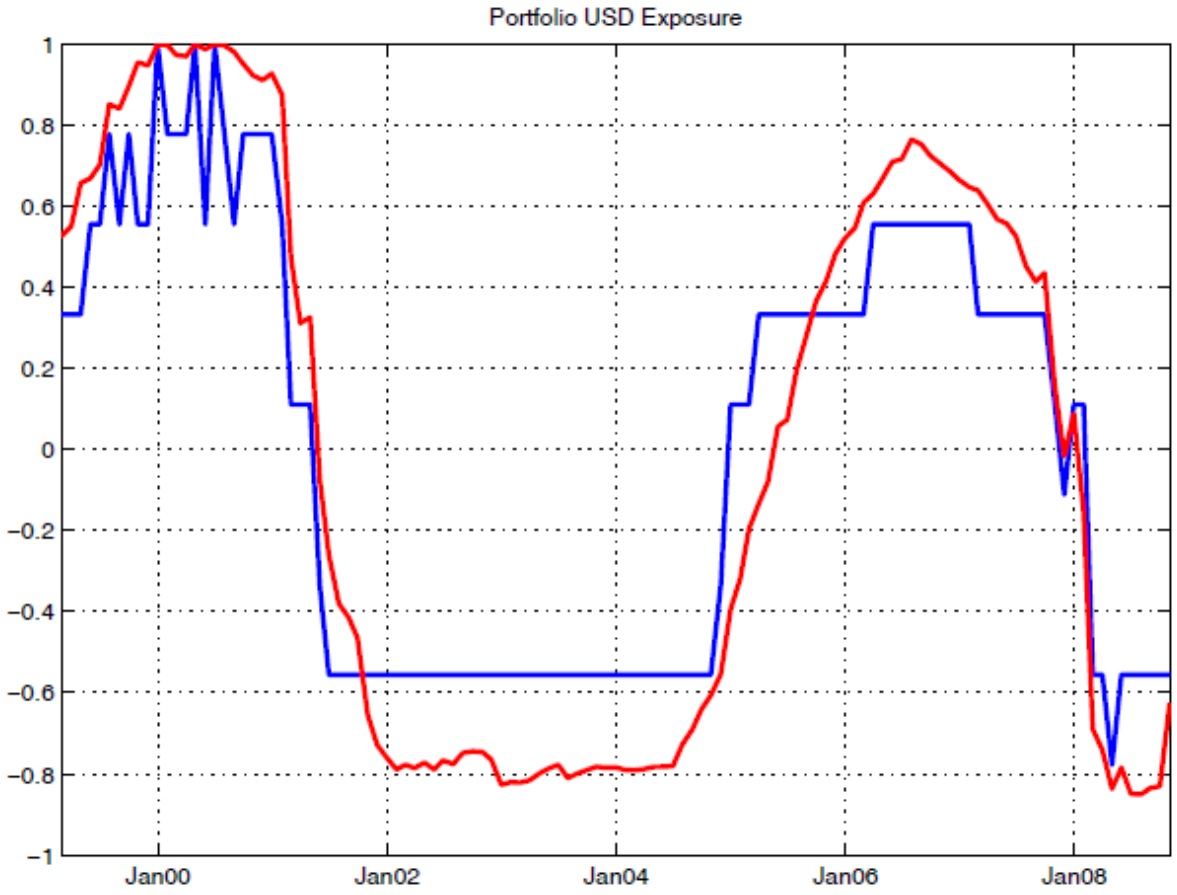
**1999:1-2008:10 (N = 117)**

| | EQL | SPR | EQL-$N | SPR-$N |
|---|---|---|---|---|
| Mean | 0.0266 | 0.0482 | 0.0119 | 0.0291 |
| t-stat | 1.46 | 2.10 | 0.48 | 0.90 |
| Std. dev. | 0.0568 | 0.0717 | 0.0778 | 0.1014 |
| Skewness | -2.08 | -1.68 | -2.29 | -2.06 |
| Kurtosis | 12.95 | 10.00 | 14.11 | 12.66 |
| Min | -0.0958 | -0.1098 | -0.1360 | -0.0651 |
| Max | 0.0375 | 0.0455 | 0.0369 | 0.0651 |
| Carry | 0.0202 | 0.0310 | 0.0352 | 0.0491 |
| SR | 0.47 | 0.67 | 0.15 | 0.29 |

# Currency Carry Trade

## Net USD Exposure (1999:1-2008:10)

The net exposure to USD is driven primarily by aggressive U.S. monetary policy.



# Currency Carry Trade

## Historical performance decomposition

- Equal-weighted (EQL)

| | 1999:1-2007:03 | | | 2007:4-2008:10 | | |
|---|---|---|---|---|---|---|
| | Total | Long | Short | Total | Long | Short |
| Mean | 0.0560 | 0.0136 | 0.0424 | -0.1349 | -0.0428 | -0.0921 |
| t-stat | 3.88 | 0.97 | 3.10 | -1.85 | -1.66 | -1.20 |
| Std. dev. | 0.0443 | 0.0433 | 0.0419 | 0.0894 | 0.0316 | 0.0942 |
| Skewness | -0.43 | -0.33 | 0.83 | -2.29 | -1.54 | -1.96 |

# Currency Carry Trade

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| Skewness | -0.43 | -0.33 | 0.83 | -2.29 | -1.54 | -1.96 |

- Equal-weighted dollar-neutral (EQL-$N)

| | 1999:1-2007:03 | | | 2007:4-2008:10 | | |
|---|---|---|---|---|---|---|
| | Total | Long | Short | Total | Long | Short |
| Mean | 0.0435 | -0.0013 | 0.0448 | -0.1633 | -0.0656 | -0.0977 |
| t-stat | 2.46 | -0.05 | 1.73 | -1.41 | -0.94 | -0.86 |
| Std. dev. | 0.0543 | 0.0746 | 0.0793 | 0.1417 | 0.0855 | 0.1399 |
| Skewness | -0.53 | -0.46 | -0.01 | -1.56 | 0.24 | -1.09 |

Recent research on currencies borrows from the equity literature and focuses on the role of *crash risk*:

- Equities: Rietz (1988), Barro (2006), Weitzman (2007)
- Equity options: Coval and Shumway (2001), Pan (2002), Bakshi and Kapadia (2003), and Driessen and Maenhout (2006)
- Currencies: Brunnermeier, Nagel and Pedersen (2008), Farhi and Gabaix (2008), Plantin and Shin (2008)

Ongoing debate regarding whether classical asset pricing models can rationalize excess returns to the currency carry trade.

- No: Burnside (2007), Burnside, et al. (2008)
- Yes: Verdelhan and Lustig (2006, 2007), Verdelhan (2008), Lustig, Roussanov and Verdelhan (2008)

# Foreign Exchange Options

## Data

Data:

- European OTC options on X/USD exchange rates (source: J. P. Morgan)
- Cross section: X = AUD, CAD, CHF, GBP, EUR, JPY, NOK, NZD, SEK
- Time series: 1999:1 - 2008:10
- Strikes: $10\delta p$, $25\delta p$, ATM, $25\delta c$, $10\delta c$
- Tenors: 1M, 3M, 6M, 1Y
- Spot exchange rates (source: Datastream / Reuters)
- LIBOR rates (source: Datastream / Reuters)

# Foreign Exchange Options

## Data

Option prices are quoted in term of the Garman-Kohlhagen (1983) implied volatilities:

$$\begin{aligned} C_t(K,\tau) &= e^{-r_{d,t}\cdot\tau}\cdot\Big[F_{t,\tau}\cdot N(d_1) - K\cdot N(d_2)\Big] \\ P_t(K,\tau) &= e^{-r_{d,t}\cdot\tau}\cdot\Big[K\cdot N(-d_2) - F_{t,\tau}\cdot N(-d_1)\Big] \end{aligned}$$

at strike prices determined by the fixed option $\delta$ values:

$$\begin{aligned} K_{\delta_c} &= F_t\cdot\exp\left(\frac{1}{2}\sigma_t(\delta_c)^2\cdot\tau - \sigma_t(\delta_c)\cdot\sqrt{\tau}\cdot N^{-1}\left[\exp(r_{f,t}\cdot\tau)\cdot\delta_c\right]\right) \\ K_{\delta_p} &= F_t\cdot\exp\left(\frac{1}{2}\sigma_t(\delta_p)^2\cdot\tau + \sigma_t(\delta_p)\cdot\sqrt{\tau}\cdot N^{-1}\left[-\exp(r_{f,t}\cdot\tau)\cdot\delta_p\right]\right) \end{aligned}$$

# Foreign Exchange Options

## Summary statistics (Table III)

**Panel A: LIBOR and Implied Volatilities**

| Currency | $r_{f,t}$ | $10\delta$p | $25\delta$p | ATM | $25\delta$c | $10\delta$c |
|---|---|---|---|---|---|---|
| AUD | 0.0556 | 0.1239 | 0.1157 | 0.1103 | 0.1102 | 0.1138 |
| CAD | 0.0373 | 0.0849 | 0.0808 | 0.0785 | 0.0800 | 0.0833 |
| CHF | 0.0157 | 0.1083 | 0.1042 | 0.1038 | 0.1078 | 0.1145 |
| EUR | 0.0326 | 0.1047 | 0.1001 | 0.0987 | 0.1018 | 0.1077 |
| GBP | 0.0496 | 0.0909 | 0.0857 | 0.0831 | 0.0845 | 0.0887 |
| JPY | 0.0023 | 0.1048 | 0.1013 | 0.1041 | 0.1132 | 0.1264 |
| NOK | 0.0497 | 0.1146 | 0.1100 | 0.1086 | 0.1117 | 0.1177 |
| NZD | 0.0646 | 0.1385 | 0.1294 | 0.1234 | 0.1231 | 0.1269 |
| SEK | 0.0336 | 0.1146 | 0.1099 | 0.1086 | 0.1116 | 0.1175 |
| USD | 0.0368 | - | - | - | - | - |

# Foreign Exchange Options

## Summary statistics (Table III)

**Panel B: FX Option Strike Values**

| | Moneyness $\left(\frac{K_\delta}{F_t}\right)$ | | | | |
|---|---|---|---|---|---|
| Currency | $10\delta$p | $25\delta$p | ATM | $25\delta$c | $10\delta$c |
| AUD | 0.9561 | 0.9785 | 1.0006 | 1.0222 | 1.0436 |
| CAD | 0.9695 | 0.9847 | 1.0003 | 1.0159 | 1.0316 |
| CHK | 0.9613 | 0.9804 | 1.0005 | 1.0217 | 1.0439 |
| EUR | 0.9626 | 0.9812 | 1.0004 | 1.0204 | 1.0412 |
| GBP | 0.9674 | 0.9839 | 1.0003 | 1.0168 | 1.0337 |
| JPY | 0.9625 | 0.9809 | 1.0005 | 1.0229 | 1.0488 |
| NOK | 0.9592 | 0.9794 | 1.0005 | 1.0224 | 1.0451 |
| NZD | 0.9511 | 0.9760 | 1.0007 | 1.0248 | 1.0487 |
| SEK | 0.9591 | 0.9794 | 1.0005 | 1.0225 | 1.0450 |

- The *moneyness* of constant-delta options changes with the implied volatility to keep the probability of expiring in-the-money constant.

The implied volatility functions are interpolated using the **vanna-volga method** (Castagna and Mercurio (2007)).

- Static hedging argument matching partial derivatives up to second order:
  1. *vega* $\left(\frac{\partial C^{BS}}{\partial \sigma}\right)$,
  2. *volga* $\left(\frac{\partial^2 C^{BS}}{\partial^2 \sigma}\right)$,
  3. *vanna* $\left(\frac{\partial^2 C^{BS}}{\partial \sigma \partial S_t}\right)$.
- The interpolated volatilities are approximately equal to:

$$\tilde{\sigma}_t(K,\tau) \approx \frac{\ln\frac{K_2}{K}\cdot\ln\frac{K_3}{K}}{\ln\frac{K_2}{K_1}\cdot\ln\frac{K_3}{K_1}}\cdot\sigma_t(K_1,\tau) + \frac{\ln\frac{K}{K_1}\cdot\ln\frac{K_3}{K}}{\ln\frac{K_2}{K_1}\cdot\ln\frac{K_3}{K_2}}\cdot\sigma_t(K_2,\tau) + \frac{\ln\frac{K}{K_1}\cdot\ln\frac{K}{K_2}}{\ln\frac{K_3}{K_1}\cdot\ln\frac{K_3}{K_2}}\cdot\sigma_t(K_3,\tau)$$

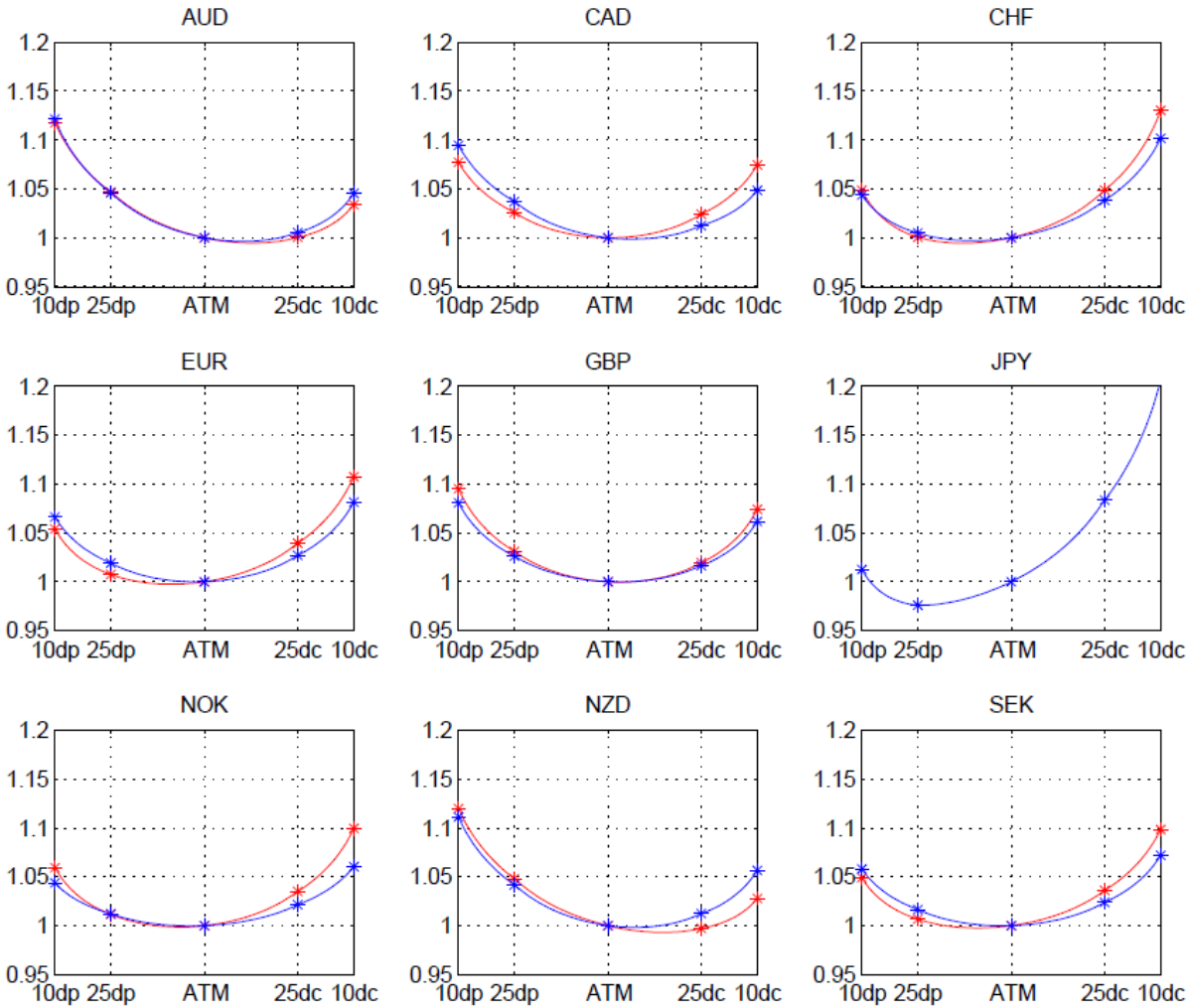
- Implied volatilities are extrapolated by appending flat tails

# Foreign Exchange Options

## Implied volatility skew

Time-series means of implied volatilities by strike and interest rate regime:

- Blue – $r_{f,t} < r_{d,t}$; Red – $r_{f,t} > r_{d,t}$



# Foreign Exchange Options

## Extracting risk-neutral moments

Options contain forward looking information about the probability of currency crashes helping address potential *peso problems*:

- The dynamics of the risk-neutral distribution can be summarized using the time-series of its risk-neutral moments (variance, skewness, kurtosis).

# Foreign Exchange Options

## Extracting risk-neutral moments

Options contain forward looking information about the probability of currency crashes helping address potential *peso problems*:

- The dynamics of the risk-neutral distribution can be summarized using the time-series of its risk-neutral moments (variance, skewness, kurtosis).
- Risk-neutral moments can be computed from (Arrow (1964), Debreu (1959), Breeden and Litzenberger (1978)):

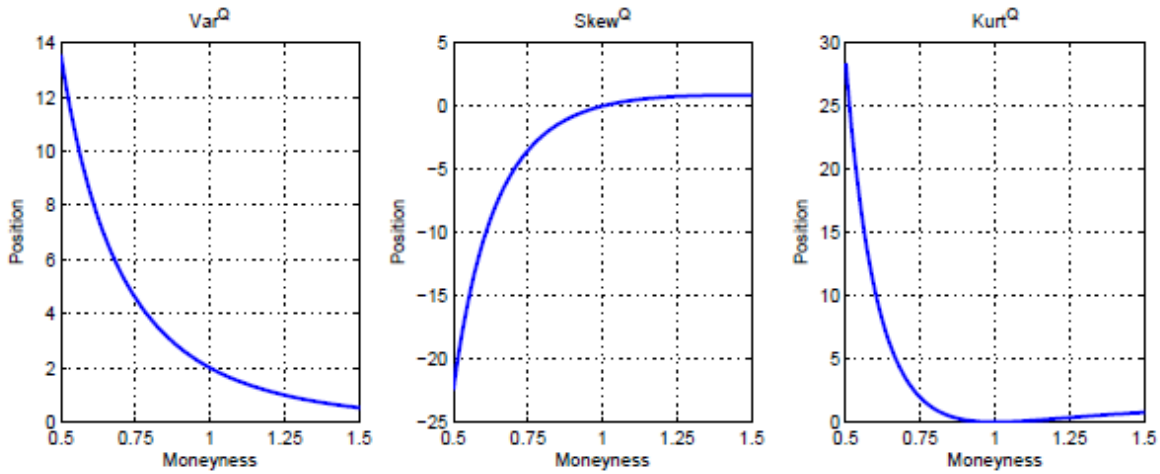
$$\begin{aligned} p_t &= \exp(-r_{d,t} \cdot \tau) \cdot \int_0^{\infty} H(S_{t+\tau}) \cdot q(S_{t+\tau}) dS_{t+\tau} \\ H(S_{t+\tau}) &= \left( \ln \frac{S_{t+\tau}}{S_t} \right)^n \end{aligned}$$

# Foreign Exchange Options

## Extracting risk-neutral moments

Any payoff function $H(S_{t+\tau}) \in \mathcal{C}^2$ with bounded expectation can be spanned using a continuum of OTM puts and calls (Bakshi and Madan (2000)):

$$\begin{aligned} p_t &= \exp(-r_{d,t} \cdot \tau) \cdot \left( H(\overline{S}) - \overline{S} \cdot H_S(\overline{S}) \right) + H_S(\overline{S}) \cdot S_t + \\ &\quad + \int_{\overline{S}}^{\infty} H_{SS}(K) \cdot C_t(K, \tau) \cdot dK + \int_0^{\overline{S}} H_{SS}(K) \cdot P_t(K, \tau) \cdot dK \end{aligned}$$

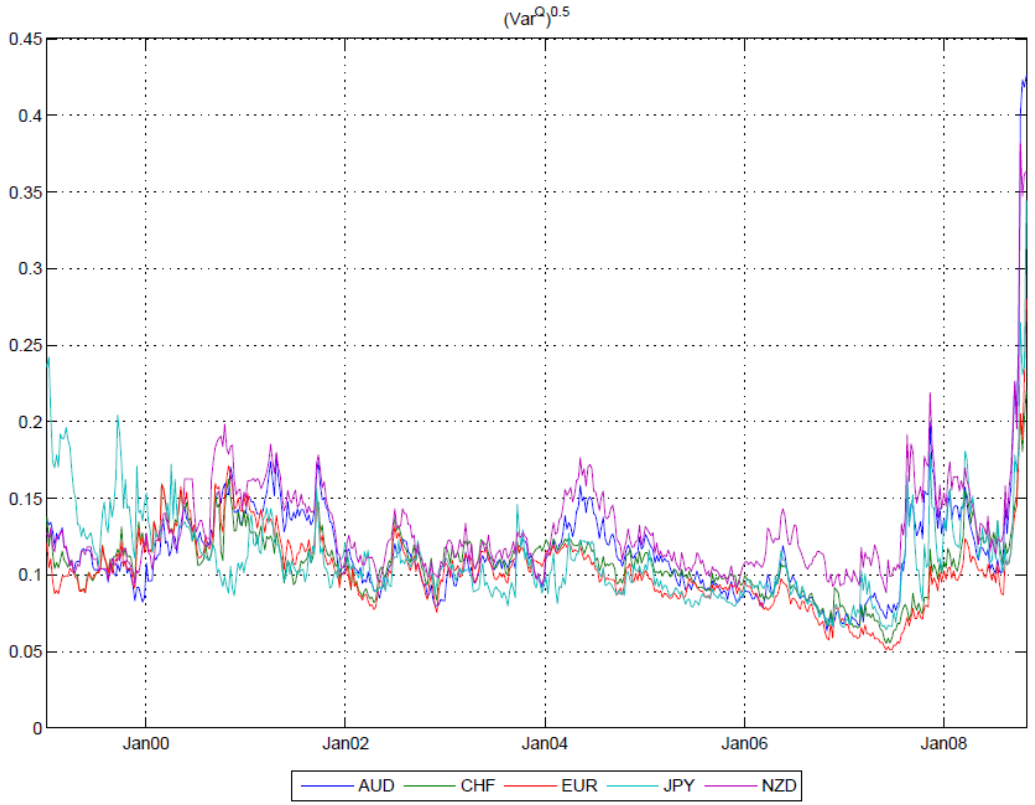


# Risk-Neutral Moments

## Volatility (Figure 6)

**Time series means and standard errors**

| | AUD | CAD | CHF | EUR | GBP | JPY | NOK | NZD | SEK |
|---|---|---|---|---|---|---|---|---|---|
| $\sqrt{\text{Var}^Q}$ | 0.1162 | 0.0824 | 0.1075 | 0.1027 | 0.0874 | 0.1113 | 0.1127 | 0.1297 | 0.1127 |
| | (0.0038) | (0.0024) | (0.0020) | (0.0026) | (0.0023) | (0.0031) | (0.0025) | (0.0032) | (0.0025) |
| $\text{Skew}^Q$ | -0.1941 | -0.0628 | 0.1105 | 0.0553 | -0.0562 | 0.4018 | 0.0548 | -0.2124 | 0.0514 |
| | (0.0162) | (0.0165) | (0.0142) | (0.0153) | (0.0171) | (0.0319) | (0.0139) | (0.0152) | (0.0136) |
| $\text{Kurt}^Q$ | 3.6835 | 3.6223 | 3.6572 | 3.6769 | 3.7166 | 4.2188 | 3.6068 | 3.6629 | 3.5980 |
| | (0.0148) | (0.0182) | (0.0207) | (0.0215) | (0.0211) | (0.0377) | (0.0204) | (0.0150) | (0.0180) |



# Risk-Neutral Moments

## Skewness (Figure 6)

**Time series means and standard errors**

| | AUD | CAD | CHF | EUR | GBP | JPY | NOK | NZD | SEK |
|---|---|---|---|---|---|---|---|---|---|
| $\sqrt{\text{Var}^Q}$ | 0.1162 | 0.0824 | 0.1075 | 0.1027 | 0.0874 | 0.1113 | 0.1127 | 0.1297 | 0.1127 |
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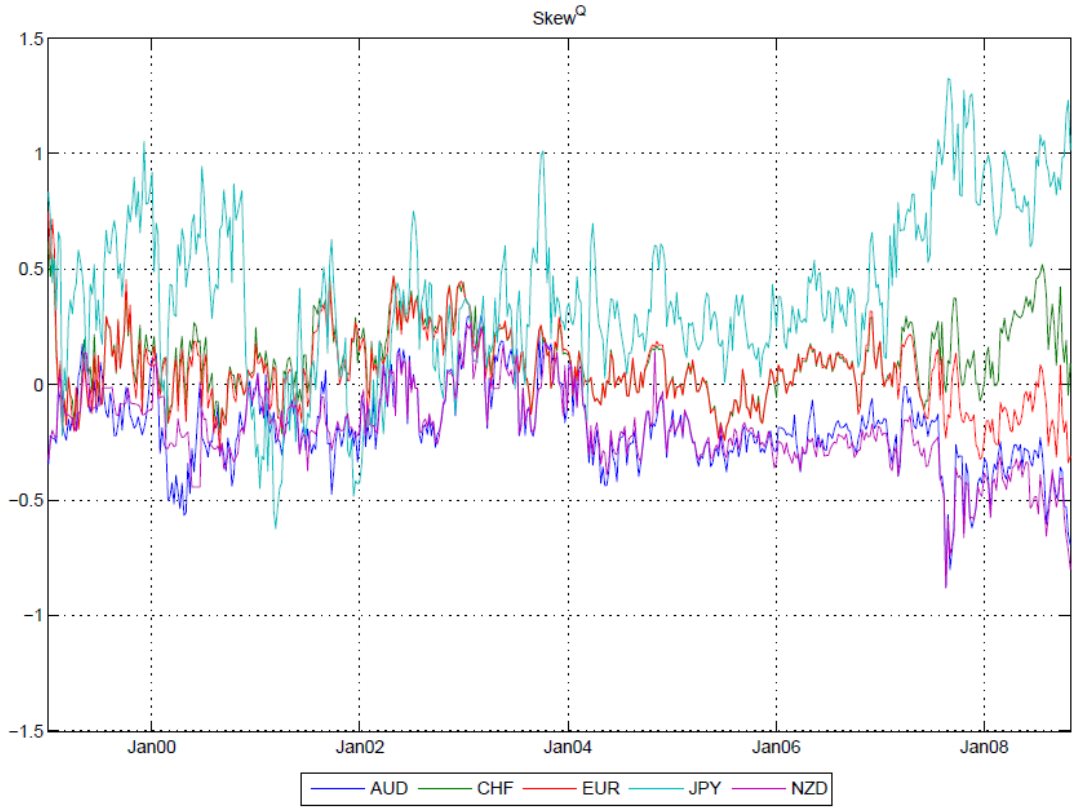


# Skewness

## Forecasting currency crashes (Table V)

| | Option-implied skewness | | | | | |
|---|---|---|---|---|---|---|
| | $xs_t$ | $r_{f,t} - r_{d,t}$ | $\text{Skew}_t^P$ | $\text{Skew}_t^Q$ | $R^2$ | $R^2_{\text{NFE}}$ |
| $\text{Skew}_{t+1}^Q$ | 3.7361 | | | | 0.5776 | 0.3198 |
| | (0.4601) | | | | | |
| $\text{Skew}_{t+1}^Q$ | | 2.1440 | | | 0.4044 | 0.0410 |
| | | (1.0279) | | | | |
| $\text{Skew}_{t+1}^Q$ | | | 0.0212 | | 0.3810 | 0.0032 |
| | | | (0.0171) | | | |
| $\text{Skew}_{t+1}^Q$ | | | | 0.5865 | 0.5912 | 0.3418 |
| | | | | (0.0293) | | |
| $\text{Skew}_{t+1}^Q$ | 3.3864 | -0.1951 | 0.0270 | 0.5606 | 0.7628 | 0.6180 |
| | (0.4922) | (0.5574) | (0.0078) | (0.0336) | | |

- Potential evidence of price pressure in FX option market; consistent with Brunnermeier, Nagel and Pedersen (2008).
- Majority of relation between risk-neutral skewness and the interest rate differential is driven by the cross-section.

# Skewness

## Forecasting currency crashes (Table V)

| | Realized skewness | | | | | |
|---|---|---|---|---|---|---|
| | $xs_t$ | $r_{f,t} - r_{d,t}$ | $\text{Skew}_t^P$ | $\text{Skew}_t^Q$ | $R^2$ | $R^2_{\text{NFE}}$ |
| $\text{Skew}_{t+1}^P$ | -2.9845 | | | | 0.0522 | 0.0199 |
| | (0.7471) | | | | | |
| $\text{Skew}_{t+1}^P$ | | -5.4556 | | | 0.0590 | 0.0269 |
| | | (1.8684) | | | | |
| $\text{Skew}_{t+1}^P$ | | | -0.0185 | | 0.0382 | -0.0008 |
| | | | (0.0483) | | | |
| $\text{Skew}_{t+1}^P$ | | | | -0.4746 | 0.0542 | 0.0220 |
| | | | | (0.1557) | | |
| $\text{Skew}_{t+1}^P$ | -1.2211 | -4.6725 | -0.0281 | -0.2742 | 0.0732 | 0.0416 |
| | (0.9328) | (1.9204) | (0.0506) | (0.1597) | | |

- Realized skewness, $\text{Skew}_{t+1}^P$, is computed using daily returns within the month.
- Currencies that have relatively high interest rates or have been targets of successful currency carry trades are more likely to crash.
- But ... protection is "cheap" precisely when it is most valuable.

# Crash-Neutral Currency Carry Trades

**Crash-neutral currency carry** trades are constructed to:

1. Eliminate exposure to exchange rate movements beyond a pre-specified threshold;
2. Match the *ex ante* currency exposure of the standard carry trade:
   - Necessary for expected return comparisons.
   - Focusing on Sharpe ratios may be inappropriate due to non-linearity.

*Example:* AUD/USD CNCT

- Borrow in USD ($r_{d,t}$), lend in AUD ($r_{f,t}$)
- Buy $q_p$ put options with a strike price of $K_p$ on the AUD/USD exchange rate and delta hedge the option overlay
- Purchase of the option overlay is financed at the domestic rate
- Positions established at the end of month $t$, and held until the end of month $t+1$ (buy-and-hold).

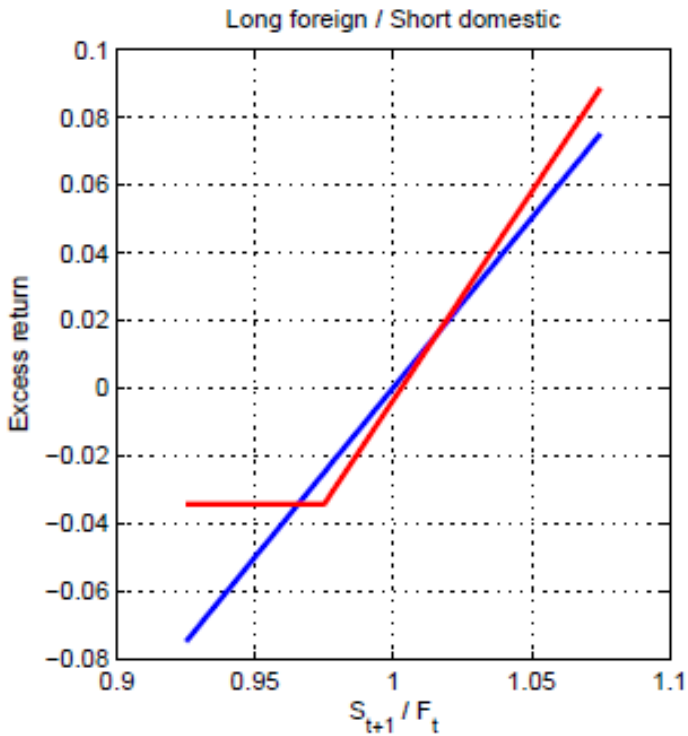
# Crash-Neutral Currency Carry Trades

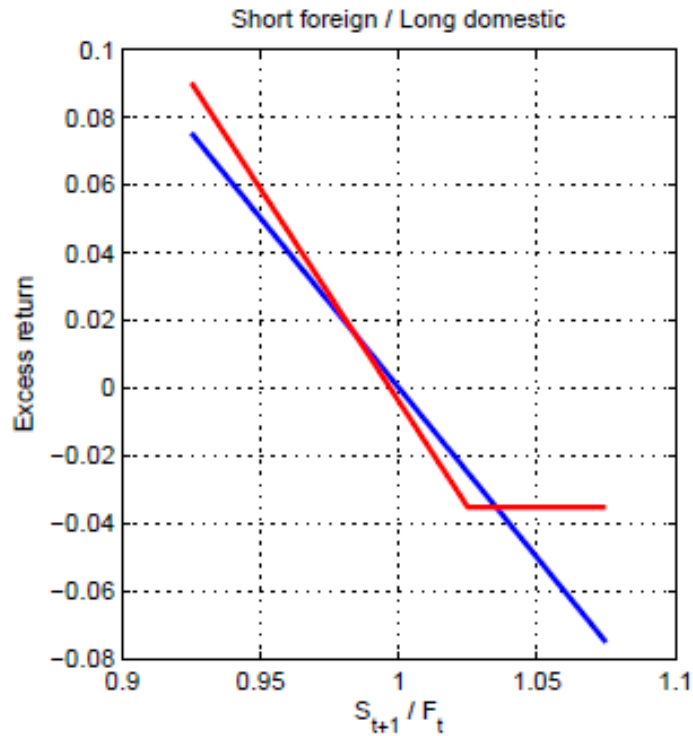
## Payoff diagram (Figure 4)

The payoff to the crash-neutral currency carry trade is given by:

$$\widetilde{CT}^{CN}_{t+1}(r_{f,t} > r_{d,t}) = q_p \cdot \max(K_p, \tilde{S}_{t+1}) - \exp(r_{d,t} \cdot \tau) \cdot ((1 - q_p \cdot \delta_p) \cdot S_t + q_p \cdot P_t(K_p, \tau))$$

$$\widetilde{CT}^{CN}_{t+1}(r_{d,t} > r_{f,t}) = \exp(r_{d,t} \cdot \tau) \cdot ((1 + q_c \cdot \delta_c) \cdot S_t - q_c \cdot C_t(K_c, \tau)) - q_c \cdot \min(K_c, \tilde{S}_{t+1})$$



# Crash-Neutral Currency Carry Trades

## Historical performance – portfolio strategies (Table VII)

Excess returns to the crash-neutral currency carry trade:

- continue to be positive and statistically significant, but;
- experience a statistically significant decline relative to their unhedged counterparts that is monotonically related to the amount of protection sought.

**Panel A: Non-dollar-neutral portfolios (1999:1-2007:3)**

| | CNCT($10\delta$) | | CNCT($25\delta$) | | CNCT(ATM) | |
|---|---|---|---|---|---|---|
| | EQL | SPR | EQL | SPR | EQL | SPR |
| Mean | 0.0459 | 0.0720 | 0.0369 | 0.0582 | 0.0193 | 0.0320 |
| t-stat | 3.13 | 3.77 | 2.77 | 3.40 | 1.81 | 2.37 |
| Std. dev. | 0.0421 | 0.0549 | 0.0382 | 0.0492 | 0.0306 | 0.0387 |
| Skewness | -0.23 | -0.12 | 0.22 | 0.30 | 0.89 | 0.90 |
| Kurtosis | 3.51 | 3.04 | 3.37 | 2.90 | 3.63 | 3.42 |
| Min | -0.0312 | -0.0351 | -0.0290 | -0.0275 | -0.0154 | -0.0157 |
| Max | 0.0359 | 0.0435 | 0.0327 | 0.0398 | 0.0289 | 0.0355 |
| SR | 1.09 | 1.31 | 0.97 | 1.18 | 0.63 | 0.83 |
| Mean (diff) | -0.0101 | -0.0124 | -0.0191 | -0.0262 | -0.0367 | -0.0524 |
| t-stat (diff) | -4.6 | -6.23 | -3.55 | -4.66 | -3.98 | -5.02 |

# Crash-Neutral Currency Carry Trades

## Historical performance – portfolio strategies (Table VII)

Excess returns to the crash-neutral currency carry trade remain positive and statistically significant for spread-weighted strategy even after the sample is extended through Oct. 2008.

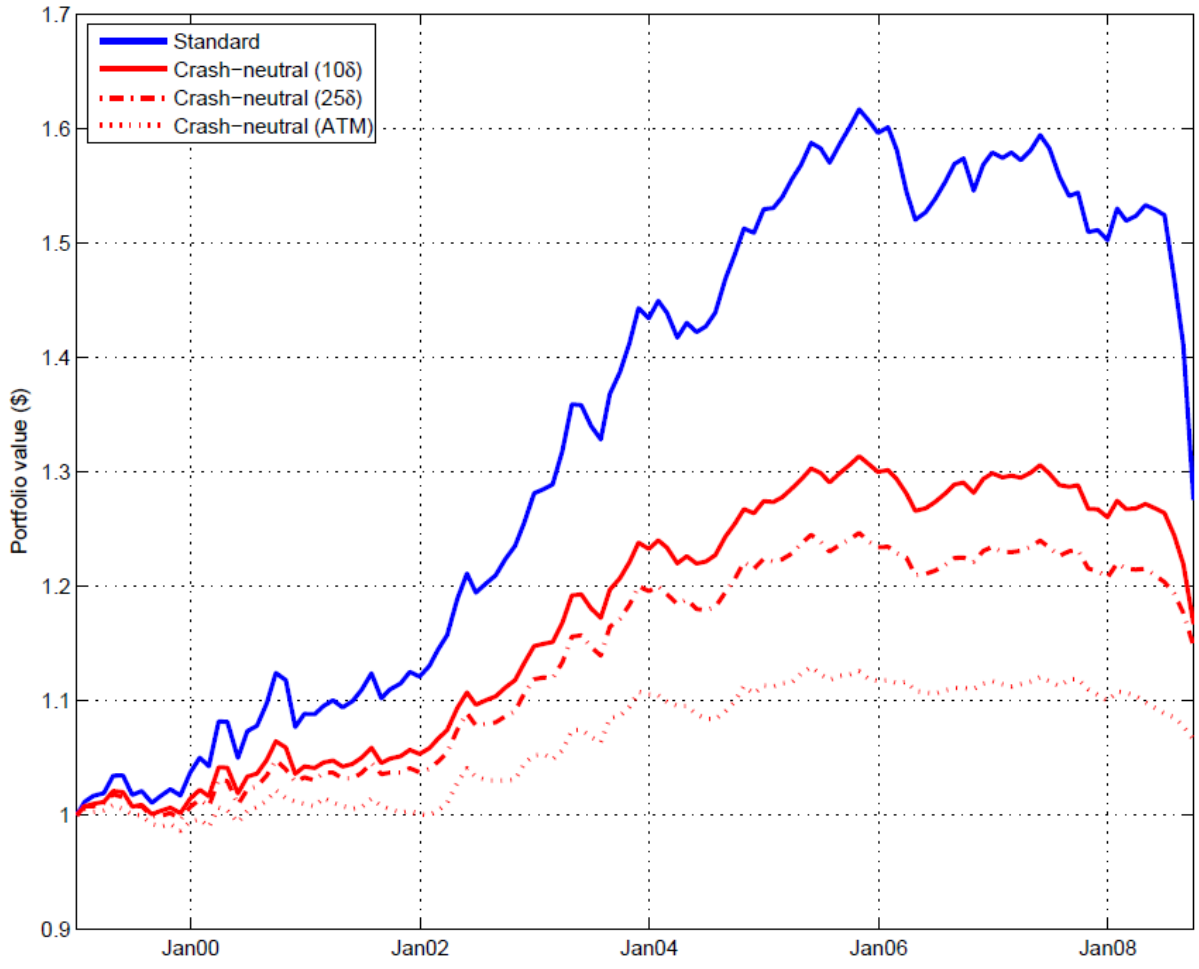
**Panel A: Non-dollar-neutral portfolios (1999:1-2008:10)**

| | CNCT($10\delta$) | | CNCT($25\delta$) | | CNCT(ATM) | |
|---|---|---|---|---|---|---|
| | EQL | SPR | EQL | SPR | EQL | SPR |
| Mean | 0.0237 | 0.0432 | 0.0210 | 0.0370 | 0.0098 | 0.0190 |
| t-stat | 1.55 | 2.16 | 1.65 | 2.22 | 1.01 | 1.55 |
| Std. dev. | 0.0476 | 0.0625 | 0.0398 | 0.0517 | 0.0302 | 0.0383 |
| Skewness | -0.94 | -0.82 | 0.01 | 0.04 | 0.90 | 0.91 |
| Kurtosis | 6.28 | 5.50 | 3.78 | 3.38 | 3.77 | 3.55 |
| Min | -0.0618 | -0.0763 | -0.0354 | -0.0443 | -0.0154 | -0.0198 |
| Max | 0.0805 | 0.0435 | 0.0327 | 0.0398 | 0.0289 | 0.0355 |
| SR | 0.50 | 0.69 | 0.53 | 0.72 | 0.32 | 0.50 |
| Mean (diff) | -0.0029 | -0.0050 | -0.0057 | -0.0111 | -0.0169 | -0.0292 |
| t-stat (diff) | -0.64 | -1.06 | -0.65 | -1.15 | -1.30 | -1.94 |

# Crash-Neutral Currency Carry Trades

## Total return indices

An equal-weighted portfolio of crash-neutral currency carry trades delivers statistically significant excess returns with positive skewness.



# Crash-Neutral Currency Carry Trades

## Historical performance – portfolio strategies (Table VII)

Excess returns to the **dollar-neutral** crash-neutral currency carry trade are generally indistinguishable from zero once the sample is extended through Oct. 2008.

**Panel B: Dollar-neutral portfolios (1999:1-2008:10)**

| | CNCT($10\delta$) | | CNCT($25\delta$) | | CNCT(ATM) | |
|---|---|---|---|---|---|---|
| | EQL | SPR | EQL | SPR | EQL | SPR |
| Mean | 0.0030 | 0.0171 | 0.0051 | 0.0165 | -0.0056 | -0.0007 |
| t-stat | 0.14 | 0.61 | 0.30 | 0.72 | -0.39 | -0.04 |
| Std. dev. | 0.0654 | 0.0881 | 0.0538 | 0.0714 | 0.0436 | 0.0548 |
| Skewness | -1.28 | -1.29 | -0.33 | -0.31 | 0.43 | 0.58 |
| Kurtosis | 7.60 | 8.52 | 3.40 | 4.36 | 2.83 | 3.04 |
| Min | -0.0944 | -0.1308 | -0.0516 | -0.0759 | -0.0239 | -0.0315 |
| Max | 0.0414 | 0.0624 | 0.0353 | 0.0571 | 0.0332 | 0.0469 |
| SR | 0.05 | 0.19 | 0.10 | 0.23 | -0.12 | -0.01 |
| Mean (diff) | -0.0089 | -0.0120 | -0.0068 | -0.0127 | -0.0175 | -0.0298 |
| t-stat (diff) | -1.32 | -1.74 | -0.48 | -0.83 | -0.82 | -1.21 |

# Crash-Neutral Currency Carry Trades

## Historical performance – portfolio strategies (Table VII)

Results are robust to hedging in fixed delta or moneyness space. The returns to crash-hedged carry trade portfolios with protection that is roughly 2.5% OTM are similar to $25\delta$ hedging.

**Constant moneyness hedging (1999:1-2008:10)**

| | | CNCT(5% OTM) | | |
|---|---|---|---|---|
| | EQL | SPR | $N-EQL | $N-SPR |
| Mean | 0.0221 | 0.0390 | 0.0029 | 0.0152 |
| t-stat | 1.71 | 2.32 | 0.17 | 0.67 |
| Std. dev. | 0.0403 | 0.0524 | 0.0534 | 0.0709 |
| Skewness | 0.12 | 0.13 | -0.21 | -0.06 |
| Kurtosis | 3.20 | 2.91 | 2.79 | 3.44 |
| Min | -0.0287 | -0.0345 | -0.0399 | -0.0569 |
| Max | 0.0327 | 0.0408 | 0.0322 | 0.0595 |
| SR | 0.55 | 0.74 | 0.05 | 0.21 |
| Mean (diff) | -0.0045 | -0.0092 | -0.0090 | -0.0139 |
| t-stat (diff) | -0.51 | -0.92 | -0.65 | -0.88 |

# Conclusions

1. The time-series dynamics of realized and risk-neutral skewness indicate that protection against currency crashes is relatively cheap in periods in which it is most valuable.

2. At most 30-40% of the excess return stemming from exploiting UIP violations can be attributed to exposure to currency crashes (Jan. 1999 - Mar. 2007).

3. In order to rationalize the entirety of the excess returns to currency carry trades exploiting violations in UIP, would require implied volatilities on foreign exchange options to be 2-4x their actual values, indicating a massive mispricing in the currency option market.

4. Asymmetry with respect to dollar exposure is an important determinant of performance. Spread-weighted carry trades, which are not dollar-neutral, remain profitable even after hedging crash risk using $10\delta$ and $25\delta$ options.