Behavioral Game Theory

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Outline

Behavioral Game Theory I

Definition:

- A study of *actual* individual's behaviors in strategic situations (or games), in which an individual's success in making decision depends on the choices of others.

Behavioral game theory aims to predict how people actually behave.

Nash equilibria I

NE in behavioral game theory studies assumes:

- Error-free decision making process.
- Consistency of decisions and actions.

But in reality, sometimes it fails:

- Nash equilibrium fails to predict some experimental results.
- Why does it fail? (Intuitively, people are not so rational.)
- The chance of winning depends on the choice that other people actually make.

One-shot games I

In this particular paper [GH01], the authors present one-shot games as tools for studying people behaviors.

Why one-shot games?

- No narrow learning (We want to observe the general behavior).
- No attempt to manipulate others (very short).
- There is no complex interaction as multi-move games (chess, checker).
- It insulates behavior from the incentives for cooperation and reciprocity that are present in repeated games.

One-shot games II

- Interesting results when we change the payoff structures in one-shot games (no effect on NE), which cannot be explained by the standard game theory.
- Very prone to confusion (confusion is good?).

Outline

Traveller's Dilemma I

- A and B simultaneously choose integers between 180 and 300
- Each player is paid the lower of the two values and plus a transfer R ($R > 1$) from the player with higher number to the one with lower number
- E.g. if one is 210 and other is 300, the lower gets $210 + R$ and the higher gets $210 - R$
- Let us play two rounds, one $R = 5$ and the other $R = 180$

Traveller's Dilemma II

- NE is 180
- But the result with J. Georee et al. is

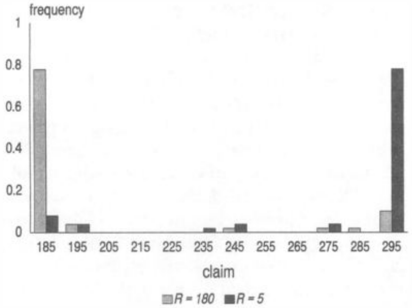


FIGURE 1. CLAIM FREQUENCIES IN A TRAVELER'S DILEMMA FOR R = 180 (LIGHT BARS) AND R = 5 (DARK BARS)

Matching Pennies I

Game 1	Left	Right
Top	80,40	40,80
Bottom	40,80	80,40

Game 2	Left	Right
Top	320,40	40,80
Bottom	40,80	80,40

Game 3	Left	Right
Top	44,40	40,80
Bottom	40,80	80,40

Matching Pennies II

- In a mixed strategy equilibrium, a player's own decision probabilities should be such that the other player is made indifferent between the two alternatives.
- The prediction is "50:50".

TABLE 1—THREE ONE-SHOT MATCHING PENNIES GAMES (WITH CHOICE PERCENTAGES)

		Left (48)	*Right* (52)
Symmetric matching pennies	*Top* (**48**)	80, 40	40, 80
	Bottom (**52**)	40, 80	80, 40
		Left (16)	*Right* (84)
Asymmetric matching pennies	*Top* (**96**)	320, 40	40, 80
	Bottom (**4**)	40, 80	80, 40
		Left (80)	*Right* (20)
Reversed asymmetry	*Top* (**8**)	44, 40	40, 80
	Bottom (**92**)	40, 80	80, 40

Kreps Game I

Basic	Left	Middle	Non-Nash	Right
Top	200,50	0,45	10,30	20,-250
Bottom	0,-250	10,-100	30,30	50,40

Basic	Left	Middle	Non-Nash	Right
Top	500,350	300,345	310,330	320,50
Bottom	300,50	310,200	330,330	350,340

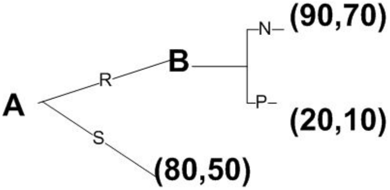
Kreps Game II

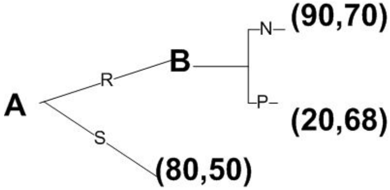
- NE is (Top,Left) and (Bottom, Right)
- However...

TABLE 3—TWO VERSIONS OF THE KREPS GAME (WITH CHOICE PERCENTAGES)

Basic game		*Left* (26)	*Middle* (8)	*Non-Nash* (68)	*Right* (0)
	Top (68)	200, 50	0, 45	10, 30	20, −250
	Bottom (32)	0, −250	10, −100	30, 30	50, 40
Positive payoff frame		*Left* (24)	*Middle* (12)	*Non-Nash* (64)	*Right* (0)
	Top (84)	500, 350	300, 345	310, 330	320, 50
	Bottom (16)	300, 50	310, 200	330, 330	350, 340

Trust and Threat I





Trust and Threat II

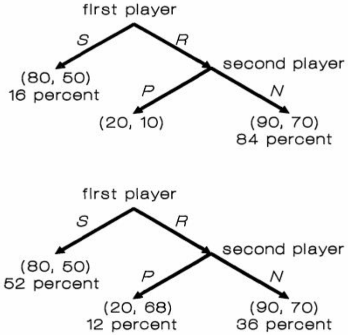


FIGURE 3. SHOULD YOU TRUST OTHERS TO BE RATIONAL?

Subgame Perfectness

In game theory, a subgame perfect equilibrium (or subgame perfect Nash equilibrium) is a refinement of a Nash equilibrium used in dynamic games. A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game.

Trust and Threat III

- There are two subgame NE, S and N.
- But we can see we do not trust the second player as before.

Signaling I

- Two players, senders and responders.
- Sender is of two types A or B (50-50).
- The sender know his own type and send a signal Left or Right.
- The responder only see the signal (Left or Right), but not the sender's type. He will respond with C, D or E.

Signaling II

TABLE 7—SIGNALING WITH A SEPARATING EQUILIBRIUM (MARKED BY ASTERISKS) (SENDER'S PAYOFF, RESPONDER'S PAYOFF)

	Response to *Left* signal				Response to *Right* signal		
	C	*D*	*E*		*C*	*D*	*E*
Type A sends *Left*	300, 300	0, 0	500, 300	Type A sends *Right*	**450**, 900 (*)	**150**, 150	**1,000**, 300
Type B sends *Left*	**500**, 500 (*)	**300**, 450	**300**, 0	Type B sends *Right*	450, 0	0, 300	0, 150

TABLE 8—SIGNALING WITHOUT A SEPARATING EQUILIBRIUM (SENDER'S PAYOFF, RESPONDER'S PAYOFF)

	Response to *Left* signal				Response to *Right* signal		
	C	*D*	*E*		*C*	*D*	*E*
Type A sends *Left*	300, 300	0, 0	500, 300	Type A sends *Right*	**450**, 900	**150**, 150	**1,000**, 300
Type B sends *Left*	**300**, 300	**300**, 450	**300**, 0	Type B sends *Right*	450, 0	0, 300	0, 150

Signaling III

- There is a separating Nash equilibrium, and the signal reveals the sender's type.
- In the latter experiment, 10 of 13 type A sender did chose Right, and 9 of 11 type B choose left. But there is no NE here, all equilibrium is this contradiction treatment involve *Pooling*, with both types sending the same signal.

Hypotheses I

In order to model these behaviors, many hypotheses have been proposed following heuristic.

- Economic aspect [GH01]
  - Inequality aversion
  - Maximim
  - The logit rule
- Psychological aspect [KST82]
  - Anchoring
  - Representativeness
  - Availability

Outline

1. Introduction
2. Examples
3. Economic aspect
4. Psychological aspect
5. Summary

Inequality aversion I

Obviously,

- Envy when earning less.
- Feel guilt when earning more (especially, when they know you).

For example, consider Trust and Threat game (keep in mind that this game is played in parallel).

- In the threat game, the increasing trend of non-severe punishment suggests the sign of envy.
- In the second trust game, most second players are fair enough.

Inequality aversion II

However, this model cannot explain the observed behavior in matching pennies.

- Consider the “320” version of the game. If the column players are averse to the $(320, 40)$ outcome, they will random only when the row players play *Bottom* more often than 0.5 probability. Consequently, the row players should play bottom choice more if they prefer fair game. But the statistic shows contradiction.

Security I

That higher profits come greater risks. But some people prefer a safe ground.

- They make decisions that maximize their security level, the most secure choice.

For example, in the Kreps game and the traveller's dilemma

- The frequently observed *Non-Nash* decision implies the possibility of this model.
- When the R is high, people tend to lower their bet.

However, this model does not apply to the matching pennies game

- For each person, the minimum payoff is the same for both decisions.

The logit rule

Trying to model player's thought processes; one thing to be sure is that the magnitude of payoff affects the choice probabilities.

$$p_i = \frac{e^{\pi_i^e/\mu}}{\sum_j^m e^{\pi_j^e/\mu}}, \quad i = 1, \ldots, m \tag{1}$$

- π_i^e is an expected payoff for playing choice i.
- μ determines how sensitive choice probabilities are to payoff differences.
- This term is computed recursively.
- "Logit best response"

Rationality levels

The expected payoff π_i^e is computed from the belief probability p_i from the previous step. This suggests rationality level.

- Level one rationality: postulate that each of the others' actions are equally likely. People use this in the traveller's dilemma and the Kreps game.
- Level two rationality: level one plus one more step. Observed in Matching pennies.
- And so on... (in minimax fashion)
- When to stop? the number of iterations can be weakly bounded by uncertainty. Namely, put some noise in the higher level. This is when the μ in Eq (1) comes into consideration.

The final form

From Eq (1), if we let

$$p = \phi_\mu(p_0)$$

For an n-level rationality,

$$p = \lim_{n \to \infty} \phi_{\mu_1}(\phi_{\mu_2}(\cdots \phi_{\mu_n}(p_0))) \tag{2}$$

- where $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_n$ with μ_∞ converging to infinity.
- uniform distribution at start and converges to an extreme policy.
- when μ approaches to infinity, Eq (1) produces the uniform distribution.

Fitting the model

- Try fitting Eq (2) in 37 simple matrix games.
- Using $\mu_n = \mu t^n$ and try to optimize t.
- The result shows that $t = 4.1$ and thus confirms that there is more noise for higher levels of rationality.

Outline

Anchoring I

Behavior:

- Estimate the expected quantity from an initual value or bias.
- Different starting points yield different estimates.
- Similar to the concept of the initial seed and the local optimum.
- We can also interpret it as a small-n logit rule with some initial value as the starting belief.

Example 1: Product of number sequences

- $8 \times 7 \times 6 \ldots \times 1 \approx 512$
- $1 \times 2 \times 3 \ldots \times 8 \approx 2250$

Anchoring II

Example 2:

- In a partially observed random process of generating a real number.
- Asking for a number at 90th percentile from the process.
- Given a number, approximate the percentile.

Anchoring III

In one-shot games:

- The results of the traveller's dilemma game may suggest that people use anchoring. Namely, they come up with the starting choice of what others may choose, then derive their actions base on these beliefs.
- In Kreps game, perhaps the column players choose to play *Non-Nash* more because they believe that the probability of row players playing each choice is uniformly distributed (level one rationality). Thus, when compared with the payoff they may get from other choices, the *Non-Nash* seems to be the most reasonable one.

Representativeness I

Behavior:

- A heuristic that involves similarity.
- When A represents B, $p(A|B)$ is usually assumed to be high.
- "Steve is very shy and withdrawn, invariably helpful, but with little interest in people." There is a higher chance that Steve is a librarian than a farmer.
- In short, comparing the properties of A to the stereotype of B.

Representativeness II

Drawbacks:

- Insensitive to priors.
- Insensitive to sample size.
- Misconcept of chance.
- There are several others in the paper, but I am not convinced that they are important.

Insensitivity to priors

- In Steve example, it is quite obvious that the number of librarians is far less than farmers.
- In matching pennies game, the row players fail to anticipate the possibility that the column players may choose to play *Right* more.

Insensitivity to sample size I

- Intuitively (statisticians), experiments with large sample size are more believable.
- "Imagine an urn filled with balls, of which $2/3$ are of one color and $1/3$ of another. One individual has drawn 5 balls from the urn, and found that 4 were red and 1 was white. Another individual has drawn 20 balls and found that 12 were red and 8 were white. Which of the two individuals should feel more confident that the urn contains $2/3$ red balls and $1/3$ white balls, rather than the opposite? What odds should each individual give?"
- Most people feel that the first sample provides much stronger evidence for the hypothesis.

Insensitivity to sample size II

- $\frac{p(4{:}1|2{:}1)}{p(4{:}1|1{:}2)} = \frac{(2/3)^4(1/3)}{(1/3)^4(2/3)} = 8/1$
- $\frac{p(12{:}8|2{:}1)}{p(12{:}8|1{:}2)} = \frac{(2/3)^{12}(1/3)^8}{(1/3)^{12}(2/3)^8} = 16/1$

Misconceptions of chance I

- Asking to give a random sequence between H and T, $p(\text{H-T-H-T-T-H}) \geq p(\text{H-H-H-T-T-T})$
- In another view, Gambler's fallacy. Given enough time, the probability may "regress toward the mean". But it will never "correct" any thing wrong on the early of the run.
- The law of small number. Namely, many researchers use only few experiments to confirm their hypothesis, according to which even small samples are highly representative of the populations. The crucial component is only few samples are taken into account.

Misconceptions of chance II

- In the signaling game, the "pooling" can be considered as this fallacy. Namely, people use "average" as the simple representation of a complicate set. They trust that most of elements in the set are closed to the mean. And even though some deviates from the mean, they might not change the entire payoff structure, which is not true in this case.

Availability I

- When people facing difficult task of judging probability, they use limited number of heuristics to reduce the complexity.
- Availability is about the psychological mechanism by which people evaluate the frequency of classes or the likelihood of events.
- E.g. The chance of heart attack of middle-aged people.
- Availability uses strength of association as a basis for the judgment of probability.
- Since frequent event is easier to recall and imagine than infrequent ones, so Availability is an ecologically valid clue for the judgment of frequency.
- The use of availability heuristics will lead to systematic biases.

Availability II

Construction:

- When people cannot construct and enumerate all instances, they judge the overall probability by an assessment of the ease with which instances could be brought to mind.

Retrieval:

- When people cannot recall and count all instances, he attempts to recall some instances and judge overall frequency by availability, i.e., by the ease whith which instances come to mind.

Availability for Construction I

- Classes whose instances are easy to construct or imagine are easy to construct or imagine will be perceived as more frequent than the same classes whose instances are less available.

Availability for Construction II

A problem:

- Consider the letter R.
- Is R more likely to appear in:
  - The first position?
  - The third position?

  - My estimation for the ration of thesee two values is x:1

Availability for Construction III

Result:

- Among 152 subjects, 105 judges the first place to be more likely 47 judged the third position to be more likely for the majority
- What about the position of letter K?
- The median estimation is 2:1 seems constant
- Actually the third is more frequent then the first position for those letters

Availability for Retrival I

- Classes whose instances are readily recalled will be judged more numerous than classes of the same size but are less available.

Availability for Retrival II

- "A list consists of names of famous people of both sexes. After listening to the list, some subjects judges whether it contained more names of men or of women. In the name list, some are very famous, and some are less, and we assume generally famous names are easier to recall. The first names of all personalities always permitted an unambiguous identification of sex."
- The experiment consists of 4 listed, two of entertainers and two of public figures. Each list includes of 39 names recorded at a rate of one name every 2 seconds. Two are composed of 19 very famous women and 20 less famous men. The two others are opposite.

Availability for Retrival III

Recall:

- On average, subjects recalled 12.3 of the 19 famous names and 8.4 of the less famous names. On the 86 subjects in the four recall groups, 57 recalled more famous than no famous names, and only 13 recalled fewer famous than less famous names.
- Frequency : among 99 subjects, the error rate is 80% on average.

Retrieval of Occurrences and Construction of Scenarios I

- Consider the probability of divorce, an economic recession, a successful medical operation, they cannot be evaluated by a simple tally of instances. But we apply availability heuristics to evaluate them.
- For evaluation the probability of divorce, one method is to recall the similar couples in one's memory, the other is to construct scenarios lead to a divorce. The ease with which they come to mind, provides basis for the judgment of probability.

Retrieval of Occurrences and Construction of Scenarios II

- Some events are perceived as so unique that past history does not seems relevant to the evaluation of its likelihood. We generally construct scenarios, i.e., stories that lead from the present situation to the target event.
- If no reasonable scenario comes to mind, the event is deemed impossible or highly unlikely.
- If enough scenarios come to mind or one scenario seems quite compelling, we assign high probability to this event.

Retrieval of Occurrences and Construction of Scenarios III

- Some events whose likelihood people wish to evaluate depend on several interrelated factors. And we only consider the simplest and the most available scenarios. For example, people tend to produce scenarios in which many factors do not vary at all only the most obvious variations take place.
- This approach may have particularly salient effects in situations of conflict. When we play chess, the player may tend to regard the opponent's strategy constant and independent of his own moves, which cannot guarantee winning.

Biases due to Retrievability

- As we show before, people tend to use information that is easily retrieved.
- In the previous experiment of famous names, people easily made mistake due to the biases in retrievability.

Biases due to effectiveness of a search set

Problem:

- The frequency of (thought, love) vs. (door, water)
- Since abstract is easier to think about, we say (thought, love) has a relative high frequency even we do not search all the literature.
- So abstract words are judged to appear in much greater variety of context than concrete words

Biases of Imaginability

- As we mentioned before, imaginability is quite important in real-life likelihood evaluation, such as retrieval of occurrences and construction of scenarios.
- Most of the time, we underestimate the risk since some danger is difficult to conceive of, or simply does not come to mind.

Illusory Correlation I

Clinical experiment:

- The data for each patient consists of a clinical diagnosis and a drawing of a person made by the patient.
- Later the judges estimate the frequency with which each diagnosis had been accompanied various features of the drawing.

Illusory Correlation II

- The subjects markedly overestimate the frequency of co-occurrence of natural associates, such as suspiciousness and peculiar eyes. This effect is labeled illusory correlation.
- It is very resistant to contradictory data and persists even when the correlation between the symptom and diagnosis is actually negative, preventing judges from detecting relationships that were in fact present.

Egocentric Biases I

- To allocate responsibility for a joint endeavor, well-intentioned participants presumably attempt to recall the contributions each made to the final product. So a person may recall a greater proportion of his own contributions than would other participants.
- An egocentric bias in availability of information in memory, in turn, could produce biased attributions of responsibility for a joint product.

Egocentric Biases II

- Consider the Trust and Threats example, since the payoff for palyer B acting P changes from (20,10) to (20,68), which is only 2 less than the subgame NE.
- Although (90,70) is obtained by cooperation, the second player may have egocentric biases, which leads him to choose P.

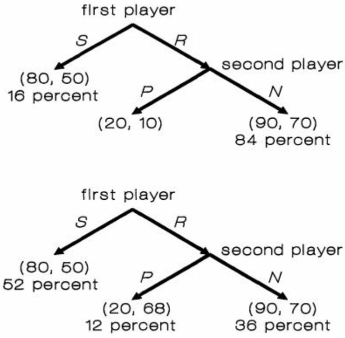


FIGURE 3. SHOULD YOU TRUST OTHERS TO BE RATIONAL?

Outline

Conclusion

- Behavioral game theory tries to understand human behaviors in games.
- The paper uses one-shot games as tools..
- To explain the phenomena in the games, people propose behavior models.
- The paper says that most people choice are not reasonable.
- However, it does not mean that what we do are bad. We are good in general (like the Watson in jeopardy compared to other computers).
- "Game theory is for proving theorems, not for playing games" – Reinhard Selten
- Game theory says "How to play" not what people actually play.

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