33 Months	3.5 months	late
36 Months	.5 months	late
38 Months	1.5 months	early
40 Months	3.5 months	early
42 Months	5.5 months	early
44 Months	7.5 months	early

If waves of about these lengths were present, their sums after such phase shifts would not give a true result.

Another objection to the use of 9-C and 9-D is that they do not give as good elimination of other waves as 9-A and 9-B. Two graphs are appended (Figure 8) showing a comparison of the action of 9-C with 9-A, and 9-D with 9-B. This shows that Methods 9-C and 9-D have very peaked bottoms to the elimination curve, and would not be very effective in removing waves close to the one being fully eliminated. Use of Methods 9-C and 9-D would be very desirable to give results up to the current date, if they could be relied upon. It is entirely unsatisfactory for this purpose if the wave being developed is a group instead of a single one. If used at all, full consideration must be given to the phase shift, and the fact that the result may be inaccurate.

The key to the use of Methods 9-A and 9-B, and any accompanying process, such as a moving periodic table, and moving average combinations, is the preparation of an amplitude table for a wide range of wave lengths, such as the one on page 89. It should be assumed that very many waves may be present, and that all should be nearly eliminated except the one to be shown. It is rare indeed that a single operation will show a wave with any degree of fidelity, and a properly prepared amplification table changes such operations from guess work to scientific planning.

501 Financial Center Building
Los Angeles 14, California

TABLE 1

AMPLIFICATION TABLE FOR METHOD 9-A*

WAVE	0-3	0-4	0-5	0-6	0-7	0-8	0-9	0-10	0-11	0-12	0-13	0-14	0-15	0-1…
3	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	1.50
3⅓	.191	.691	2.000	.691	.191	1.809	1.309	.000	1.309	1.809	.191	.691	2.000	.69
4	1.000	.000	1.000	2.000	1.000	.000	1.000	2.000	1.000	.000	1.000	2.000	1.000	.00
5	1.809	.691	.000	.691	1.809	1.809	.691	.000	.691	1.809	1.809	.691	.000	.69
5½	1.959	1.144	.159	.159	1.144	1.940	1.656	.586	.000	.586	1.656	1.940	1.144	1.5
6	2.000	1.500	.500	.000	.500	1.500	2.000	1.500	.500	.000	.500	1.500	2.000	1.5
6⅔	1.987	1.675	.754	.054	.212	1.082	1.879	1.879	1.082	.212	.054	.754	1.675	1.9
7	1.899	1.899	1.223	.377	.000	.377	1.223	1.899	1.899	1.223	.377	.000	.377	1.22
8	1.707	2.000	1.707	1.000	.293	.000	.293	1.000	1.707	2.000	1.707	1.000	.293	.00
9	1.500	1.940	1.940	1.500	.826	.234	.000	.234	.826	1.500	1.940	1.940	1.500	.82
10	1.309	1.809	2.000	1.809	1.309	.691	.191	.000	.191	.691	1.309	1.809	2.000	1.80
10⅔	1.195	1.707	1.982	1.924	1.556	1.000	.446	.076	.018	.293	.805	1.375	1.850	2.00
11	1.143	1.653	1.959	1.959	1.653	1.143	.585	.159	.000	.159	.585	1.143	1.653	1.95
12	1.000	1.500	1.866	2.000	1.866	1.500	1.000	.500	.134	.000	.134	.500	1.000	1.5
12⅔	.917	1.403	1.789	1.987	1.946	1.674	1.246	.758	.323	.054	.014	.210	.549	1.08
13	.880	1.356	1.748	1.971	1.971	1.748	1.356	.880	.432	.115	.000	.115	.432	.88
14	.777	1.223	1.623	1.899	2.000	1.899	1.623	1.223	.777	.377	.099	.000	.099	.37
15	.691	1.104	1.500	1.809	1.978	1.978	1.809	1.500	1.104	.691	.331	.086	.000	.08
16	.617	1.000	1.383	1.707	1.924	2.000	1.924	1.707	1.383	1.000	.617	.293	.076	.00
17	.554	.909	1.273	1.603	1.851	1.983	1.983	1.851	1.603	1.273	.909	.554	.259	.06
18	.500	.826	1.175	1.500	1.766	1.940	2.000	1.940	1.766	1.500	1.175	.825	.500	.23
19	.454	.754	1.081	1.403	1.676	1.879	1.986	1.986	1.879	1.676	1.403	1.081	.754	.4
20	.412	.691	1.000	1.309	1.588	1.809	1.951	2.000	1.951	1.809	1.588	1.309	1.000	.69
21	.377	.635	.925	1.223	1.500	1.733	1.897	1.989	1.989	1.897	1.733	1.500	1.223	.92
22	.347	.585	.857	1.143	1.414	1.653	1.834	1.959	2.000	1.959	1.834	1.653	1.414	1.14
23	.318	.540	.795	1.064	1.335	1.577	1.775	1.918	1.990	1.990	1.918	1.775	1.577	1.33
24	.293	.500	.740	1.000	1.258	1.500	1.707	1.866	1.966	2.000	1.966	1.867	1.707	1.50
25	.271	.464	.691	.937	1.187	1.426	1.637	1.809	1.930	1.992	1.992	1.936	1.809	1.63
26	.252	.432	.645	.880	1.119	1.356	1.567	1.748	1.886	1.972	2.000	1.975	1.886	1.74
27	.234	.403	.604	.826	1.056	1.285	1.500	1.684	1.834	1.940	1.993	1.993	1.940	[illegible]
28	.218	.377	.566	.777	1.000	1.223	1.431	1.625	1.782	1.900	1.975	2.000	1.975	[illegible]
29	.204	.353	.532	.732	.945	1.161	1.368	1.561	1.725	1.857	1.947	1.994	1.994	1.94
30	.191	.331	.500	.691	.895	1.104	1.309	1.500	1.669	1.809	1.913	1.979	2.000	1.97
32	.169	.293	.446	.617	.805	1.000	1.195	1.383	1.556	1.707	1.831	1.923	1.976	2.00
34	.151	.259	.397	.554	.726	.909	1.091	1.273	1.445	1.603	1.739	1.851	1.932	1.98
36	.134	.234	.358	.500	.658	.826	1.000	1.175	1.341	1.500	1.642	1.766	1.865	1.94
38	.120	.209	.322	.454	.597	.754	.916	1.084	1.245	1.403	1.546	1.675	1.788	1.87
40	.109	.191	.293	.412	.546	.691	.843	1.000	1.156	1.309	1.454	1.588	1.707	1.8
41	.104	.182	.280	.393	.522	.661	.807	.962	1.113	1.264	1.409	1.542	1.666	1.77
42	.099	.173	.267	.377	.500	.635	.777	.925	1.075	1.224	1.367	1.500	1.623	1.73
44	.090	.159	.244	.347	.459	.585	.717	.857	1.000	1.143	1.283	1.414	1.540	1.65
46	.083	.146	.226	.318	.423	.540	.665	.795	.926	1.069	1.203	1.335	1.460	1.57
48	.076	.134	.207	.293	.391	.500	.617	.740	.870	1.000	1.130	1.258	1.383	1.50
50	.070	.124	.191	.271	.362	.464	.574	.691	.813	.937	1.063	1.187	1.309	1.42
52	.065	.115	.177	.250	.336	.432	.535	.645	.760	.880	1.000	1.119	1.240	1.35
54	.060	.107	.165	.234	.313	.403	.500	.604	.714	.825	.942	1.059	1.175	1.2
57	.054	.096	.148	.210	.284	.363	.452	.545	.649	.753	.862	.975	1.083	1.19
60	.049	.086	.134	.191	.257	.331	.412	.500	.593	.691	.792	.895	1.000	1.10
63	.044	.079	.122	.174	.234	.302	.376	.457	.544	.635	.728	.826	.926	1.02
66	.041	.072	.111	.159	.214	.276	.344	.420	.500	.586	.673	.763	.858	.95
70	.036	.064	.099	.141	.191	.246	.309	.378	.448	.527	.606	.691	.777	.86
73	.033	.059	.091	.130	.176	.228	.285	.348	.417	.488	.564	.642	.726	.80
76	.031	.054	.084	.121	.163	.211	.265	.322	.386	.454	.524	.598	.676	.75
80	.028	.049	.076	.109	.147	.191	.240	.293	.348	.412	.480	.546	.614	.6
84	.025	.044	.069	.099	.134	.174	.218	.267	.320	.377	.438	.500	.566	.63
88	.023	.041	.063	.090	.122	.159	.199	.244	.293	.346	.402	.459	.522	.58
92	.021	.037	.058	.083	.112	.146	.183	.224	.269	.320	.369	.423	.481	.54
96	.019	.034	.053	.076	.103	.134	.168	.206	.248	.293	.341	.392	.445	.50
100	.018	.031	.049	.070	.095	.124	.156	.191	.229	.271	.315	.362	.412	.46
104	.016	.029	.045	.066	.088	.114	.144	.177	.213	.252	.293	.334	.383	.43
108	.015	.027	.042	.060	.082	.107	.134	.164	.198	.233	.273	.313	.358	.4
114	.014	.024	.038	.054	.073	.096	.120	.143	.178	.211	.246	.283	.323	.3
120	.012	.022	.034	.049	.066	.086	.109	.134	.161	.191	.223	.256	.293	[illegible]

*COLUMN HEADINGS ARE ABBREVIATED. 0 - 3 IS 0 - ½(3A + 3B)

17	0-18	0-19	0-20	0-21	0-22	0-23	0-24	0-25	0-26	0-27	0-28	0-29	0-30	WAVE
00	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	3
41	1.809	1.309	.000	1.309	1.809	.191	.691	2.000	.691	.191	1.809	1.309	.000	3⅓
00	2.000	1.000	.000	1.000	2.000	1.000	.000	1.000	2.000	1.000	.000	1.000	2.000	4
09	1.809	.691	.000	.691	1.809	1.809	.691	.000	.691	1.809	1.809	.691	.000	5
59	1.144	1.940	1.656	.586	.000	.586	1.656	1.940	1.144	.159	.159	1.144	1.940	5½
00	.000	.500	1.500	2.000	1.500	.500	.000	.500	1.500	2.000	1.500	.500	.000	6
03	.454	.000	.454	1.403	1.987	1.675	.754	.054	.212	1.082	1.879	1.879	1.082	6⅓
99	1.899	1.223	.377	.000	.377	1.223	1.899	1.899	1.223	.377	.000	.377	1.223	7
93	1.000	1.707	2.000	1.707	1.000	.293	.000	.293	1.000	1.707	2.000	1.707	1.000	8
34	.000	.234	.826	1.500	1.940	1.940	1.500	.826	.234	.000	.234	.826	1.500	9
09	.691	.191	.000	.191	.691	1.309	1.809	2.000	1.809	1.309	.691	.191	.000	10
50	1.375	.805	.293	.018	.076	.446	1.000	1.555	1.924	1.982	1.707	1.195	.623	10½
59	1.653	1.143	.585	.159	.000	.159	.585	1.143	1.653	1.959	1.959	1.653	1.143	11
66	2.000	1.866	1.500	1.000	.500	.134	.000	.134	.500	1.000	1.500	1.866	2.000	12
46	1.879	2.000	1.879	1.546	1.083	.597	.210	.014	.054	.323	.753	1.246	1.674	12⅔
56	1.748	1.971	1.971	1.748	1.356	.880	.432	.115	.000	.115	.432	.880	1.356	13
77	1.223	1.623	1.899	2.000	1.899	1.623	1.223	.777	.377	.099	.000	.099	.377	14
31	.691	1.104	1.500	1.809	1.978	1.978	1.809	1.500	1.104	.691	.331	.086	.000	15
76	.293	.617	1.000	1.383	1.707	1.924	2.000	1.924	1.707	1.383	1.000	.617	.293	16
00	.067	.259	.554	.909	1.273	1.603	1.851	1.983	1.983	1.851	1.603	1.273	.909	17
60	.000	.060	.234	.500	.826	1.175	1.500	1.766	1.940	2.000	1.940	1.766	1.500	18
09	.054	.000	.054	.209	.454	.754	1.081	1.403	1.676	1.879	1.986	1.986	1.879	19
12	.191	.049	.000	.049	.191	.412	.691	1.000	1.309	1.588	1.809	1.951	2.000	20
35	.377	.178	.044	.000	.044	.173	.377	.635	.925	1.223	1.500	1.733	1.897	21
57	.586	.347	.159	.040	.000	.039	.159	.347	.586	.857	1.142	1.414	1.653	22
69	.795	.540	.318	.146	.037	.000	.037	.146	.318	.540	.795	1.069	1.335	23
58	1.000	.740	.500	.293	.134	.033	.000	.033	.133	.293	.500	.740	1.000	24
26	1.187	.937	.691	.464	.271	.124	.031	.000	.031	.124	.271	.464	.691	25
47	1.356	1.119	.880	.646	.432	.252	.115	.030	.000	.030	.115	.252	.432	26
4	1.500	1.285	1.056	.826	.604	.403	.234	.107	.027	.000	.027	.107	.234	27
2	1.623	1.431	1.224	1.000	.777	.566	.377	.218	.099	.025	.000	.025	.099	28
57	1.725	1.561	1.368	1.161	.945	.732	.532	.353	.204	.092	.023	.000	.023	29
13	1.809	1.669	1.500	1.309	1.104	.895	.691	.500	.331	.191	.088	.022	.000	30
66	1.924	1.831	1.707	1.557	1.383	1.195	1.000	.805	.618	.446	.293	.169	.077	32
00	1.982	1.932	1.850	1.738	1.603	1.445	1.273	1.091	.908	.726	.554	.397	.260	34
85	2.000	1.985	1.940	1.865	1.765	1.642	1.500	1.341	1.175	1.000	.826	.658	.500	36
46	1.986	2.000	1.986	1.946	1.878	1.788	1.677	1.546	1.402	1.245	1.082	.916	.752	38
91	1.951	1.988	2.000	1.988	1.951	1.891	1.809	1.707	1.588	1.454	1.309	1.156	1.000	40
61	1.928	1.974	1.997	1.997	1.974	1.927	1.861	1.771	1.662	1.542	1.409	1.266	1.117	41
27	1.902	1.956	1.989	2.000	1.989	1.956	1.903	1.827	1.733	1.623	1.500	1.367	1.224	42
55	1.836	1.909	1.959	1.990	2.000	1.990	1.959	1.909	1.836	1.755	1.653	1.540	1.418	44
84	1.775	1.855	1.918	1.963	1.990	2.000	1.990	1.963	1.918	1.855	1.775	1.684	1.572	46
08	1.707	1.794	1.867	1.924	1.966	1.991	2.000	1.991	1.965	1.924	1.867	1.794	1.707	48
36	1.637	1.729	1.809	1.876	1.930	1.969	1.992	2.000	1.992	1.969	1.930	1.876	1.809	50
65	1.568	1.663	1.749	1.823	1.887	1.935	1.972	1.993	2.000	1.992	1.971	1.935	1.885	52
97	1.500	1.598	1.686	1.766	1.836	1.894	1.940	1.973	1.993	2.000	1.993	1.973	1.940	54
98	1.403	1.500	1.543	1.676	1.754	1.823	1.879	1.926	1.962	1.987	1.998	1.998	1.987	57
08	1.309	1.406	1.500	1.588	1.669	1.743	1.809	1.866	1.913	1.951	1.978	1.994	2.000	60
25	1.224	1.321	1.413	1.500	1.586	1.662	1.733	1.798	1.854	1.901	1.940	1.969	1.989	63
38	1.144	1.236	1.327	1.414	1.500	1.582	1.657	1.725	1.786	1.840	1.891	1.930	1.960	66
55	1.044	1.135	1.222	1.309	1.392	1.472	1.551	1.623	1.692	1.753	1.809	1.859	1.901	70
93	.978	1.065	1.150	1.236	1.320	1.399	1.475	1.549	1.618	1.685	1.743	1.801	1.848	73
32	.918	1.000	1.082	1.164	1.246	1.325	1.403	1.476	1.546	1.616	1.675	1.737	1.789	76
67	.844	.922	1.000	1.078	1.156	1.233	1.309	1.383	1.454	1.522	1.587	1.649	1.707	80
05	.777	.850	.925	1.000	1.075	1.150	1.224	1.296	1.363	1.433	1.500	1.562	1.623	84
51	.718	.788	.856	.928	1.000	1.073	1.143	1.212	1.281	1.351	1.414	1.481	1.541	88
03	.665	.731	.797	.863	.931	1.000	1.068	1.138	1.204	1.270	1.334	1.400	1.461	92
57	.617	.679	.741	.803	.868	.935	1.000	1.065	1.130	1.195	1.260	1.321	1.382	96
18	.574	.632	.691	.751	.813	.875	.937	1.006	1.063	1.125	1.187	1.249	1.309	100
83	.535	.589	.645	.703	.760	.820	.880	.938	1.000	1.060	1.120	1.180	1.240	104
51	.500	.552	.603	.658	.714	.770	.826	.883	.941	1.000	1.059	1.115	1.175	108
07	.454	.500	.549	.598	.649	.703	.754	.808	.862	.917	.972	1.026	1.082	114
71	.412	.455	.500	.546	.593	.652	.691	.741	.792	.844	.895	.948	1.000	120

TABLE ONE Cond't.*

WAVE	0-31	0-32	0-33	0-34	0-36	0-38	0-40	0-41	0-42	0-44	0-45	0-46	[illegible]
3	1.500	1.500	.000	1.500	.000	1.500	1.500	1.500	.000	1.500	.000	1.500	.0
3½	1.309	1.809	.191	.691	.691	1.809	.000	1.309	1.809	.691	2.000	.691	1.80
4	1.000	.000	1.000	2.000	.000	2.000	.000	1.000	2.000	.000	1.000	2.000	.00
5	.691	1.809	1.809	.691	.691	1.809	.000	.691	1.809	.691	.000	.691	1.80
5½	1.656	.586	.000	.586	1.940	.159	1.144	1.440	1.656	.000	.586	1.656	1.14
6	.500	1.500	2.000	1.500	.000	1.500	1.500	.500	.000	1.500	2.000	1.500	.00
6½	.212	.054	.754	1.675	1.403	.000	1.403	1.487	1.675	.054	.212	1.082	1.87
7	1.899	1.899	1.223	.377	.377	1.899	1.223	.377	.000	1.223	1.899	1.899	.37
8	.293	.000	.293	1.000	2.000	1.000	.000	-.293	1.000	2.000	1.707	1.000	.0
9	1.940	1.940	1.500	.826	.000	.826	1.940	1.440	1.500	.234	.000	.234	1.50
10	.191	.691	1.309	1.809	1.809	.691	.000	.191	.691	1.809	2.000	1.809	.69
10½	.170	.000	.170	.623	1.707	1.924	1.000	.446	.076	.293	.805	1.375	2.00
11	.585	.159	.000	.159	1.143	1.959	1.653	1.143	.585	.000	.159	.585	1.65
12	1.866	1.500	1.000	.500	.000	.500	1.500	1.866	2.000	1.500	1.000	.500	.00
12½	1.946	1.987	1.789	1.403	.453	.000	.453	.917	1.403	1.987	1.946	1.674	.75
13	1.748	1.971	1.971	1.748	.880	.115	.115	.432	.880	1.748	1.971	1.971	.35
14	.777	1.223	1.623	1.899	1.844	1.223	.377	.099	.000	.327	.776	1.222	1.8
15	.086	.331	.691	1.104	1.809	1.978	1.500	1.104	.691	.086	.000	.086	.69
16	.076	.000	.076	.293	1.000	1.707	2.000	1.924	1.707	1.000	.617	.293	.00
17	.554	.259	.067	.000	.259	.409	1.603	1.851	1.983	1.851	1.603	1.273	.55
18	1.175	.826	.500	.234	.000	.234	.826	1.175	1.500	1.940	2.000	1.940	1.50
19	1.676	1.403	1.081	.754	.209	.000	.209	.454	.754	1.403	1.676	1.879	1.98
20	1.951	1.809	1.588	1.309	.691	.191	.000	.049	.191	.691	1.000	1.309	1.80
21	1.989	1.987	1.899	1.733	1.223	.635	.173	.044	.000	.173	.377	.635	1.2
22	1.836	1.959	2.000	1.959	1.653	1.142	.586	.345	.159	.000	.040	.159	.5
23	1.577	1.775	1.918	1.990	1.918	1.577	1.069	.795	.540	.146	.037	.000	.14
24	1.258	1.500	1.707	1.867	2.000	1.867	1.500	1.258	1.000	.500	.293	.134	.00
25	.937	1.187	1.426	1.637	1.930	1.992	1.809	1.637	1.426	.937	.691	.464	.12
26	.645	.880	1.119	1.356	1.748	1.970	1.971	1.885	1.748	1.356	1.119	.880	.43
27	.403	.604	.826	1.056	1.500	1.834	1.993	1.993	1.940	1.684	1.500	1.285	.82
28	.218	.377	.568	.777	1.224	1.623	1.900	1.975	2.000	1.900	1.782	1.623	1.22
29	.092	.204	.353	.532	.945	1.368	1.725	1.857	1.947	1.994	1.947	1.857	[illegible]
30	.022	.086	.190	.331	.691	1.104	1.500	1.668	1.809	1.979	2.000	1.979	[illegible]
32	.019	.000	.019	.076	.293	.618	1.000	1.195	1.383	1.707	1.831	1.924	2.00
34	.151	.067	.017	.000	.067	.260	.554	.727	.908	1.273	1.445	1.603	1.85
36	.358	.234	.134	.060	.000	.060	.234	.357	.500	.826	1.000	1.175	1.50
38	.597	.454	.322	.209	.054	.000	.054	.121	.210	.454	.592	.754	1.08
40	.843	.691	.546	.412	.191	.049	.000	.012	.049	.191	.293	.412	.69
41	.962	.807	.661	.522	.280	.104	.012	.000	.012	.104	.182	.280	.52
42	1.075	.925	.777	.635	.377	.175	.044	.011	.000	.044	.099	.173	.37
44	1.283	1.143	1.000	.857	.585	.347	.159	.090	.039	.000	.010	.039	.15
46	1.460	1.335	1.203	1.069	.795	.540	.318	.226	.146	.037	.009	.000	.03
48	1.608	1.500	1.383	1.258	1.000	.740	.500	.392	.293	.134	.076	.033	.00
50	1.729	1.637	1.536	1.426	1.187	.937	.691	.574	.464	.271	.191	.134	.03
52	1.824	1.748	1.663	1.568	1.356	1.120	.880	.760	.646	.432	.337	.250	.11
54	1.894	1.836	1.766	1.687	1.500	1.285	1.059	.941	.826	.604	.500	.403	.2
57	1.962	1.926	1.879	1.823	1.675	1.500	1.297	1.190	1.082	.862	.754	.649	.45
60	1.996	1.978	1.951	1.914	1.809	1.669	1.500	1.405	1.309	1.105	1.000	.895	.69
63	1.999	1.999	1.989	1.969	1.901	1.788	1.661	1.586	1.500	1.321	1.223	1.125	.9
66	1.982	1.996	2.000	1.996	1.960	1.890	1.786	1.725	1.656	1.582	1.414	1.328	1.14
70	1.936	1.964	1.984	1.996	1.995	1.964	1.901	1.859	1.809	1.690	1.623	1.551	1.39
73	1.891	1.926	1.956	1.988	1.999	1.992	1.955	1.926	1.891	1.801	1.744	1.685	1.54
76	1.837	1.879	1.917	1.946	1.987	2.000	1.987	1.969	1.946	1.879	1.836	1.790	1.67
80	1.761	1.809	1.853	1.891	1.951	1.988	2.000	1.997	1.988	1.951	1.924	1.891	1.80
84	1.680	1.733	1.782	1.827	1.901	1.956	1.989	1.997	2.000	1.989	1.976	1.956	1.90
88	1.599	1.657	1.707	1.755	1.841	1.909	1.960	1.977	1.990	2.000	1.998	1.990	1.96
92	1.520	1.578	1.633	1.684	1.776	1.854	1.918	1.941	1.963	1.991	1.998	2.000	1.99
96	1.442	1.500	1.557	1.609	1.707	1.794	1.867	1.897	1.925	1.965	1.982	1.992	2.00
100	1.368	1.426	1.482	1.536	1.637	1.729	1.809	1.844	1.876	1.930	1.951	1.969	1.99
104	1.297	1.356	1.411	1.465	1.568	1.664	1.749	1.787	1.823	1.885	1.912	1.935	1.97
108	1.230	1.286	1.343	1.396	1.500	1.598	1.685	1.728	1.767	1.836	1.867	1.892	1.94
114	1.138	1.191	1.233	1.297	1.402	1.500	1.593	1.636	1.675	1.754	1.788	1.802	1.87
120	1.053	1.104	1.156	1.208	1.309	1.406	1.500	1.546	1.588	1.669	1.707	1.743	1.80

*COLUMN HEADING 0 - 31 IS 0 - ½(31A+31B). ETC. FOR METHOD 9-B SUBTRACT THESE FIGURES FROM 2.000.

0-[illegible]	0-52	0-54	0-56	0-58	0-60	0-62	0-64	0-66	0-70	0-76	0-80	0-84	WAVE
.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	1.500	.000	3
.000	1.809	.691	.691	1.809	.000	1.809	.691	.691	.000	.691	.000	.691	3⅓
2.000	.000	2.000	.000	2.000	.000	2.000	.000	2.000	2.000	.000	.000	.000	4
.000	1.809	.691	.691	1.809	.000	1.809	.691	.691	.000	.691	.000	.691	5
.159	1.940	.586	.586	1.940	.159	1.144	1.656	.000	1.144	.586	1.940	1.144	5½
1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	.000	1.500	1.500	1.500	.000	6
.212	.754	1.987	.454	.454	1.987	.754	.212	1.879	.054	.000	1.675	1.082	6⅔
.377	1.899	1.223	.000	1.223	1.899	.377	.377	1.899	.000	.377	1.899	.000	7
.000	2.000	1.000	.000	1.000	2.000	1.000	.000	1.000	1.000	2.000	.000	2.000	8
1.940	.826	.000	.826	1.940	1.500	.234	.234	1.500	.826	1.940	.234	1.500	9
.000	.691	1.809	1.809	.691	.000	.691	1.809	1.809	.000	1.809	.000	1.809	10
1.375	.293	.076	1.000	1.924	1.707	.623	.000	.623	1.924	.293	2.000	.293	10⅔
1.959	1.143	.159	.159	1.143	1.959	1.653	.585	.000	1.653	.159	1.143	1.653	11
.500	1.500	2.000	1.500	.500	.000	.500	1.500	2.000	.500	1.500	1.500	.000	12
.054	.210	1.083	1.879	1.879	1.083	.210	.054	.753	1.987	.000	1.403	1.674	12⅔
.432	.000	.432	1.356	1.971	1.748	.880	.115	.116	1.748	.432	.432	1.971	13
.899	1.223	.377	.000	.377	1.223	1.899	1.899	1.223	.000	1.899	1.223	.000	14
1.500	1.978	1.809	1.104	.331	.000	.331	1.104	1.809	1.500	1.809	1.500	1.809	15
.293	1.000	1.707	2.000	1.707	1.000	.293	.000	.293	1.707	1.000	.000	1.000	16
.067	.067	.554	1.273	1.851	1.983	1.603	.909	.259	.259	1.983	1.273	.067	17
.826	.234	.000	.234	.826	1.500	1.940	1.940	1.500	.234	.826	1.940	1.500	18
1.676	1.081	.454	.054	.054	.454	1.081	1.676	1.986	1.403	.000	.754	1.879	19
2.000	1.809	1.309	.691	.191	.000	.191	.691	1.309	2.000	1.309	.000	.691	20
1.733	1.989	1.899	1.500	.925	.377	.044	.044	.377	1.500	1.733	.635	.000	21
.142	1.653	1.960	1.960	1.653	1.142	.586	.159	.000	.585	1.959	1.653	.585	22
.540	1.069	1.577	1.918	1.990	1.775	1.335	.795	.318	.037	1.335	1.990	1.577	23
.133	.500	1.000	1.500	1.866	2.000	1.867	1.500	1.000	.134	.500	1.500	2.000	24
.000	.120	.464	.937	1.426	1.809	1.992	1.930	1.637	.691	.031	.691	1.637	25
.115	.000	.115	.432	.880	1.356	1.748	1.971	1.972	1.356	.115	.115	.880	26
.403	.107	.000	.107	.403	.826	1.285	1.684	1.940	1.834	.604	.027	.234	27
.777	.377	.099	.000	.099	[illegible]	.777	1.223	1.623	2.000	1.223	.377	.000	28
[illegible]	.732	.353	.092	.000	.092	.353	.732	1.161	1.857	1.725	.945	.204	29
[illegible]	1.104	.691	.331	.088	.000	.088	.331	.691	1.500	1.979	1.500	.691	30
1.924	1.707	1.383	1.000	.618	.293	.077	.000	.077	.617	1.707	2.000	1.707	32
1.982	1.982	1.850	1.603	1.273	.908	.554	.259	.067	.067	.909	1.603	1.983	34
1.765	1.940	2.000	1.940	1.766	1.500	1.175	.826	.500	.060	.234	.826	1.500	36
1.402	1.677	1.878	1.986	1.986	1.878	1.675	1.403	1.082	.454	.000	.210	.754	38
1.000	1.309	1.588	1.809	1.951	2.000	1.951	1.809	1.588	1.000	.191	.000	.191	40
.808	1.113	1.409	1.666	1.861	1.974	1.997	1.974	1.771	1.264	.393	.046	.044	41
.634	.925	1.223	1.500	1.733	1.903	1.989	1.989	1.902	1.500	.635	.173	.000	42
.347	.585	.857	1.143	1.414	1.653	1.836	1.959	2.000	1.836	1.143	.585	.159	44
.146	.318	.540	.795	1.069	1.335	1.577	1.775	1.918	1.990	1.577	1.069	.540	46
.034	.134	.293	.500	.740	1.000	1.258	1.500	1.707	1.966	1.867	1.500	1.000	48
.000	.031	.124	.271	.464	.691	.937	1.187	1.426	1.809	1.992	1.809	1.426	50
.030	.000	.030	.115	.250	.432	.646	.878	1.120	1.568	1.971	1.972	1.748	52
.107	.027	.000	.027	.107	.234	.403	.604	.826	1.281	1.836	1.994	1.940	54
.284	.148	.054	.006	.006	.054	.148	.284	.454	.862	1.500	1.823	1.987	57
.500	.331	.191	.086	.022	.000	.022	.086	.191	.500	1.105	1.500	1.809	60
.728	.543	.377	.234	.120	.044	.005	.005	.044	.234	.728	1.125	1.500	63
.951	.763	.585	.420	.276	.159	.071	.018	.000	.072	.420	.764	1.143	66
1.223	1.044	.865	.691	.527	.376	.247	.141	.064	.000	.141	.376	.691	70
1.397	1.236	1.064	.893	.725	.564	.416	.285	.176	.033	.033	.176	.417	73
1.547	1.403	1.246	1.083	.917	.755	.597	.454	.322	.121	.000	.054	.211	76
1.707	1.588	1.454	1.309	1.156	1.000	.844	.691	.546	.293	.049	.000	.049	80
1.827	1.733	1.623	1.500	1.363	1.223	1.075	.925	.777	.500	.174	.044	.000	84
1.909	1.841	1.755	1.656	1.541	1.414	1.281	1.144	1.000	.720	.346	.156	.041	88
1.963	1.918	1.854	1.776	1.683	1.578	1.461	1.335	1.204	.933	.540	.315	.146	92
1.992	1.965	1.925	1.867	1.794	1.707	1.608	1.500	1.383	1.130	.740	.500	.293	96
2.000	1.992	1.969	1.930	1.876	1.809	1.729	1.637	1.536	1.309	.937	.691	.464	100
1.993	2.000	1.993	1.971	1.935	1.885	1.822	1.748	1.664	1.465	1.118	.880	.645	104
1.974	1.993	2.000	1.993	1.974	1.940	1.892	1.836	1.767	1.548	1.286	1.056	.826	108
1.926	1.962	1.986	1.998	1.998	1.986	1.962	1.924	1.879	1.754	1.500	1.297	1.082	114
1.866	1.913	1.951	1.978	1.994	2.000	1.994	1.978	1.951	1.866	1.669	1.500	1.309	120

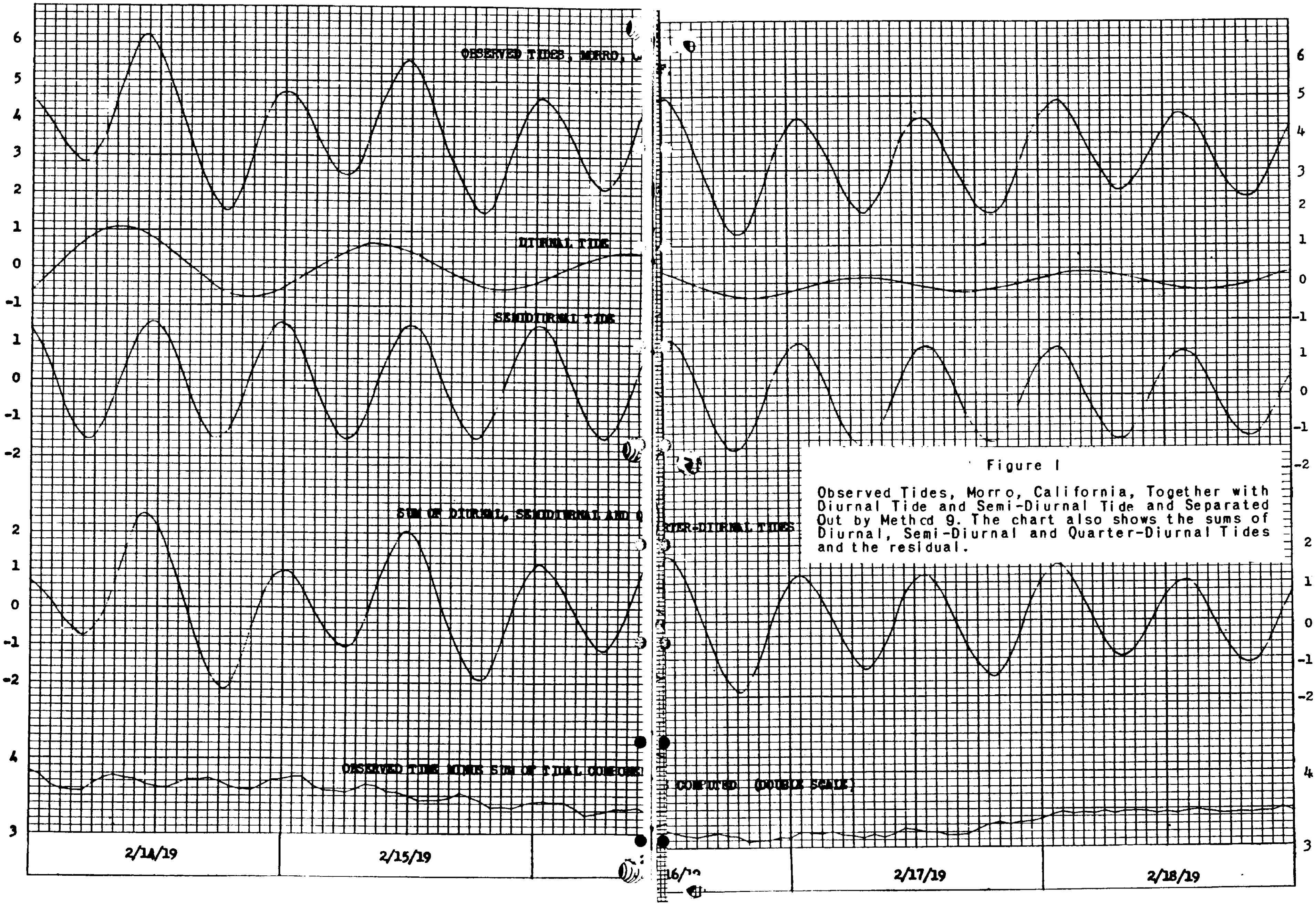


Figure I

Observed Tides, Morro, California, Together with Diurnal Tide and Semi-Diurnal Tide and Separated Out by Method 9. The chart also shows the sums of Diurnal, Semi-Diurnal and Quarter-Diurnal Tides and the residual.

Table 2

Separation of Three Waves, (11, 17, and 24) by Method 9.

11	17	24	SUM	0 · ½ (12A+12B)	0 · ½ (17A+17B) 3.606	(11)	0 · ½ (24A+24B)	0 · ½ (11A+11B) 2.966	(17)	0 · ½ (11A+11B)	0 · ½ (17A+17B) 2.475	(24)
.000	.000	.000	.000									
.541	.361	.259	1.161									
.910	.674	.500	2.084									
.990	.895	.707	2.592									
.757	.996	.866	2.619									
.282	.962	.966	2.210									
.282	.798	1.000	1.516									
.757	.526	.966	.735									
.990	.184	.866	.060									
.910	.184	.707	.367									
.541	.526	.500	.567									
.000	.798	.259	.539							.778		
.541	.962	.000	.421	.247						1.541		
.910	.996	.259	.345	.952						2.155		
.990	.895	.500	.405	1.173						2.417		
.757	.674	.707	.624	.403						[illegible]		
.282	.361	.866	.945	.257						2.261		
.282	.000	.966	1.248	.320						1.699		
.757	.361	1.000	1.396	1.131						1.387		
.990	.674	.966	1.282	[illegible]						.519		
.910	.895	.866	.881	1.076						.269		
.541	.996	.707	[illegible]	[illegible]						.206		
.000	.962	.500	.462	.649						.558		
.541	.798	.259	[illegible]	1.575						.770		
.910	.526	.000	1.436	2.057			1.505			.844		
.990	.184	.259	1.433	1.967			.919			.803		
.757	.184	.500	1.073	1.259			.103			.688		
.282	.526	.707	.463	.138			[illegible]			.545		
.282	.798	.866	.314	1.044			1.692			.423	2.143	.866
.757	.962	.966	.753	2.092	2.738	.757	2.222			.357	2.389	.966
.990	.996	1.000	.986	1.552	3.528	.991	2.421			.270	2.475	1.000
.910	.895	.966	[illegible]	2.325	3.283	.910	[illegible]			.464	2.389	.966
.541	.674	.866	[illegible]	1.485	1.451	.541	[illegible]			.622	2.143	.866
.000	.361	.707	.346	.212	.000	.000	[illegible]			.811	1.749	.707
.541	.000	.500	1.041	.996	1.452	.541	.316			.983	1.237	.500
.910	.361	.259	1.530	1.932	[illegible]	.910	1.888	1.072	.361	1.088	.641	.259
.990	.674	.000	1.664	2.313	3.573	.990	1.825	1.997	.674	1.080	.000	.000
.757	.895	.259	1.393	2.043	2.729	.757	2.099	2.654	.895	.926	.640	.259
.282	.996	.500	.778	1.249	1.019	.282	2.000	2.453	.996	.613	1.237	.500
.282	.962	.707	.027	.180	1.017	.282	1.614	2.150	.962	.151	1.750	.707
.757	.798	.866	.825	.813	2.728	.757	1.033	2.365	.798	.429	2.144	.866
.990	.526	.966	1.430	1.441	3.528	[illegible]	.245	1.560	.526	1.055	2.390	.966
.910	.184	1.000	1.726	1.542	3.283	[illegible]	.191	.545	.184	1.671	2.475	1.000
[illegible]	.532	.866	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.545	[illegible]	[illegible]	2.390	.866
.541	.798	.707	[illegible]	.416	1.452	[illegible]	1.160	2.366	.798	2.669	1.749	.707
.910	.962	.500	[illegible]	.996	2.323	[illegible]	1.242	2.852	.962	2.574	1.237	.500
.990	.996	.259	[illegible]	1.100	3.273	.990	[illegible]	2.952	.996	2.105	.641	.259
.757	.895	.000	.138	.743	2.729	.757	1.213	2.654	.895	1.434	.000	.000
.282	.674	[illegible]	.130	.030	1.019	.282	1.021	1.997	.674	.571	.641	.259
.353	.361	[illegible]	.143	.781	1.017	.282	.833	1.071	.361	.404	1.237	.500
[illegible]	.000	.707	[illegible]	1.393	2.728	.757	[illegible]	[illegible]	.000	1.390	1.750	.707
.990	.361	.866	.237	1.561	3.573	.990	.090	1.072	.361	2.281	2.143	.866
.910	.674	.966	.730	1.166	3.283	.910	.715	1.997	.674	2.479	2.391	.966
.541	.895	1.000	1.354	.242	1.921	.541	1.340	2.654	.895	3.400	2.475	1.000
.000	.996	.966	1.962	.724	.000	.000	1.843	2.853	.996	3.445	2.390	.966
.541	.962	.866	2.369	1.645	1.452	.541	2.097	2.853	.962	3.244	2.143	.866
.910	.798	.707	2.415	2.255	3.283	.910	2.008	2.346	.798	2.669	1.750	.707
.990	.526	.500	2.016	2.206	3.573	.990	1.552	1.561	.524	1.826	1.237	.500
.757	.184	.259	1.200	1.526			.784	.545	.188	.804	.641	.259
.282	.184	.000	.098	.386			.175	.545	.184	.344	.000	.000
.282	.526	.259	1.067	.952			1.139			1.352		
.757	.798	.500	2.055	1.973			1.920			2.262		
.990	.962	.707	2.659	2.223			2.159			2.921		
.910	.996	.866	2.772	2.399			2.374			3.298		
.541	.895	.966	2.402	1.646			1.972			3.333		
.000	.674	1.000	1.674	.481			1.247			2.946		
.541	.361	.966	.782	[illegible]			.358			2.472		
.910	.000	.866	.044	1.675			.531			1.702		
.990	.361	.707	.644	2.085			1.246			.811		
.757	.674	.500	.931	1.882			1.689			.047		
.282	.895	.259	.918	1.169			1.821			.485		
.282	.996	.000	.714	.204						1.596		
.757	.962	.259	.464	.694						2.050		
.990	.798	.500	.308	.244						2.362		
.910	.526	.707	.323	[illegible]						2.333		
.541	.184	.866	.509							1.997		
.000	.184	.966	.782							1.604		
.541	.526	1.000	1.015							1.188		
.910	.798	.966	1.078							.488		
.990	.962	.866	.894							.161		
.959	.996	.707	.460							.306		
.282	.895	.500	.113							.451		
.282	.674	.259	.647							.571		
.757	[illegible]	.000	1.118							.579		
.990	.000	.259	1.249									
.110	.361	.500	1.649									
.541	.674	.707	.529									
.000	.895	.866	.024									
.541	.996	.966	.571									
.910	.962	1.000	.872									
.990	.798	.966	.888									
.757	.526	.866	.417									
.282	.184	.707	.241									
.282	.184	.500	.966									
[illegible]	[illegible]	.259	1.542									

This test shows that three waves, bearing non-harmonic relationship, can be accurately separated by Method #9, if not too nearly of the same length.

Table 3

Trial Separation of Waves and Trend Line by Methods 9-A and 9-B.

TREND LINE	WAVES 20	36	SUM	0 - 1 (10A+10B)	0 + 1 (18A+18B)	÷ 3.62	0 - 1 (20A 20B)	÷ 1.94	0 + 1 (10A+10B) ÷ 2	0 + 1 (18A+18B) ÷ 2
						(20)		(36)		Trend Line
80.0	10.0	.0	90.0							
80.5	9.5	1.7	91.7							
81.0	8.1	3.4	92.5							
81.5	5.9	5.0	92.4							
82.0	3.1	6.4	91.5							
82.5	.0	7.7	90.2							
83.0	3.1	8.7	88.6							
83.5	5.9	9.4	87.0							
84.0	8.1	9.8	85.7							
84.5	9.5	10.0	85.0							
85.0	10.0	9.8	84.8	5.50					89.05	
85.5	9.5	9.4	85.4	[illegible]					89.32	
86.0	8.1	8.7	86.6	[illegible]					89.60	
86.5	5.9	7.7	88.3	[illegible]					89.67	
87.0	3.1	6.4	90.3	1.35					89.62	
87.5	.0	5.0	92.5	5.85					89.57	
88.0	3.1	3.4	94.5	10.15					89.42	
88.5	5.9	1.7	96.1	13.80					89.20	
89.0	8.1	.0	97.1	16.20					89.00	
89.5	9.5	1.7	97.3	17.00					88.80	
90.0	10.0	3.4	96.6	16.05			6.60	3.4	88.57	
90.5	9.5	5.0	95.0	13.15			9.70	5.0	88.42	
91.0	8.1	6.4	92.7	8.65			12.45	6.4	88.37	
91.5	5.9	7.7	89.7	2.75			14.90	7.7	88.32	
92.0	3.1	8.7	86.4	4.00			16.80	8.7	88.40	
92.5	.0	9.4	83.1	11.05			18.25	9.4	88.62	
93.0	3.1	9.8	80.1	17.70			19.05	9.8	88.95	
93.5	5.9	10.0	77.6	23.60			19.40	10.0	89.35	
94.0	8.1	9.8	76.1	27.70	29.30	8.1	19.05	9.8	89.95	94.0
94.5	9.5	9.4	75.6	30.05	34.40	9.5	18.25	9.4	90.62	94.5
95.0	10.0	8.7	76.3	30.20	36.20	10.0	16.80	8.7	91.40	95.0
95.5	9.5	7.7	78.3	28.05	34.40	9.5	14.90	7.7	92.32	95.5
96.0	8.1	6.4	81.5	23.75	29.30	8.1	12.45	6.4	93.37	96.0
96.5	5.9	5.0	85.6	17.60	21.30	5.9	9.70	5.0	94.42	96.5
97.0	3.1	3.4	90.5	10.15	11.20	3.1	6.60	3.4	95.57	97.0
97.5	.0	1.7	95.8	2.00	.00	.0	3.35	1.7	96.80	97.5
98.0	3.1	.0	101.1	6.00	11.20	3.1	.00	.0	98.00	98.0
98.5	5.9	1.7	106.1	13.80	21.30	5.9	3.35	1.7	99.20	98.5
99.0	8.1	3.4	110.5	20.15	29.30	8.1	6.60	3.4	100.42	99.0
99.5	9.5	5.0	114.0	24.85	34.40	9.5	9.70	5.0	101.57	99.5
100.0	10.0	6.4	116.4	27.35	36.20	10.0	12.45	6.4	102.62	100.0
100.5	9.5	7.7	117.7	28.05	[illegible]	9.5	14.90	7.7	103.67	100.5
101.0	8.1	8.7	117.8	26.40	29.30	8.1	16.80	8.7	104.60	101.0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
102.0	3.1	9.8	114.9	17.70	11.20	3.1	19.05	9.8	106.05	102.0
102.5	.0	10.0	112.5	11.70	.00	.0	19.40	10.0	106.65	102.5
103.0	3.1	9.8	109.7	[illegible]	[illegible]	3.1	19.05	9.8	107.05	103.0
103.5	5.9	9.4	107.0	.75	21.30	5.9	18.25	9.4	107.37	103.5
104.0	8.1	8.7	104.6	6.00	29.30	8.1	16.80	8.7	107.60	104.0
104.5	9.5	7.7	102.7	[illegible]	34.40	9.5	14.90	7.7	107.67	104.5
105.0	10.0	6.4	101.4	[illegible]	36.20	10.0	12.45	6.4	107.62	105.0
105.5	9.5	5.0	101.0	13.15	34.40	9.5	9.70	5.0	107.57	105.5
106.0	8.1	3.4	101.3	[illegible]			6.60	3.4	107.42	
106.5	5.9	1.7	102.3	9.20			3.35	1.7	107.20	
107.0	3.1	.0	103.9	[illegible]			.00	.0	107.00	
107.5	.0	1.7	105.8	2.00			3.35	1.7	106.80	
108.0	3.1	3.4	107.7	2.25			6.60	3.4	106.57	
108.5	5.9	5.0	109.4	5.95			9.70	5.0	106.42	
109.0	8.1	6.4	110.7	8.65			12.45	6.4	106.37	
109.5	9.5	7.7	111.2	9.95			14.90	7.7	106.32	
110.0	10.0	8.7	111.3	9.80					106.40	
110.5	9.5	9.4	110.6	7.95					106.62	
111.0	8.1	9.8	109.3	4.70					106.95	
111.5	5.9	10.0	107.4	.10					107.35	
112.0	3.1	9.8	105.3	5.30					107.95	
112.5	.0	9.4	103.1	11.05					108.62	
113.0	3.1	8.7	101.2	16.40					109.40	
113.5	5.9	7.7	99.9	[illegible]					110.32	
114.0	8.1	6.4	99.5	[illegible]					111.37	
114.5	9.5	5.0	100.0	[illegible]					112.42	
115.0	10.0	3.4	101.6							
115.5	9.5	1.7	104.3							
116.0	8.1	.0	107.9							
116.5	5.9	1.7	112.3							
117.0	3.1	3.4	117.3							
117.5	.0	5.0	122.5							
118.0	3.1	6.4	127.5							
118.5	5.9	7.7	132.1							
119.0	8.1	8.7	135.8							
119.5	9.5	9.4	138.4							

Column 4 consists of two waves and straight trend line. The first wave is separated in Column 7, the second in Column 9, and the trend line in Column 11. The trend line could, of course, be separated by subtracting the two waves from Col. 4.

Table 4
Amplitude Reduction of 60 Months Sine Waves
By Moving Average of Various Lengths.

1	1.0000	61	.0164	121	.0082	181	.0055	241	.0041	301	.0033
2	.9986	62	.0322	122	.0164	182	.0110	242	.0082	302	.0066
3	.9963	63	.0474	123	.0243	183	.0163	243	.0123	303	.0099
4	.9932	64	.0621	124	.0320	184	.0216	244	.0163	304	.0131
5	.9891	65	.0761	125	.0396	185	.0267	245	.0202	305	.0162
6	.9841	66	.0895	126	.0469	186	.0317	246	.0240	306	.0193
7	.9782	67	.1022	127	.0539	187	.0366	247	.0277	307	.0223
8	.9714	68	.1143	128	.0607	188	.0413	248	.0313	308	.0252
9	.9638	69	.1257	129	.0672	189	.0459	249	.0348	309	.0281
10	.9554	70	.1365	130	.0735	190	.0503	250	.0382	310	.0308
11	.9461	71	.1466	131	.0794	191	.0545	251	.0415	311	.0334
12	.9359	72	.1560	132	.0851	192	.0585	252	.0446	312	.0360
13	.9260	73	.1647	133	.0904	193	.0623	253	.0475	313	.0384
14	.9132	74	.1728	134	.0954	194	.0659	254	.0503	314	.0407
15	.9007	75	.1801	135	.1001	195	.0693	255	.0530	315	.0429
16	.8875	76	.1868	136	.1044	196	.0724	256	.0555	316	.0450
17	.8732	77	.1928	137	.1084	197	.0754	257	.0577	317	.0468
18	.8588	78	.1982	138	.1122	198	.0781	258	.0599	318	.0486
19	.8434	79	.2028	139	.1153	199	.0805	259	.0618	319	.0502
20	.8274	80	.2068	140	.1182	200	.0827	260	.0636	320	.0517
21	.8107	81	.2102	141	.1207	201	.0847	261	.0652	321	.0530
22	.7934	82	.2129	142	.1229	202	.0864	262	.0666	322	.0542
23	.7756	83	.2149	143	.1247	203	.0879	263	.0678	323	.0552
24	.7572	84	.2163	144	.1262	204	.0891	264	.0688	324	.0561
25	.7382	85	.2171	145	.1273	205	.0900	265	.0696	325	.0568
26	.7188	86	.2173	146	.1280	206	.0907	266	.0702	326	.0573
27	.6990	87	.2169	147	.1284	207	.0912	267	.0706	327	.0577
28	.6787	88	.2159	148	.1284	208	.0914	268	.0709	328	.0579
29	.6580	89	.2144	149	.1281	209	.0913	269	.0709	329	.0580
30	.6369	90	.2123	150	.1274	210	.0910	270	.0708--	330	.0579
31	.6155	91	.2097	151	.1264	211	.0904	271	.0704	331	.0576
32	.5938	92	.2065	152	.1250	212	.0896	272	.0699	332	.0573
33	.5719	93	.2029	153	.1233	213	.0886	273	.0691	333	.0567
34	.5497	94	.1988	154	.1214	214	.0873	274	.0682	334	.0560
35	.5273	95	.1943	155	.1191	215	.0858	275	.0671	335	.0551
36	.5048	96	.1893	156	.1165	216	.0841	276	.0658	336	.0540
37	.4821	97	.1839	157	.1136	217	.0822	277	.0644	337	.0529
38	.4593	98	.1781	158	.1104	218	.0801	278	.0627	338	.0516
39	.4365	99	.1720	159	.1071	219	.0777	279	.0610	339	.0502
40	.4137	100	.1655	160	.1034	220	.0752	280	.0591	340	.0486
41	.3908	101	.1587	161	.0995	221	.0725	281	.0570	341	.0470
42	.3680	102	.1515	162	.0954	222	.0696	282	.0548	342	.0452
43	.3453	103	.1442	163	.0911	223	.0666	283	.0525	343	.0433
44	.3227	104	.1365	164	.0866	224	.0634	284	.0500	344	.0413
45	.3002	105	.1287	165	.0819	225	.0600	285	.0474	345	.0391
46	.2779	106	.1206	166	.0770	226	.0566	286	.0447	346	.0369
47	.2558	107	.1124	167	.0720	227	.0530	287	.0419	347	.0347
48	.2340	108	.1040	168	.0668	228	.0492	288	.0390	348	.0323
49	.2124	109	.0955	169	.0616	229	.0454	289	.0360	349	.0298
50	.1911	110	.0868	170	.0562	230	.0415	290	.0329	350	.0273
51	.1701	111	.0781	171	.0507	231	.0375	291	.0298	351	.0247
52	.1495	112	.0694	172	.0452	232	.0335	292	.0266	352	.0221
53	.1292	113	.0606	173	.0396	233	.0294	293	.0234	353	.0193
54	.1093	114	.0518	174	.0339	234	.0252	294	.0201	354	.0167
55	.0899	115	.0430	175	.0283	235	.0210	295	.0168	355	.0139
56	.0709	116	.0342	176	.0225	236	.0168	296	.0134	356	.0111
57	.0524	117	.0255	177	.0169	237	.0126	297	.0101	357	.0084
58	.0344	118	.0169	178	.0112	238	.0084	298	.0067	358	.0056
59	.0169	119	.0084	179	.0056	239	.0042	299	.0033	359	.0028
60	.0000	120	.0000	180	.0000	240	.0000	300	.0000	360	.0000

This table may be used for a wave of any length by proportioning it to 60 months length. Interpolations can also be used; such as a 41 months moving average of 120 months being equal to 20½ months of 60.

Table 5
Amplitude Amplification and Reduction, of Sine Waves of Various Lengths from 10 to 132, for 3 Cycle Moving Periodic Tables of Various Lengths.

WAVE LENGTH	-38 + 0 + 38	-40 + 0 + 40	-41 + 0 + 41	-42 + 0 + 42	-44 + 0 + 44	-45 + 0 + 45
10	1.618	3.000	2.618	1.618	-.618	-1.000
11	-.921	-.313	.237	1.828	3.000	2.681
12	2.000	.000	-.732	-1.000	.000	1.000
13	2.770	2.640	2.134	1.243	-.486	-.942
14	1.552	2.246	2.802	3.000	2.248	1.420
15	-.954	.000	.791	2.618	2.827	3.000
16	-.414	-1.000	-.848	-.414	1.000	1.765
17	1.181	-.206	-.702	-.965	-.702	-.206
18	2.533	1.352	.648	.000	-.880	-1.000
19	3.000	2.579	2.094	1.493	.197	-.352
20	2.618	3.000	2.902	2.618	1.618	1.000
21	1.729	2.652	2.912	3.000	2.652	2.248
22		1.829	2.309	2.681	3.000	2.929
23	-.255	.729	1.408	1.919	2.709	2.926
24	-.734	.000	.478	1.000	2.000	2.414
25	-.984	-.618	-.275	.148	1.126	1.618
26	-.954	-.942	-.770	-.487	.392	.759
27	-.669	-.987	-.983	-.880	-.371	.000
28	-.246	-.802	-.950	-1.000	-.801	-.56[illegible]
29	.264	-.449	-.712	-.895	-.989	-.893
30	.799	.000	-.317	-.618	-.956	-1.000
31	1.300	.501	.130	-.226	-.750	-.909
32	1.775	1.000	.610	.235	-.414	-.661
33	2.169	1.473	1.098	.717	.000	-.311
34	2.472	1.889	1.546	1.186	.454	.109
36	2.880	2.540	2.283	2.000	1.348	1.000
38	3.000	2.892	2.758	2.556	2.092	1.807
40	2.902	3.000	2.975	2.902	2.618	2.414
42	2.652	2.911	2.978	3.000	2.911	2.803
44	2.311	2.680	2.818	2.919	3.000	2.980
46	1.922	2.365	2.551	2.709	2.926	2.982
48	1.519	2.000	2.212	2.414	2.732	2.848
50	1.126	1.618	1.852	2.072	2.458	2.618
52	.762	1.238	1.475	1.709	2.133	2.328
54	.430	.885	1.116	1.348	1.792	2.000
57	.000	.406	.615	.834	1.276	1.493
60	-.336	.000	.183	.382	.791	1.000
63	-.596	-.323	-.167	.000	.360	.569
66	-.782	-.573	-.446	-.313	.000	.171
70	-.924	-.802	-.718	-.618	-.372	-.246
73	-.984	-.910	-.850	-.778	-.597	-.491
76	-1.000	-.973	-.938	-.892	-.759	-.675
80	-.975	-1.000	-.994	-.975	-.902	-.848
84	-.910	-.977	-.994	-1.000	-.977	-.950
88	-.818	-.920	-.953	-.979	-1.000	-.995
92	-.709	-.835	-.884	-.924	-.981	-.995
96	-.588	-.731	-.793	-.848	-.931	-.961
100	-.458	-.618	-.688	-.753	-.860	-.902
104	-.328	-.495	-.573	-.647	-.770	-.823
108	-.196	-.370	-.456	-.533	-.668	-.734
114	.000	-.186	-.274	-.360	-.509	-.580
120	.189	.000	-.092	-.176	-.338	-.414
123	.275	.092	.000	-.083	-.255	-.332
126	.363	.177	.086	.000	-.165	-.250
132	.537	.287	.258	.156	.000	-.082

Table 6

Amplitude and Phase Shift Resulting from Operations by Method 9-C.

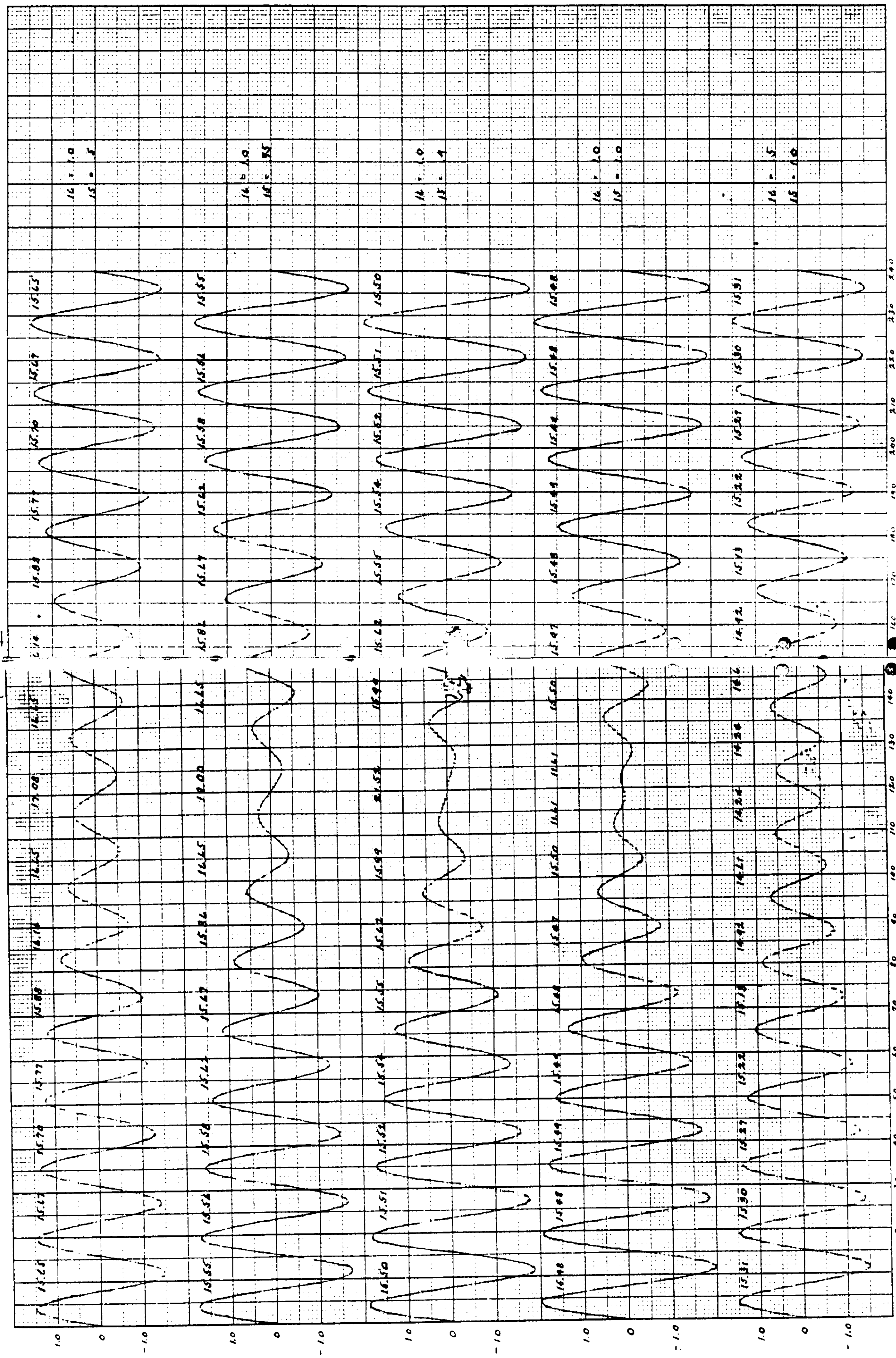
WAVE	0-10 AMPL	0-10 PHASE SHIFT	0-20 AMPL	0-20 PHASE SHIFT	0+20 AMPL	0+20 PHASE SHIFT	0-30 AMPL	0-30 PHASE SHIFT	0+30 AMPL	0+30 PHASE SHIFT
2.00	.000	—								
2.22	1.902	.17								
2.67	1.414	.42								
4.0	.000	—								
4.4	1.607	.45			.000	—				
4.5	1.732	.37			.348	1.00				
5.0	1.902	.25	.000	—	2.000	.00	.000	—	2.000	.00
5.2	1.665	.50	.331	.92	1.770	.45	1.322	.79	1.446	.60
5.5	.952	.44	1.819	.37	.830	1.00	1.980	.12	.282	1.25
5.714	.624	1.14	2.000	.00	.000	—	1.417	.71	1.414	.71
6.0	.000	—	1.732	.50	1.000	1.00	.000	—	2.000	.00
6.5	[illegible]	1.12	.483	1.37	1.942	.25	1.828	.37	.707	1.25
6.67	1.176	1.00	.000	—	2.000	.00	2.000	.00	.000	—
7.0	1.564	.75	.869	1.25	1.801	.50	1.565	.75	1.248	1.00
7.5	1.902	.37	1.732	.62	1.000	1.25	.000	—	2.000	.00
8.0	2.000	.00	2.000	.00	.000		1.414	1.00	1.414	1.00
8.5	1.924	.57	1.790	.62	.891		1.491	.12	.180	2.00
9.0	1.732	.75	1.286	1.25	1.532		1.732	.75	1.000	1.50
9.5	1.491	1.12	.684	1.87	1.892		.452	1.62	1.758	.75
10	1.176	1.50	.000	—	2.000		.000	—	2.000	.00
11	.565	2.25	1.083	1.75	1.682	1.00	1.510	1.25	1.310	1.50
12	.000	—	1.732	1.00	1.000	2.00	2.000	.00	.000	—
13	.478	2.75	1.486	.25	.249	3.00	1.641	1.35	1.136	2.00
13.33	.618	2.67	2.000	.00	.000	—	1.414	1.67	1.414	1.67
14	.869	2.50	1.950	.50	.446	3.00	.869	2.50	1.801	1.00
15	1.176	2.25	1.732	1.75	1.000	2.50	.000	—	2.000	.00
16	1.414	2.00	1.414	2.10	1.414	2.00	.765	3.00	1.848	1.00
17	1.682	1.75	1.052	2.25	1.700	1.50	1.346	2.25	1.478	2.00
18	1.732	1.50	.686	3.50	1.880	1.00	1.732	1.50	1.000	3.00
19	1.831	1.25	.325	4.25	1.978	.50	1.439	.75	.490	4.00
20	1.902	1.00	.000	—	2.000	.00	2.000	.00	.000	—
21	1.950	.75	.308	4.75	1.978	.50	1.950	.75	.446	4.50
22	1.980	.50	.565	4.50	1.919	1.00	1.819	1.50	.824	4.00
24	2.000	.00	1.000	4.00	1.732	2.00	1.414	3.00	1.414	3.00
26	1.985	.50	1.328	3.50	1.497	3.00	.930	4.50	1.777	2.00
28	1.950	1.00	1.563	3.00	1.247	4.00	.447	[illegible]	1.944	1.00
30	1.902	1.50	1.732	2.50	1.000	5.00	.000	—	2.000	.00
32	1.848	2.00	1.848	2.00	.765	6.00	.387	7.00	1.962	1.00
34	1.790	2.50	1.924	1.50	.546	7.00	.724	6.50	1.865	2.00
36	1.732	3.00	1.932	1.00	.342	8.00	1.000	6.00	1.732	3.00
38	1.674	3.50	1.943	.50	.166	9.00	1.228	5.50	1.574	4.00
40	1.618	4.00	2.000	.00	.000	—	1.414	5.00	1.414	5.00
42	1.564	4.50	1.944	.50	.151	10.0	1.564	4.50	1.247	6.00
44	1.512	5.00	1.989	1.00	.283	10.0	1.682	4.00	1.080	7.00
46	1.463	5.50	1.958	1.50	.406	10.0	1.776	3.50	.920	8.00
50	1.369	6.50	1.902	2.00	.618	10.0	1.902	2.50	.618	10.00
54	1.286	7.50	1.835	3.00	.742	10.0	1.970	1.50	.348	12.00
60	1.176	9.00	1.732	5.00	1.000	10.0	2.000	.00	.000	—
66	1.081	10.50	1.629	6.50	1.160	10.0	1.980	1.50	.286	15.0
76	.951	13.00	1.472	9.00	1.355	10.0	1.892	4.70	.648	15.0
84	.865	15.00	1.360	11.00	1.466	10.0	1.801	6.00	.868	15.0
96	.765	18.00	1.216	14.00	1.586	10.0	1.662	9.00	1.111	15.0
108	.687	21.00	1.090	17.00	1.671	10.0	1.532	12.00	1.884	15.0
120	.618	24.00	1.000	20.00	1.732	10.0	1.414	15.00	1.414	15.0

FORMULAS FOR PHASE SHIFT FOR 0 + X

WAVE LENGTH		
∞	to 2X	Phase shift = $\frac{1}{2}X$
2X	to $\frac{2X}{3}$	Phase shift = $\frac{1}{2}(X - W)$
$\frac{2X}{3}$	to $\frac{2X}{5}$	Phase shift = $\frac{1}{2}(X - 2W)$
$\frac{2X}{5}$	to $\frac{2X}{7}$	Phase shift = $\frac{1}{2}(X - 3W)$
$\frac{2X}{7}$	to $\frac{2X}{9}$	Phase shift = $\frac{1}{2}(X - 4W)$

FORMULAS FOR PHASE SHIFT FOR 0 - X

∞	to X	Phase shift = $\frac{1}{2}(2X - W)$
X	to X/2	Phase shift = $\frac{1}{2}(2X - 3W)$
X/2	to X/3	Phase shift = $\frac{1}{2}(2X - 5W)$
X/3	to X/4	Phase shift = $\frac{1}{2}(2X - 7W)$



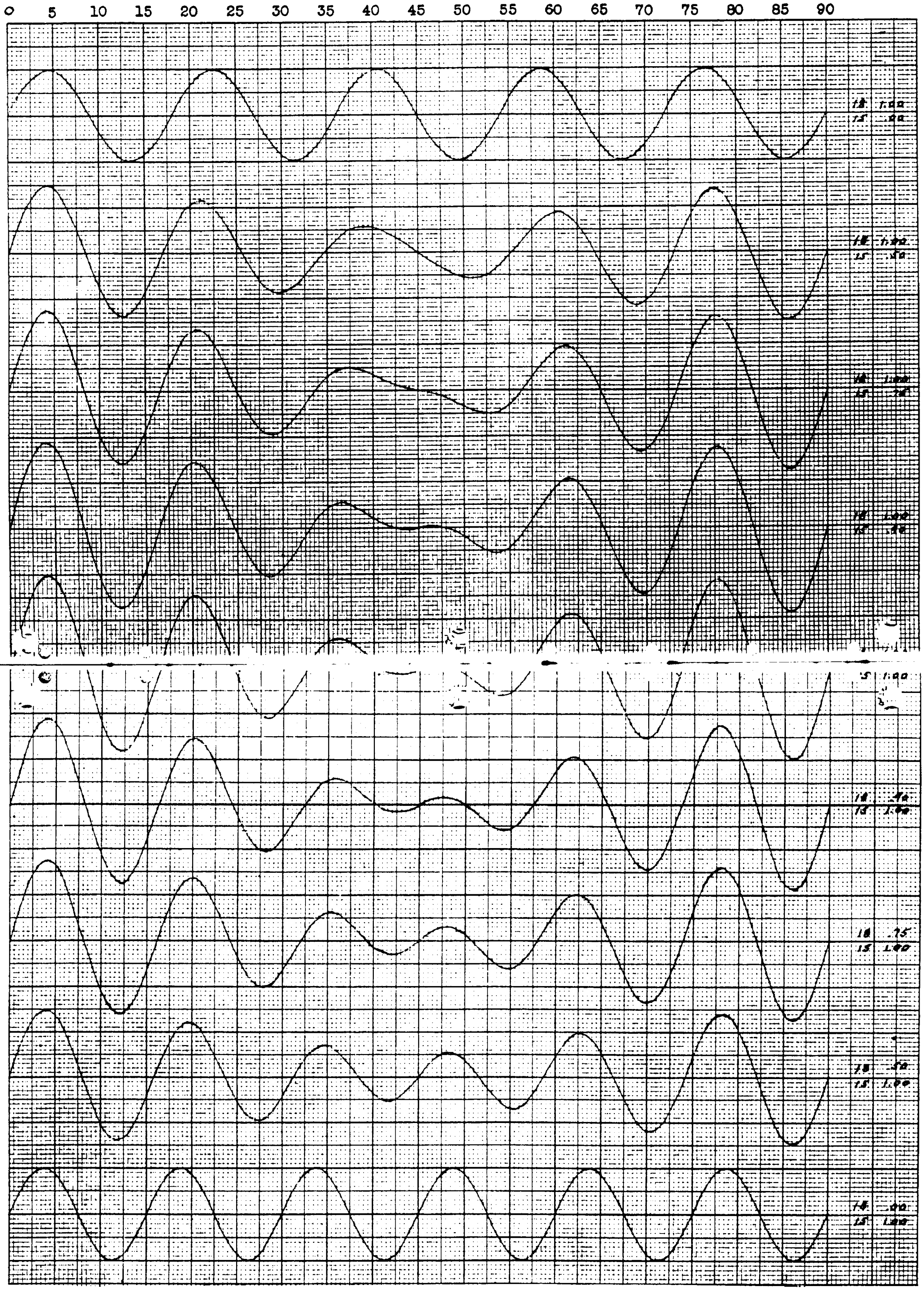


Figure 3

The Summation Sine Waves 18 Units and 15 Units long of Various Relative Amplitudes as Indicated for their Full Synodic Period of 90 Units.

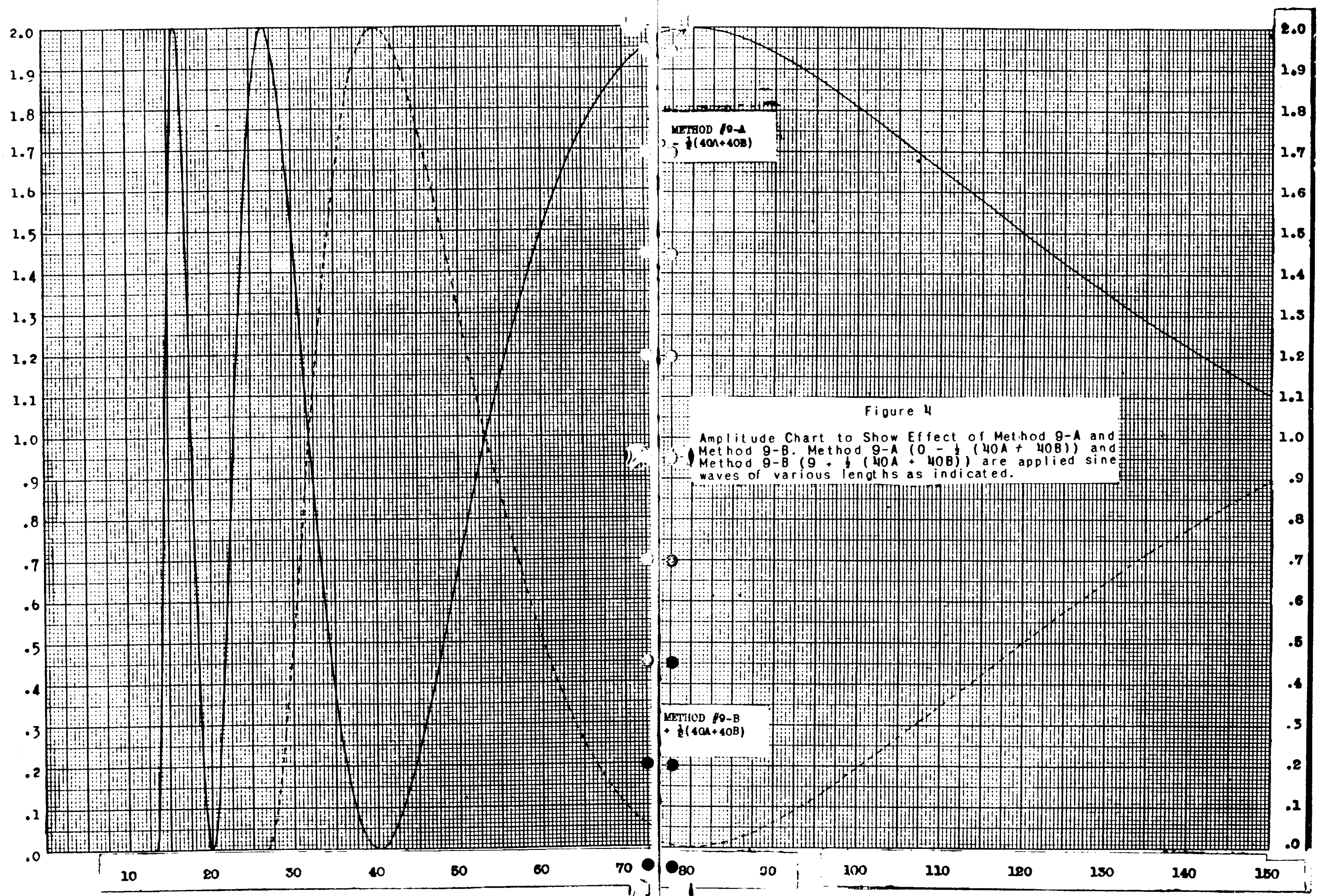


Figure 4

Amplitude Chart to Show Effect of Method 9-A and Method 9-B. Method 9-A (0 - ½ (40A + 40B)) and Method 9-B (9 + ½ (40A + 40B)) are applied sine waves of various lengths as indicated.

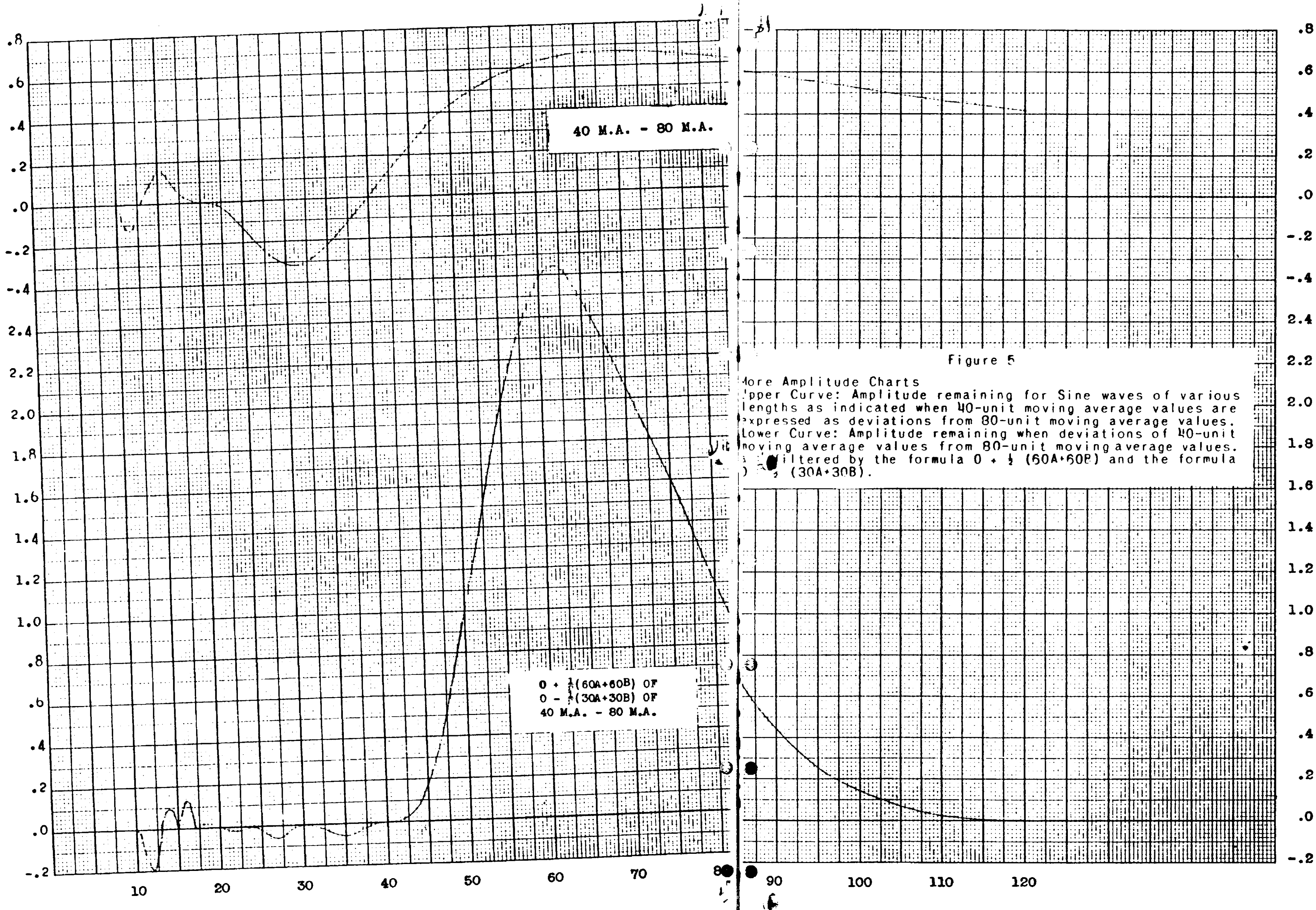


Figure 5

More Amplitude Charts
Upper Curve: Amplitude remaining for Sine waves of various lengths as indicated when 40-unit moving average values are expressed as deviations from 80-unit moving average values.
Lower Curve: Amplitude remaining when deviations of 40-unit moving average values from 80-unit moving average values is filtered by the formula 0 + ½ (60A+60B) and the formula 0 - ½ (30A+30B).

3 CYCLE MOVING PERIODIC TABLES

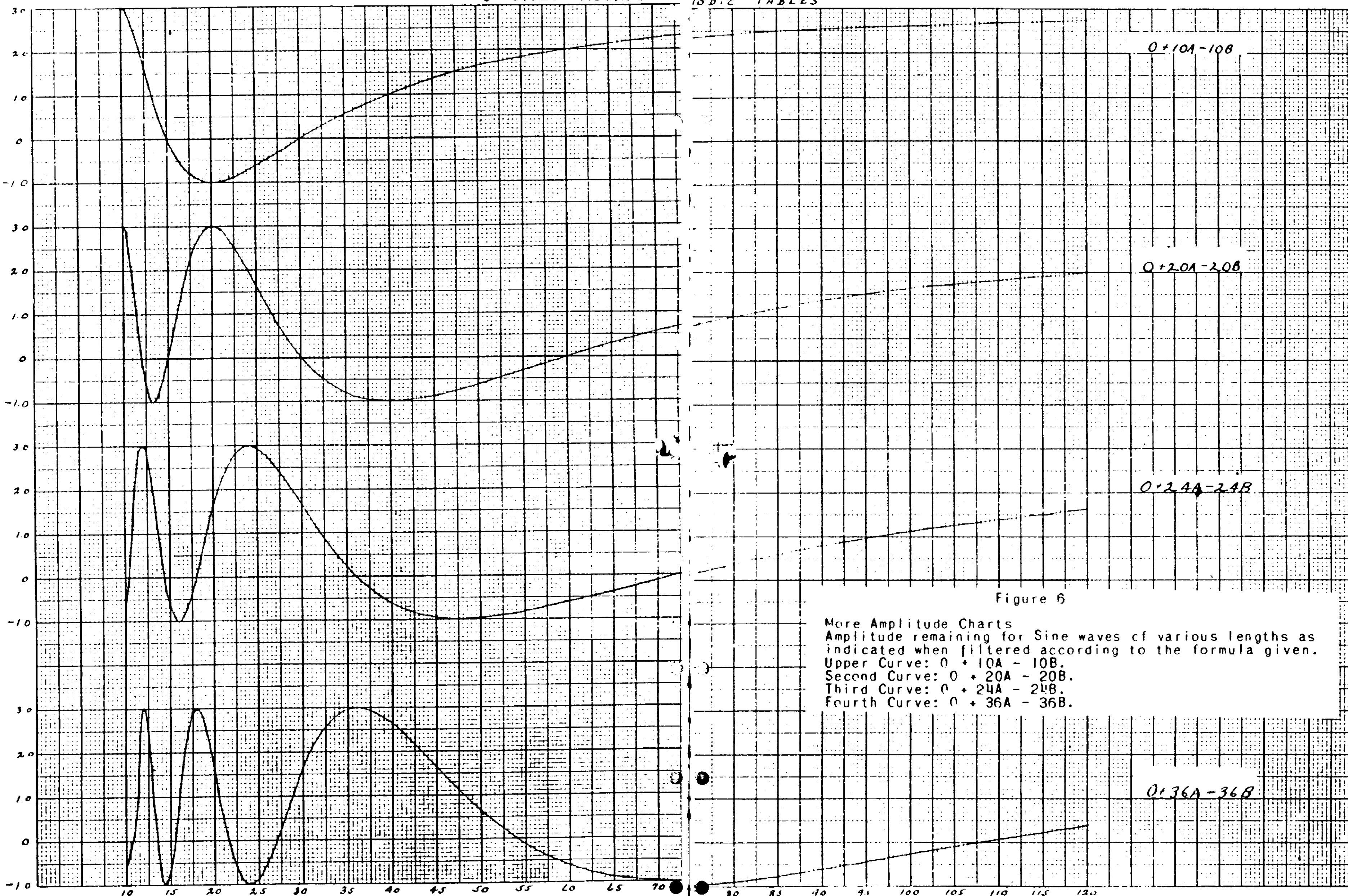


Figure 6

More Amplitude Charts
Amplitude remaining for Sine waves of various lengths as indicated when filtered according to the formula given.
Upper Curve: 0 + 10A - 10B.
Second Curve: 0 + 20A - 20B.
Third Curve: 0 + 24A - 24B.
Fourth Curve: 0 + 36A - 36B.

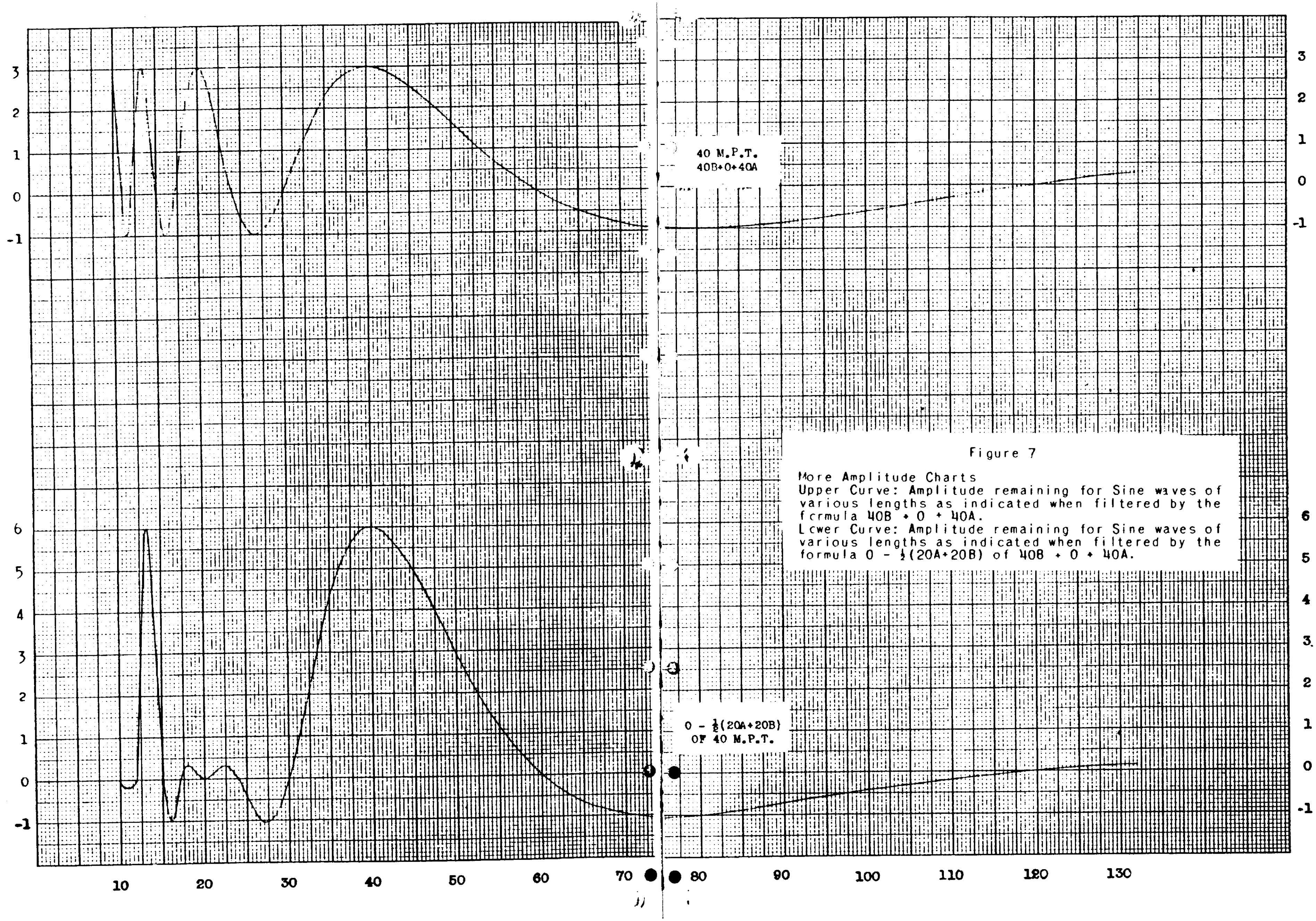


Figure 7

More Amplitude Charts
Upper Curve: Amplitude remaining for Sine waves of various lengths as indicated when filtered by the formula 40B + 0 + 40A.
Lower Curve: Amplitude remaining for Sine waves of various lengths as indicated when filtered by the formula 0 - ½(20A+20B) of 40B + 0 + 40A.

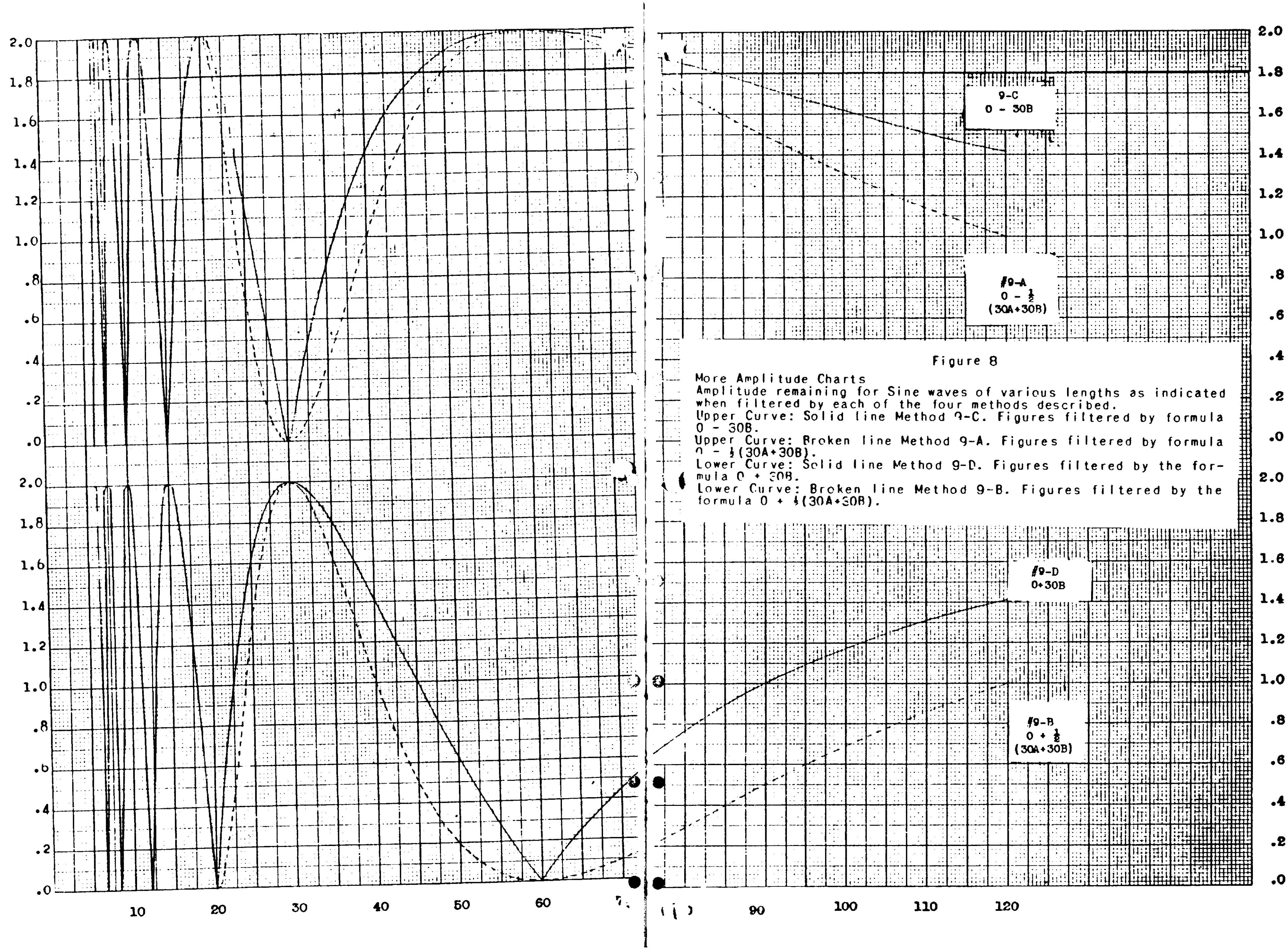


Figure 8

More Amplitude Charts
Amplitude remaining for Sine waves of various lengths as indicated when filtered by each of the four methods described.
Upper Curve: Solid line Method 9-C. Figures filtered by formula 0 - 30B.
Upper Curve: Broken line Method 9-A. Figures filtered by formula 0 - ½(30A+30B).
Lower Curve: Solid line Method 9-D. Figures filtered by the formula 0 + 30B.
Lower Curve: Broken line Method 9-B. Figures filtered by the formula 0 + ½(30A+30B).

LESSON XXI

The Streiff Method of Cycle Analysis

Mr. Abraham Streiff, a consulting engineer of New York City, has developed a method of cycle analysis which I find useful. The essence of the method is that, better than many other methods, it permits the waves of the component cycles to vary in length and in amplitude.

I find the method useful to help me to understand the basic structure of a series of figures. For purposes of forecasting, however, it should be supplemented by other methods. Cycles of irregular length and of irregular amplitude are hard to project!

Mr. Streiff's method breaks a curve into four parts: "Short cycle," "middle cycle," "long cycle," and "trend." These four parts, when added together, exactly reproduce the original curve.

A Bird's Eye Picture of the Method

To start with, you compute a short moving average of the data. I'll tell you later just how to do this, i.e., what kind of a moving average to compute.

In order to tie this birds eye picture into the charts and into the detailed description to follow I'm going to letter the various computations to conform to the charts and to their various columns on the worksheets. In accordance with this plan I shall call the data "A" and call the short moving average "F."

Next you express your data (A) as deviations from your short moving average (F). These deviations (G) are the first approximation of your short cycle. They contain randoms, short cycles, and vestiges of your middle cycle.

Now you compute a long moving average (L). This moving average is a mixture of trend and the first approximation of your long cycle.

Next you express your short moving average (F) as deviations from your long moving average (L). These deviations (M) constitute the first approximation of your middle cycle.

After this you figure out what part of the first approximation of your short cycle (G) should be removed and added to the first approximation of your middle cycle (M). Call this part N. I'll show you later how to figure it out.

Take N out of G. What is left after you take N out of G is your short cycle, O.

Add N to M. What you have after you add these vestiges (N), to M, is a second approximation of your middle cycle (P).

Now figure out what part of P belongs to the long cycle. Call this part Q.

Take Q out of P and add it to L. When Q is out of P, what is left is your middle cycle (R). When Q is added to L you have a combination of long cycle and trend. This combination is S.

You then estimate trend (T), and remove it from S. What is left is your long cycle (U).

What Data to Use

The cycles which are brought out depend somewhat upon the period covered by each datum. If you apply the method to daily data your short cycle will bring out waves from about 2 to about 4 or 5 days long; your middle cycle will bring out waves from about 5 to about 8 or 9 days long; your long cycle will bring out waves from about 9 to about 13 or 14 days long; your "trend" will be a mixture of true trend and longer cycles.

Similarly, if you use weekly data, your short cycle will bring out cycles from about 2 to about 4 or 5 weeks long, etc. If you use monthly data the short cycle will bring out cycles from about 2 to about 4 or 5 months long. If you use semi-annual data your short cycle will bring out cycles from about 2 to about 4 or 5 half years (1 to 2 or 2½ years) long. And so on.

Thus, by regulating the period covered by your data from daily to annual, or even to 2- or 5- or 10-year averages, you can use the method to isolate cycles in any range you wish.

To make it easy for you to understand the method, I shall first give you the procedure, step by step. Then I shall work out an example for you in Table A below. Then, I shall follow the example through for you, column by column.

Procedure

First step: Post the data onto a tabulation sheet.

The data used in the example are logs of semi-annual averages of some sales figures. They are posted into Column A. And charted as Curve A.

Second step: Make a 2-item moving total of the data. Post it in Column B, centered. That is, write your first total midway between first and second lines. As there are an even number of terms to your moving total, exact centering puts the moving total between the lines instead of on a line.

(If you find it more convenient you may, of course, post the moving total on either the first or on the second line. If you do this, it puts the moving total 1/2 of a line out of proper timing--1/2 of a line early or 1/2 of a line late, as the case may be--and you must keep this distortion in mind. Mr. Streiff posts to the second line. However, to keep things simple I have suggested you plot between the lines.)

Third step: Make a 2-item moving total of the values in Column B. Post into Column C, centered. Write your first total on the second line (line G).

__Fourth step__: Make a 2-item moving total of the values in Column C. Post into Column D, centered. Write your first total between the 2nd and 3rd lines (between lines b and c).

__Fifth step__: Make a 2-item moving total of the values in Column D. Post into Column E, centered. Write your first total on the third line down from the top (line c).

The fifth step gives you a 2-item of a 2-item of a 2-item of a 2-item moving __total__ of the values in Column A.

__Sixth step__: Divide every value in Column E by 16. Post the result in Column F on the same line as the number in Column E which is being divided. (That is, the first dividend would be posted on the third line down from the top of the page.

In actual practice it would be easier to multiply each value in Column E by the reciprocal of 16. The reciprocal of 16 is of course .0625. As you know, you can always compute the reciprocal of a number by dividing 1 by the number ($\frac{1}{16} = .0625$) or by looking up the reciprocal of the number in a table of reciprocals such as the one starting on page 548 of your test.

The sixth step gives you a 2-item of a 2-item of a 2-item of a 2-item moving __average__ of the values in Column A.

A 2-item of a 2-item of a 2-item of a 2-item moving average is a weighted __moving average__ of the following formula:

$$C, \text{3rd term} = \frac{a + 4b + 6c + 4d + e}{16}$$

(or, $1/16a + 1/4b + 3/8c + 1/4d + 1/16e$, which is the same thing),

where a = first term, b = second term, c = third term, d = fourth term, etc.

You can see how it builds up if you letter each term as follows:

	Col. A	Col. B	Col. C	Col. D	Col. E	Col. F
(1st term)	a					
		a+b				
(2nd term)	b		a+2b+c			
		b+c		a+3b+3c+d		
(3rd term)	c		b+2c+d		a+4b+6c+4d+e	$\frac{a+4b+6c+4d+e}{16}$
		c+d		b+3c+3d+e		
(4th term)	d		c+2d+e			
		d+e				
(5th term)	e					

Obviously the 4th term would be $\frac{b + 4c + 6d + 4e + f}{16}$, etc.

<u>Seventh step</u>: Now get the deviations of the data, A, from the weighted moving average, F. Post the result in Column G.

"Deviations," you remember, are the amounts by which the data are above or below (deviate from) the moving average or other trend.

You will notice that in the above instruction I said in effect, "Subtract Column F from Column A." I think that by this time you should be sufficiently familiar with procedures so that I do not need to spell it out. That is, I no longer need to say, "From each value in Column A substract the corresponding value of Column F."

The results in Col. G could be expressed as + and - numbers. However, as the data are in logs, it is convenient to add 2.000 to the results so as to keep the values all positive around 2.000 as an axis. In practice you add the 2.000 mentally before you put the values of Column A in the machine.

The values in Col. G are the first approximation of your short cycle.

<u>Eighth step</u>: Make a 2-item moving total of the <u>alternate</u> values in Column F. That is, total first and third terms, second and fourth terms, etc. Post the results into Column H, centered. Write the first total on the fourth line of Column H, (line d) midway between the two values you have added.

What you have here is another kind of a weighted moving <u>total</u> with weights of 1, 0, and 1. That is, the first term of your moving total added the value in Col. F line c with a weight of 1, Col. F, lined with a weight of 0 (therefore, you did not include it) and in Col. F line c with a weight of 1. The next item of your moving total added line d with a weight of 1, line e with a weight of 0, and line f with a weight of 1. Etc.

<u>Ninth step</u>: Make a 2-item moving total of the alternate values in Column H and post into Column I, centered. That is, write your first total on the fifth line of Column I.

<u>Tenth step</u>: Make a 2-item moving total of the alternate values in Column I and post into Column J. Write the first total on the sixth line of Column J.

<u>Eleventh step</u>: Make a 2-item moving total of the alternate values in Column J. Post into Column K. Write the total on the seventh line of Column K.

<u>Twelfth step</u>: Divide each value in Column K by 16 and post the result at the corresponding position in Column L. This is your long moving average.

What you have here is a 2-item weighted moving <u>average</u> (weights 1/2, 0, 1/2) of a 2-item weighted moving average (weights 1/2, 0, 1/2) of a 2-item weighted moving average (weights 1/2, 0, 1/2) of a 2-item weighted moving average (weights 1/2, 0, 1/2) of the values in Column F.

(The moving <u>totals</u> had weights of 1, 0, 1; the moving <u>average</u> has weights of 1/2, 0, 1/2.)

If you wish you can build the formula up for yourself out of letters as I did for you for Column F.

Thirteenth step: Compute the deviation of the values in Column F from the corresponding values in Column L. Add 2.000 to avoid negative numbers. Post the results in Column M. These values are the first approximation of your middle cycle.

Fourteenth step: Chart the values of Column G. (See Curve G)

Fifteenth step: Chart the values of Column M immediately below the chart of Column G. (See Curve M).

Sixteenth step: Run a median line through the waves of Column G. See Curve N. Let it approximate Column M. Call this median N. Read off values and record them in Column N. I'll discuss later just how to compute this median.

Seventeenth step: Subtract each value of the median N from the corresponding value of G. Record the result in Column O. The result is your short cycle. (See Curve O).

Eighteenth step: Add the median N to the deviation M. Record in Column P. This is the second approximation of your middle cycle.

Nineteenth step: Chart Column P. (See Curve P)

Twentieth step: Chart the values of Column L below the chart of Column P. (See Curve L)

Twenty-first step: Run a median line through Column P. (See Curve Q) Let this median resemble Column L. Read off values of median and post in Column Q.

Twenty-second step: Subtract Column Q from Column P. Post result in Column R. These values constitute your middle cycle. (See Curve R)

Twenty-third step: Add Column Q to Column L. Record results in Column S. This gives you the combination of long cycle and trend. (See Curve S)

Twenty-fourth step: Run a median line through S. (See Curve T) Record values in Column T. These values are your trend.

Twenty-fifth step: Subtract Column T from Column S. Post result in Column U. These values constitute your long cycle. (See Curve U)

Twenty-sixth step: As a check, add values in Column T (trend), Column U (long cycle), Column R (middle cycle), and Column O (short cycle). Sum should equal values in Column A, the figures with which you started.

Twenty-seventh step: Chart your short cycle (Column O), your middle cycle (Column R), your long cycle (Column U) and your trend (Column T).

Twenty-eighth step: Project each of the four curves above, as best you can, in the way in which they have been going. Combine. This is your forecast.

Twenty-ninth step: Compare the projection with actual behavior over the period that has been lost by the process of manipulation. If necessary revise the projection so that, as well as possible, it fits the actual behavior.

Thirtieth step: Do the job over using basic data of some other length (for example, annual data instead of semi-annual data, or vice versa).

Resume

Column G of the above process is merely the deviations from a relatively short moving average. The fact that it is a weighted moving average does not change the essential character of Curve G.

The fact that any moving average cuts off the tops of longer cycles, and fills up their troughs, automatically means that any deviations from a moving average contain a certain vestige of the longer cycles.

The Streiff method attempts, by means of the median line N, to capture this vestige of the next longer cycles. Then, by subtracting N from G, the Streiff method attempts to show the short cycle freed from distortion by the vestiges of the middle cycle. These vestiges are not only taken from G, where they don't belong, but added to M, where they do belong.

The first approximation of the middle cycle M is derived by obtaining the deviations of the short moving average G from the middle moving average L. To this basic pattern, M, you then add, as said above, the vestiges of the middle cycle which were in the first approximation of the short cycle, and which, as N, you took out of G, to get P. In addition to adding the vestiges of the middle cycle which has been in the short cycle from P you take out Q, the vestiges of the long cycle still remaining in the middle cycle.

These vestiges of the long cycle Q are added to L to give you your long cycle and trend.

All this is very simple but I am afraid it may sound complicated. Perhaps it would help if I said it over again in other words.

G is the first approximation of your first cycle. To get the actual short cycle (O) from G you subtract N, the vestige of the middle cycle remaining in G.

M is the first approximation of the middle cycle. It's merely the deviations of the short moving average from the long moving average. You get the middle cycle from M by adding the vestiges (N) of the middle cycle which you took out of G and subtracting the vestiges of the long cycle Q.

L, your longer moving average, is the first approximation of your long cycle and trend. From L you get your long cycle by adding the vestiges of the long cycle (Q), found in the first approximation of the middle cycle (P) and by subtracting trend.

TABLE A

An Example of a Cycle Analysis
by the Methods of Mr. Abraham Streiff
as Applied to the Shipments of Product X

Line	Date	A Logs of Data	B A 2-item moving total of data, centered	C A 2-item moving total of Col. B centered	D A 2-item moving total of Col. C centered	E A 2-item moving total of Col. D centered	F Short mov. average. (A 2-item of a 2-item of a 2-item of a 2-item m.z. of data, centered) (Col. E ÷ .6)
a	1st H '35	1.998					
			4.113				
b	2nd H '35	2.115		8.390			
			4.277		17.090		
c	1st H '36	2.162		8.700		34.808	2.176
			4.423		17.718		
d	2nd H '36	2.261		9.018		35.706	2.232
			4.595		17.988		
e	1st H '37	2.334		8.970		35.515	2.220
			4.375		17.527		
f	2nd H '37	2.041		8.557		34.501	2.156
			4.182		16.974		
g	1st H '38	2.141		8.417		33.862	2.116
			4.235		16.888		
h	2nd H '38	2.094		8.471		33.977	2.124
			4.236		17.089		
i	1st H '39	2.142		8.618		34.563	2.160
			4.382		17.474		
j	2nd H '39	2.240		8.856		35.443	2.215
			4.474		17.969		
k	1st H '40	2.234		9.113		35.567	2.285
			4.639		18.598		
l	2nd H '40	2.405		9.485			
			4.846				
m	1st H '41	2.441					

TABLE A--Continued

Line	G	H	I	J	K	L	M
	1st. approx. of short cycle. Dev. of data from the short mov. av. (Col. A + 2.000 - Col. F)	A 2-item wtd. mov. total of Col F, centered. (Weights 1,0,1)	A 2-item wtd. mov. total of Col H, centered. (Weights 1,0,1)	A 2-item wtd. mov. total of Col I, centered. (Weights 1,0,1)	A 2-item wtd. mov. total of Col J, centered. (Weights 1,0,1)	Long mov. Av. (A 2-item of a 2-item of a 2-item of a 2-item wtd. m.a. of short m.a. cent. (Col. K ÷ 16)	1st approx. of middle cycle. Dev. of short wtd. m.a. (Col. F) from long wtd. m.a. (Col. L) (Col. F + 2.000 - Col. L)
a							
b							
c	1.986						
d	2.029	4.396					
e	2.114	4.388	8.732				
f	1.885	4.336	8.668	17.344			
g	2.005	4.280	8.612	17.287	34.677	2.167	1.949
h	1.970	4.276	8.619	17.333			
i	1.982	4.339	8.721				
j	2.025	4.445					
k	1.949						
l							
m							

TABLE A--Continued

Line	Median of waves in Col. G determined graph-ically	Short cycle (Col. G ∤ 2.000 - Col N)	Second approxi-mation of middle cycle. (Col. M ∤ Col. N - 2.000)	Median of waves in Col. P deter-mined graph-ically	Middle cycle (Col. P ∤ 2.000 - Col Q)	Long cycle & Trend (Col. L ∤ Col Q -2.000)	Trend. Median of waves in Col. S deter-mined graph-ically	Long cycle (Col. S ∤ 2.000 - Col. T)
a								
b								
c	2.020	1.966						
d	2.043	1.986						
e	2.027	2.087						
f	1.975	1.910						
g	1.965	2,060	1.914	1.995	1.919	2.162	2.190	1.972
h								
i								
j								
k								
l								
m								

Details of Computation

Column B: The first value, 4.113, is obtained by adding the first two values in Column A, namely 1.998 and 2.115.

The next value in Column B is obtained by adding the second and third values in Column A, namely 2.115 and 2.162. Etc.

Note that the values in Column B are recorded halfway between the values of Column A from which they are computed.

Column C: The values in Column C are obtained by adding successive pairs of figures in Column B. Thus 8.390 is the sum of 4.113 and 4.277. 8.700 is the sum of 4.277 and 4.423.

Note that as the values in Column C are posted midway between the values in Column B, that the values in Column C fall on the lines instead of between them.

Column D: Column D is obtained similarly. Thus 17.090, the first value in Column D is obtained by adding 8.390 and 8.700 in Column C, etc.

Column E: The values in this column are obtained in the same way. 34.808 is the sum of 17.090 and 17.718, etc. By now the various values in Column A have been taken 16 times (2 x 2 x 2 x 2).

Column F: Each value in Column E is now divided by 16 to reduce the totals of Column E to an average. 34.808 in Line c divided by 16 and rounded equals 2.176, etc.

Column G: The values in this column are obtained by subtracting the values in Column F from the corresponding value in Column A. And, to avoid negative numbers, adding 2.000. Thus, the first value 1.986 is obtained by starting with 2.162 from Column A and subtracting 2.176 from Column F and adding 2.000. In practice you subtract 2.176 from 4.162 to get your answer.

Column H: The first value 4.396 on Line d is obtained by adding the values of Lines c and e in Column G, namely 1.986 and 2.114. The second value in Column H, 4.388 is obtained by adding 2.029 on Line d of Column G and 1.885 on Line f of Column G and so on down the page.

Column I: Column I is obtained from Column H in the same way that Column H was obtained from Column G. 8.712 is the sum of 4.396 and 4.336.

Column J: Column J is obtained from Column I in the same way that Column I was obtained from Column H. 17.334 is the sum of 8.732 plus 8.612.

Column K: Column K is obtained likewise. 34.677 is the sum of 17.334 and 17.333.

The next value in Column K would be the sum of 17.287 on Line g of Column J plus whatever value belongs on Line i of Column J.

Column L: The values here are the values in Column K divided by 16. Thus, 34.677 divided by 16 is, when rounded, 2.167. We divide by 16 for the same reason that we divided Column E by 16 to get Column F. The process of addition has built up a total of 16 terms in Column K.

Column M: The value of 1.949 in Line g is 2.116 in Column F minus 2.167 in Column L plus 2.000, to keep the values above and below 2.000.

Column N: These values are obtained graphically from Column G and naturally start in Line c because Column G can be computed back to Line c.

Column N needs a little bit of discussion. To draw your median line N in the chart of Column G, you make a mark on each leg of each cycle halfway between the top and bottom of that leg.

Where the top and/or bottom is flat it is a good plan to estimate the top at the place it would be if it could be plotted between the lines. Use dotted lines to let the cycle go up or down to its estimated top or bottom between the line value.

Now connect these points that you have marked on the legs of the cycles in Curve G with a curved line, to approximate as nearly as possible the waves in Column M, which, according to instructions, you have plotted beneath Column G.

The idea is to take out of Column G the vestiges of Column M that have been left in it by method of construction, and to add these back into Column M where they belong.

When you have drawn your median line N, read the values from the chart and plot them in Column N.

Column O: The values here are obtained by subtracting from each value in Column G the corresponding value in Column N and adding 2.000 to keep the values positive. The first value in Column O, 1.966, is obtained by subtracting 2.020, the first value in Column N from 1.986, the first value in Column G, and adding 2.000.

The other values are obtained similarly.

Column P: The value of 1.914, the first value in this column, is obtained by adding the value in Column M to the value in Column N and subtracting 2.000.

We put into M the same values that we took out of G.

Column Q: These values are obtained graphically from Column P in the same way that Column N was obtained graphically from Column G.

As the values in Column P cannot be computed prior to Line G the values of Column Q start in Line g also.

All the values in Column Q are read from the chart.

Column Q is drawn in such a way as to resemble as much as possible Column L.

Column R: The value of 1.919 is obtained by adding 2.000 to the corresponding value of Column P, namely 1.914, and subtracting the corresponding value of Column Q, namely 1.995.

Column S: The first value in this column, namely 2.162, is obtained by adding 1.995 from Column Q to 2.167 from Column L and subtracting 2.000.

Column T: Trend is determined by running a median line with a French curve or otherwise through the midpoint of the legs of the various cycles in Column S. The values are then read from the curve so drawn.

Column U: The value of 1.972 with which this column starts is 2.162 from Column S plus 2.000 minus 2.190 from Column T.

Check: To check your result you add Column U, Column T, Column R, and Column O (less of course 6.000) to get the corresponding value in Column A. In this case the arithmetic is 1.972 plus 2.190 plus 1.919 plus 2.060 less 6.000 to give us an answer of 2.141 which checks.

This matter of adding or subtracting 2.000 may be a little confusing. Remember you are dealing with logarithms. When you add it is equivalent to multiplication, when you subtract it is equivalent to division. Adding or subtracting 2.000 merely keeps your values in terms of logs of percentages instead of ratios.

I have not bothered to fill in the rest of the computations, partly to save work at this end but chiefly to keep from confusing you.

Discussion

The value of the Streiff methods of analysis are that they, for me, help to clarify the main structure of a series. For purposes of forecasting I feel that more detailed studies should be made of each of the component cycles. The usefulness of the method as I see it is that it concentrates your attention upon those areas where further work is likely to be most productive.

References

Mr. Streiff has explained the mathematical justification for his methods and has given several examples of their use in two articles which have appeared in the Monthly Weather Review as follows:

Monthly Weather Review, July, 1926, Pages 289--296.

Monthly Weather Review, March, 1928, Pages 98--99.

The Monthly Weather Reviews in which these articles are printed can be bought for 25¢ apiece from Superintendent of Documents, Washington, D. C.

PROBLEMS

Problem 1. Make a Streiff analysis of the Standard and Poor's combined annual stock market data or any other series of figures in which you are interested.

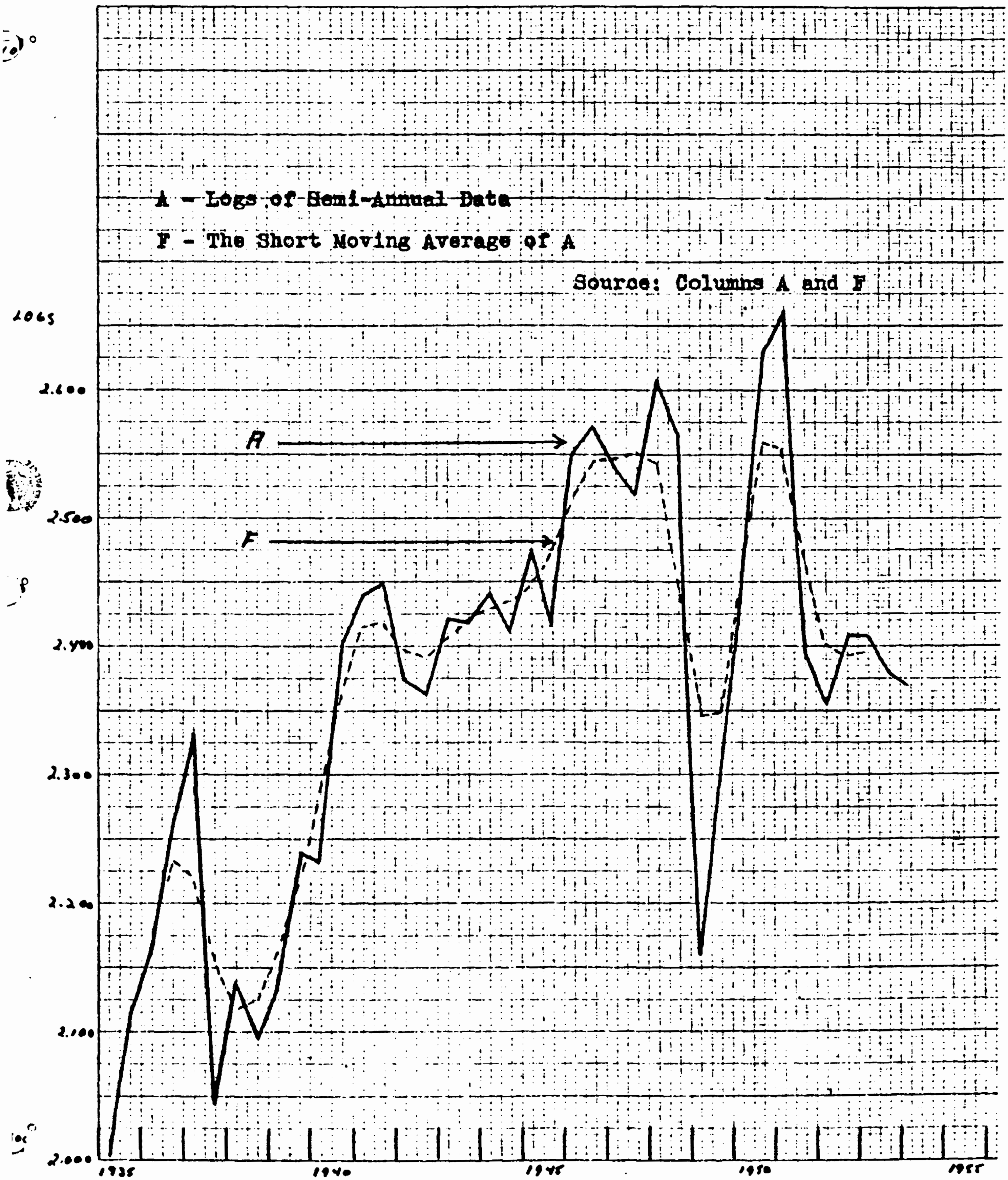


Fig. 1 A Series of Sales Figures

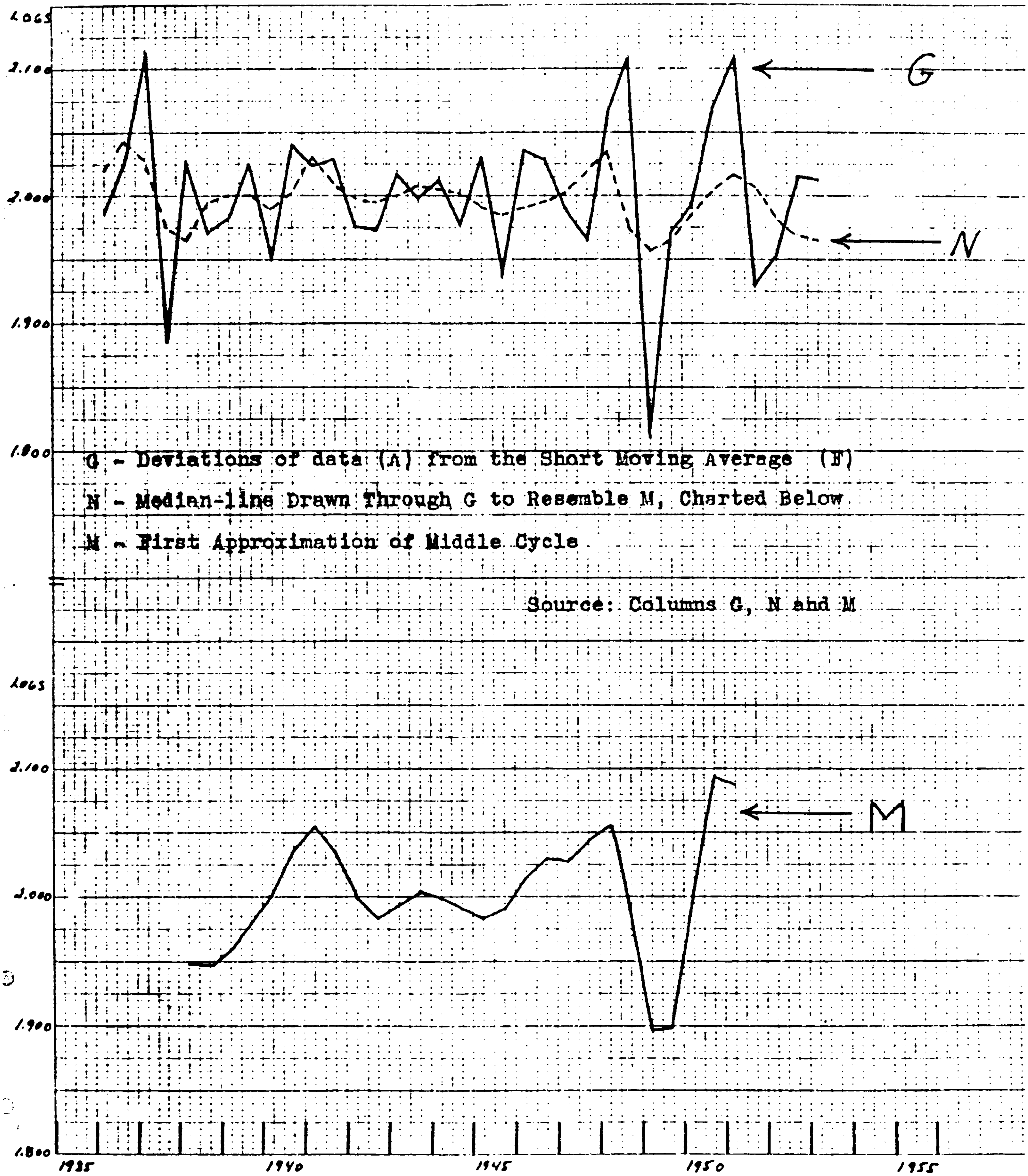


Fig. 2 Development of the First Median

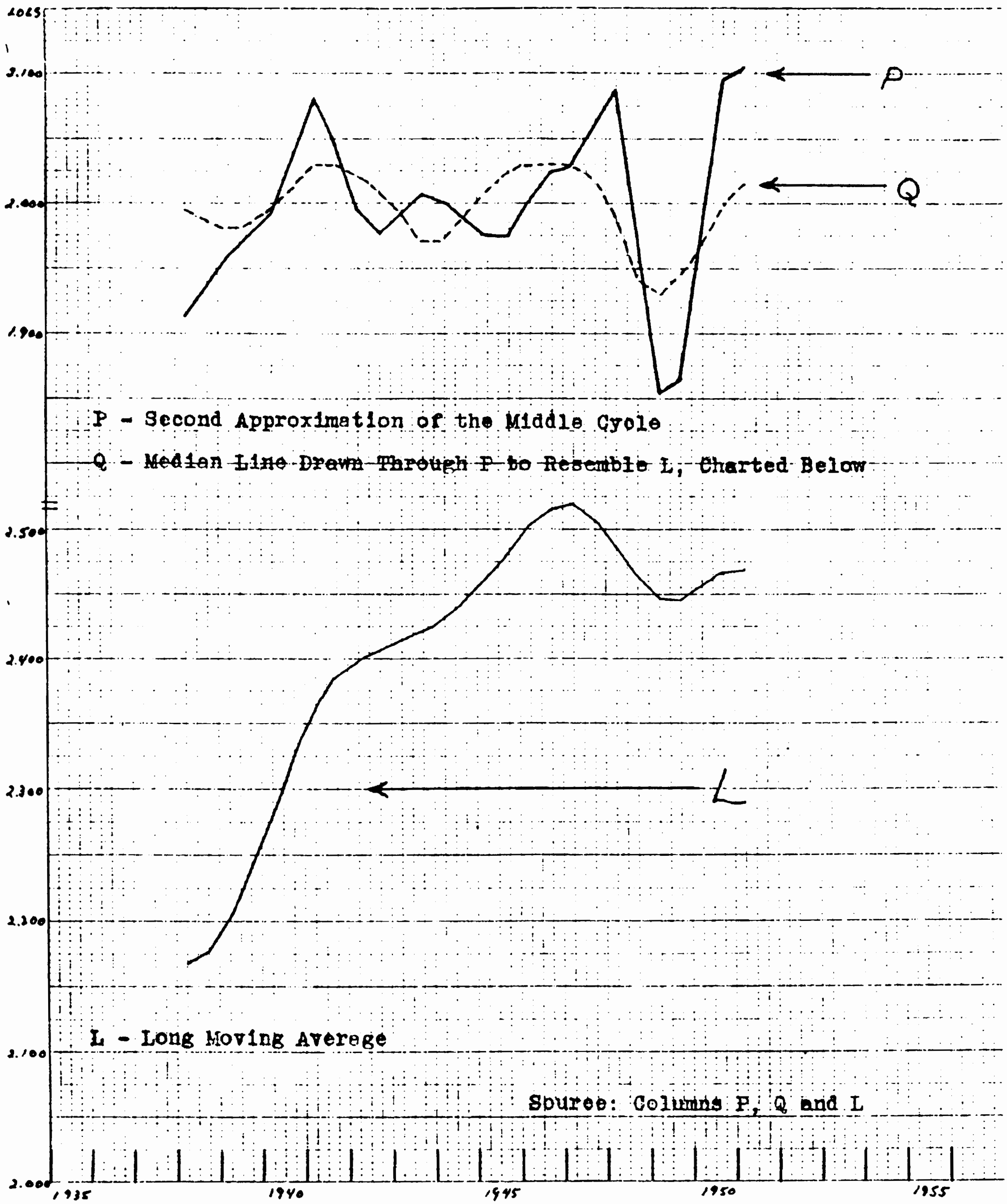


Fig. 3 Development of the Second Median

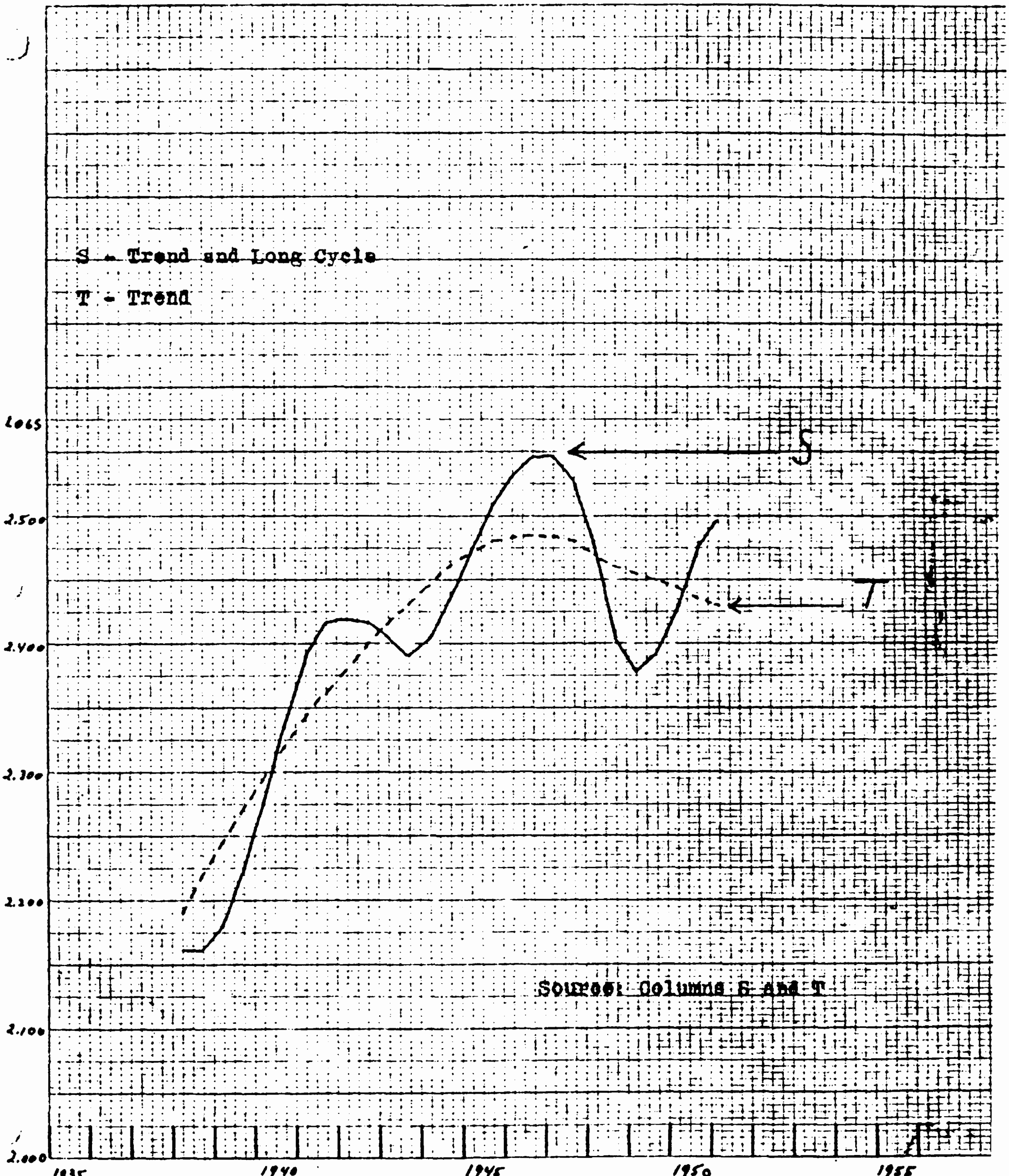


Fig. 4 Determination of Trend

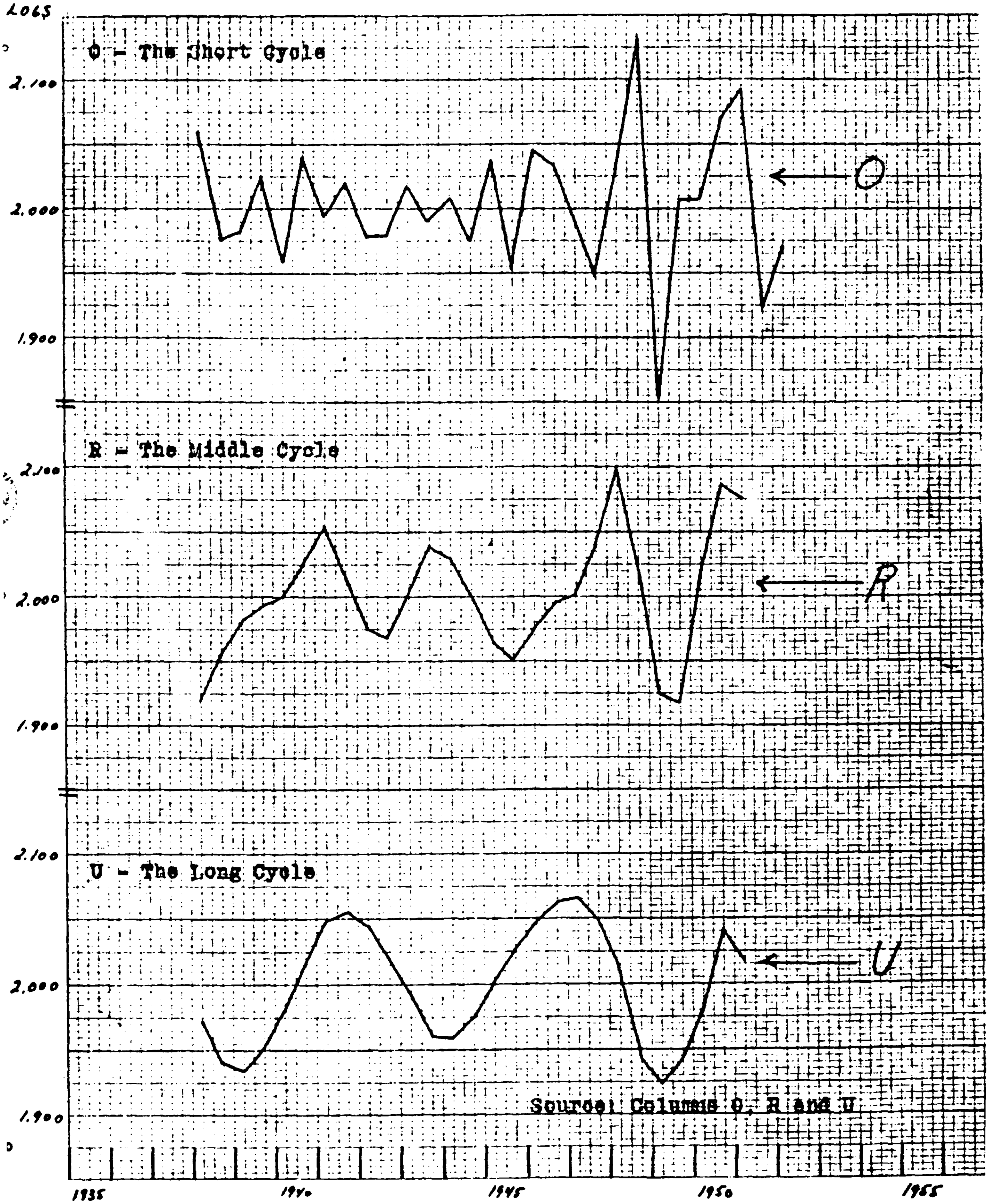


Fig. 5 The Three Cycles (These three irregular cycles, plus trend (T), equal date (A)).

LESSON XXII

HOW TO DETERMINE AND PROJECT TREND

As you know, trend is the tendency of a series of figures to increase (or decrease) gradually over a period of time.

To determine the trend: (1) First you remove the cycles. (2) Then, from the remainder, you idealize the trend. (3) Then you determine the characteristic of your idealized trend. (4) Finally, you project your idealizated trend according to its characteristics. It is as simple as that!

(1) To Remove the Cycles

You do not need to be told how to remove the cycles. You (a) find them, (b) idealize them, and (c) adjust your series for them one after the other. What is left is (a) trend (b) randoms, (c) undiscovered cycles and (d) artifacts introduced by any erroneous cycle determinations.

(2) To Idealize the Remainder

First of all, plot the values of the remainder on ratio scale, unless your remainder is in logs, in which event plot it on arithmetic scale. (In this lesson the term "remainder" is used to denote the values of the data after the removal of all cycles.)

In most instances the main structure of your curve will be smoothly concave downward. For the moment let us assume that, in the imaginary case we are considering, it is. (Later we can consider those instances where it is not.)

Your problem now is to get rid of your randoms and to leave your trend. **Easy.** (a) Smooth by a moving average. A moving average will largely eliminate your randoms; will have but very little effect upon your trend. (b) Or fit a French Curve to the chart of the remainder. Draw your trend. Read off your values. (c) Or draw in your trend freehand. Read off your values.

(a) Smoothing by a Moving Average

You will remember that moving averages lie below the trend when the trend is concave downward. The longer the moving average the more it lies below the trend. The more concave the curve the more the moving average will lie below the curve.

As every curve will be different I can give you no rules to follow to determine the correction you must apply to your moving average to reconstitute the trend. You must proceed by trial and error. That is, you must make a guess as to the true trend, take the same moving average of it as you took of your remainder, see if the two moving averages correspond. If not, make another guess, and so on until the correspondence is close enough.

One way to make your first guess is to make a moving average of the moving average. Suppose you made a 9-item moving average of the remainder. Make a 9-item moving average of the 9-item moving average. If your second 9-item moving average lies uniformly .005 below your first moving average you could come very close to your trend by adding .005 to each value of your first moving average. The same principle would apply even in those instances where the interval between your moving averages was variable. You make each term of your first guess of the trend as much above your first moving average as the corresponding term of your second moving average is below the corresponding term of your first moving average.

Of course this method will cost you values at each end of the series. These will have to be extrapolated (extended) graphically.

(b) Smoothing by a French Curve

Another way of idealizing the remainder is to lay a French Curve on the chart of the remainder, slide it around until it fits, draw in your trend, and read the values from the chart.

A French Curve is a curved ruler used by draftsmen. It is usually transparent.

A modification of this method is to lay a flexible ruler edgeways on the remainder, bend it until it fits, draw the trend, read off the values.

(c) Smoothing Freehand

Still another way to idealize your trend is to draw it in freehand. Be careful to draw a smooth line which changes direction smoothly and consistently.

General

In all instances be sure that the variations of the remainder above your trend are approximately equal to the variations of the remainder below the trend.

In making these determinations it is sometimes wise to make use of the principle of limited data, already explained in earlier lessons. That is, occasional freak highs and lows should be "cut down to size" before being averaged or used to determine variations from trend.

When the Trend is Not a Curve Smoothly Concave Downward.

Occasionally you will find a trend which, when plotted on ratio scale or in logs, is not smoothly concave downward. Reference is made to Cycles for December 1955 which will be sent on request without charge as a supplement to each person taking the course who has not already received it as a member of the Foundation.

For example ——

Growth may be killed, as with horse drawn vehicles.

Growth may be cut back, as with cotton production.

Growth may have reached its upper asymptote, as seems to be true of lumber production. (The asymptote is the limit toward which a curve approaches.)

Growth may have experienced a rebirth, as with lead production, petroleum production, coffee production, the population of Germany.

Growth may be irregular because of lack of homogeneity of the data. For example sales may suddenly have doubled due to a merger. Sales in dollars may have been distorted due to inflation.

Growth may be concave upward, as with the debt of the Federal Government. (Such a circumstance cannot continue indefinitely. Eventually the trend will change or we will have repudiation. My guess favors the probability of repudiation, but not until after the next major war.)

Growth, may be a straight line.

In all such instances the trend must be idealized as it actually is. However be very careful lest what seems to be a discontinuity is really a long cycle. For example I once thought the trend of shipbuilding was sharply downward 1855 to 1895; discontinuous from 1895 to 1905; sharply downward 1905 to 1935. It was not until the pattern began to repeat--discontinuity 1935 to 1945, downward from 1945 onward--that I realized that what we have in shipbuilding is a practically level trend with a 50-year cycle cresting about 1853, 1903, 1953, etc.

Where growth is irregular due to lack of homogeneity in the data you should not try to force the trend into a smooth pattern. Let the discontinuity show in the trend.

Growth figures in currency dollars should be adjusted for inflation. There is no entirely satisfactory way to do this. One way is given in a supplement to this lesson. If the growth and inflation factors are separated each can be projected separately.

(3) <u>To Determine the Characteristics of Your Idealization</u>

The first thing to do at this juncture is to compute the 1-item moving difference of the logs of your trend. (The 1-item moving difference is called the first difference.) If your trend is not in logs compute the 1-item moving percentage.

The first difference should change values smoothly and regularly. For example: .010, .010, .009, .008, .008, .007, .007, .006, .006, .005, .004, .004 etc. It should not jump around. For example: .010, .010, .008, .009, .008 etc. If it does jump around you must change the values of the trend so that it does not.

In the example given you would raise the fourth term of your trend by .001, viz:

	1st Term	2nd Term	3rd Term	4th Term	5th Term
Trend	1.560	1.570	1.580	1.588	1.597
1st Difference	----	.010	.010	.008	.009
Change	1.560	1.570	1.580	1.589	1.597
1st Difference	----	.010	.010	.009	.008

Of course there will be some irregularity because you will not ordinarily have enough decimals to give you a perfectly smooth first difference. In the first example given you have two .010's one .009, two .008's, .007's, .006's, one .005 etc. This cannot be helped unless you use four or five place logs.

To see if your 1st differences are smooth you can compute the second differences (1st differences of the 1st differences). However you can usually see by inspection how smooth they are.

It's a good plan to plot the 1st differences. If they go up and down in waves you may get a hint of a previously undetected cycle. Such a cycle can be isolated and removed or smoothed out by a different (or additional) moving average.

The first differences of your trend must be smooth, but do not get the idea that they must plot as a straight line. For example they might run .060, .050, .041, .033, .026, .020, .015, in which case the 2nd differences would be plotted as a straight line. (The second differences would be .010, .009, .008, .007, .006, .005, etc.) However the 2nd differences don't need to be a straight line either.

The thing is, you must figure out how the trend has been behaving. You must get a trend that varies according to some regular law. When you have done this you can project it according to the same law.

Once you have found the law you can devise a mathematical formula to express it, if your mind runs to that sort of thing. But a formal mathematical expression is not necessary.

Conversely it may happen that one of the well known mathematical curves—a straight line, a parabola or a logistic—fitted to the remainder (after the cycles are removed) or to the logs of the remainder, will describe the behavior of the remainder (or its logs). If so, well and good. However remember that no matter how "mathematical" your curve it is merely a <u>description</u>, and essentially no better—and certainly no more objective—than the homely methods I have suggested earlier.

You should not be completely unaware of these mathematic curves, however, I am therefore going to ask you to read as part of this lesson, pages 334-351 of your text book, Spurr, Kellogg and Smith.

In reading these pages you will note that the method suggested for fitting a straight line trend by means of least squares is slightly different from the one I gave you in Lesson XVIII. I like my method better. I think it is easier to explain, to understand, and to use. However both methods come out to the same place in the end. Use either one you like.

Although it is desirable to fit a trend to <u>all</u> of the remainder, the really important thing, from the standpoint of projection, is to see, as best you can, how the trend has been behaving in the immedate past. If the trend has been increasing by .001 an interval (month, day, year, or whatever) for the past ten intervals it is reasonable to assume that the trend for the next ten intervals will behave likewise. That is, if the last ten values of trend have been 2.191, 2.192, 2.193, 2.194, etc. up to 2.200 it is reasonable to assume that the next few values will be 2.201, 2.202, 2.203 etc. If the last few values are right it does not matter so much about the others.(from the standpoint of projection).

(4) <u>Projecting the Trend</u>

The discussion of projecting the trend has been largely anticipated by the last paragraph. You merely find out how the trend has been behaving and project it on the same basis.

Perhaps the only things that need to be said further are: Don't project the trend any farther than you have to. Revise your projection at frequent intervals.

Synthesizing Trend and Cycles

It's a good plan to multiply the trend and cycles together (or add them together if they are in logs). You already know how to do this. You can then compare the synthesis with the original figures to see how well they correspond. This will show you how good you are.

Don't think from the fact that they correspond pretty well that you nedessarily have anything. You may merely have done a good job of description.

You should compute and chart the differences between logs of the synthesis of trend and cycle and the logs of the original data. This will show you how bad you are. It is important to know this. These differences should be the same as the differences between logs of the remainder after removal of all the cycles and the logs of trend.

Projecting Trend and Cycles

Having projected your trend you multiply each value of the trend by the appropriate value of each of the cycles. This is your projection.

If your cycles have been determined correctly, if they are significant, if they continue, and if your trend has been determined and projected correctly, the projection of cycles and trend should forecast the future with no more error than is shown by the divergence of actual figures from the synthesis of cycles and trend. (If, if, if, and if!!!)

LESSON XXIII

TESTS FOR THE SIGNIFICANCE OF CYCLES

If you have a series of numbers in which each number is totally unrelated to the number before and the number following, it is relatively easy to evaluate the significance of any cycles you find.

Significance means meaningfulness. In other words, significance is the extent to which the behavior is the result of chance (random forces), or isn't.

More exactly, the significance of the behavior refers to the number of times out of a hundred or a thousand or some other number that the behavior could have come about purely as a result of chance. If the behavior could not have come about by chance more than once in 20 times it is ordinarily considered "significant" (meaningful). In other words, "Prick up your ears." If the behavior could not have come about as a result of chance more than once in 100 times it is ordinarily considered "highly significant." In other words, "Its a good bet." If it could not have come about as a result of chance more than once in 1,000 times it is ordinarily considered "extremely significant." In other words, "You can almost bank on it."

Now, as I said above, for numbers successively unrelated to each other—like the numbers in a telephone book for instance—it is quite easy to determine the significance of cycles. But--and here's the rub--the numbers we are interesed in are related to each other.

Consider pig iron prices for example. During the past 180 years the average annual price of pig iron (adjusted) has varied from $12 a ton to $53 a ton. But not all at once. Prices have worked themselves up, then worked themselves down again. Prices go up or down from where they are. If they are 14 this year they can't possibly go up to 50 next year, then down to 15 the next. This interrelationship from term to term is called serial correlation. To eliminate serial correlation--that is to get pig iron prices in random order—you would have to write the price for each year on a card, put the cards in a hat, shake the hat, and draw the cards out one by one. The series you would get this way would have the same prices but it would not look like any economic time series you ever saw. It would jump around every which way in pure random order.

All economic time series I can think of have serial correlation. So do climatological series, biological series, medical series, and many others. And the presence of serial correlation invalidates almost all of the tests of significance that have so far been worked out! This fact makes it unnecessary for us to go into detail in regard to these tests, or for you to learn them. They wouldn't do you any good if you did learn them.

Professor Hotelling--one of our leading mathematical statisticians--told me he though (sic!) that, if he had the time (which he hadn't), he could modify some of these tests so that serial correlation would probably (sic!) make no difference. The application of the modified test to a series I wanted him to test would cost about $20,000, he said. Not much help there!

There is one test however, which [illegible] limitations, seems to be on the right track. The test was [illegible] on of

Washington. In the words of C. E. Armstrong, who wrote the account of it being sent you herewith, "among the various devices used in testing for significance, Bartel's technique seems to give perhaps the most reasonable results."

I think that by reading Armstrong's paper you will be able to apply Bartel's test without difficulty, especially as he works out an example for you. However, to be on the safe side I'll work another example for you.

Let us use controlled data. Let us use the first 40 values of Curve E of Lesson V. These values are repeated for you herewith in Table I. As you will recall, Curve E represents a perfectly regular 8-year cycle with amplitude ± 20, and randoms. The randoms obscure the cycle so that you can barely see it.

Table 1

The Data

A perfectly regular zigzag with amplitude of ± 20 to which random numbers have been added.

Source: Col. E of Table 2 of Lesson V (page 18 for figures, page 24 for chart).

Year	Value	Year	Value	Year	Value	Year	Value	Year	Value
1	1	9	2	17	17	25	-58	33	-1
2	19	10	2	18	-9	26	22	34	2
3	35	11	-38	19	19	27	12	35	-1
4	-8	12	35	20	5	28	-32	36	21
5	-20	13	66	21	-9	29	26	37	10
6	-2	14	-2	22	-5	30	-22	38	5
7	-44	15	-37	23	-24	31	-72	39	-11
8	-13	16	-4	24	-19	32	4	40	20

Lets pretend Curve E represents some real behavior and that we suspected an 8-year cycle. How many times could you get by chance alone a cycle as good as this in a series of figures as long as this?

The answer, according to Bartels, is once in about 10 times. The arithmetic is fully worked out in Table II, Table III, and the few computations which follow.

Table II

Sine-cosine curves fitted to
the first five 8-item cycles of Curve E

Source: Table I

(a)	(b)	(c)	(d)	1st Cycle			2nd Cycle		
	θ	Sin θ	Cos θ	Y	Y sin θ	Y cos θ	Y	Y sin θ	Y cos θ
Year	(a + 8)				A	B		A	B
1	12.5	+ .707	+ .707	1.	+ .71	+ .71	2	+ 1.41	+ 1.41
2	25.	+1.000	000	19	+19.	.0	2	+ 2.	0.
3	37.5	+ .707	- .707	35	+24.75	-24.75	-38	-26.87	+26.87
4	50	.000	-1.000	-8	0.	+ 8.	35	0.	-35.
5	62.5	- .707	- .707	-20	+14.14	+14.14	66	-46.66	-46.66
6	75	-1.000	000	-2	+ 2.	0.	-2	+ 2.	0
7	87.5	- .707	+ .707	-44	+31.11	-31.11	-37	+26.16	-26.16
8	100.0	.000	+1.000	-13	0.	-13.	-4	0.	- 4.
			Total		+91.71	-46.01		-41.96	-83.54
			+ 8/2		+22.93	-11.50		-10.49	-20.89

Table II continued.

(a)	3rd Cycle			4th Cycle			5th Cycle		
	Y	Y sin θ	Y cos θ	Y	Y sin θ	Y cos θ	Y	Y sin θ	Y cos θ
Year		A	B		A	B		A	B
1	17	+12.02	+12.02	-58	-41.01	-41.01	-1	- .71	- .71
2	-9	- 9.	0.	22	+22.00	0.	2	+ 2.00	0.
3	19	+13.43	-13.43	12	+ 8.48	- 8.48	-1	- .71	+ .71
4	5	0.	- 5	-32	0.	+32.00	21	0.	-21.00
5	-9	+ 6.36	+ 6.36	26	-18.38	-18.38	10	- 7.07	- 7.07
6	-5	+ 5.	0.	-22	+22.00	0.	5	- 5.	0.
7	-24	+16.97	-16.97	-72	+50.90	-50.90	-11	+ 7.78	- 7.78
8	-19	0.	-19.	4	0.	+ 4.	20	0.	+20.
Total		+44.78	-36.02		+43.99	-82.77		- 3.71	-15.85
+ 8/2		+11.20	- 9.01		+11.00	-20.69		- .93	- 3.96

Table III

Tabulation of A and B values
and the sum of their squares

Source: Table II

Cycle	A	B	A^2	B^2	$A^2 + B^2$
1	+22.93	-11.50	525.78	132.25	658.03
2	-10.49	-20.89	110.04	436.39	546.43
3	+11.20	- 9.01	125.44	81.18	206.62
4	(+) -11.00	-20.69	121.00	428.08	549.08
5	- .93	- 3.96	.86	15.68	16.54
Sum	+11.71	-66.05			1976.70
Average	+ 2.34	-13.21			395.34

33.71

+6.74

Computations

Average amplitude——— $= \sqrt{A^2 + B^2}$

(Substituting from Table III) $= \sqrt{(2.34)^2 + (-13.21)^2}$

(Simplifying) $= \sqrt{5.48 + 174.50}$

(Simplifying) $= \sqrt{179.98} = 13.42$

Quadratic mean (square root of the mean of the squares) of the five individual amplitudes——— $= \sqrt{\frac{\Sigma (A^2 + B^2)}{N}}$, where N equals the number of cycles

(Substituting from Table III) $= \sqrt{395.34}$

(Simplifying) $= 19.88$

Quadratic mean divided by the square root of the number of cycles——— $= \left(\frac{\text{Quadratic Mean}}{\sqrt{N}}\right)$

(Substituting from above) $= \frac{19.88}{\sqrt{5}}$

(Simplifying) $= \frac{19.88}{2.236} = 8.891$

Square of the average amplitude divided by the quadratic mean divided by the square root of the no. of cycles——— $= \left(\frac{\text{Average Amplitude}}{\frac{\text{Quadratic Mean}}{\sqrt{N}}}\right)^2$

(Substituting from above) $= \left(\frac{13.42}{8.891}\right)^2$

(Simplifying) $= (1.5094)^2 = 2.278$

Reciprocal of e raised to a power equal to the square of the average amplitude divided by the quadratic mean divided by the square root of the number of cycles——— $= \frac{1}{e^{2.278}}$

(Substituting) $= \frac{1}{(2.71828)^{2.278}}$

(Simplifying) $= \frac{1}{9.76}$

Hence, an 8-item cycle as regular as this in a series as long as this (40 terms) can be found by chance about once every 10 times (9.69). It's not very significant.

Explanation and Comments.

Table I: Note that the data contain cycle and randoms only. If trend had been present you would have had to remove it.

Table II: (Review Lesson XV, Sine and Cosine Curves and how to fit them.)

As the cycle is 8 items long there are 8 points to fit.

If the cycle had been of a fractional length, such as 8.2 items long there would still have been only 8 points to fit. However five cycles of an 8.2-item cycle would have required 41 terms. Your Y Columns would then have corresponded to the first 8 columns of an 8.2-item periodic table. In such a table the 3rd line would have been 9 items long (-58 being in the 9th column), and your 4th line would have started with 22, Consequently your Y values for the 4th cycle would have been 22, 12, -32, 26, -22, -72, 4, and -1; your Y values for the 5th cycle would have started with ≁2 in position 1, etc.

If the cycle had been over 8 1/2 items long (say 8.8 items) you would have had to call it 9 items long. Just as in the 9th column of an 8.8-item Table you would have had to use certain items twice.

The angle θ (theta) in an 8-item cycle is $360^\circ \div 8$ or 45°. Or, if you use 100% to represent your whole circle it is 100% ÷ 8 or 12.5. The angles in Col. (b) have been figured on a percentage basis. They are in each case 12.5 times the values in Col. (a).

If you use percentages in Col. (b) you have to look your sines and cosines up in a percentage table of sines. Such a table is supplied with the supplement to this lesson. It was also supplied with Lesson XV.

If you had used degrees your Col. (b) would have read 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°.. You would have looked sines and cosines up in a degree table. In either event the sine and cosine values entered into Col. (c) and (d) would have been the same.

I think the rest of the computations in Table II are quite clear, if you mastered Lesson XV. For each cycle you multiply the Y values by sin θ and by cos θ, total, and divide by 4 (half the number of terms in the cycle).

Armstrong left this last step out (see his Footnote 3), but I prefer to include it. However it is not important. You get the same answer either way.

Table III: I can't see much of anything to talk about here. Your A's and B's come from Table II. Your squares come from the table of squares starting on page 548 of Spurr, Kellogg, and Smith, The table starting on page 548 also gives the square root values needed in the computations to follow.

Computations: I can't see anything here which isn't self evident, except perhaps the last figure of 9.76. The table of exponentials in the supplement gives $e^{2.2}$ as 9.02, $e^{2.3}$ as 9.97. The difference is 9.97-9.02 or .95. Consequently $e^{2.278}$ would be (approximately) 9.02 + 78% of .95 or 9.02 + .74 = 9.76.

<u>To find the number of cycles for any desired degree of significance</u>.

According to the Bartel's test, how many cycles <u>of this degree of regularity</u> would we need to reduce the possibility of chance down to 1 in 100?

From our computations above we see that——

exponent of e $= 2.277 = \left(\frac{13.42}{8.89}\right)^2$

For the denominator 8.89 we can substitute $\frac{19.88}{\sqrt{N}}$ to get $\left(\frac{13.42}{\frac{19.88}{N}}\right)^2$

$$\text{Hence} \left(\frac{13.42}{\frac{19.88}{N}}\right)^2 = \text{exponent of e.}$$

Taking square root of both sides we get——

$$\frac{13.42}{\frac{19.88}{\sqrt{N}}} = \sqrt{\text{exponent of e}}$$

(Simplifying) $\frac{13.42\sqrt{N}}{19.88} = \sqrt{\text{exponent of e}}$

(Transposing) $\sqrt{N} = \frac{19.88}{13.42} \cdot \sqrt{\text{exponent of e}}$

(Simplifying) $\sqrt{N} = 1.48137 \cdot \sqrt{\text{exponent of e}}$

(Squaring we get) $N = (1.48137)^2 \cdot \text{exponent of e}$

(Simplifying) $N = 2.19446 \cdot \text{exponent of e}$

From this equation you can substitute any value of the exponent of e to find the corresponding N, or vice versa.

From your table of exponents you see that $e^{4.6} = 99.5$, which is close enough to 100 for our purposes.

Hence solve for

$$N = 2.19446 \cdot 4.6$$
$$= 10.09$$

Therefore, if this cycle continues with the same regularity for 10 cycles, the behavior could not happen by chance more than once in 100 times, according to Bartel's test.

To get the times this cycle would have to continue with equal regularity to meet the 1 in 1000 test we refer again to our table of exponentials. We see that an exponent of 6.9 will give us 1 in 992 times, which is good enough.

$$N = 2.19446 \cdot 6.9$$
$$= 15.14$$

Therefore this cycle could not continue with this same regularity for 15 repetitions purely as a result of chance more than once in a thousand times, according to the Bartel's test.

Evaluation of the Bartel's test.

I think Armstrongs comment, given above, farily evaluates the Bartel's test; "it gives perhaps the most reasonable results" of any of the tests. This is not enough. We wish to know the probability of chance. Bartel's test may tell us this, but, as far as I am concerned, this is not certain.

The Bartel's test has at least three flaws:

(a) The Bartel's test presupposes sine-cosine waves whereas the waves we deal with are usually zigzag.

(b) The Bartel's test makes no distinction between the significance of predetermined wave lengths and wave lengths determined from internal evidence. It is surely harder, in a series of random numbers, to find a cycle of some given length—say 45-items—than to find the best cycle that is there.

(c) The Bartel's test makes no distinction between cycles which keep on coming true after discovery and those which are found in past experience alone. A cycle which repeats ten times and which then, after discovery, repeats ten more times, is obviously more significant than one which repeated twenty times all in the past.

However, in spite of its flaws the Bartel's test is the best test I know about. In fact it is the only test I know about which is not invalidated by the serial correlation present in all most all natural time series.

Further Comment

Don't be misled into thinking that because a cycle is highly significant there is any certainty that it will continue. Cycles are funny things. They come and go. They reappear. They are not always dependable. There are periods when they do not operate.

The regularity of the choo-choo of the old steam locomotive could not conceivably come about by chance, but what happens to the sound when the train goes around the hill? or into a tunnel? or when the engineer shuts off the power? It's the same way with cycles.

Problems

1. What is the significance of the 9.15-year cycle in the Standard & Poor's Corporation Index of common stock prices?

Note: Remember that you must remove the trend. Use deviations from a 9-year moving average.

A TABLE OF SINES AND COSINES OF ANGLES EXPRESSED AS PERCENTAGES OF PERIGONS

Lesson XV
Supplement 2

(360° = 100%)

Sin	.0	.2	.4	.6	.8	Cos
%		(All Figures	Positive)			%
0	0	.012	.025	.038	.050	75
1	.063	.075	.088	.100	.113	76
2	.125	.138	.150	.163	.175	77
3	.187	.200	.212	.224	.236	78
4	.249	.261	.273	.285	.297	79
5	.309	.321	.333	.345	.356	80
6	.368	.380	.391	.403	.414	81
7	.426	.437	.448	.460	.471	82
8	.482	.493	.504	.514	.525	83
9	.536	.546	.557	.567	.578	84
10	.588	.598	.608	.618	.628	85
11	.637	.647	.656	.666	.675	86
12	.684	.694	.702	.712	.720	87
13	.729	.737	.746	.754	.762	88
14	.770	.778	.786	.794	.802	89
15	.809	.816	.823	.831	.838	90
16	.844	.851	.857	.864	.870	91
17	.876	.882	.888	.894	.899	92
18	.905	.910	.915	.920	.925	93
19	.930	.934	.939	.943	.947	94
20	.951	.955	.958	.962	.965	95
21	.968	.972	.974	.977	.980	96
22	.982	.984	.987	.989	.990	97
23	.992	.994	.995	.996	.997	98
24	.998	.999	.999	1.000	1.000	99
25	1.000	1.000	1.000	.999	.999	0
26	.998	.997	.996	.995	.994	1
27	.992	.990	.989	.987	.984	2
28	.982	.980	.977	.974	.972	3
29	.968	.965	.962	.958	.955	4
30	.951	.947	.943	.939	.934	5
31	.930	.925	.920	.915	.910	6
32	.905	.899	.894	.888	.882	7
33	.876	.870	.864	.857	.851	8
34	.844	.838	.831	.823	.816	9
35	.809	.802	.794	.786	.778	10
36	.770	.762	.754	.746	.737	11
37	.729	.720	.712	.702	.694	12
38	.684	.675	.666	.656	.647	13
39	.637	.628	.618	.608	.598	14
40	.588	.578	.567	.557	.546	15
41	.536	.525	.514	.504	.493	16
42	.482	.471	.460	.448	.437	17
43	.426	.414	.403	.391	.380	18
44	.368	.356	.345	.333	.321	19
45	.309	.297	.285	.273	.261	20
46	.249	.236	.224	.212	.200	21
47	.187	.175	.163	.150	.138	22
48	.125	.113	.100	.088	.075	23
49	.063	.050	.038	.025	.012	24

Sin	.0	.2	.4	.6	.8	Cos
%		(All Figures	Negative)			%
50	0	.012	.025	.038	.050	25
51	.063	.075	.088	.100	.113	26
52	.125	.138	.150	.163	.175	27
53	.187	.200	.212	.224	.236	28
54	.249	.261	.273	.285	.297	29
55	.309	.321	.333	.345	.356	30
56	.368	.380	.391	.403	.414	31
57	.426	.437	.448	.460	.471	32
58	.482	.493	.504	.514	.525	33
59	.536	.546	.557	.567	.578	34
60	.588	.598	.608	.618	.628	35
61	.637	.647	.656	.666	.675	36
62	.684	.694	.702	.712	.720	37
63	.729	.737	.746	.754	.762	38
64	.770	.778	.786	.794	.802	39
65	.809	.816	.823	.831	.838	40
66	.844	.851	.857	.864	.870	41
67	.876	.882	.888	.894	.899	42
68	.905	.910	.915	.920	.925	43
69	.930	.934	.939	.943	.947	44
70	.951	.955	.958	.962	.965	45
71	.968	.972	.974	.977	.980	46
72	.982	.984	.987	.989	.990	47
73	.992	.994	.995	.996	.997	48
74	.998	.999	.999	1.000	1.000	49
75	1.000	1.000	1.000	.999	.999	50
76	.998	.997	.996	.995	.994	51
77	.992	.990	.989	.987	.984	52
78	.982	.980	.977	.974	.972	53
79	.968	.965	.962	.958	.955	54
80	.951	.947	.943	.939	.934	55
81	.930	.925	.920	.915	.910	56
82	.905	.899	.894	.888	.882	57
83	.876	.870	.864	.857	.851	58
84	.844	.838	.831	.823	.816	59
85	.809	.802	.794	.786	.778	60
86	.770	.762	.754	.746	.737	61
87	.729	.720	.712	.702	.694	62
88	.684	.675	.666	.656	.647	63
89	.637	.628	.618	.608	.598	64
90	.588	.578	.567	.557	.546	65
91	.536	.525	.514	.504	.493	66
92	.482	.471	.460	.448	.437	67
93	.426	.414	.403	.391	.380	68
94	.368	.356	.345	.333	.321	69
95	.309	.297	.285	.273	.261	70
96	.249	.236	.224	.212	.200	71
97	.187	.175	.163	.150	.138	72
98	.125	.113	.100	.088	.075	73
99	.063	.050	.038	.025	.012	74

C. E. Armstrong

CONCLUSION

Attached are two separate studies which illustrate first, the techniques involved in the isolation and definitization of a single cycle, and second, the result of an effort to make a more thorough study, definitizing several cycles, combining and projecting them to compare against actual experience.

The first study is "The 9.18-Year Cycle in Railroad Stock Prices, 1831-1955 and the Techniques Used in Its Definitization." This is reprinted from the Journal of Cycle Research for July, 1956.

This study gives a detailed account of the steps taken and the work done to isolate the 9.18-year cycle. All the figure work is included in the account.

The second reprint is "Trend and Cycles in Stock Prices" together with "The Stock Market Cycle Synthesis."

This summarizes the work done in isolating and combining eleven different waves and the trend to arrive at a synthesis line with which to compare actual stock prices. We are currently in the middle of auditing this study, which was originally done in 1944.

As a rough measure it can be said that the second study represents at least 11 times the work done in the first study. Those of you who are tempted to use daily figures should reflect a little about the time and effort used for this work. Remember also that we do not consider this a complete answer.

As you become interested in a particular series of data, or in a particular cycle length in any series, you may want to check and see if any work has already been done in the series or the length, you are studying.

For reference we are also including here the "Index to 'Cycles' - Vols. I - V".

Good luck.

TESTING CYCLES FOR STATISTICAL SIGNIFICANCE

Applying the Bartels Test of Significance to a Time Series Cycle Analysis

by Charles E. Armstrong

In the cycle analysis of time series, the question almost invariably arises as to whether or not a given period shows statistical evidence of reality. Among the various devices used in testing for significance, Bartels' technique seems to give perhaps the most reasonable results. This test of significance, when applied to the cycle analysis of time series, reflects the degree to which a particular periodicity is consistently present throughout the series under study, as well as the persistence of the cycle, i.e., the number of waves contained in the series. In this test the cycle curve for the given period under test is fitted to the entire series, and the amplitude of the resulting measured cycle is compared with the amplitude which might be expected to occur as a result of chance factors alone. This expected amplitude is determined by fitting the cycle curve for the given period separately to successive segments of the series, each segment being one period in length. From the n individual amplitudes so obtained, an estimate of the expected average amplitude is computed in accordance with the rule used in determining the dispersion of the mean, i.e., by dividing the quadratic mean of the individual amplitudes by $\sqrt{n}$. By comparing the measured average amplitude as obtained by fitting the cycle curve to the entire series with the expected average amplitude, we have a direct means, through the use of standard probability formulae, of arriving at a mathematical measure of the genuineness of the observed cycle. As will be seen, the resulting measure of genuineness will be high in cases where individual cycles exhibit stability in both amplitude and timing, and low where the opposite conditions obtain. Also, the value of n, the number of individual periods contained in the series, has a positive effect on this measure. Of course, for n=1 the test is meaningless.

Since periods other than the particular one under analysis affect both the measured and expected average amplitudes in the same way and to the same degree, the final measure of genuineness is not affected by such disturbing elements. Likewise, the effects of any serial correlation present in the series is nullified by appearing in both sides of the ratio of measured to expected amplitude. Hence, the test may with safety be applied to series of deviations from moving averages or from other trend or smoothing curve devices, the use of which may change the amount of serial correlation present in the series.

The Bartels test is not designed primarily as a means of locating the periods of cycles present in a series. Its chief value lies in its application as a test of significance after the period has been located by some other means. However, in the process of locating the exact period, it is often possible to set up the work processes and papers in such a way as to facilitate the application of the Bartels test after the period has been determined. The following tables and chart illustrate the application of the test to the results of a harmonic analysis of a typical economic series.

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