Column 7 is the product of the values in Columns 4 (the cosine curve and 5 (the data).

Now you total columns 5, 6, and 7.

What you are striving to do is to find the sine curve and the cosine curve which, when combined into a sine-cosine curve, will most nearly fit your data. You can't change the shape of either sine curve or cosine curve, but you can change their amplitudes. By getting the proper amplitude for the sine curve and for the cosine curve, not only will the sine-cosine curve have the proper amplitude, but it will have the proper phase as well.

I'll tell you how to get these amplitudes in a minute, but for now let's go on with a description of the table.

A is the symbol uded to denote the amplitude of the sine. It turns out to be, in this example, 15.519. Column 8 then is merely 15.519 times the various values in Column 3 (the values of the sine curve with an amplitude of 1). Column 8 is therefore merely a sine curve with an amplitude of 15.519.

B is the amplitude of the cosine. It turns out to be, in this example, 4.9133. Column 9 is merely 4.9133 times the various values in Column 4. Column 9 is therefore merely a cosine curve with an amplitude of 4.9133.

Column 10 is the combination of the sine curve of proper (15.519) amplitude (Col. 8) and the cosine curve of proper (4.9133) amplitude (Col. 9) and the average value of the Y values (100). It is your answer. It is plotted in Fig. 9 by means of a broken line.

Now, how do you get your A and your B?

A is the sum of the sines times the Y values (186.23) divided by half the number of terms (12).

$$\frac{186.23}{12} = 15.519$$

B is the sum of the cosines times the Y values (58.96) divided by half the number of terms (12).

$$\frac{58.96}{12} = 4.9133$$

Easy, isn't it?

The Shape of Sine and Cosine Curves

When I spoke of the shape of sine and of cosine curves I referred to their mathematical characteristics. Visually one sine curve (or one cosine curve) may look very different from another. See Fig. 10. All of the curves in Fig. 10 are sines, but of different period and/or amplitude.

Sines and Cosines as Percentages of Perigons

Instead of using 360° to represent the entire wave it is often easier to consider the whole wave as 100%. When you do this, 25% corresponds to 90°, 50% to 180°, etc. The whole 360° is known as a perigon.

A table of sines and cosines expressed as percentages of perigons is appended as a supplement.

Amplitude of Sine-Cosine Curves

The overall amplitude of the sine-cosine curve is twice the square root of the sum of the squares of the amplitude of the sine and cosine curves which were combined to create it. The formula $2\sqrt{A^2 + B^2}$. In the example given $32.6 = 2\sqrt{(15.519)^2 + (4.9133)^2}$.

The value of 32.6 is the overall amplitude (116.3 - 83.7 = 32.6).

For Those Who Like Formulae
(Skip if you wish)

The formula for the sine-cosine curve is as follows:

$$Y_c = \bar{Y} + A \sin\left(\frac{360}{T} X\right)^\circ + B \cos\left(\frac{360}{T} X\right)^\circ$$

Where

Y_c = any computed Y value
$\bar{Y}$ = the average of the Y values
A = the constant of the sine component (at 90° this constant equals the amplitude of the sine curve).
B = the constant of the cosine component (at 0° or 360° this constant equals the amplitude of the cosine curve).
T = the periodicity
X = a variable

$$A = \frac{2}{T} \Sigma \left[Y \sin\left(\frac{360}{T} X\right)^\circ\right]$$

$$B = \frac{2}{T} \Sigma \left[Y \cos\left(\frac{360}{T} X\right)^\circ\right]$$

In the example given $T = 24$ (24 months) and $\bar{Y} = 100.0$. Hence

$$Y_c = 100 + A \sin(15\,X)^\circ + B \cos(15\,X)^\circ$$

$$A = \frac{\Sigma(Y \sin(15\,X)^\circ)}{12}$$

$$B = \frac{\Sigma(Y \cos(15\,X)^\circ)}{12}$$

$$A = \frac{186.23}{12} = 15.519$$

$$B = \frac{58.96}{12} = 4.9133$$

Hence

$$Y_c = 100 + 15.519 \sin(15\,X)^\circ + 4.9133 \cos(15\,X)^\circ$$

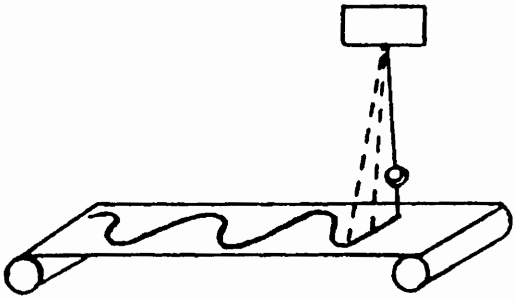


Fig. 1

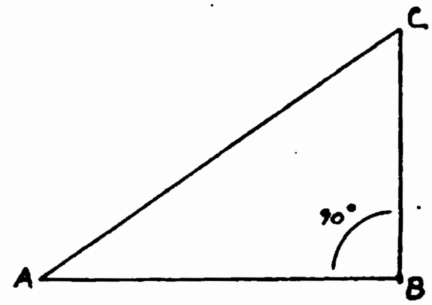


Fig. 2

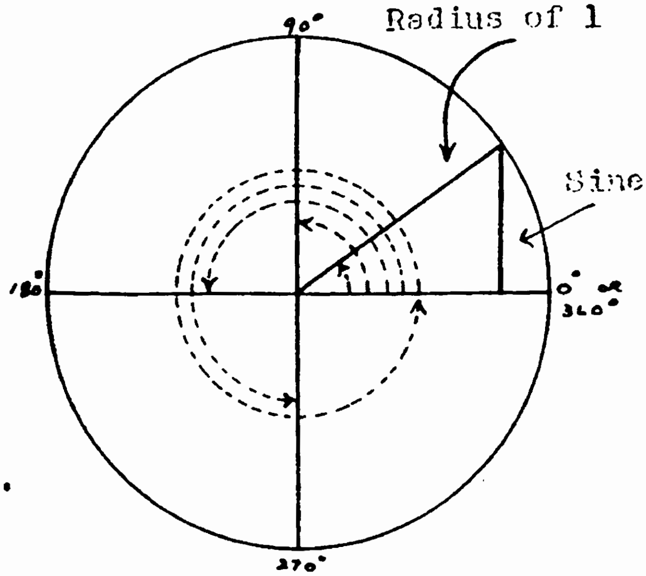


Fig. 3

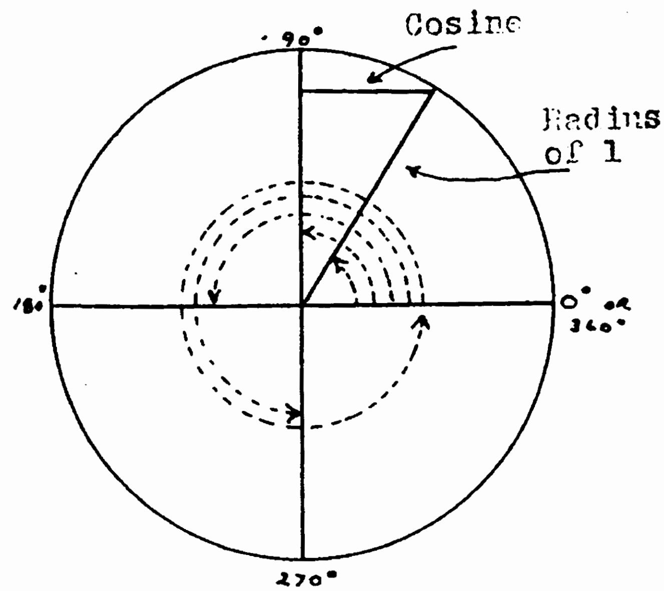


Fig. 4

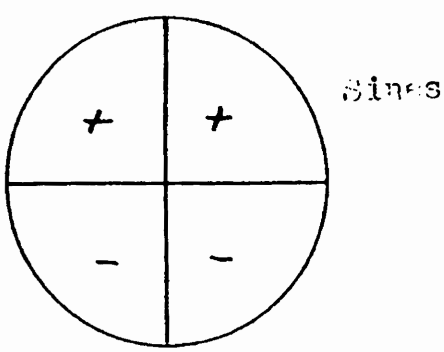


Fig. 5

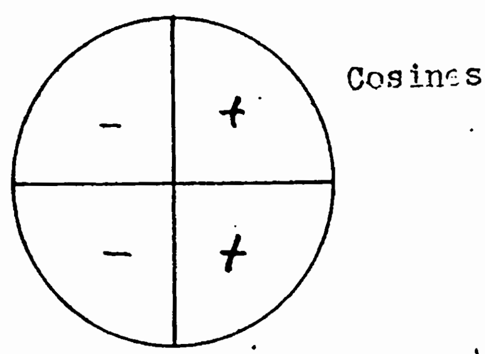


Fig. 6

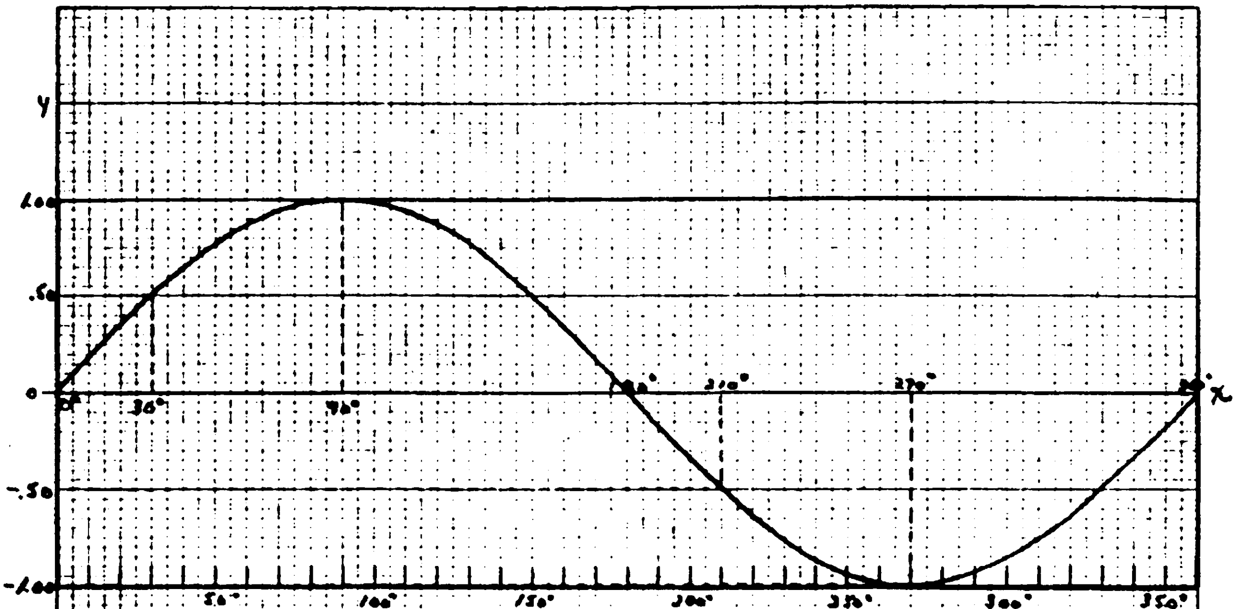


Fig. 7 A Sine Curve

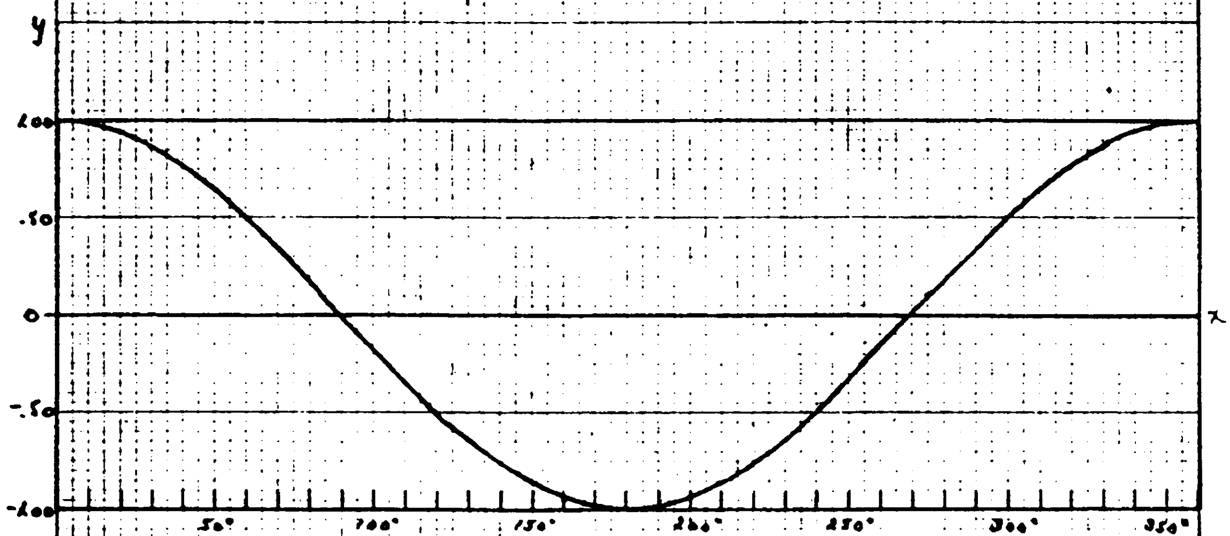


Fig. 8 A Cosine Curve

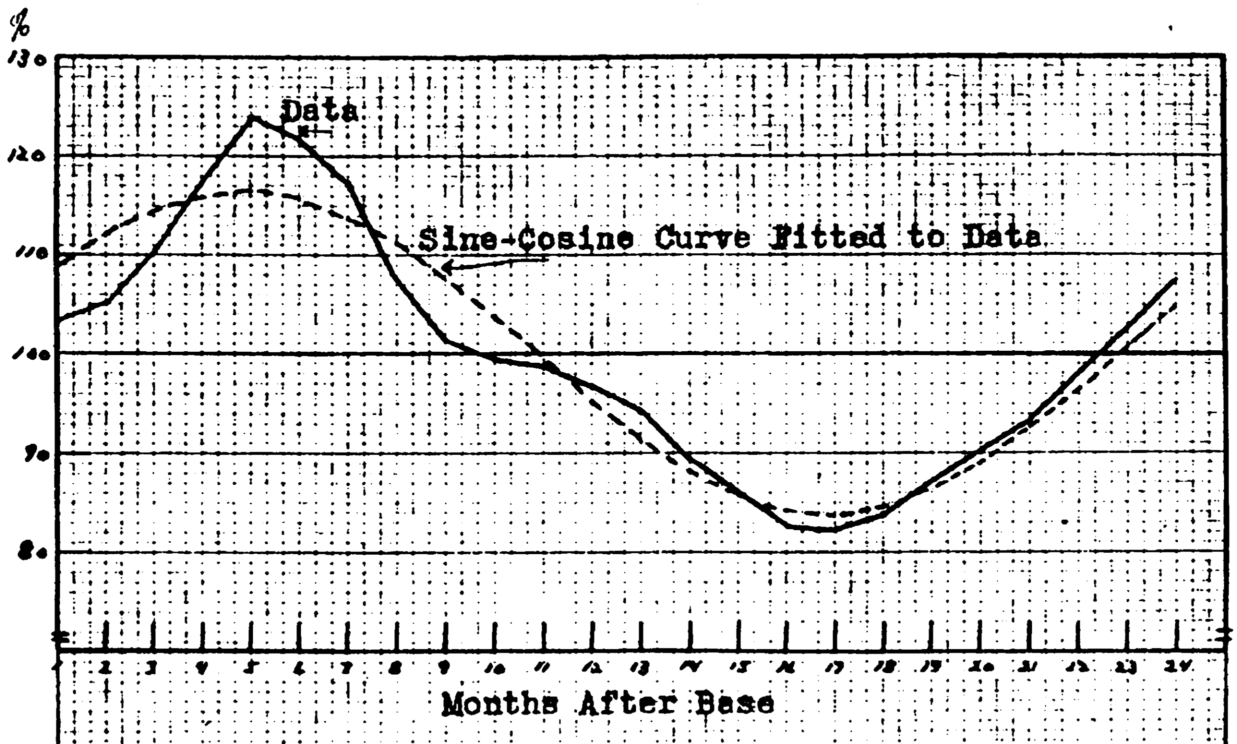
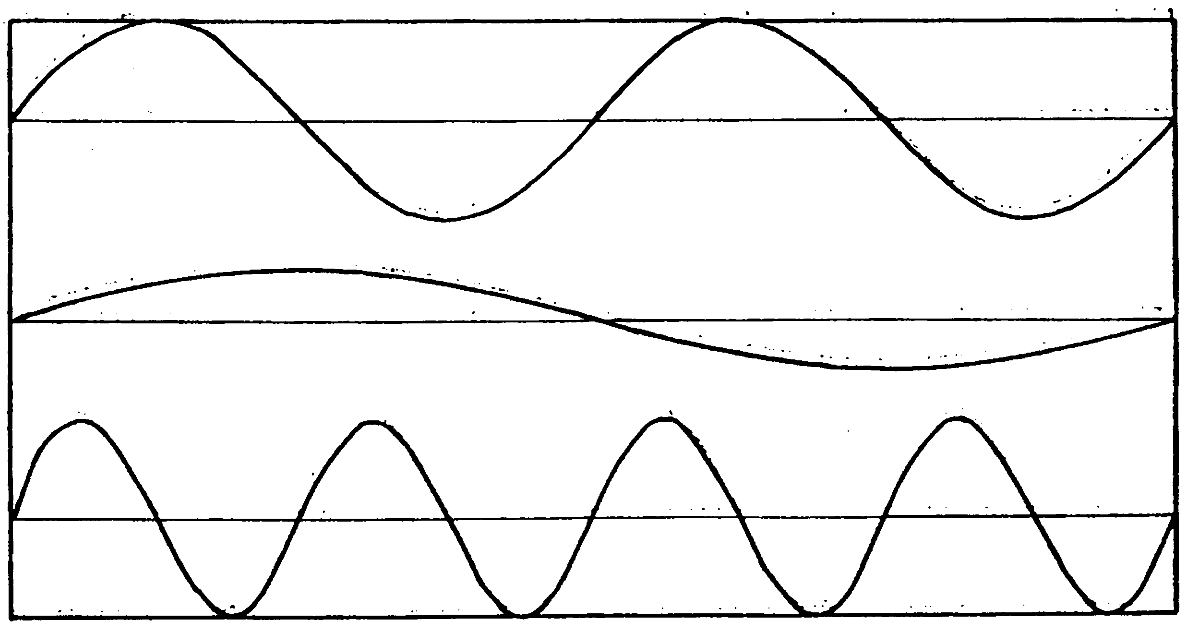
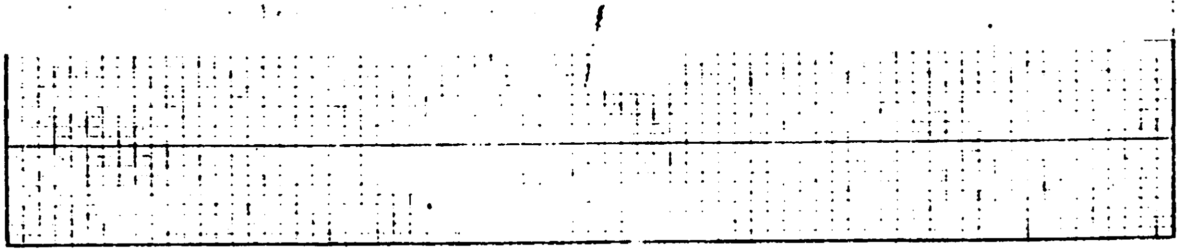


Fig. 9 Averages of 24 Month Periodic Table of Non-Acetate Rayon Deliveries, January, 1923 - December, 1934 And Fitted Sine-Cosine Curve





Problems

Problem 1. Fit a sine cosine curve to a 9-year cycle of the following characteristics:

Position	1	93
"	2	87
"	3	90
"	4	100
"	5	110
"	6	125
"	7	102
"	8	98
"	9	95

Plot the sine curve, the cosine curve, and the sine-cosine curve.

Brief Table of Sines and Cosines

To find the sine or cosine of any angle greater than 90°, subtract the angle from some integral multiple of 180° and use the table below. Signs to be prefixed to table values are as follows:

Quadrant	Angle (degrees)	Sin	Cos
I	0-90	+	+
II	90-180	+	-
III	180-270	-	-
IV	270-360	-	+

Degree	Sine	Cosine	Degree	Sine	Cosine	Degree	Sine	Cosine
0	.0000	1.0000	30	.5000	.8660	60	.8660	.5000
1	.0175	.9998	31	.5150	.8572	61	.8746	.4848
2	.0349	.9994	32	.5299	.8480	62	.8829	.4695
3	.0523	.9986	33	.5446	.8387	63	.8910	.4540
4	.0698	.9976	34	.5592	.8290	64	.8988	.4384
5	.0872	.9962	35	.5736	.8192	65	.9063	.4226
6	.1045	.9945	36	.5878	.8090	66	.9135	.4067
7	.1219	.9925	37	.6018	.7986	67	.9205	.3907
8	.1392	.9903	38	.6157	.7880	68	.9272	.3746
9	.1564	.9877	39	.6293	.7771	69	.9336	.3584
10	.1736	.9848	40	.6428	.7660	70	.9397	.3420
11	.1908	.9816	41	.6561	.7547	71	.9455	.3256
12	.2079	.9781	42	.6691	.7431	72	.9511	.3090
13	.2250	.9744	43	.6820	.7314	73	.9563	.2924
14	.2419	.9703	44	.6947	.7193	74	.9613	.2756
15	.2588	.9659	45	.7071	.7071	75	.9659	.2588
16	.2756	.9613	46	.7193	.6947	76	.9703	.2419
17	.2924	.9563	47	.7314	.6820	77	.9744	.2250
18	.3090	.9511	48	.7431	.6691	78	.9781	.2079
19	.3256	.9455	49	.7547	.6561	79	.9816	.1908
20	.3420	.9397	50	.7660	.6428	80	.9848	.1736
21	.3584	.9336	51	.7771	.6293	81	.9877	.1564
22	.3746	.9272	52	.7880	.6157	82	.9903	.1392
23	.3907	.9205	53	.7986	.6018	83	.9925	.1219
24	.4067	.9135	54	.8090	.5878	84	.9945	.1045
25	.4226	.9063	55	.8192	.5736	85	.9962	.0872
26	.4384	.8988	56	.8290	.5592	86	.9976	.0698
27	.4540	.8910	57	.8387	.5446	87	.9986	.0523
28	.4695	.8829	58	.8480	.5299	88	.9994	.0349
29	.4848	.8746	59	.8572	.5150	89	.9998	.0175
						90	1.0000	.0000

A TABLE OF SINES AND COSINES OF ANGLES EXPRESSED AS PERCENTAGES OF PERIGONS

(360° = 100%)

Sin	.0	.2	.4	.6	.8	Cos
%	(All Figures Positive)					%
0	0	.012	.025	.038	.050	75
1	.063	.075	.088	.100	.113	76
2	.125	.138	.150	.163	.175	77
3	.187	.200	.212	.224	.236	78
4	.249	.261	.273	.285	.297	79
5	.309	.321	.333	.345	.356	80
6	.368	.380	.391	.403	.414	81
7	.426	.437	.448	.460	.471	82
8	.482	.493	.504	.514	.525	83
9	.536	.546	.557	.567	.578	84
10	.588	.598	.608	.618	.628	85
11	.637	.647	.656	.666	.675	86
12	.684	.694	.702	.712	.720	87
13	.729	.737	.746	.754	.762	88
14	.770	.778	.786	.794	.802	89
15	.809	.816	.823	.831	.838	90
16	.844	.851	.857	.864	.870	91
17	.876	.882	.888	.894	.899	92
18	.905	.910	.915	.920	.925	93
19	.930	.934	.939	.943	.947	94
20	.951	.955	.958	.962	.965	95
21	.968	.972	.974	.977	.980	96
22	.982	.984	.987	.989	.990	97
23	.992	.994	.995	.996	.997	98
24	.998	.999	.999	1.000	1.000	99
25	1.000	1.000	1.000	.999	.999	0
26	.998	.997	.996	.995	.994	1
27	.992	.990	.989	.987	.984	2
28	.982	.980	.977	.974	.972	3
29	.968	.965	.962	.958	.955	4
30	.951	.947	.943	.939	.934	5
31	.930	.925	.920	.915	.910	6
32	.905	.899	.894	.888	.882	7
33	.876	.870	.864	.857	.851	8
34	.844	.838	.831	.823	.816	9
35	.809	.802	.794	.786	.778	10
36	.770	.762	.754	.746	.737	11
37	.729	.720	.712	.702	.694	12
38	.684	.675	.666	.656	.647	13
39	.637	.628	.618	.608	.598	14
40	.588	.578	.567	.557	.546	15
41	.536	.525	.514	.504	.493	16
42	.482	.471	.460	.448	.437	17
43	.426	.414	.403	.391	.380	18
44	.368	.356	.345	.333	.321	19
45	.309	.297	.285	.273	.261	20
46	.249	.236	.224	.212	.200	21
47	.187	.175	.163	.150	.138	22
48	.125	.113	.100	.088	.075	23
49	.063	.050	.038	.025	.012	24

Sin	.0	.2	.4	.6	.8	Cos
%	(All Figures Negative)					%
50	0	.012	.025	.038	.050	25
51	.063	.075	.088	.100	.113	26
52	.125	.138	.150	.163	.175	27
53	.187	.200	.212	.224	.236	28
54	.249	.261	.273	.285	.297	29
55	.309	.321	.333	.345	.356	30
56	.368	.380	.391	.403	.414	31
57	.426	.437	.448	.460	.471	32
58	.482	.493	.504	.514	.525	33
59	.536	.546	.557	.567	.578	34
60	.588	.598	.608	.618	.628	35
61	.637	.647	.656	.666	.675	36
62	.684	.694	.702	.712	.720	37
63	.729	.737	.746	.754	.762	38
64	.770	.778	.786	.794	.802	39
65	.809	.816	.823	.831	.838	40
66	.844	.851	.857	.864	.870	41
67	.876	.882	.888	.894	.899	42
68	.905	.910	.915	.920	.925	43
69	.930	.934	.939	.943	.947	44
70	.951	.955	.958	.962	.965	45
71	.968	.972	.974	.977	.980	46
72	.982	.984	.987	.989	.990	47
73	.992	.994	.995	.996	.997	48
74	.998	.999	.999	1.000	1.000	49
75	1.000	1.000	1.000	.999	.999	50
76	.998	.997	.996	.995	.994	51
77	.992	.990	.989	.987	.984	52
78	.982	.980	.977	.974	.972	53
79	.968	.965	.962	.958	.955	54
80	.951	.947	.943	.939	.934	55
81	.930	.925	.920	.915	.910	56
82	.905	.899	.894	.888	.882	57
83	.876	.870	.864	.857	.851	58
84	.844	.838	.831	.823	.816	59
85	.809	.802	.794	.786	.778	60
86	.770	.762	.754	.746	.737	61
87	.729	.720	.712	.702	.694	62
88	.684	.675	.666	.656	.647	63
89	.637	.628	.618	.608	.598	64
90	.588	.578	.567	.557	.546	65
91	.536	.525	.514	.504	.493	66
92	.482	.471	.460	.448	.437	67
93	.426	.414	.403	.391	.380	68
94	.368	.356	.345	.333	.321	69
95	.309	.297	.285	.273	.261	70
96	.249	.236	.224	.212	.200	71
97	.187	.175	.163	.150	.138	72
98	.125	.113	.100	.088	.075	73
99	.063	.050	.038	.025	.012	74

C. E. Armstrong

LESSON XVI

How to Make a Simple Harmonic Analysis

You hear so much about harmonic analysis that you should know about it.

This is how you make a simple harmonic analysis:

Take a series of figures.

Fit a sine-cosine curve to them.

Make a periodic table 1/2 the length of the series (2nd harmonic).

Fit a sine-cosine curve to the averages of this table (or, if you like, fit two consecutive sine-cosine curves to your series.)

Make a periodic table 1/3 the length of the series (3rd harmonic).

Fit a sine-cosine curve to the averages of this table (or, if you like, fit three consecutive sine-cosine curves to your series.)

Continue the process as far as you like.

As you get more and more sine-cosine curves their sum will come closer and closer to reproducing the original curve.

If you carry the process far enough (50 or 75 harmonics) the synthesis of the various harmonic sine-cosine curves will be almost as good as a photostat of a curve of your data.

If you project the various sine-cosine curves into the future you will get an almost exact duplication of the curve you started with. But it would be a lot easier and cheaper to get a photostat and paste it next to the original curve.

Harmonic analysis is useful in the analysis of sound waves (hence its name). It is also useful in tide analysis, in engineering, and in other special instances. Harmonic analysis is a method of breaking a curve into a number of mathematical building blocks. However, these building blocks have <u>no necessarily relation to the real cycles</u> which may be present in the original data, unless these cycles happen to be unit fractions of the length of your curve. Harmonic analysis is a method of <u>description</u>. HARMONIC ANALYSIS, AS SUCH, IS ABSOLUTELY NO GOOD FOR FORECASTING.

Why then do I devote a lesson to it? To keep you from burning yourself! And because a modification of harmonic analysis, to be discussed, <u>is</u> of value.

The results of a harmonic analysis are often recorded on a periodogram. Harmonic analysis is therefore sometimes (erroneously) called periodogram analysis.

Now, because harmonic analysis is a well known mathematical technique, it is fully described in many textbooks. One of the clearest and simplest descriptions is that contained in a book called <u>The Science of Musical Sounds</u>. This book, now out of print, was written in 1937 by the late Dayton C. Miller, Professor of Physics, Case School of Applied Science, Cleveland, Ohio. Excerpts from that book, are enclosed herewith as supplemental material.

A harmonic analysis of a curve can be made mechanically. Material being sent you herewith will tell you something about this method of making a harmonic analysis. Harmonic analyses can also be made electronically.

Multiple or Compound Harmonic Analysis

If you have access to a mechanical or an electronic analyzer you can make what is known as a multiple or compound harmonic analysis. This is merely a number of harmonic analyses of shorter and shorter sections of the curve. The result is a periodogram with points between the harmonic intervals of the original curve as well as at them.

Multiple harmonic analysis is of real help in cycle analysis because it gives you, as exactly as you wish, the length of all the cycles which are present on the average, whether they are harmonics or not.

You don't really need a harmonic analyzer to make multiple harmonic analysis. You can achieve approximately the same result by making periodic tables of closely consecutive lengths, fitting sine-cosine curves to the averages of the various columns of the tables, and plotting the amplitudes of these various sine curves to a periodogram. The advantage of a mechanical or an electrical gadget is it is easier.

I have made several multiple harmonic analyses. Some of these have been reported upon in Cycles. I believe that if you study the accounts of these analyses you will get as good an idea of this process as I am able to give you.

Start your lesson by studying the paper called "Multiple Harmonic Analysis Applied to Cotton Prices, 1731-32---1939-40," reprinted from Cycles of November 1950, pages 20-26, and continued in Cycles of April 1951, pages 141-42.

The article referred to on page 20 is also reprinted as part of this supplement, as are the actual cotton prices.

A picture of the Mico Instrument Company's latest model Henrici type analyzer is likewise shown.

I am sorry that I have never had time to work up for publication the results of this analysis as it applies to cycles of $5\frac{1}{2}$ to 20 years in length.

Study next the article "Possible Cycles in Railroad Stock Prices, 1831-1950" reprinted from the April 1952 issue of Cycles, pages 123-129, and a supplement from page 140 of the same issue.

Next study the article called "Possible Cycles in Industrial Stock Prices, 1871-1950" reprinted from Cycles for May 1952, pages 166-178.

Finally read the last article of this series called "Possible Cycles in Pig Iron Prices, 1784-1951" reprinted from Cycles for June 1952, pages 207-9. This supplement also includes a chart of pig iron prices 1784-1951 taken from Cycles for May 1952, pages 164-5.

In addition to telling you about multiple harmonic analysis the supplements should prove of assistance to you for reference purposes.

EXCERPTS FROM

THE SCIENCE OF MUSICAL SOUNDS

by Dayton C. Miller

Simple Harmonic Motion and Curve[1]

The simplest possible type of vibration which a particle of elastic matter of any kind may have is called <u>simple harmonic motion</u>; it takes place in a straight line, the middle of which is the position of rest of the particle; when the particle is displaced from this position, elasticity develops a force tending to restore it, which force is <u>directly proportional to the amount of the displacement</u>; if the displaced particle is now freely released, it will vibrate to and fro with simple harmonic motion. The name originated in the fact that musical sounds in general are produced by complex vibrations which can be resolved into component motions of this type.

Other forces than those of elasticity may act in the manner described, as for instance the action of the force of gravity on the bob of a pendulum; if the bob is considered as swinging in a straight line, it has simple harmonic motion, which is also called <u>pendular motion</u>.

Simple harmonic motion has several evident features: it takes place in a straight line; it is vibratory moving to and fro; it is periodic, repeating its movements regularly; there are instants of rest at the two extremes of the movement; starting from rest at one extreme the movement quickens till it reaches its central point, after which it slackens in reverse order, till it comes to rest at the other extreme. The speed of the particle so moving, the rate at which the speed changes, and other features are very important in a complete study of simple harmonic motion, but for our purpose we need give only a few simple definitions.

The <u>frequency</u> of a simple harmonic motion is the number of complete vibrations to <u>and</u> fro per second; the <u>period</u> is the time required for one complete vibration; the <u>amplitude</u> is the range on one side or the other from the middle point of the motion, therefore it is half the extreme range of vibration; the <u>phase</u> at any instant is the fraction of a period which has elapsed since the point last passed through its middle position in the direction chosen as positive.

Simple harmonic motion is approximated in various mechanical movements, while a few simple machines reproduce it exactly ....

A simple harmonic motion combined with a uniform motion of translation traces a <u>simple harmonic curve</u>; this condition is illustrated by a pendulum swinging from a fixed point (See Fig. 1 of Lesson XV), and leaving a trace on a sheet of paper moving underneath. The simple harmonic curve (See Fig. 7 of Lesson XV), is perfectly simple, regular, and symmetrical; in mathematical study it is frequently referred to as a <u>sine curve</u>; a curve of the same form but differing in phase by a quarter period, or 90°, is a <u>cosine curve</u> (See Fig. 8 of Lesson XV) ....such a curve is an instantaneous representation of the condition of motion in a simple wave. Various terms used with regard to simple harmonic motion are also applicable to the curve; the amplitude is the height of a crest above the axis; the period is the time required to trace one wave length consisting of a crest and trough; the frequency is the number of periods, or wave lengths traced, per second; the phase varies along the axis, passing through a complete cycle in one wave length; the velocity is the rate of translation and is equal to the wave length multiplied by the number of waves per second.

[1] Chapter I, Pages 6-12

Harmonic Analysis[2]

Curves and wave forms such as those obtained with the phonodeik are representative not only of sound, but of many other physical phenomena, and their study is of general importance in science. While inspection and simple measurement will often give some information concerning these curves, as will be explained later, they are in general too complicated for interpretation in their original forms, and several methods of analysis have been developed which greatly assist in our understanding of them.

In the wave method of analysis, often used in optics, the attention is directed to the speed and direction of propagation of the waves in the medium and to their combined effects; in the harmonic method consideration is given primarily to the vibratory character of the movements of the medium, these vibrations being regarded as compounded of a series of motions, which may be infinite in number, but each of which is of a simple definite type.

For the investigation of the complex curves of the sounds of music and speech, the harmonic method of analysis is the most suitable and convenient; it is based upon the important mathematical principle known as <u>Fourier's Theorem</u>, the statement and proof of which was first published in Paris, in 1822, by Baron J. B. J. Fourier. For the present purpose Fourier's theorem may be stated as follows: If any curve be given, having a wave length <u>l</u>, the same curve can always be reproduced and in one particular way only, by compounding simple harmonic curves of suitable amplitudes and phases, in general infinite in number, having the same axis, and having wave lengths of <u>l</u>, 1/2 <u>l</u>, 1/3 <u>l</u>, and successive aliquot parts of <u>l</u>; the given curve may have any arbitrary form whatever, including any number of straight portions, provided that the ordinate of the curve is always finite and that the projection on the axis of a point describing the curve moves always in the same direction. Many of the curves studied by this method can be exactly reproduced by compounding a limited number of the simple curves; for sound waves the number of components required is often more than ten, and rarely as many as thirty; in some arbitrary mathematical curves, a finite number of components gives only a more or less approximate representation, while an exact reproduction requires the infinite series of components.

Fourier's theorem may be stated in mathematical form in the <u>Fourier Equation</u> as follows:

$$y=\frac{1}{l}\int_0^l y\,dx + \left\{\begin{array}{l} \left[\frac{2}{l}\int_0^l y\sin\frac{2\pi x}{l}dx\right]\sin\frac{2\pi x}{l} + \left[\frac{2}{l}\int_0^l y\sin\frac{4\pi x}{l}dx\right]\sin\frac{4\pi x}{l} + \dots\dots\dots\dots \\ \left[\frac{2}{l}\int_0^l y\cos\frac{2\pi x}{l}dx\right]\cos\frac{2\pi x}{l} + \left[\frac{2}{l}\int_0^l y\cos\frac{4\pi x}{l}dx\right]\cos\frac{4\pi x}{l} + \dots\dots\dots\dots \end{array}\right. \qquad \text{I}$$

In this equation <u>y</u> is the ordinate of the original complex curve at any specified point <u>x</u> on the base line, and <u>l</u> is the fundamental wave length. The principal part of this equation is a trigonometric series of sines and cosines and this (or the whole equation) is often referred to as <u>Fourier's Series</u>.

The Fourier equation may be given a simpler appearance by writing it in a second symbolic form:

$$y=a_0 + \left\{\begin{array}{l} a_1\ \sin\theta + a_2\ \sin 2\theta + a_3\ \sin 3\theta + \dots\dots\dots\dots\dots\dots \\ b_1\ \cos\theta + b_2\ \cos 2\theta + b_3\ \cos 3\theta + \dots\dots\dots\dots\dots\dots \end{array}\right. \qquad \text{II}$$

[2] Chapter IV, Pages 92-97

The term a_0 is a constant and is equal to the distance between the chosen base line and the true axis of the curve; if the base line coincides with the axis, $a_0 = 0$, and this term does not appear in the equation of the curve. Since this term has no relation to the shape of the curve, its value is not required in sound analysis...

The other terms of the equation occur in pairs, as $a_1 \sin\theta$, $b_1 \cos\theta$, etc., and each, whether a sine or cosine term, represents a simple harmonic curve. The successive simple curves of the sine series evidently repeat themselves with frequencies of 1, 2, 3, etc., that is, they have wave lengths in the proportions of 1, 1/2, 1/3, etc., and the same is true of the cosine series.

Each of the coefficients a_1, b_1, a_2, b_2, etc., is a number or factor indicating how much of the ocrresponding simple harmonic curve enters into the composite; that is, it shows the amplitude, or height, of the simple wave. For the reproduction of a given curve it may happen that certain of the simple curves are not required, and the corresponding coefficients then have the value zero and their terms do not appear in the Fourier equation of the curve.

A sine and a cosine curve of the same frequency but with independent amplitudes, such as the pairs of curves in the Fourier equation, can be compounded into a single sine (or cosine) curve of like frequency which starts on the axis at a point different from that of the component curves, and which has an amplitude dependent upon the amplitudes of the components. The relation of the starting point of the new curve to that of its components is called its phase............ This principle may be stated in symbols as follows, *a* and *b* being the amplitudes of the given curves, and *A* that of the resultant, and *P* the phase of the new curve:

$$a \sin\theta + b \cos\theta = A \sin(\theta + P),$$

when $$A = \sqrt{a^2 + b^2},$$

and $$\tan P = \frac{b}{a}.$$

If the amplitudes *a* and *b* are made the base and altitude, respectively, of a right triangle, Fig. 73, then the hypothenuse is the amplitude *A* of the resultant curve and the angle which the hypothenuse makes with the base is the phase.

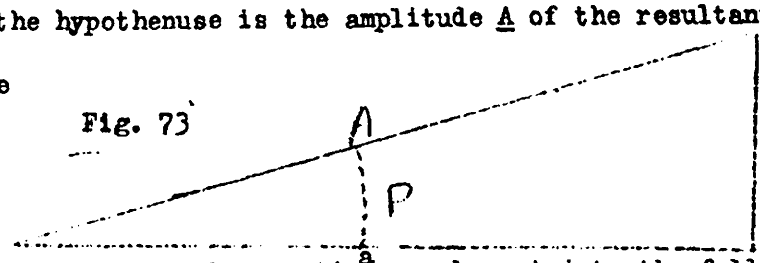


Fig. 73

If each pair of sine and cosine terms of the general Fourier equation is reduced in this manner, and if the origin is on the axis of the curve, the equation may be put into the following equivalent form, consisting of a single series of sines:

$$y = A_1 \sin(\theta + P_1) + A_2 \sin(2\theta + P_2) + A_3 \sin(3\theta + P_3) + \ldots \qquad \text{III}$$

In this equation A_1 is the amplitude of the first component (the fundamental tone) and P_1 is its phase; while A_2 and P_2 determine the second component (first overtone or octave), etc.

Form III of the Fourier equation is most suitable for representing the results of the physical analysis of a sound, though the actual mmerical analysis is obtained in the first form, I, of the equation.

Technical

Multiple Harmonic Analysis

APPLIED TO COTTON PRICES 1731-32—1939-40

IN the Research Department of this issue I reported upon some hints of long cycles in cotton prices.* These hints were revealed by a multiple harmonic analysis of those prices.

An assistant and I made the analysis on an Henrici type mechanical analyzer manufactured and placed at our disposal for the purpose through the courtesy of the Mico Instrument Company of Cambridge, Massachusetts.

This article will tell you just how that analysis was made. In other words, it will work out for you an example of the technique of multiple harmonic analysis. I would suggest that before reading this article you read the other one on page 17 for background.

When you make a harmonic analysis with an Henrici type machine, you trace with a stylus the curve to be analyzed, Fig. 1 on pages 18 and 19. Then you read the sine-cosine values from the dials on the machine. From these values, which are in centimeters, you can easily compute an index of the amplitude, or an index of the slope of the wave. Values for five harmonics can be read at one time. I think the procedure will be clear to you as we proceed.

We first traced the curve with the machine set for harmonics 1, 2, 3, 4, and 5.

As the curve was 209 years long, the readings for harmonic 1, the fundamental, enabled us to compute the amplitude of a sine curve fitted to the entire series of data—that is, to all of Fig. 1. This value turned out to be 6.6. This wave has no meaning whatever for forecasting purposes. It merely means that on the average, one 104½-year sections of the curve we are analyzing is higher than the other 104½ years.

The lengths of the five harmonics, and their relative amplitudes, are given in the table below.

Table 1

A Name of Harmonic	B Length as a Fraction of the Fundamental	C Length in Years	D Relative Amplitude of Fitted Sine Curve, (Centimeters)
Fundamental	1	209	6.58
2nd	1/2	104.5	2.92
3rd	1/3	69.7	3.80
4th	1/4	52.25	3.89
5th	1/5	41.8	3.84

What does this table tell you? It tells you that IF there are waves of the indicated length, and if they are sine shaped, that they have the relative amplitude shown.

It does not tell you that waves of this length are real, or if real, that they are repetitive.

After we took our readings for the first five harmonics, we retraced the curve with the gear wheels changed so that the machine would give us readings for the 6th to the 10th harmonics (1/6 of 209 years down to 1/10 of 209 years; in other words, from 34.8 years down by four steps to 20.9 years.)

Then we retraced the curve with the machine set to give us readings for the 11th to the 15th harmonics, and so on, down

*Reprinted at the end of this article.

to the 30th, 31st, 32nd, 33rd, 34th and 35th harmonics. (The 35th harmonic would have a length of 1/35 of 209 years, or 5.971 years.)

What we found as we went down the scale is too long a story to report upon here, but I shall give you the results of it another time. Let us therefore get back to a consideration of the first five harmonics, which is the subject of this report.

I want to make sure at this point that you understand just what the machine did.

I think you understand about the first harmonic or fundamental, but I am not sure you understand about the other harmonics. To understand about them, let us forget about the machine for a moment.

There are 209 years in our series. Let us call it 208 to avoid the complication of the odd number. Now let us make a 104-year periodic table of our 208 numbers. We do this by writing the first 104 numbers on a line, each number in a column, and the next 104 numbers on another line beneath them. The 105th number will thus be under the first number, the 106th number under the second number, etc., and we will have 104 columns with two numbers (two lines) in each.

We now average each of the 104 columns in our periodic table and get a series of 104 averages. In all probability, the average of some consecutive 52 numbers out of the 104 will be higher or lower than the average of the other numbers. Therefore, if we fit a sine curve to the entire series of 104 numbers, the sine curve will have some height or amplitude. If you go through all this work for the figures we are dealing with, and if you used the same vertical scale I did, you will find that the amplitude of the sine wave fitted to this series of 104 figures will be approximately 2.92 centimeters. (You will recall that this is the value given in Table 1 above for the second harmonic.)

The number does not mean anything in particular. It depends upon the scale you chose for your chart. If you knew the scale you could convert it into logs, and thence into percent, but there is no particular use, as all we wish at this point is a relative number so as to compare one wave with another.

If you now make a 69-month periodic table of our 209 numbers, again disregarding fractions for the purposes of the example, you will have three lines of 69 figures each. A sine curve fitted to an average of each of these 69 columns gives you a relative amplitude of 3.90) or it would if you took the fractions into account, as the machine does).

Similarly, throwing our 209 numbers into a periodic table of 52 columns and four lines would give you another series of average figures. A sine curve fitted to them would have a relative amplitude of 3.89.

And finally, a sine curve fitted to the averages of a 52-column table of five lines will have, on the same scale, an amplitude of 3.84.

(The machine does all this work for you at one clip in the two or three minutes it takes to trace the curve with the stylus.)

It should be obvious that the lengths are meaningless as far as indicating any rhythms. They are purely arbitrary halves, thirds, fourths, fifths, etc., of a fundamental length which, in its turn, is a purely accidental length, depending on the length of our series of figures. If we had done this work 49 years earlier, our fundamental would have been 160 years long, and the second, third, fourth, and fifth harmonics would have been 80, 53 1/3, 40, and 32 years respectively.

The fact that the sine waves fitted to the averages of our periodic table have amplitude merely means that some groups of our figures are bigger than other groups. The consequence of this fact is that the cycle analyst is not interested in simple harmonic analysis as such for the same reason that a certain man was not interested in horse racing. You will remember that this fellow said, "Anyone knows that one horse can run faster than another." One group of figures is almost always higher or lower than some other group. Therefore the amplitude of the various harmonic functions does not mean that there are rhythms or repetitive waves of this length in the figures.

To see how this relatively meaningless information can become valuable you will have to bear with me a little longer.

To start with, let us concentrate our attention for a moment upon the second harmonic, the average of the sections 104 years long (ignoring fractions, which we shall leave to the machine to bother with).

It is obvious that IF there really did happen to be a regular wave of 104 years in these figures, the crest of this wave in both the first and second lines—that is, in both the first and the second 104-year periods—would fall under each other, and so would the troughs. In fact, if the series of 208 figures consisted of nothing but two perfectly regular 104-year waves, the figures in each column of the second line would be identical with the figures above it in the first line, and the average would, of course, be the same too.

Just for convenience of discussion, let us assume that this series of 208 figures does consist merely of two perfectly regular 104-year waves. Imagine the low in position 1, that is, in the first column, and the high 52 years later in column 53.

Suppose now that in addition to your 104-year periodic table you made a shorter table 103 years in length. In such a table the lows and highs of the second 104-year wave would fall, not under the low of the first 104-year wave, but one year later, and the amplitude of the average would be less than when both waves were exactly under each other.

If we made a longer periodic table of 105 years, the lows and highs of the second cycle would fall one year earlier, and of course, the amplitude would decrease likewise.

The longer or shorter we made our periodic table, the less would be the amplitude.

A wave of 104 terms is too long for purposes of illustration, but suppose I work out an example for you using a shorter wave.

Consider a series of 8-year waves having these values: -4, -2, 0, +2, +4, +2, 0, -2, and repeat thus: -4, -2, 0, +2, +4, +2, 0, -2, etc.

Throw these values into an 8-year periodic table:

Table 2

	Position							
	1	2	3	4	5	6	7	8
1st Cycle	-4	-2	0	+2	+4	+2	0	-2
2nd Cycle	-4	-2	0	+2	+4	+2	0	-2
Total	-8	-4	0	+4	+8	+4	0	-4
Average	-4	-2	0	+2	+4	+2	0	-2

Now throw the same values into a 7-year periodic table:

Table 3

	Position						
	1	2	3	4	5	6	7
1st Cycle	-4	-2	0	+2	+4	+2	0
2nd Cycle	-2	-4	-2	0	+2	+4	+2*
Total	-6	-6	-2	+2	+6	+6	+2
Average	-3	-3	-1	+1	+3	+3	+1

(*With 0 and -2 left over)

It is clear that the amplitude has shrunk from 4 to 3 by reason of the failure of the two crests and the two troughs to match up under each other.

If you had slipped the second wave the other way by making a 9-year periodic table, you would have had the same reduction of amplitude. Here we have worked out a 9-year periodic table of the perfectly regular 8-year wave:

Table 4

	Position								
	1	2	3	4	5	6	7	8	9
1st Cycle	-4	-2	0	+2	+4	+2	0	-2	-4
2nd Cycle	-2	0	+2	+4	+2	0	-2	-4*	-2*
Total	-6	-2	+2	+6	+6	+2	-2	-6	-6
Average	-3	-1	+1	+3	+3	+1	-1	-3	-3

(*Two more figures are borrowed from the next cycle.)

Now let's get back to our 104-year wave.

By now it should be clear that, if there really is a regular 104-year wave in these figures, it will show its maximum amplitude only when you make your periodic table exactly this length. If you make the table longer or shorter than the true length of the wave, the amplitude will reduce.

Conversely—and this is important—if you do not know whether or not there are waves present, you can make a series of periodic tables with lengths close to one another. If you find that the indicated amplitude gets larger and larger as the length of the table gets larger and larger until a point, after which as the length of the table gets still larger and larger the indicated amplitude gets less and less, we are justified in assuming that there MAY be a rhythm just exactly as long as the length of the periodic table at the point where the amplitude is greatest.

That is, in the little example above, when you make a 7-year periodic table you get an amplitude of 3; when you make an 8-year periodic table you get an amplitude of 4; and when you make a 9-year periodic table your amplitude is back to 3 again. This tells you that there may be an 8-year rhythm in the values that go to make up the periodic table.

In Table 1 you have some values for waves of different lengths, but the lengths are not close enough together for their progression to mean anything.

As a matter of fact, the circumstance that we have used harmonic intervals means that there is no chance of any wave that might be present at one of our chosen lengths having any of its strength included in the values for the interval next higher or lower.

That is, none of the 3.90 amplitude present for the third harmonic of 69.7 years could be the result of real waves of the length 104.5 years or 52.25 years, the length of the 2nd and 4th harmonics respectively. (However, any real wave between 104.5 years and 52.25 years would have an effect upon the amplitude of the third harmonic at 69.7 years.)

How can you use the machine to get eight or ten values between the harmonic intervals? In other words, how can you use it to give you a multiple harmonic analysis?

Very easy! Chop a little off the end of your series of figures and run your stylus over the remaining curve. Then chop off a little more and retrace the curve again. Continue this procedure time after time and every run will give you five more values for your periodogram.

(A periodogram is merely a chart on which you record the amplitudes of the sine waves fitted to the averages of the various periodic tables.)

For example, after we had made the run over the full series of 209 years, we made another run using only 201 years. This gave us values for 201 years; for 1/2 of 201 years, or 100½ years; 1/3 of 201 years or 67 years; 1/4 of 201 years or 50¼ years, and 1/5 of 201 years or 40 1/5 years.

Then we made another run using the first 197 years as our fundamental. Then a run using 191 years as our fundamental, and so on, until we had made a total of ten runs.

Here is a table showing the length of the fundamentals we used and the lengths of the first five harmonics of each.

Table 5

Length of Fundamental	Length of Harmonics 2nd	3rd	4th	5th
209	104.5	69.6	52.25	41.7
201	100.5	67.0	50.25	40.2
197	98.5	65.6	49.25	39.2
191	95.5	63.6	47.75	38.2
185.5	92.75	61.83	46.38	37.1
181	90.5	60.3	45.25	36.2
175	87.5	58.3	43.75	35.0
168.3	84.16	56.11	42.08	33.6
165	82.5	55.0	41.25	33.0
162	81.0	54.0	40.5	32.4

You will notice that this procedure gives us 39 different wave lengths between our shortest original length of 41.7 years and 209 years, and in addition, 11 more readings from 41.7 years down to 32.4 years, which fills the gap down to (and slightly past) the 6th harmonic of our 209-year series, which is 34.83 years in length.

Also you will notice that the coverage between harmonics is not entirely smooth.

For example, there is a gap of 57½ years between the length of the shortest fundamental and the length of the longest 2nd harmonic, a gap of 11.3 years between the length of the shortest 2nd harmonic and the longest 3rd harmonic.

Between the shortest 3rd harmonic and the longest 4th harmonic, however, there is no especial gap, and between the shortest 4th harmonic and the longest 5th harmonic, there is an actual overlap.

This is merely one of the characteristics of fractions and there is nothing one can do about it except to make enough runs of the machine to get the lengths you want, and to discard any excess values you do not need.

Let us now arrange all these lengths in order of size and list the amplitude of each so as to have the values for our periodogram. Table 6 gives the rank list and the figure on page 25 gives the periodogram based upon them.

Table 6

Period in Years		Amp.	Period in Years		Amp.
209	Fundamental	6.6	60.3	3rd Harmonic	4.06
201	"	7.6	58.3	"	4.20
197	"	8.1	56.11	"	4.13
191	"	8.0	55.0	"	3.66
185.5	"	8.0	54.0	"	3.43
181	"	7.7	52.25	4th Harmonic	3.86
175	"	7.9	50.25	"	3.37
168.3	"	7.8	49.25	"	3.13
165	"	7.2	47.75	"	2.98
162	"	7.7	46.38	"	2.50
104.5	2nd Harmonic	2.55	45.25	"	2.85
100.5	"	2.51	43.75	"	3.58
98.5	"	2.85	42.08	"	4.30
95.5	"	3.55	41.25*	"	4.43
92.75	"	4.55	40.5*	"	4.47
90.5	"	4.50	41.7	5th Harmonic	3.82
87.5	"	4.50	40.2	"	2.94
84.16	"	4.40	39.2	"	2.60
82.5	"	4.30	38.2	"	2.56
81.0	"	3.90	37.1	"	2.70
69.6	3rd Harmonic	3.90	36.2	"	2.48
67.0	"	4.73	35.0	"	1.72
65.6	"	4.72	33.6*	"	.86
63.6	"	4.53	33.0*	"	.92
61.83	"	4.16	32.4*	"	.92

*Note overlap with next harmonic.

What does this periodogram tell us? It tells us that, from half the length of the series down, at every peak there MAY be a repetitive wave.

For example, the periodogram shows strength in the neighborhood of 90 years. This fact means that on the average there is strength when the curve is cut into sections of this length. This average strength could be the result of two 90-year waves coming together in the periodic table as we change the length of the sections from 80 years to 100 years, or it could be the result of something else.

Assuming that there really are two 90-year waves, this fact may or may not have significance. There are bumps on any fluctuating curve. Two of them happen to come 90 years apart. So what? They have to be some distance apart. This does not mean that there will be another bump 90 years later. We need many repetitions of a wave before we can be sure that the rhythm has significance.

From the standpoint of rhythm analysis, we might just as well ignore all of the longer waves. That explains why, in making this analysis, we did not bother to fill in the gaps between 70 and 80 years, and between 105 and 160 years.

As we go down in length however from 70 years, the peaks have more potential significance.

There are five such peaks on the periodogram: At 66½ years, at 57 years, at about 52¼ years, at about 41 years, and at 36¾ years.

There is relative strength at each of these lengths, and this relative strength MAY be due to rhythms of these lengths present in the series, and, if there are rhythms of these lengths, the rhythms MAY have significance—that is, the relative strength may have arisen as a result of continuing force as distinct from a series of unrelated forces, which just happened to come at approximately equal intervals.

It cannot be repeated too often that by itself a periodogram analysis or a multiple harmonic analysis gives you only hints of rhythms that may be present. It clears the underbrush, in other words, and tells you of lengths at or near which you may find rhythms.

Before I close our consideration of Fig. 2, I would like to call your attention to one more aspect of the periodogram. You will note that the overlapping sections of the fourth and fifth harmonics do not agree. As you take fourth harmonics of shorter and shorter fundamentals the curve on the periodogram keeps on rising from 42.08 to 41.25 to 40.5, indicating an average wave at 40.5 or shorter.

On the other hand, the fifth harmonics start at 41.7 and get continuously shorter, indicating an average at 41.7 or longer. Why is this?

There are four possible explanations. First, in one run or the other, the curve may not have been traced perfectly. Second, in one run or the other, the dials on the instrument may not have been read exactly right. (Explanations one and two are unlikely.) Third, the curves were not adjusted for trend, which fact introduces a small error. Fourth, the figures dealt with were not entirely the same. In the fifth harmonic of 209 years, we used all the figures, in the fourth harmonic of 162 years we ignored all figures after 1892.

Now, one last point before we close. In the article in the Research section there is a table giving an index of slope so that you can see which waves will prevail over the others. The following table gives details of the calculation which are as follows: The amplitude times two, to give the overall move from crest to trough, divided by half the wave length to give the time it takes to go up from trough to peak, or vice versa.

Table 7

A	B	C	
		Index of Slope (2B ÷ ½ A)	
Wave Length	Average Amplitude	Calculation	Result
90	4.5	(9.0 ÷ 45.00=)	.200
66½	4.9	(9.8 ÷ 33.25=)	.295
57	4.35	(8.7 ÷ 28.5=)	.305
52½	3.9	(7.8 ÷ 26.25=)	.297
41	3.85	(7.7 ÷ 20.5=)	.376
36 3/4	2.8	(5.6 ÷ 18.375=)	.305

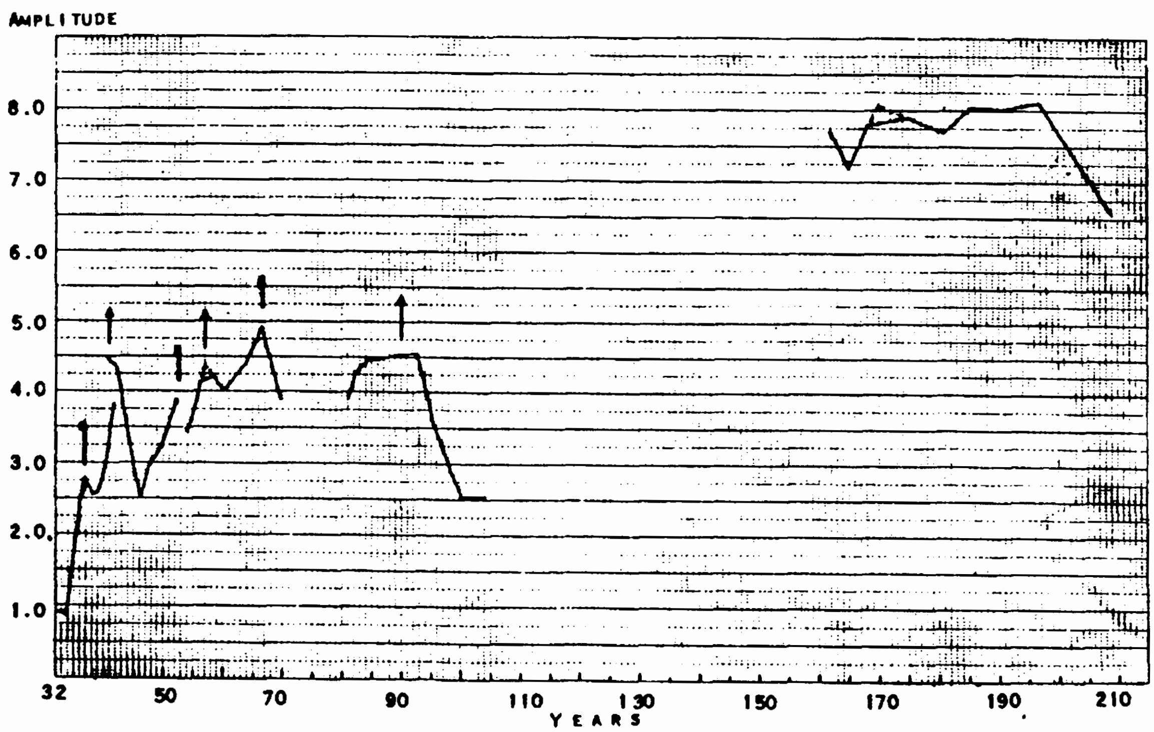


FIG. 2. PERIODOGRAM SHOWING THE RESULTS OF A MULTIPLE HARMONIC ANALYSIS OF COTTON PRICES. 1731-32 TO 1939-40. FIRST FIVE HARMONICS ONLY.

From this table we see that although the 66⅓-year waves (if there are such) have a greater amplitude than the 41-year waves (if there are such), the 41-year waves are steeper than the 66⅓-year waves. The 41-year waves go from 3.85 below the axis to 3.85 above the axis in only 20½ years (half of 41) or .376 a year, whereas the 66⅓-year waves require 33⅙ years for the move of 9.8 or an average of only .295 a year.

In this connection however, it should be noted again (in case you did not read the article on page 17) that any two waves closer than harmonic intervals from each other affect each other in the periodogram. For example, part of the apparent strength of the 66⅓-year wave (if there is one) is due to the strength of a possible 57-year wave only 10⅓ years longer. As 66⅓ is a third harmonic of 199½ years, any waves shorter than 99¾ years (the 2nd harmonic) or longer than 49.8 years (the 4th harmonic) will influence the average amplitude of any wave of 66⅓ years. You must be on the lookout for overlaps of this sort.

It is also well to remember that if you have a regular symmetrical wave of a certain length, it will be revealed in periodic tables of any odd fraction of that length. For instance, if a series of figures has a perfectly regular symmetrical wave 60 years long and nothing else, it will show up in periodic tables of 20 years (1/3 of 60) 12 years, (1/5 of 60) 8.57 years, (1/7 of 60) and so on.

Conversely, any average wave revealed in a periodogram may be in whole or in part a reflection of some other wave some odd multiple of the indicated length. For instance, here is a 4-year periodic table of a perfectly regular 12-year wave, starting from a low of -3, going up to +3, and then falling off to -3 again.

Table 8

	Position			
	1	2	3	4
1st Cycle	-3	-2	-1	0
2nd Cycle	+1	+2	+3	+2
3rd Cycle	+1	0	-1	-2
Total	-1	0	+1	0
Average	-1/3	0	+1/3	0

An average wave of 4 years appears, but of course it is not rhythmic, as we know, for there was no 4-year wave in the arbitrary figures with which we started.

There is another little matter that one must look out for, but perhaps a discussion of it hardly belongs here as multiple harmonic analysis is merely supposed to give hints of rhythms that may be present. Comments such as the above probably belong more properly in another discussion that would deal with how to tell whether or not the hints revealed by multiple harmonic analysis in any given case have any significance. As soon as possible, I will get into this aspect of the subject also.

Technical

Harmonic Analysis of Cotton Prices Continued

THIS article continues the technical article on the multiple harmonic analysis of cotton prices from 1731-32 to 1939-40 which appeared in the November 1950 report, pages 20 to 26 inclusive. This article will tell you what was revealed by a determination of the values of the 6th to the 10th harmonics, inclusive. That is, it gives us what light we can hope for through the use of this method in regard to cycles in cotton prices from 19 to 35 years in length.

Tables 1, 5, 6, and 7 below continue tables of the same numbers in earlier reports.

Fig. 2 below continues the periodogram from 36 years down to 18¼ years, but uses different horizontal and vertical scales.

Although the behavior of the 5th harmonics do not indicate it, the 6th harmonics of various fundamentals suggest a cycle at 33¼ years. This could be the well-known Brückner cycle.

The 7th harmonics very clearly indicate average strength at 27¾ years.

The 9th harmonics show a peak at 21¾ years. Possibly this strength is the result of the well-known 22 1/5-year cycle.

I do not know why all the 9th harmonic values are so low in relation to the 8th and 10th harmonic values, but I assume it is because of distortions introduced by the fact that this method of analysis makes it impossible, or at least impractical, to make proper correction for trend.

The 8th and 10th harmonics do not give us any indication of cycles.

Taking the 10th harmonic as an example, the strength at 18¼ years indicates a cycle at this length or less, and we cannot tell which unless we study the 11th harmonics.

We can see from the 9th harmonic that the line from 20½ years to 21 years keeps going right on up to 22 years, and I therefore attach no significance to the strength of the 10th harmonic at 21 years.

Similarly for the 8th harmonic, I feel that the strength indicated at 23 years and 26 years could very easily be the result of strength at 22 and 28 years and that the 8th harmonics, like the 10th, probably fail to indicate anything of value.

In closing there are two more things I would like to say:

First of all, the periodogram shown in Fig. 2 is drawn in the usual conventional form. There is a much better way of drawing a periodogram, as I shall explain in an early issue.

Second, there are severe limitations upon the usefulness of the periodic table, (i.e. harmonic analysis) in unscrambling two or more waves. Some work of Mr. W. C. Yeatman illustrates this beautifully. I shall show you this work just as soon as possible.

Table 1, Continued

A Name of Harmonic	B Length as a Fraction of the Fundamental	C Length in Years	D Relative Amplitude of Fitted Sine Curve, (Centimeters)
6th	1/6	34.83	1.78
7th	1/7	29.86	1.77
8th	1/8	26.13	1.85
9th	1/9	23.22	.88
10th	1/10	20.9	2.06

Table 5, Continued

Length of Fundamental	Length of Harmonics 6th	7th	8th	9th	10th
209	34.83	29.86	26.13	23.22	20.90
205	34.16	29.29	25.63	22.77	20.50
201	33.50	28.71	25.13	22.33	20.10
197	32.83	28.14	24.63	21.88	19.70
193	32.16	27.57	24.13	21.44	19.30
189	31.50	27.0	23.63	21.0	18.90
185	30.83	26.43	23.13	20.55	18.50

Note: Fundamentals do not necessarily agree in length with the fundamentals of Table 5 on page 23 of the November 1950 report. Some fundamentals have been dropped and others have been added.

Table 7, Continued

A Wave Length	B Average Amplitude	C Index of Slope (2 x col.B + ½col.A) Calculation	Result
33½	1.82	(3.64 + 16.875)	.216
27¾	2.41	(4.82 + 13.875)	.347
21¾	1.56	(3.12 + 10.875)	.288

Table 6, Continued

Period in Years		Amp.	Period in Years		Amp.
34.83*	6th Harm.	1.78	24.63	8th Harm.	1.44
34.16*	"	1.79	24.13	"	1.75
33.5*	"	1.76	23.63	"	1.90
32.83*	"	1.48	23.13	"	2.13
32.16	"	1.09	23.22**	9th Harm.	.88
31.5	"	.25	22.77	"	1.11
30.83	"	.19	22.33	"	1.26
29.86	7th Harm.	1.77	21.88	"	1.50
29.29	"	2.02	21.44	"	1.42
28.71	"	2.11	21.0	"	1.23
28.14	"	2.28	20.55	"	1.22
27.57	"	2.29	20.9**	10th Harm.	2.06
27.0	"	1.93	20.5	"	1.95
26.43	"		20.1	"	1.42
26.13	8th Harm.	1.85	19.7	"	1.18
25.63	"	1.82	19.3	"	1.05
25.13	"	1.66	18.9	"	.27
			18.5	"	1.17

Note: Fundamentals do not necessarily agree in length with the fundamentals of Table 6 on page 24 of the November 1950 report. Some fundamentals have been dropped and others have been added.
*Overlap with the 5th harmonics recorded in Table 6 on page 24 of the November 1950 issue.
**Note overlap with the next harmonic.

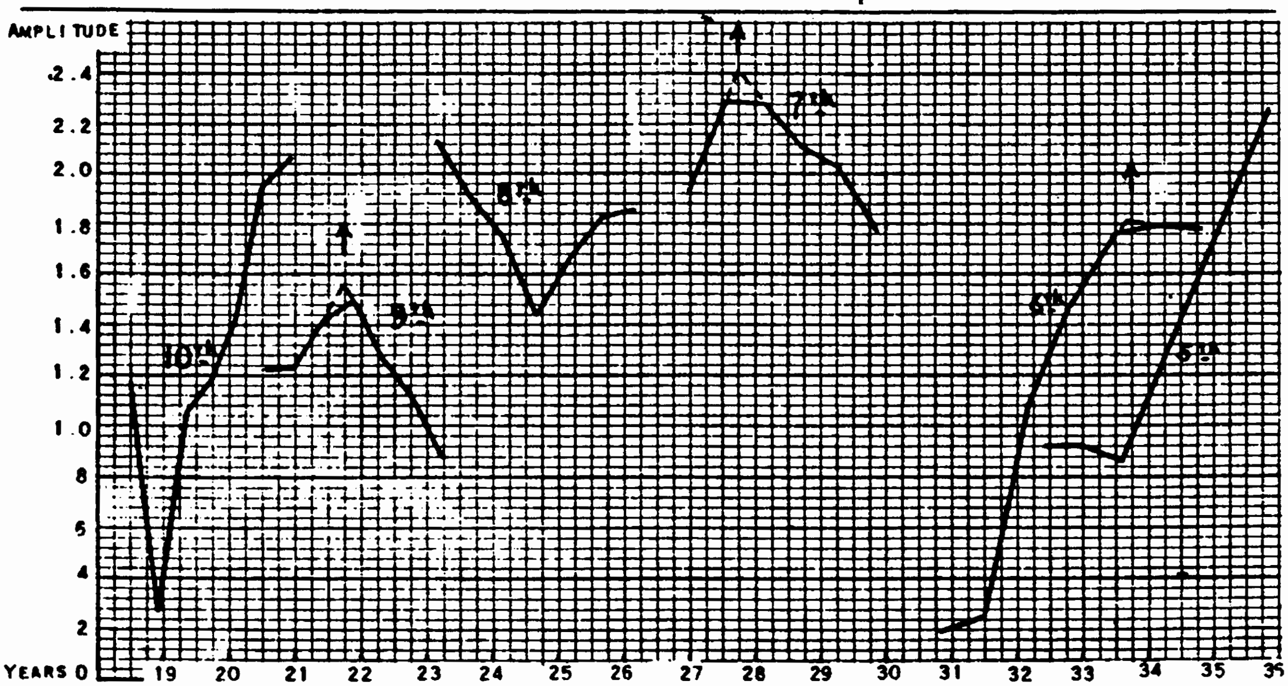


FIG. 2. PERIODOGRAM SHOWING THE RESULTS OF A MULTIPLE HARMONIC ANALYSIS OF COTTON PRICES, 1731-32 TO 1939-40 6TH TO 10TH HARMONICS INCLUSIVE PART OF THE OVERLAPPING 5TH HARMONICS APPEAR AT THE EXTREME RIGHT

TABLE 1

AVERAGE ANNUAL SPOT PRICE OF COTTON PER POUND
CROP YEATS 1731-2 TO 1953-4
(VALUES WHICH HAVE UNFOLDED FROM THE TIME OF THE ORIGINAL STUDY ARE SHOWN IN BOLDFACE TYPE.)

Crop Year	Cents per Pound
1731-32	12.7
-33	14.3
-34	13.6
-35	14.3
-36	13.8
-37	15.5
-38	15.1
-39	17.8
-40	17.8
-41	16.7
1741-42	13.4
-43	13.3
-44	14.0
-45	17.5
-46	21.5
-47	25.7
-48	25.8
-49	21.2
-50	24.0
-51	26.8
1751-52	27.9
-53	22.1
-54	22.4
-55	18.7
-56	19.7
-57	24.0
-58	19.2
-59	19.8
-60	18.5
-61	18.6
1761-62	23.4
-63	25.1
-64	29.2
-65	27.6
-66	26.3
-67	27.5
-68	23.4
-69	20.2
-70	17.1
-71	17.3
1771-72	16.1
-73	20.0
-74	19.1
1774-75	23.8
1775-76	29.4
-77	-
-78	-
-79	-
-80	-
-81	-
1781-82	30.5
-83	38.2
-84	32.0
-85	31.4
-86	35.9
-87	40.2
-88	38.2
-89	30.5
-90	26.3
-91	21.9
1791-92	30.6
-93	34.5
-94	31.6
-95	32.2
-96	31.8
-97	28.5
-98	29.4
-99	29.8
-00	29.9
-01	28.7
1801-02	24.00
-03	18.30
-04	18.08
-05	22.00
-06	23.50
-07	22.08
-08	17.75
-09	15.00
-10	14.83
-11	13.58
1811-12	9.33
-13	13.00
-14	25.50
-15	23.08
-16	28.33
-17	27.05
1817-18	30.99
1818-19	24.67
-20	16.52
-21	14.88
1821-22	14.69
-23	11.18
-24	14.65
-25	17.92
-26	13.38
-27	10.20
-28	10.34
-29	9.89
-30	9.72
-31	10.14
1831-32	9.12
-33	11.40
-34	13.06
-35	16.29
-36	16.72
-37	14.36
-38	10.02
-39	13.22
-40	9.41
-41	9.58
1841-42	8.13
-43	7.24
-44	7.82
-45	5.88
-46	7.77
-47	10.89
-48	8.54
-49	7.24
-50	12.02
-51	12.57
1851-52	9.30
-53	10.98
-54	11.01
-55	10.31
-56	10.33
-57	13.18
-58	12.46
-59	12.12
-60	11.27
1860-61	12.27
1861-62	27.29
-63	46.98
-64	54.15
-65	46.98
-66	31.40
-67	22.94
-68	17.48
-69	20.92
-70	20.72
-71	15.16
1871-72	19.54
-73	17.64
-74	15.51
-75	13.87
-76	11.51
-77	10.94
-78	10.95
-79	10.82
-80	12.13
-81	11.36
1881-82	12.09
-83	10.81
-84	10.87
-85	10.74
-86	9.47
-87	9.91
-88	10.15
-89	10.44
-90	11.27
-91	9.48
1891-92	7.68
-93	8.45
-94	7.75
-95	6.38
-96	8.10
-97	7.71
-98	6.40
-99	6.00
-00	8.36
-01	9.38
1901-02	8.73
-03	9.96
1903-04	12.84
1904-05	9.09
-06	11.30
-07	11.24
-08	11.53
-09	10.23
-10	14.66
-11	14.87
1911-12	10.85
-13	12.29
-14	13.21
-15	-
-16	11.98
-17	19.28
-18	29.58
-19	31.01
-20	38.29
-21	17.89
1921-22	18.92
-23	26.24
-24	31.11
-25	24.74
-26	20.53
-27	15.15
-28	20.42
-29	19.73
-30	16.50
-31	10.38
1931-32	6.34
-33	7.37
-34	11.09
-35	12.44
-36	11.75
-37	12.93
-38	8.75
-39	8.99
-40	10.34
-41	11.1
1941-42	18.3
-43	20.2
-44	20.6
-45	21.9
-46	26.0
-47	34.8
-48	34.6
-49	**32.2**
-50	**31.8**
-51	**42.6**
1951-52	**39.4**
-53	**34.5**
-54	**33.5**

Long Cycles In Cotton Prices

SEVERAL years ago an assistant and I made a multiple harmonic analysis of cotton prices to obtain hints of waves that might be present in that series of figures.

(In the Technical section of this issue you will find a description of multiple harmonic analysis and an account of the methods used in the analysis.)

The analysis was made on a Henrici type mechanical analyzer placed at our disposal through the kindness of the Mico Instrument Company of Cambridge, Massachusetts. This company makes the best harmonic analyzer I know about, and the only one that I know of adapted to multiple harmonic analysis. (For a picture of an Henrici analyzer, see page 21 of the September report.)

This report will tell you about a part of that analysis.

Cotton prices are available by crop years from 1731-32 to date.

Fig. 1. shows you these cotton prices plotted on semilog or ratio scale. The dotted lines in Fig. 1 show certain arbitrary values used for the various war periods. For the actual figures refer to Foundation Report No. 2, **Cycles in Wholesale Prices: Cotton**.

It is not argued that the values selected for the war periods are what prices really would have been if there had been no war, but they are certainly closer to such prices than either the actual war prices or zero. One must use some value when making a mechanical harmonic analysis; the values we used clearly eliminate some of the war distortions.

In making the analysis we used logs of the data, as must always be done in analyzing series of this sort.

We made analyses of the data 1731-32 to 1939-40, a span of 209 years and, as a check, of the data 1822-23 to 1939-40, a span of 118 years.

The complete analysis carried the study down to waves of 5½ years, but in this report I shall tell you only about the part dealing with waves down to 32 years. I will tell you about the shorter waves in later reports.

That section of the main periodogram giving results from the length of the fundamental (209 years) down through five harmonics to waves of 32 years in length is shown in the figure on page 25. Relative strength at about 90 years, 66½ years, 57 years, 52¼ years, 41 years, and 36¾ years, indicates the possibility of rhythmic waves at about these lengths.

(A **periodogram** is a chart on which you record the amplitude of sine (or other) waves fitted to the averages of all the columns of a periodic table.

(A **periodic table** is an arrangement of the data into columns and rows,—as many columns as there are terms in the suspected cycle, and as many rows as there are cycles in the data. Thus, if you had a series of 12 figures and were studying 3-year waves, your periodic table would contain these 12 figures in four lines of three columns each.

(Unless otherwise stated, the **fundamental** is the longest wave possible within the series. In other words, the full length of your series of figures.

(**Harmonics** are wave lengths of one-half, one-third, one-fourth, one-fifth, etc. of the fundamental.)

In the following table you will find an index of the amplitude of the average waves we found in this series of figures.

Table 1

Wave Length	Index of Amplitude
90 years	4.5
66½ years	4.9
57 years	4.35
52¼ years	3.9
41 years	3.85
36¾ years	2.8

There might be actual rhythms at or about any of these lengths, or there might not.

It should be remembered that insofar as the lengths of these waves are not separated from each other by harmonic intervals, they tend to reinforce each other.

For example, 36⅓ years is the 5th harmonic of 183.75 years. Any wave shorter than 45.93, the 4th harmonic of 183.75 years, will combine with any 36⅓-year wave that might be present to give it an apparent amplitude greater than the 36⅓-year wave really has. Part of the amplitude of the average 41-year wave is therefore included in the amplitude of the average 36⅓-year wave, and vice versa.

The amplitude of a wave is important, but you will find the average slope of the wave to be even more important. It is the slope, or rise or fall per year, that determines whether one wave will prevail over another.

The following table gives an index of the slope of the waves listed in Table 1.

Table 2

Wave Length	Index of Slope (Rise or Fall Per Year) If Waves Are Rhythmic
90 years	.200
66⅓ years	.295
57 years	.305
52⅓ years	.297
41 years	.376
36⅓ years	.305

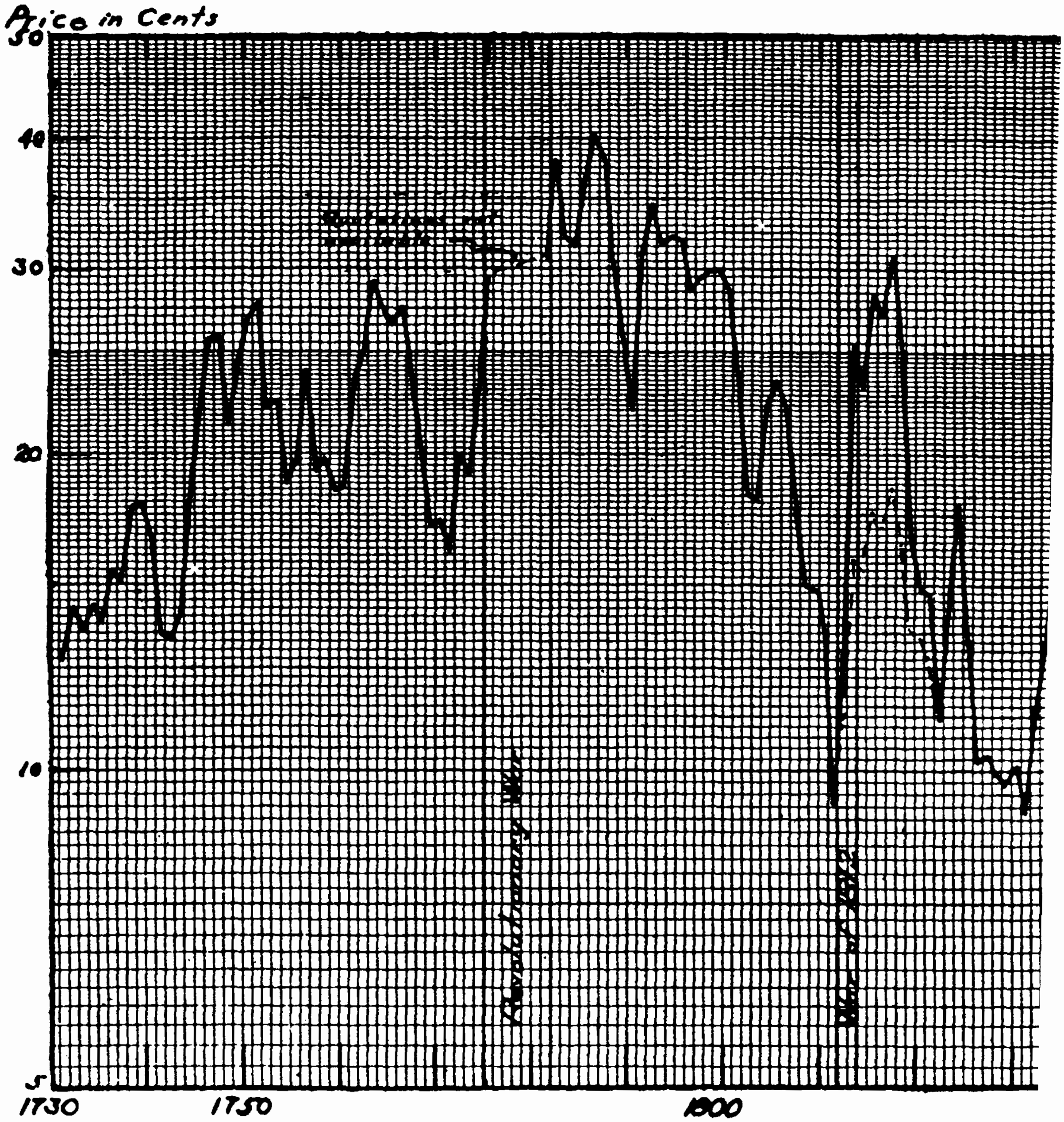
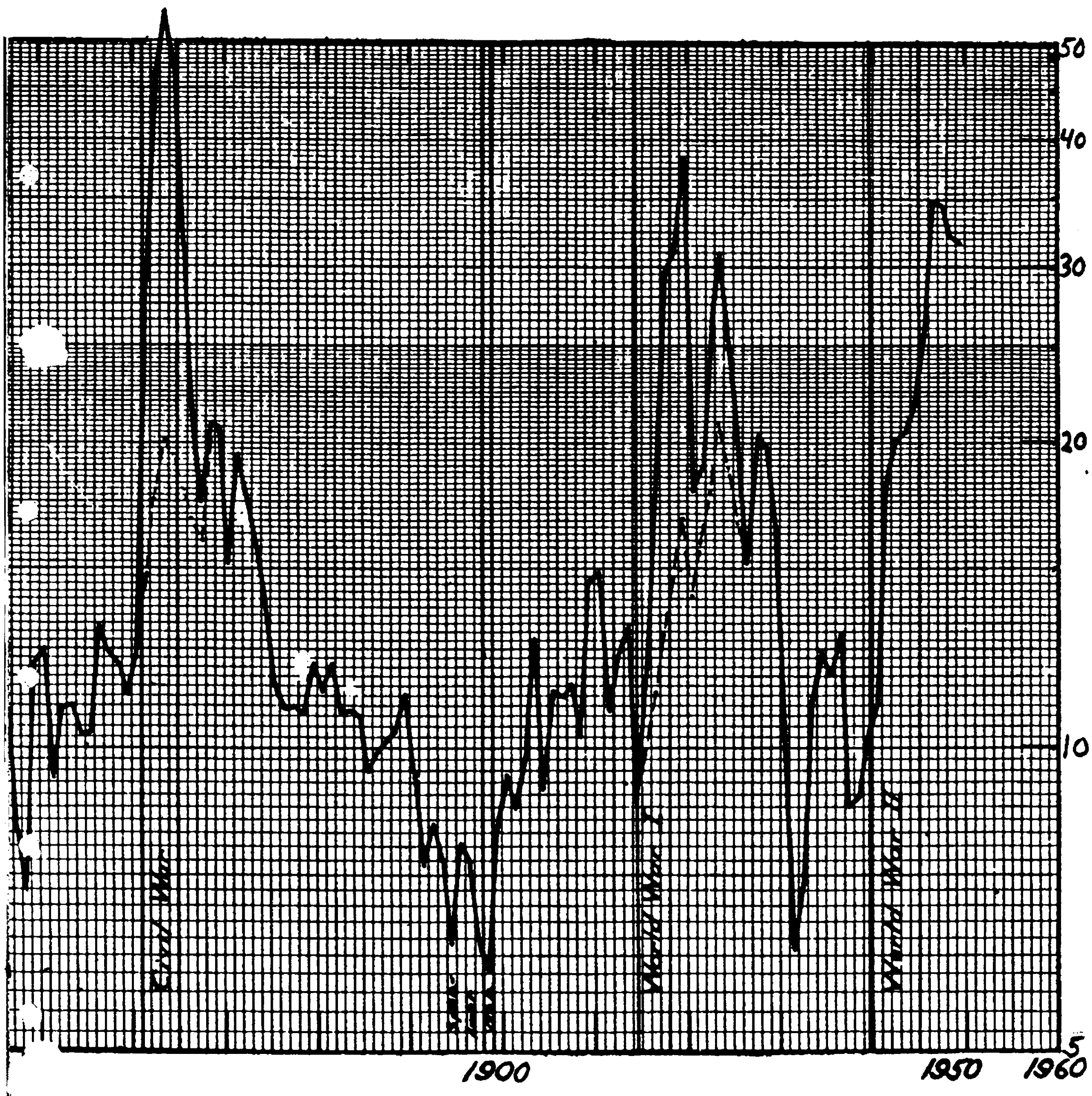


FIG. 1. AVERAGE ANNUAL PRICE OF SPOT CO

Of course, insofar as the amplitudes of any of the above waves is due to the effect of conflicting waves of another wave length, as with the 36¾- and 41-year waves discussed above, the slope will be distorted too.

What does this analysis give us? It gives us the fact that there MAY be rhythms of these lengths in cotton prices. You must go on from here by other means to see if these waves are rhythmic, and, if they are rhythmic, then to determine as well as may be the number of times out of a hundred that rhythms of the indicated amplitude, regularity, and repetitiveness could come about on the one hand as the result of random, sporadic, unrelated causes, or on the other, as the result of some underlying rhythmic cause that may be presumed to continue.

Here we are reporting only upon hints of cycles. We are, as it were, just clearing away the underbrush to explore a vein of quartz which is potentially gold bearing. As you dig deeper you may end with only hard work and disappointment, or you may find real gold.



POUND. CROP YEARS 1731-32 TO 1949-50. (RATIO SCALE.)

Research

POSSIBLE CYCLES IN RAILROAD STOCK PRICES, 1831-1950

Summary

A comprehensive reconnaissance survey of railroad stock prices, 1831 - 1950, suggests a multitude of cycles, the length and relative importance of which are given in the article.

Railroad stock prices were chosen because this series of figures is long and has the advantage of industrial homogeneity.

The task ahead is to examine in detail each of the suggested cycles to see if it is rhythmic, and to determine its various characteristics such as shape, timing, length, and amplitude.

THROUGH the generosity of Stockham Valves and Fittings, Inc. of Birmingham, Alabama, one of our members, who, as a public service, supplied the money, and the Mico Instrument Company of Cambridge, Massachusetts, who, similarly, allowed us to test out one of their newly completed harmonic analyzers, I am able to present to you a reconnaissance survey of the cycles in rail stock prices from 1831 to date.

(We also studied industrial stock prices, 1871 to date, and pig iron prices, 1784 to date. Reports on these two other series will be made to you as soon as the material can be put together.)

Railroad stock prices are particularly important for the student of cycles because they constitute the longest single relatively homogeneous series of stock prices in existence. They now cover a period of 121 years.

In contrast to this coverage, industrial common stock prices extend back only to 1871 and are confused by the fact that new industries with different cyclic patterns, or with the same cyclic patterns coming at slightly different times, are added from year to year.

Of course, the cyclic behavior of railroad stock prices may similarly have changed from time to time in accordance with the varying nature of the railroad industry, but there are reasons to believe that this may not be so.

My observations are that a change in the nature of a business usually does not change the cycle. For example, much now made by the General Electric Company was not even invented back in 1893 when the company started, and yet the orders received by that company show clear evidence of a 6-year cycle, continuously from the beginning. (See an article on this subject in the September 1950 report.)

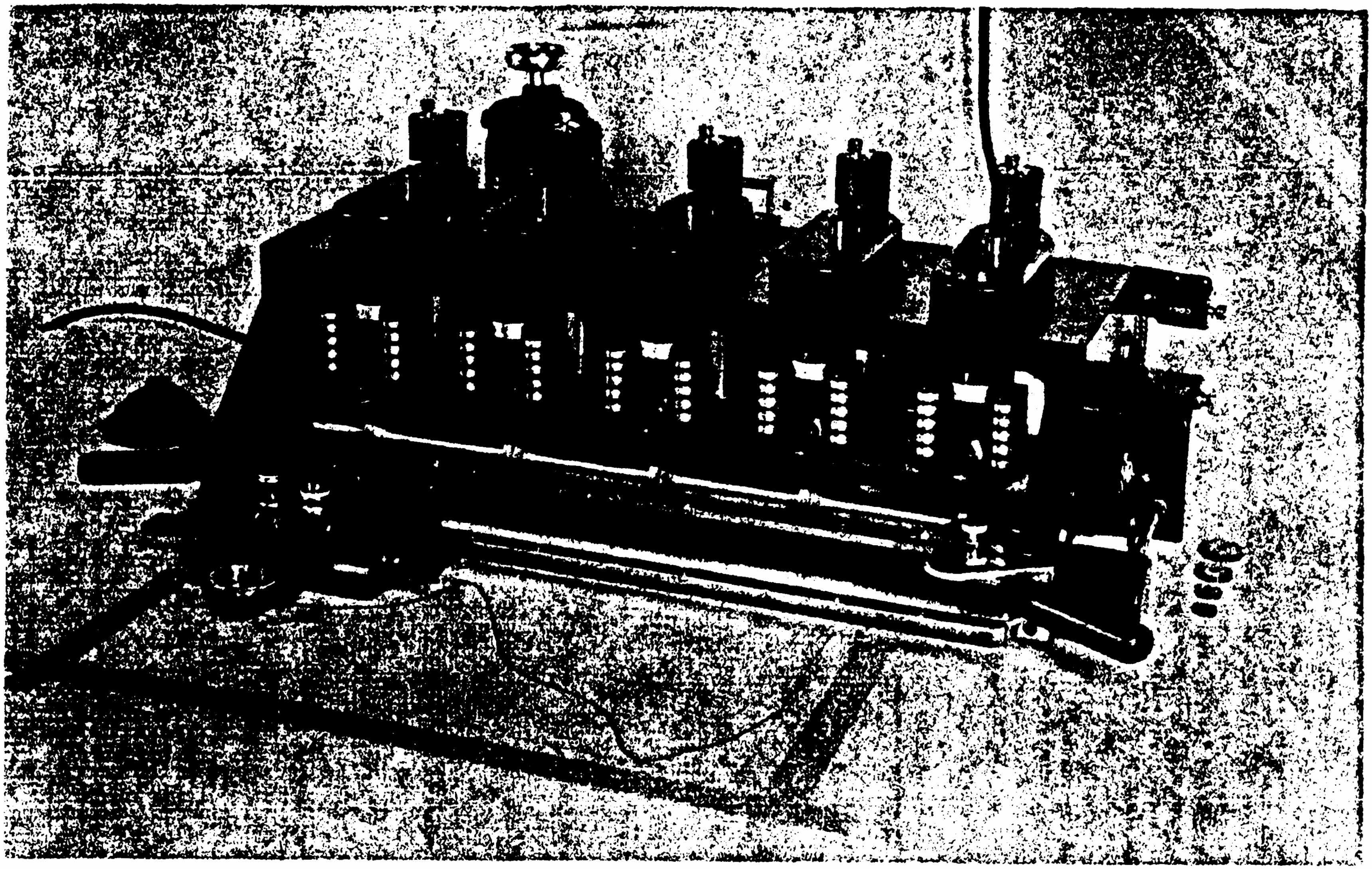
Whether this continuity of cycle length holds true for railroad stock prices I do not know, but there is certainly a better chance of continuity in such prices than in industrial stock prices.

A chart of railroad stock prices, year by year, from 1831 to date is shown in Fig. 1. This chart is reprinted from our September 1951 report, and brought up to date. You will find the sources and the actual figures on page 143 following.

In railroad stock prices the trend is so distorting that it must be removed as a prelude to cycle analysis. The trend chosen was a 19-year geometric moving average of the values, extended arbitrarily at each end as shown by the broken line in Fig. 1.

A 19-year geometric moving average does not give you the true trend. That cannot be determined until all the cycles are discovered and removed. But it is close enough to the true trend for preliminary purposes.

After the actual prices have been adjusted for trend we get the values shown in Fig. 2. These are the values we ran through the harmonic analyzer to get the results shown in the periodogram plotted herewith as Fig. 3.



Harmonic Analyzer

This is a picture of the machine we used to make the multiple harmonic analyses. The results of this analysis are shown on the preceeding pages. The machine was made by the Mico Instrument Company of Cambridge, Massachusetts.

Fig. 1.

RAILROAD STOCK PRICES, 1831 – 1951

The solid line shows the course of railroad stock prices for the past six generations. The general trend was upward from 1831 to 1910 and has been generally downward since about that time. This behavior is in contrast to industrial stock prices, the underlying trend of which has been continuously upward.

Note particularly the 19.4-year cycle which can be seen easily by inspection. The 19.4-year cycle does not seem to be present in industrial stock prices, or at least not to an important extent.

The broken line represents a 19-year moving average trend. As a 19-year moving average lacks the first and the last nine values these have been added arbitrarily by a dotted line.

Until later in the analysis we cannot tell whether or not the trend has continued to go down since 1940, or has leveled off.

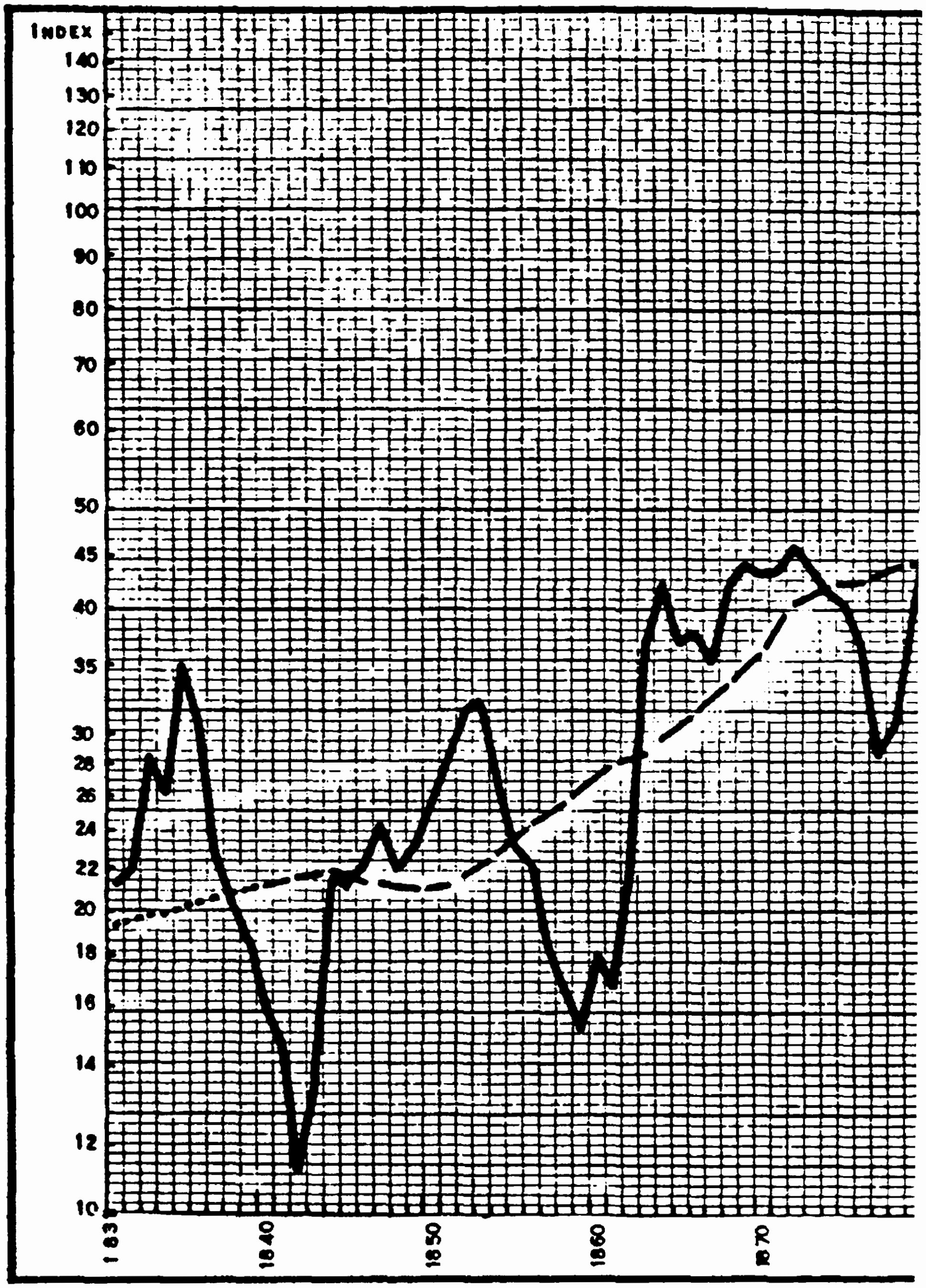
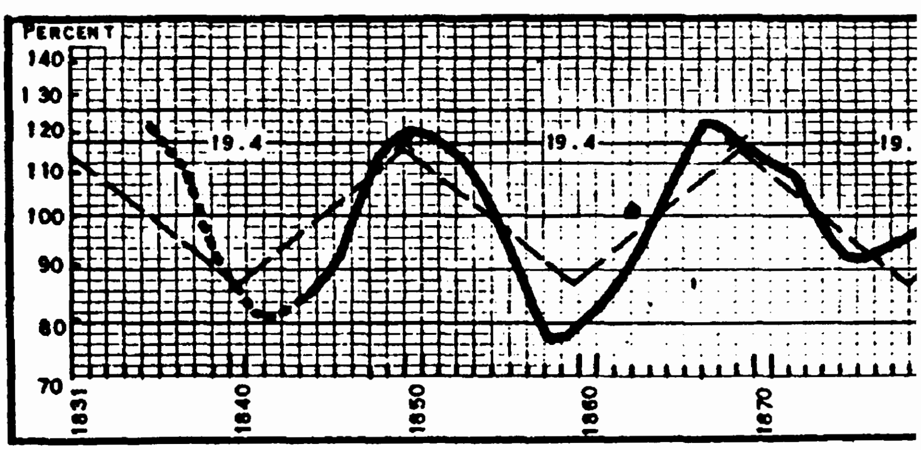
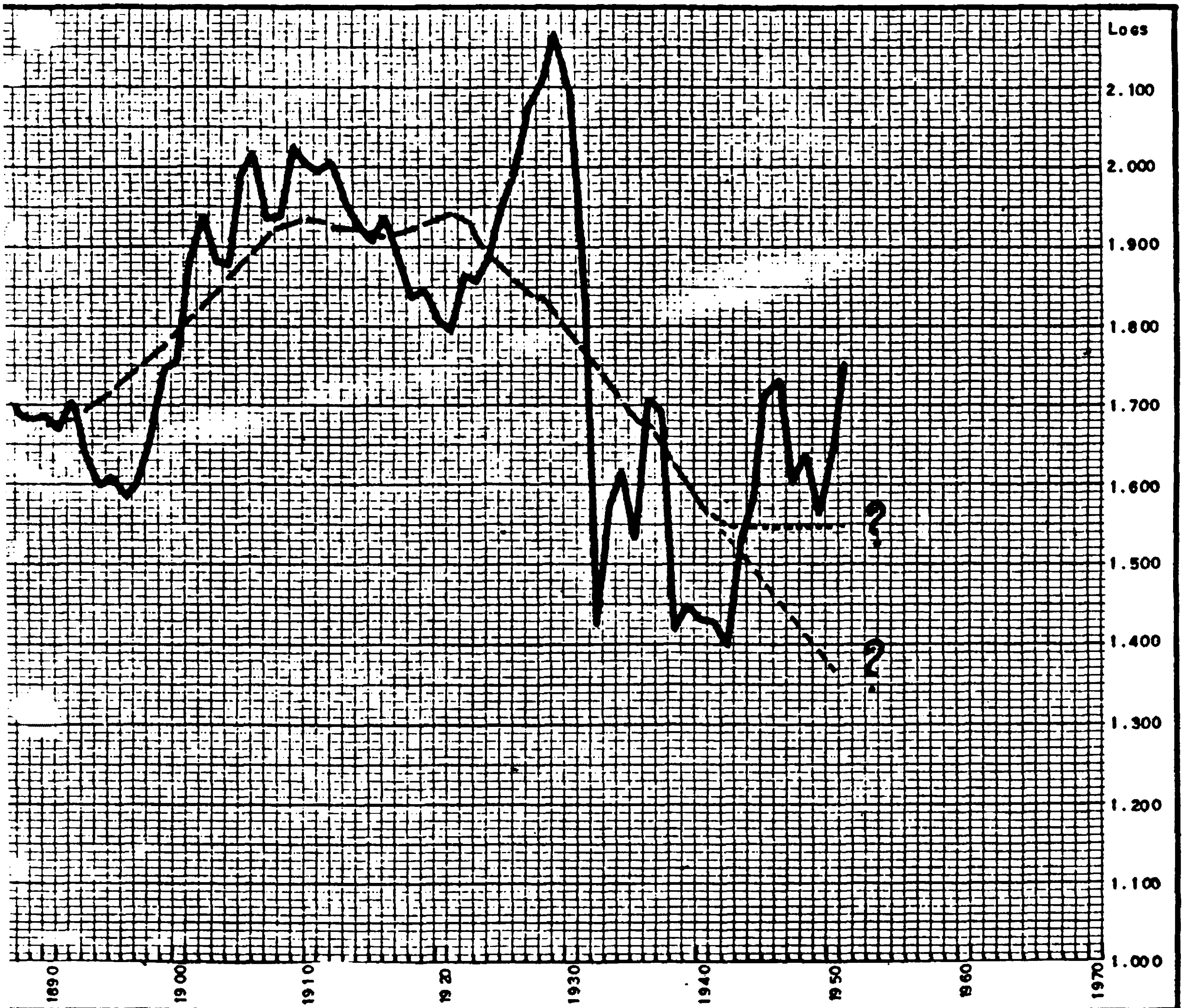


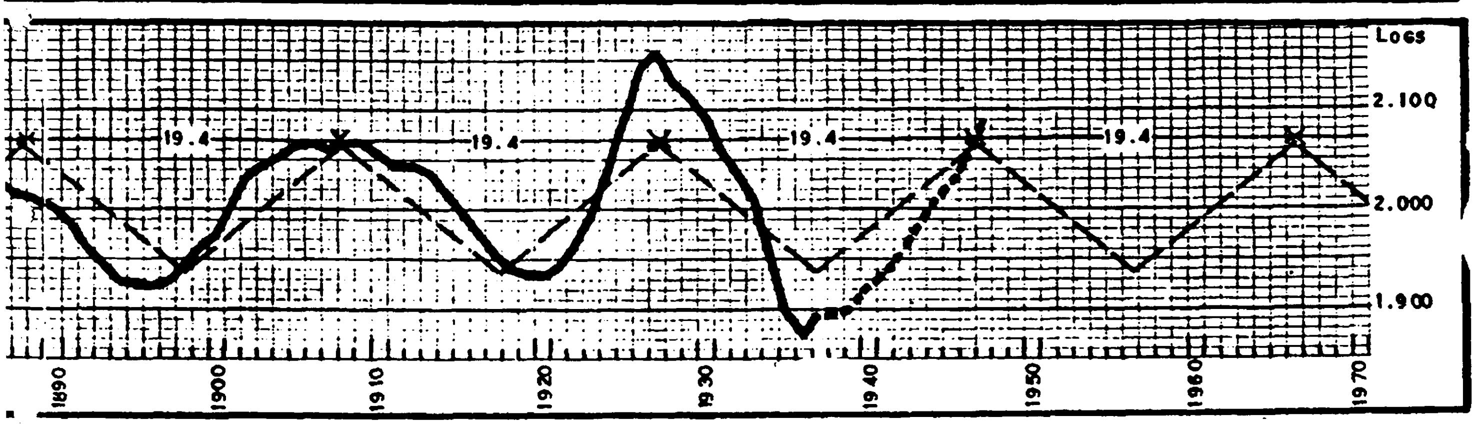
Fig. 2.

This curve shows railroad stock prices as a percentage of trend, smoothed by means of a 9-year moving average. This smoothing eliminates the 9-year cycle also present in these figures, and enables you to see the 19.4-year cycle more clearly.

I have added a series of ideal 19.4-year waves to guide your eye.




Logs
2.100
2.000
1.900
1.800
1.700
1.600
1.500
1.400
1.300
1.200
1.100
1.000
1890
1900
1910
1920
1930
1940
1950
1960
1970
?
?


Logs
2.100
2.000
1.900
19.4
19.4
19.4
19.4
1890
1900
1910
1920
1930
1940
1950
1960
1970

A periodogram is merely a "gram" or writing down of the periods, or wave-lengths chosen for study, and the strength or amplitude of each. That is, it gives you a chart of a table of wave lengths and amplitudes. Wave length is given by the horizontal scale. Amplitude is given by the vertical scale. (For a discussion of the harmonic scale used and the methods of charting see an article called "A New Form of Periodogram" starting on page 54 in the Winter 1950-51 issue of the quarterly **Journal of Cycle Research**.

Whenever the amplitude gets bigger as you increase the wave length, and then gets smaller as you increase it still further, you know that there *may* be a cycle of the wave-length which shows the greatest amplitude.

That word *may* is very important. A peak in the periodogram could mean (1) a repetitive cycle of the indicated length, or (2) that a few important accidental variations just happened to come at intervals of that length or some multiple of it, or (3) that a real cycle has been split up or that two or more real cycles have combined to give an appearance of strength at a false length. (See an article called "Limitations of the Periodogram" printed in the June 1951 report.)

In other words, a peak in a periodogram is like the wink of a pretty woman. It could mean (1) that she likes you and wants to know you better, or (2) that she has a cinder in her eye, or (3) that she is winking at someone just behind your left shoulder. With the woman you have to go on from there and investigate to find out the true state of affairs. And so it is with the peaks of a periodogram, also.

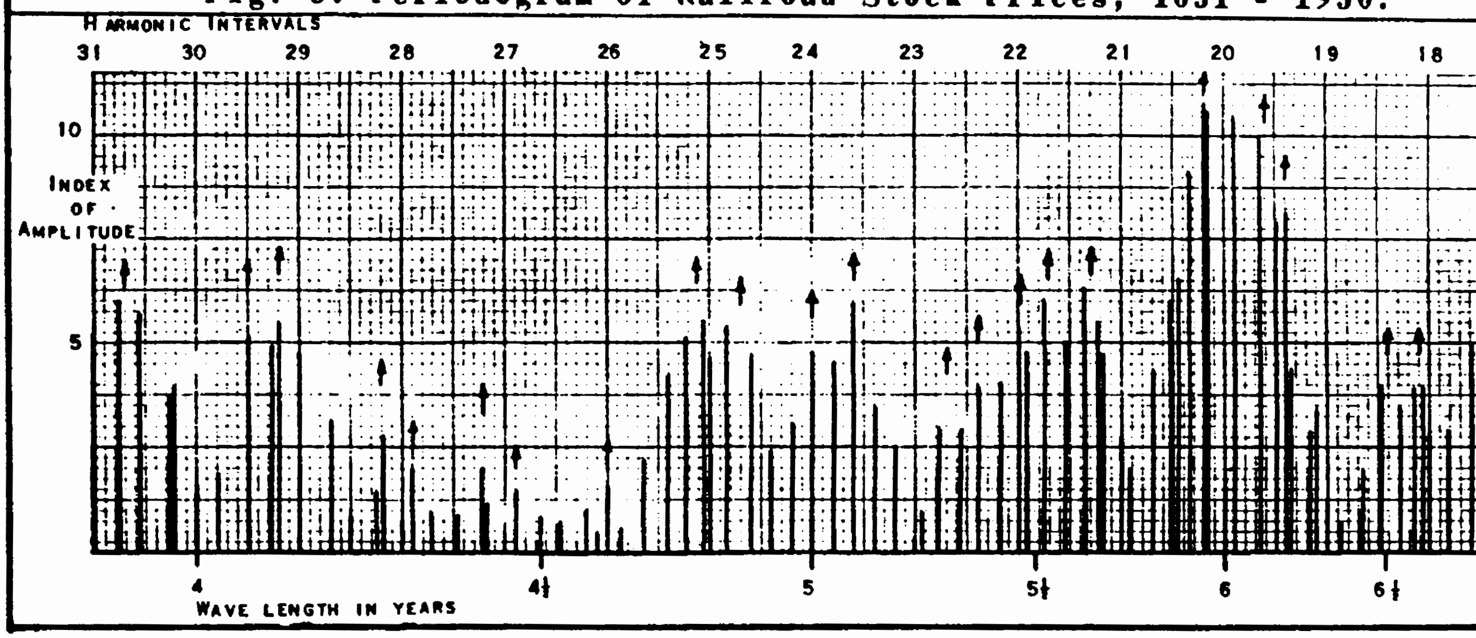
A better analogy might be to compare the peak of a periodogram with the "ping" of an Asdic or submarine detecting device of the sort used by destroyers in the last war. Such a ping *may* mean a submarine, or it may mean a whale, or a school of fishes, or even sometimes perhaps a vortex of water currents.

With these explanations I print for you the table on page 128 which indicates the wave lengths of cycles, which *may* be present in this series of railroad stock prices.

This table (Table 1) gives recognition to every little wiggle of the periodogram. Most of these little wiggles probably have no significance. That is to say, probably there are not cycles at both 4.8 years and 4.9 years. Probably there are not cycles at both 5.0 years and 5.1 years. However, as this is merely reconnaissance, it does no harm to indicate the possibility.

You will want to know how the wave lengths suggested by the periodogram compare with wave lengths suggested by my earlier work and by the work of Mr. Hughey. Such a comparison is made for you in Table

Fig. 3. Periodogram of Railroad Stock Prices, 1831 - 1950.



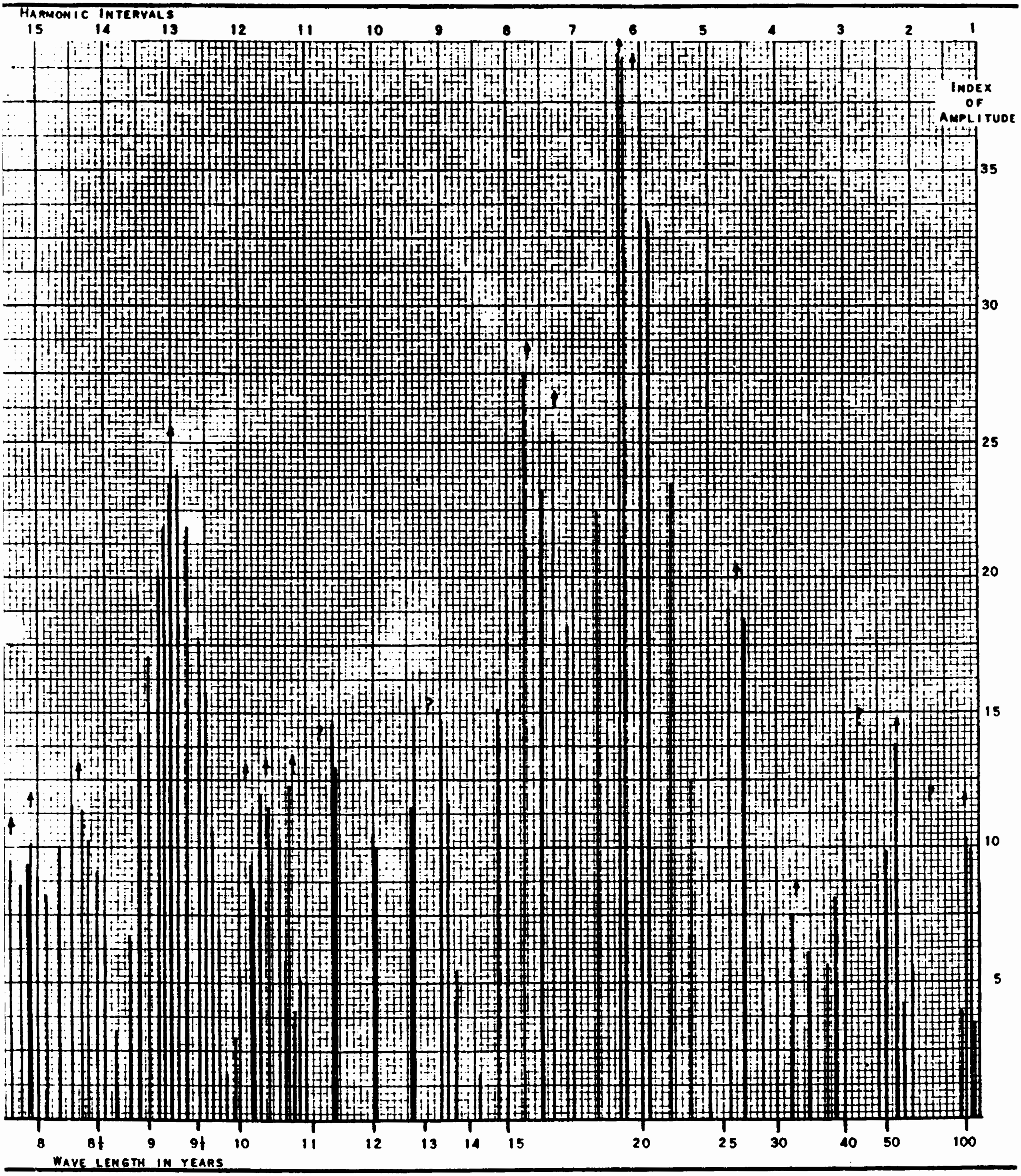

HARMONIC INTERVALS
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
INDEX OF AMPLITUDE
35
30
25
20
15
10
5
8 8½ 9 9½ 10 11 12 13 14 15 20 25 30 40 50 100
WAVE LENGTH IN YEARS

TABLE 1.

POSSIBLE CYCLES INDICATED BY PERIODOGRAM

Harmonic	Cycle Length in Years	Index of Approximate Amplitude
1.2	100.0	10.2
Between 1.3 & 2.0	Between 92.3 & 60.0	At least 6.0
2.2	54.5	10.0
Between 2.5 & 3.0	Between 48.0 & 40.0	At least 10.0
3.7	32.4	5.0
4.6	26.1	25.0 ?
*6.0 *6.3	*20.0 *19.0	**48.0
*7.3 *7.7	*16.4 *15.6	**37.0
9.0 & 9.4 or Between 9.0 & 9.4	13.3 & 12.8 or Between 13.3 & 12.8	**20.0 At least 15.0
Between 10.5 & 11.0	Between 11.4 & 10.9	At least 14.4

Harmonic	Cycle Length in Years	Index of Approximate Amplitude
*11.2 *11.6 *11.8	*10.7 *10.3 *10.2	**21.5
13.0	9.2	24.5
*14.4 *15.1 *15.4	*8.3 *7.9 *7.8	**21.0
*16.7 *17.2	*7.2 *7.0	**14.0
*18.1 *18.4	*6.6 *6.5	**5.5
*19.4 *19.6 *20.2	*6.2 *6.1 *5.9	**15.7
*21.3 *21.7 *22.0	*5.6 *5.5 *5.4	**12.0
22.4	5.3	Small
22.7	5.3	Small

Harmonic	Cycle Length in Years	Index of Approximate Amplitude
*23.6 *24.0	*5.1 *5.0	**7.3
*24.7 *25.1	*4.9 *4.8	**9.0
26.0	4.6	1.0
*26.9 *27.2	*4.5 *4.4	**2.0
*27.9 *28.2	*4.3 *4.2	**2.8
*29.2 *29.5	*4.1 *4.1	**6.9
30.7	3.9	6.0

* Suggestion of multiple cycles
** Combined effect

2. (In this connection, remember that with the exception of the 19.4-year cycle, all of the earlier work was done on combined or mixed or industrial series and that industrials and railroads behave quite differently.)

In some cases, the periodogram tends to confirm the wave lengths suggested by the earlier work. In other cases, the periodogram suggests that the cycles isolated earlier are combinations of cycles of closely related wave lengths. In still other instances, the periodogram suggests additional wave lengths that should be investigated.

If you try to compare the amplitudes given in Table 1 with the amplitudes as shown by the periodogram you will run into a certain amount of trouble because, when the periodogram indicated wave lengths very close to each other, it was necessary to make adjustments to attempt to eliminate the effect of the adjacent cycle or cycles.

Also, you will notice an uneven spacing of the bars of the periodogram, for example for lengths from 11 to 12 years or 12 to 13 years. This irregularity is due to technical difficulties in the operation of the machine that could have been overcome if we had had the time to make the additional runs to fill up these gaps.

You need to be reminded of one other fact. Namely, that deviations from a 19-year moving average minimize all cycles longer than 19 years. It is this fact which largely accounts for the decrease in amplitude as the wave lengths get longer than 19 years.

The next step is to refine the work to determine as nearly as possible the probable reality of each of these cycles, and for those which are real, or seem to be, the exact length, timing, amplitude, and shape.

When all the cycles have been pinned down it is an easy matter to project them into the future and to show their effect for the future, provided of course that they are real and that they continue.

All this we will do for you, step by step, as fast as possible.

TABLE 2.
SUGGESTED WAVE LENGTH - IN YEARS

Dewey's Work	Predix (Hughey's Work)	Harm. Anal.	Dewey's Work	Predix (Hughey's Work)	Harm. Anal.	Dewey's Work	Predix (Hughey's Work)	Harm. Anal.	Dewey's Work	Predix (Hughey's Work)	Harm. Anal.
3.8		3.9			5.6		10.0	10.2	19.4	19.5	
		4.1			5.9			10.3			20.0
		4.2	6.1	6.0	6.1		10.7	10.7	21.0		
		4.3			6.2			Between 10.9 & 11.4		23.3	
						11.0					
		4.4			6.5						26.1
						12.0	12.0				
		4.5			6.6					30.0	
		4.6	6.9					Between 12.8 & 13.3			32.4
		4.8		7.2	7.2		13.5			37.5	
4.9	4.9	4.9		7.7	7.8	14.5					Between 40.0 & 48.0
		5.0			7.9			15.6			
										51.5	
		5.1	8.2				16.0				
											54.5
		5.3			8.3			16.4			
		5.4		8.5			18.3				Between 60.0 & 92.3
5.5		5.5						19.0			100.0

Technical

TECHNIQUES INVOLVED IN THE CONSTRUCTION OF THE PERIODOGRAM OF RAILROAD STOCK PRICES 1831-1950

Using the railroad stock price index of the Foundation for the Study of Cycles (see page 143 of this issue) we proceeded as follows:

1. We converted the values to logs. We did this chiefly because logs are easier to work with.

2. We next computed a 19-year moving average of the logs in order to get an approximation of the trend line.

(The other reason for the use of logs appears at this point. A moving average of the logs gives you a geometric moving average of the data, and a geometric moving average fits the data better than would a simple arithmetic moving average. If we had not used logs our moving average would have been an arithmetic one.)

We chose a moving average trend instead of some other kind of a trend chiefly because you can so easily calculate the effect of a moving average upon cycles of every possible length. Such computations cannot be made easily, if at all, for other curved trend lines.

We chose a 19-year moving average instead of a moving average of some other length because the most important cycle in railroad stock prices has an average length of about 19 years and, therefore, a 19-year moving average gives a smoother trend than any other moving average that might have been selected.

3. We extended the 19-year moving average arbitrarily for the nine years at each end of the series lost by the moving average computations.

4. In order to keep the answers in terms of percentages and to avoid negative logarithms, we added 2.000 to each log of the data. From the sum we then subtracted the logs of the corresponding value of the moving average. The answers gave us the logs of the percentages that the original figures were of the moving average trend.

5. We plotted the result as shown in Fig. 2 on pages 124 and 125.

6. We traced this curve time after time with the stylus of the harmonic analyzer in order to get the sine—cosine values for integral harmonic intervals.

7. We then chopped short sections from each end of the curve to get a shorter over-all length and repeated the process to get sine—cosine values for the integral harmonics of the shorter length which were, on their part, fractional harmonics of the original length.

8. We then computed the square root of the sum of the squares of the sine—cosine values and recorded the result in the periodogram on pages 126 and 127.

POSSIBLE CYCLES IN INDUSTRIAL STOCK PRICES, 1871 – 1950

Summary

A comprehensive reconnaissance survey of industrial common stock prices, 1871-1950, suggests a multitude of cycles, the length and relative importance of which are shown in the article.

This study supplements the reconnaissance survey of railroad stock prices, 1831-1950, published in the April 1952 report.

As with the railroad study, the task ahead is to investigate each one of the indicated cycles to see if it is rhythmic, and if so, to determine its various characteristics such as shape, timing, length, and amplitude, and to get some idea of its probable reality.

Those cycles that "stand up" can be projected into the future.

IN the April 1952 report I presented to you the results of a multiple harmonic analysis of railroad stock prices, 1831-1950. In this report I shall give you the results of a similar analysis of industrial stock prices, 1871-1950.

This study, like the railroad stock price study, was made possible by the generosity of Stockham Valves & Fittings, Inc. of Birmingham, Alabama, who supplied the money, and the Mico Instrument Company, of Cambridge, Massachusetts, who gave free access to one of their latest model harmonic analyzers.

The actual work was done by Alexander Malinowski of the Foundation staff who laid out the work, operated the machine, and computed the final values.

As we had the use of the machine for only a limited period of time, the making of these periodograms necessitated a period of intensive activity. In order to bring these results to you Mr. Malinowski worked an average of 114 hours per week for the three weeks during which the machine was available. On at least two occasions he did not go to bed at all.

As a result of his zeal we are able to present to you a series of results which would have taken 6½ years work with paper, pencil, and an ordinary calculating machine such as a Friden, Monroe, or Marchant.

By way of background in connection with "mechanical brains" you may wish to read correspondence with Mr. R. I. Abbay, Jr. of Memphis reprinted in this report in the **Letters** section on page 188. You may also wish to refer to the railroad stock price article referred to above and to an article on page 54 of the Spring 1952 issue of the **Journal of Cycle Research** called, "A New Form of Periodogram."

The Data

For our industrial stock price study we used the Standard & Poor's Corporation series of Industrial Common Stock Prices (365 stocks) as carried backward to 1871 by Alfred Cowles III and Associates in their book, **Common Stock Indexes**, and as adjusted for trend.

We charted the actual index for you on page 87 of our March 1951 report. We charted the deviations from trend for you on pages 148 and 149 of the April 1951 report. We gave you the numerical values in logs of these deviations on pages 146 and

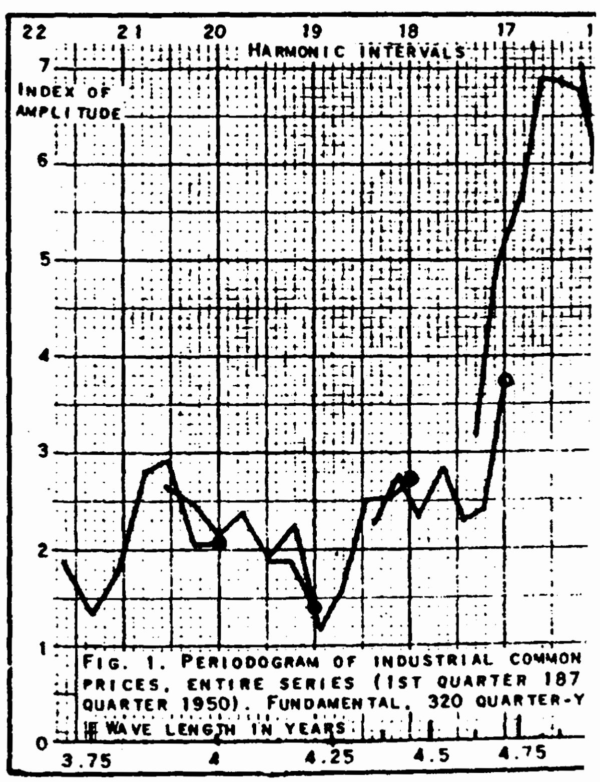
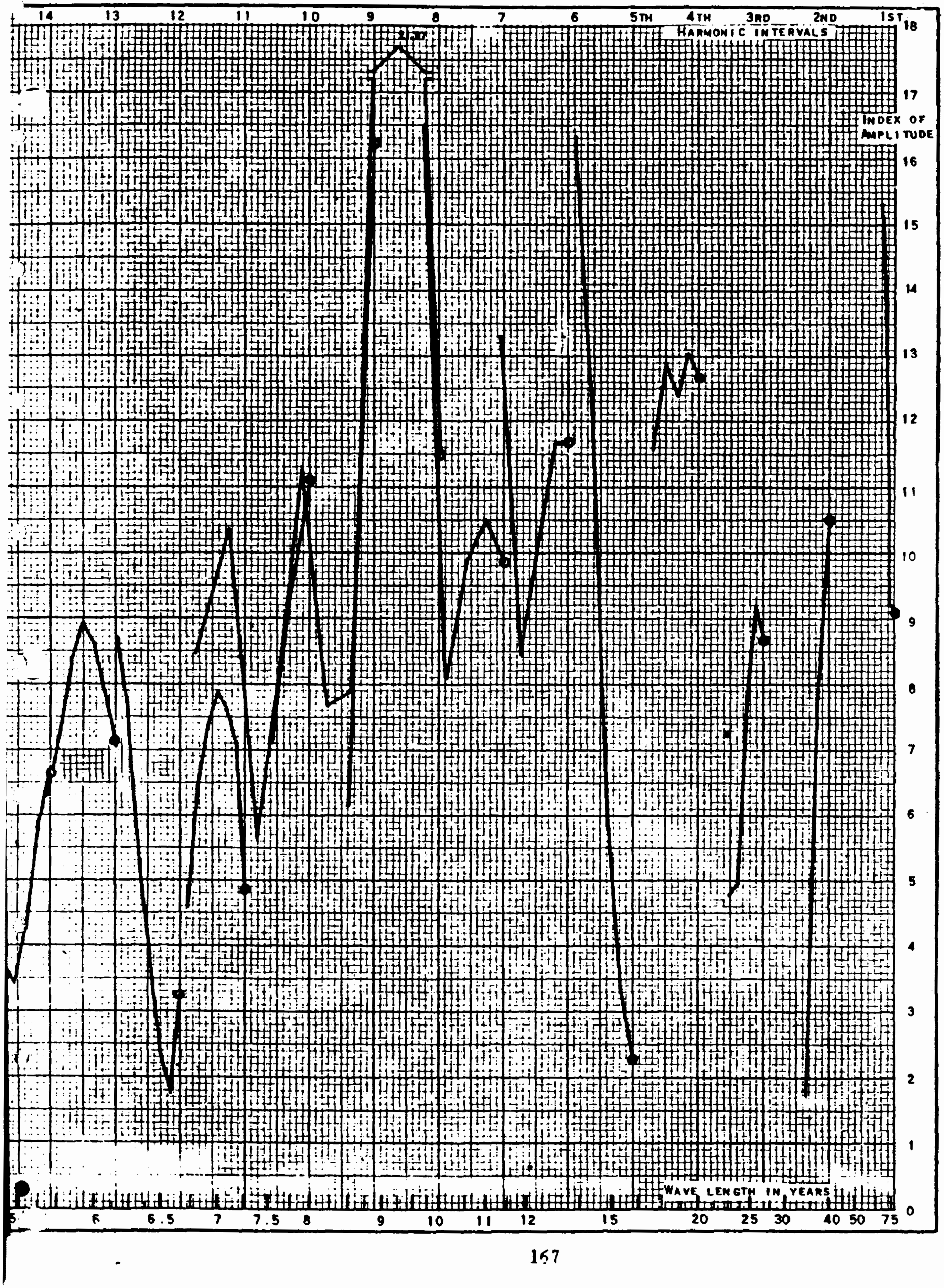


FIG. 1. PERIODOGRAM OF INDUSTRIAL COMMON PRICES, ENTIRE SERIES (1ST QUARTER 187 QUARTER 1950). FUNDAMENTAL, 320 QUARTER-Y


14
13
12
11
10
9
8
7
6
5TH
4TH
3RD
2ND
1ST
HARMONIC INTERVALS
INDEX OF AMPLITUDE
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
WAVE LENGTH IN YEARS
5
6
6.5
7
7.5
8
9
10
11
12
15
20
25
30
40
50
75

147 of that same issue. On page 86 of the March 1951 issue we printed a chart to show you the dates when the stocks of the various industries were added to the index.

Scope of the Analysis

In making the multiple harmonic analysis of industrial common stock prices, we analyzed the entire series of 80 years or 960 months down to its 21st harmonic. That is, we scanned the series for cycles of all lengths from 80 years long down to $\frac{1}{21}$ of 80 years, or 45+ months long.

We then divided the series into three separate parts, each part 320 months long. We analyzed these three parts separately down to their 31st harmonic. This work gave us a reconnaissance of all cycles that might be present in each third of the series down to wave lengths $\frac{1}{31}$ of 320 months, or 10+ months.

Finally we divided the series into sixths, each sixth being 160 months long. We intended to analyze each one of the six segments down to its 31st harmonic (wave lengths of $\frac{1}{31}$ of 160 months, or 5+ months). Unfortunately time did not permit us to run through the machine any except the last two sections, namely, the 5th sixth, from May 1924 through August 1937, and the 6th sixth, from September 1937 through December 1950.

As you can see from the foregoing description, we made six different multiple analyses of industrial common stock prices—one analysis of the series as a whole, three of the series by thirds, and two of the last two parts of the series by sixths. This is the most intensive analysis of these prices ever made, as far as I know.

Six Periodograms Obtained

From these analyses we were able to construct six periodograms which are shown herewith as Figs. 1 to 6 inclusive.

A periodogram bears the same relationship to a forecast that a recipe bears to a piece of cake. A periodogram is a highly desirable prerequisite to cycle analysis, but it will not provide you with facts you can use directly. If all you want is final

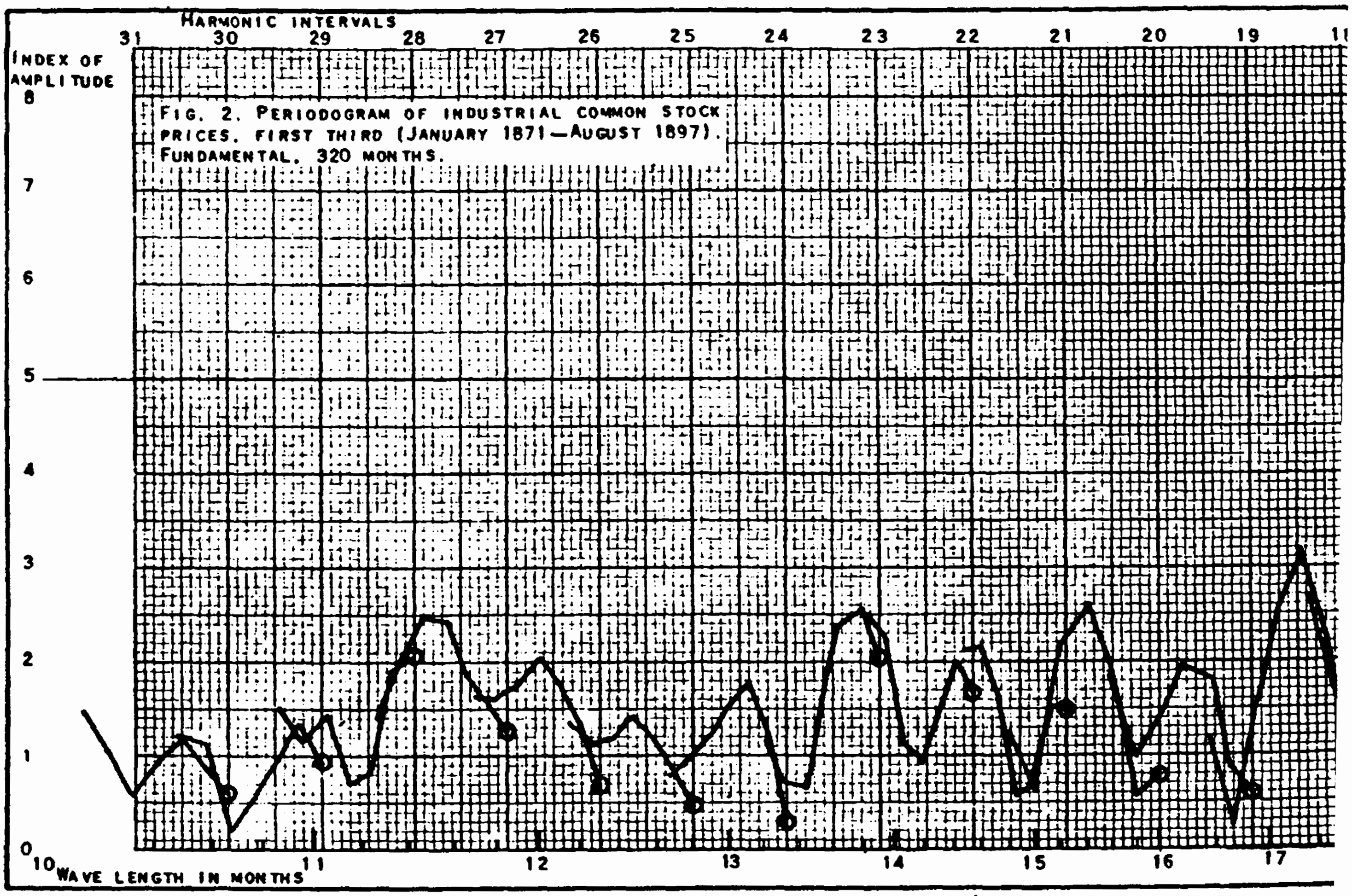


FIG. 2. PERIODOGRAM OF INDUSTRIAL COMMON STOCK PRICES, FIRST THIRD (JANUARY 1871—AUGUST 1897). FUNDAMENTAL, 320 MONTHS.

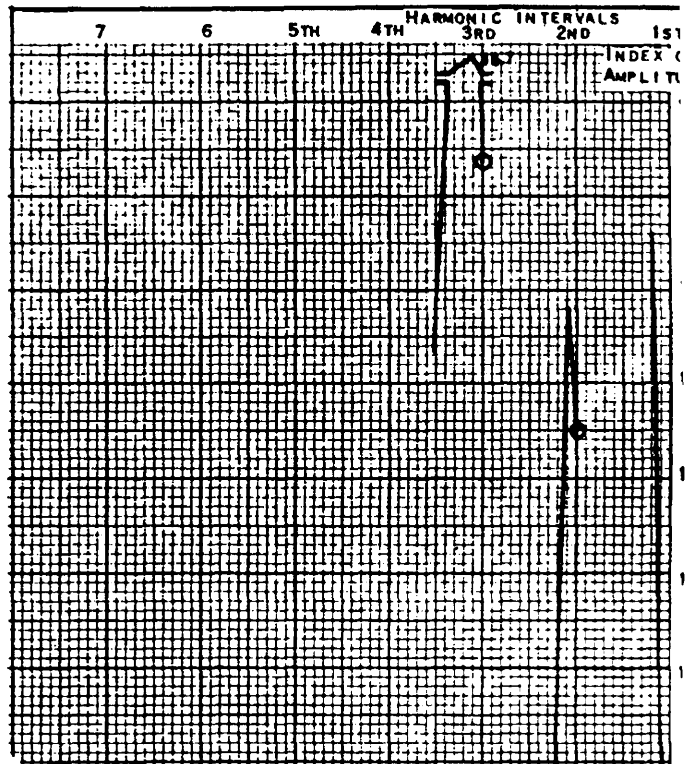
results, you had better stop reading this article at this point and await future reports where the material will be worked up into usable form. From the periodograms in their raw state you will not get even a glimmer of what is ahead in common stock prices.

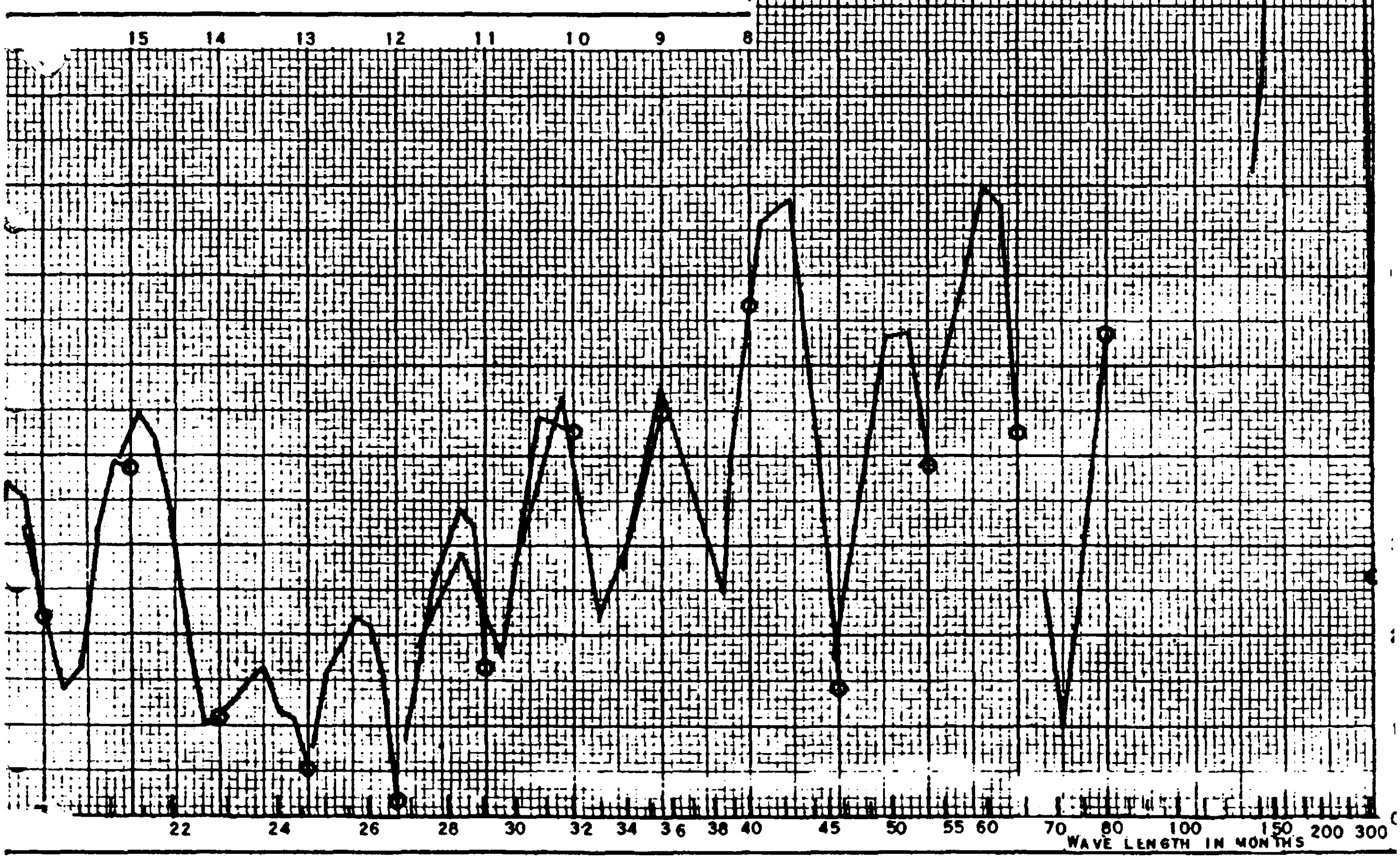
However, if you want to know a little something more about the underlying structure of the stock market, and are interested in techniques, you may wish to continue.

What is a Periodogram?

To start at the beginning, the periodogram is a curve which shows you the relative strength of cycles of various lengths. When the strength increases as the wave length increases and then decreases as the wave length increases still further, you have the hint of a cycle of the wave length of maximum strength. That is, peaks in the periodogram curve suggest possible cycles.

Thus in Fig. 1 strength increases (the curve goes up) from wave length 7½ years





to wave length 8 years and then goes down again to wave length 8½ years. This means that there m a y be a cycle at wave length 8 years (read from the bottom scale) with a strength of about 11 (read from the side scale). As we go still further to the right the curve goes up again to about wave length 9¼ and then down again to 10. This indicates that there may be a cycle with a wave length of about 9¼ years with strength or amplitude of about 22,—twice as strong as the cycle with a wave length of 8 years.

Scales Used

The vertical scale is easy to understand. However, if you notice carefully you will see that the wave lengths on the horizontal scale get closer and closer together as you go toward the right. For example, in Fig. 1, wave lengths of 20 years and 40 years are separated by no more horizontal space than are wave lengths of 8 years and 10 years. This is so because I have used a h a r m o n i c horizontal scale. A harmonic horizontal scale should a l w a y s be used for periodograms.

In a harmonic scale equal horizontal distances represent equal harmonics or unit fractions of the total length of the series being analyzed. Thus in Fig. 1, as the series being analyzed is 80 years long (1871-1950), the harmonics are—

1st harmonic (fundamental),	80 years	
2nd harmonic,	40 years	(½ of 80 years)
3rd harmonic,	26.6 years	($\frac{1}{3}$ of 80 years)
4th harmonic,	20 years	(¼ of 80 years)
5th harmonic,	16 years	($\frac{1}{5}$ of 80 years)
	And so on.	

It is these harmonics or unit fraction lengths that are represented by the heavy vertical lines, starting with the first harmonic at the right and running to the 31st harmonic at the left. The harmonic scale is shown, running from right to left, at the t o p of the periodogram.

At this point I would like to introduce the idea of fractional harmonics. In Fig. 1, if the 3rd harmonic represents $\frac{1}{3}$ of 80 years and the 4th harmonic represents ¼ of 80 years the location $\frac{1}{10}$ of the way from the 3rd harmonic to the 4th harmonic would logically represent $\frac{1}{3.1}$ of 80 years or 25.8 years. If you look at Fig. 1 you will find a hint of a cycle at

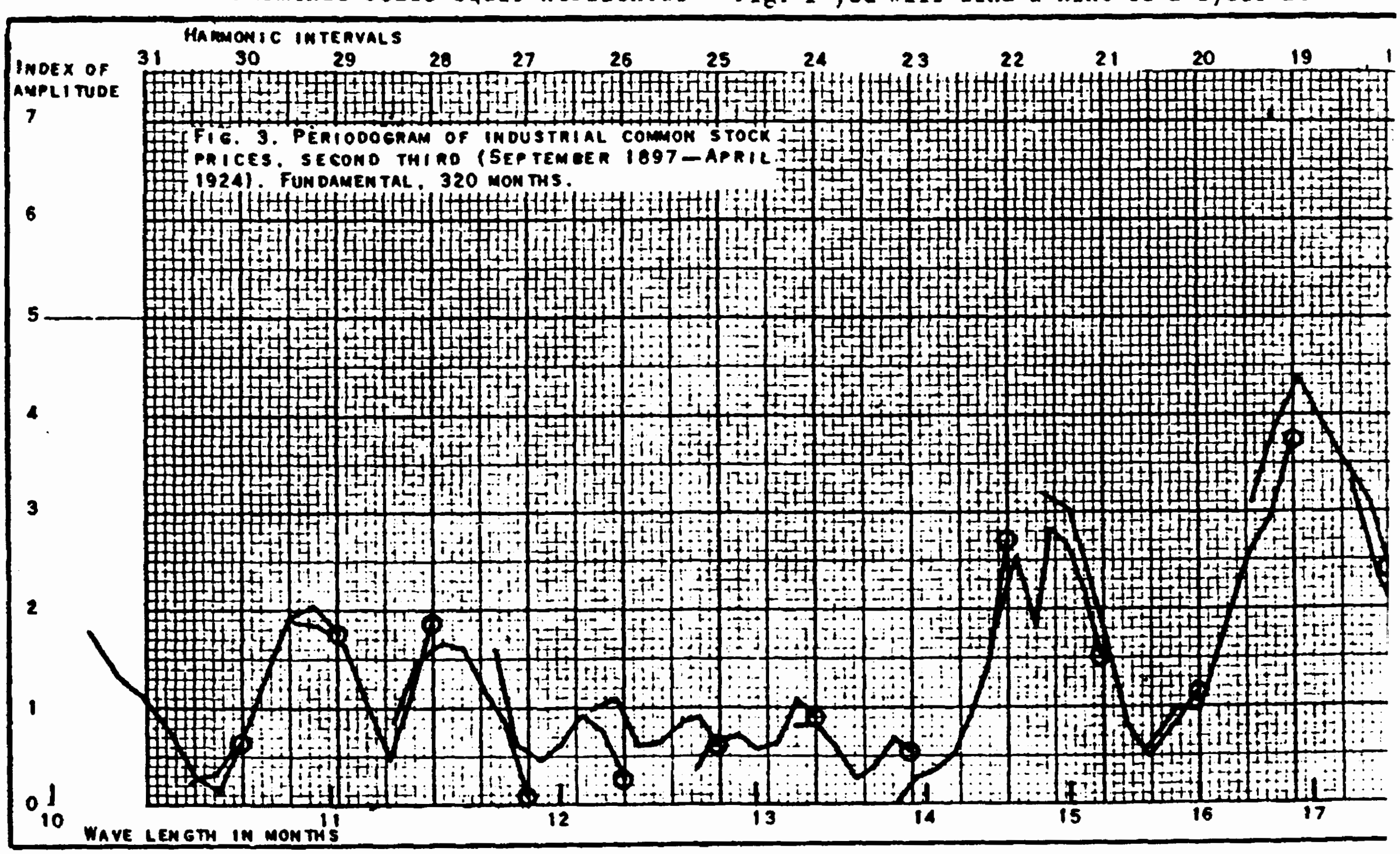


FIG. 3. PERIODOGRAM OF INDUSTRIAL COMMON STOCK PRICES, SECOND THIRD (SEPTEMBER 1897—APRIL 1924). FUNDAMENTAL, 320 MONTHS.

about this length.

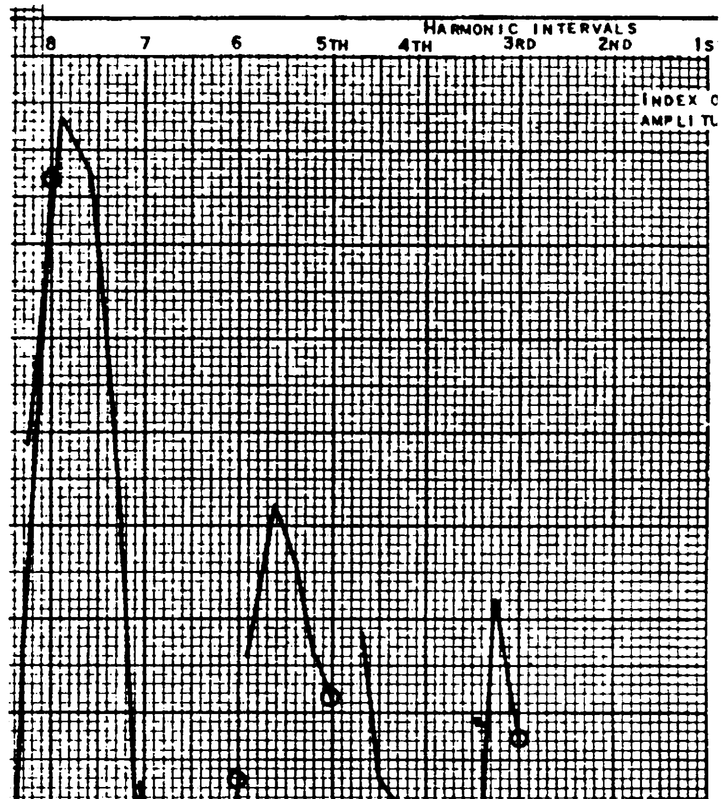
So much for the grid upon which the periodogram is plotted and its scales.

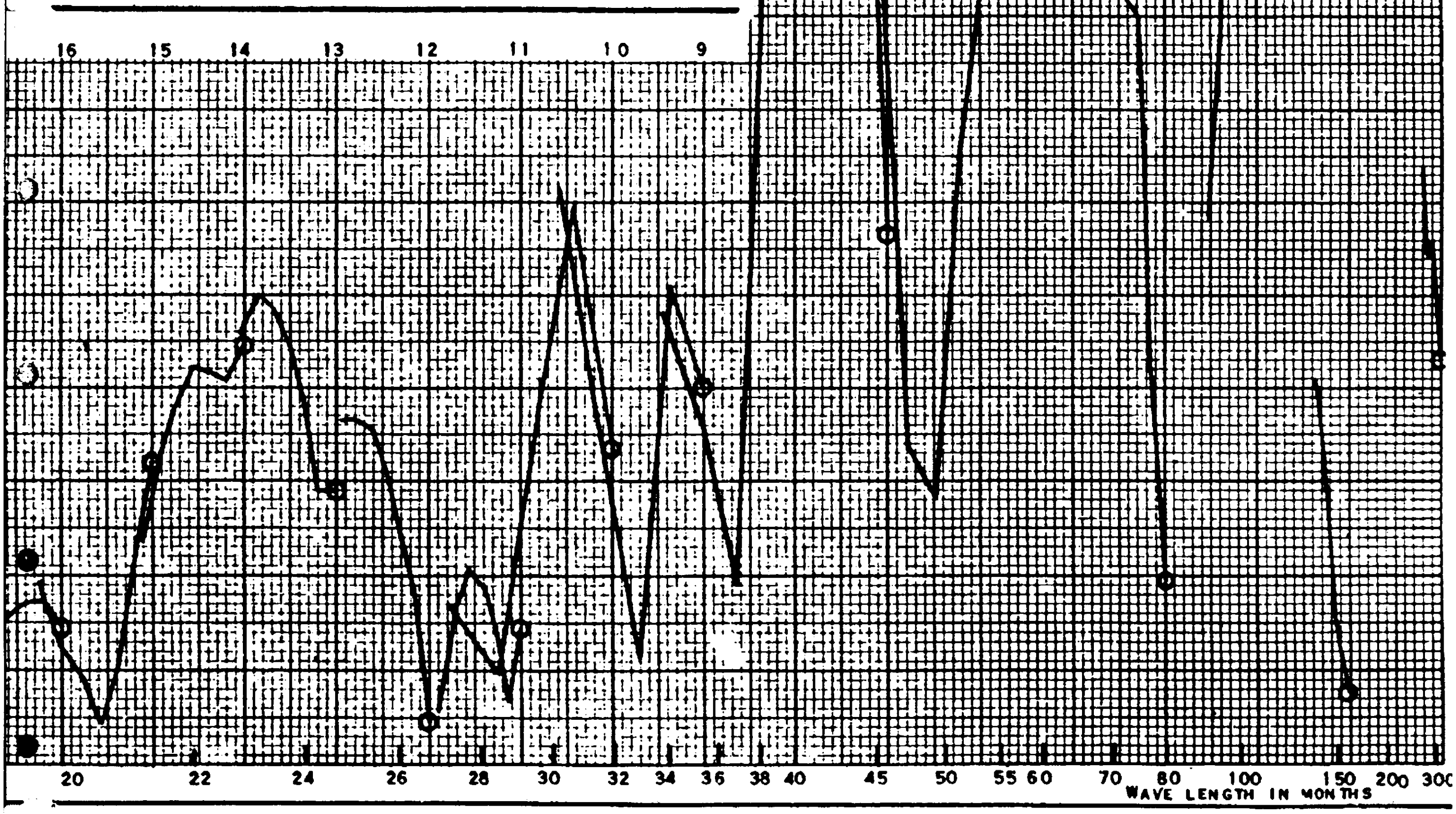
The Curves

One other matter needs explanation:

If you look at the periodograms you will see that they all consist of a number of curves each of which start at a dot with a circle around it and run to the left. These dots with circles around them all lie on the heavy vertical lines which represent the harmonics or unit fractions of the total length of the curve being analyzed. The values shown by these dots are the only values you would get by a simple harmonic analysis of the curve. These points are not close enough together to indicate wave lengths unless the actual wave lengths are exact harmonics of the entire curve being analyzed.

In multiple harmonic analysis, however, having made a simple harmonic analysis and obtained the results shown by the dots with the circles around them, you cut a little off the end of the series and make another complete simple harmonic analysis of the shorter series. You then cut a





little more off the series and do it again, time after time, until you get all the points you want.

In the periodogram of the multiple harmonic analysis of railroad stock prices I showed you all these values by means of vertical horizontal lines. In the periodograms printed herewith, charting the results of the multiple harmonic analysis of industrial stock prices, I am showing you the results by points, connected to each other.

The points are not all connected. We connected only the points representing the same harmonic. Thus, in analyzing the entire series (Fig. 1) when we came to the 12th harmonic we found the strength of the average cycle $\frac{1}{12}$ of the entire series long ($\frac{80}{12}$ or 6.6 years) to be 3.25. This value was plotted as a point with a circle around it. Then we cut about a year off the end of the series and found the 12th harmonic of the slightly shorter series; and so on six times to get six more points. It is these seven points which are connected.

By the time we had cut six sections off the original curve we had reduced its length about six years so that $\frac{1}{12}$ of the abated length was just about as long as $\frac{1}{13}$ of the original length.

Therefore on the line representing the 13th harmonic of the original series ($\frac{80}{13}$ or 6.2 years long) you had two points: one with an amplitude or strength of 8.7 representing the 12 harmonic of 80 years, the other at amplitude 7.1 representing the strength of the 13th harmonic of 73.8 years.

Reasons for Discontinuity

As the 12th harmonic of 73.8 is 6.2 years long and the 13th harmonic of 80 is 6.2 years long they both represent waves of identical length. Why, then, do we not get the same strength or amplitude?

Ideally we should, but there are two sources of error that have crept into the work which you should know about and be forewarned against. First, in the two analyses we are not dealing with the same data. Six more years of data in the longer curve can materially alter the picture. Second, a harmonic analysis should deal with data that has been fully corrected for trend. That is, the curve should end at the same value as the value at which it starts. In multiple harmonic analysis this is not always feasible. An error is introduced for this reason also.

Sources of Error

On the other hand, these errors are not usually too important, because there are two other errors inherent in harmonic analysis which are usually even worse.

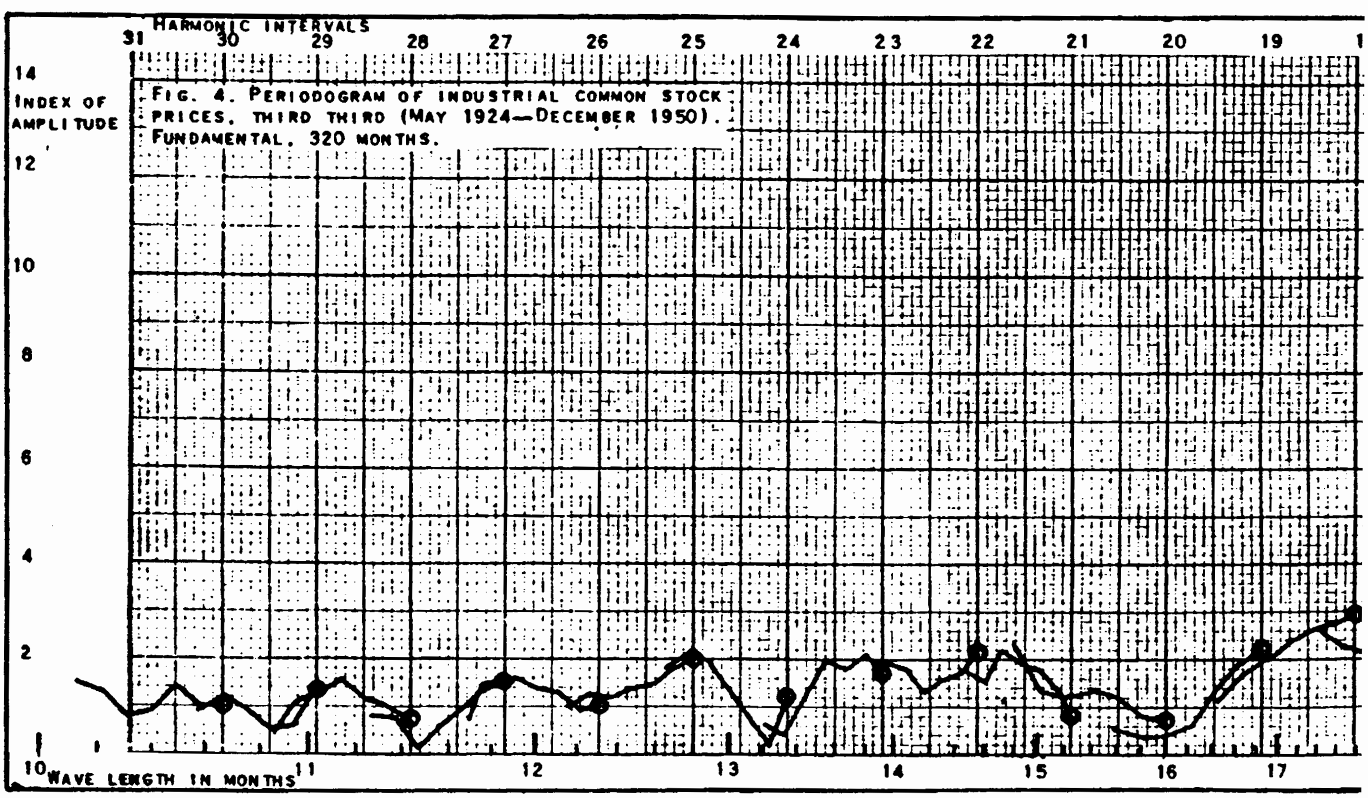
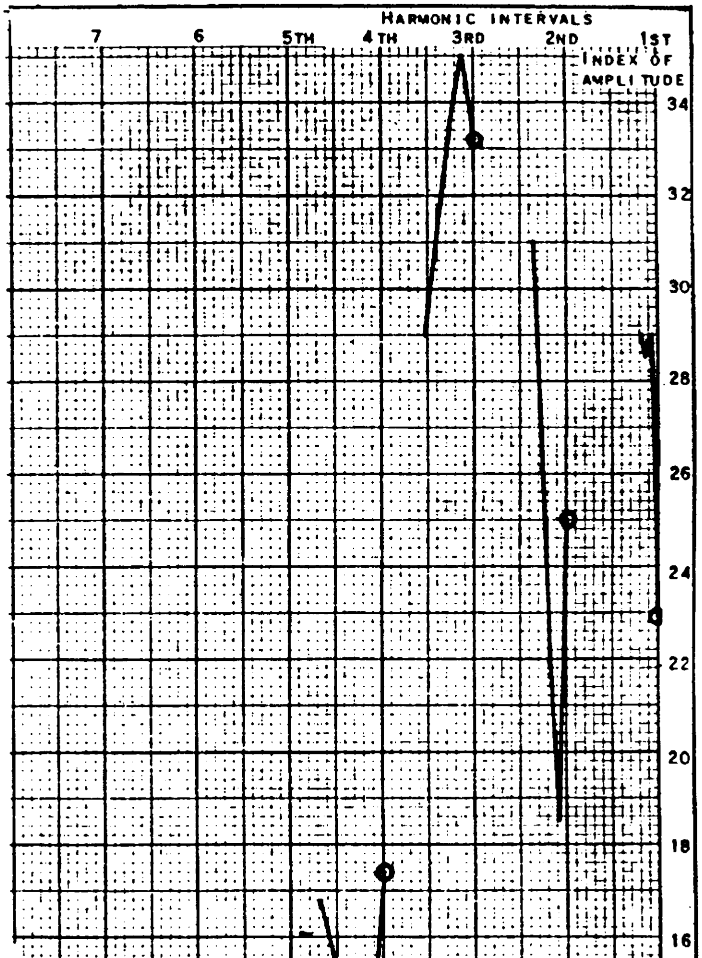


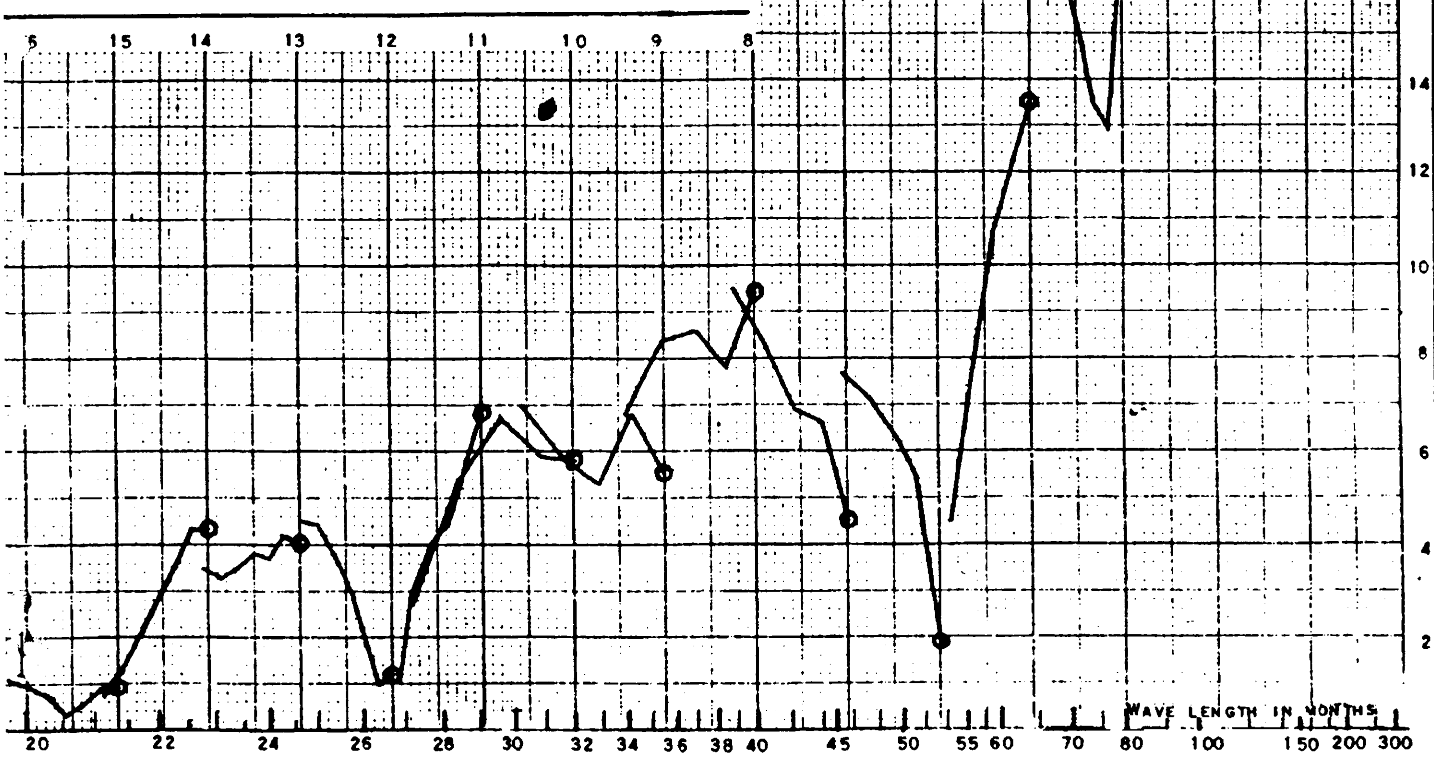
FIG. 4. PERIODOGRAM OF INDUSTRIAL COMMON STOCK PRICES, THIRD THIRD (MAY 1924—DECEMBER 1950). FUNDAMENTAL, 320 MONTHS.

Why bother too much about little errors when there are bigger errors to start with? Anyway, we are looking only for r e l a t i v e strength and this can usually be figured out. For example, in Fig. 1 at the 13th harmonic it does not much matter which of the two amplitude values is correct—there is no peak there anyway (if there were, the line from the 13th to the 14th harmonic would slope down), and peaks are all we are interested in.

The two larger errors I have just mentioned are these: First, mechanical harmonic analysis has no way of getting rid of or even minimizing random numbers. (There is a way of doing this in paper and pencil calculation, as I have mentioned from time to time and will explain fully some day.) Unless you can minimize the random numbers you get serious distortions.

Second, when more than one cycle is present, harmonic analysis puts the peaks at the wrong places unless the length of the series being examined covers the full synodic period of the two cycles—that is, unless it is long enough to let the two cycles come around into step with each other again. For example, if you had five-year cycles and six-year cycles you would not get peaks at the right places on your periodogram unless you took 30 years of data (or some unit multiple of this length,





such as 60 years or 90 years). If you took some other length of data, such as 13 years or 45 years you would get your peaks moved a bit one way or the other—and perhaps quite a bit. (In this connection, refer to "Limitations of the Periodogram," an article printed in our report for June 1951.) In view of this fact, and the utter impossibility of using enough data to give you the synodic period of all the different cycles, you are going to get errors from this source too. That is why the two other errors I spoke of at first do not loom too importantly.

Finally, there are errors of interpretation of the periodogram which arises in actual practice because you do not have enough points between harmonics. For example, in Fig. 1 we have placed an arrow at the 6.1 harmonic ($\frac{1}{10}$ of a harmonic interval to the left of the numeral six at the top of the chart). This means that we guessed that the curve might go up between the 6th harmonic and the 6.2 harmonic, cresting perhaps at 6.1. It is a guess that seems reasonable and can be rather easily justified. My own private unjustifiable hunch, however, is that the peak really lies between 6.2 and 6.5, that it is on its way down at 6.2, and is coming up again at 6.0 toward a second peak at 5.9 (all numerals referring to the harmonic scale at the top of the page), and that 6.1 probably represents a low spot instead of a peak.

Periodograms Give Merely Hints

For all these reasons you must consider the peaks of the periodograms merely as hints of cycles which may possibly be present in the series. You must go on from there.

Two other points need to be discussed: First, why are there gaps with no coverage at the right of the periodograms and overlaps with double coverage in the left section? Second, when we break the curve into sections, as the 1st, 2nd, and 3rd thirds, why are the various sections not consistent with each other? Why, for example, do we find a cycle about 23 months in the section from 1950 back to 1924 and from 1924 back to 1897, but not in the section 1897 back to 1871?

Why Are There Gaps in the Periodogram Curve?

To answer the first question first, there are gaps and overlaps because the machine computes values for five harmonics at a clip. The particular machine we worked with computed five even harmonics or five odd harmonics. That is, it would run the 1st, 3rd, 5th, 7th, and 9th harmonics at one time, the 2nd, 4th, 6th, 8th, and 10th harmonics, the next time, then the 11th, 13th, 15th, 17th, and 19th harmonics, and so on.

Now suppose that, as in Fig. 1, you decide that you will make a total of seven runs for each harmonic as we discussed above, chopping off sections of the curve so that the 19th harmonic of the shortest

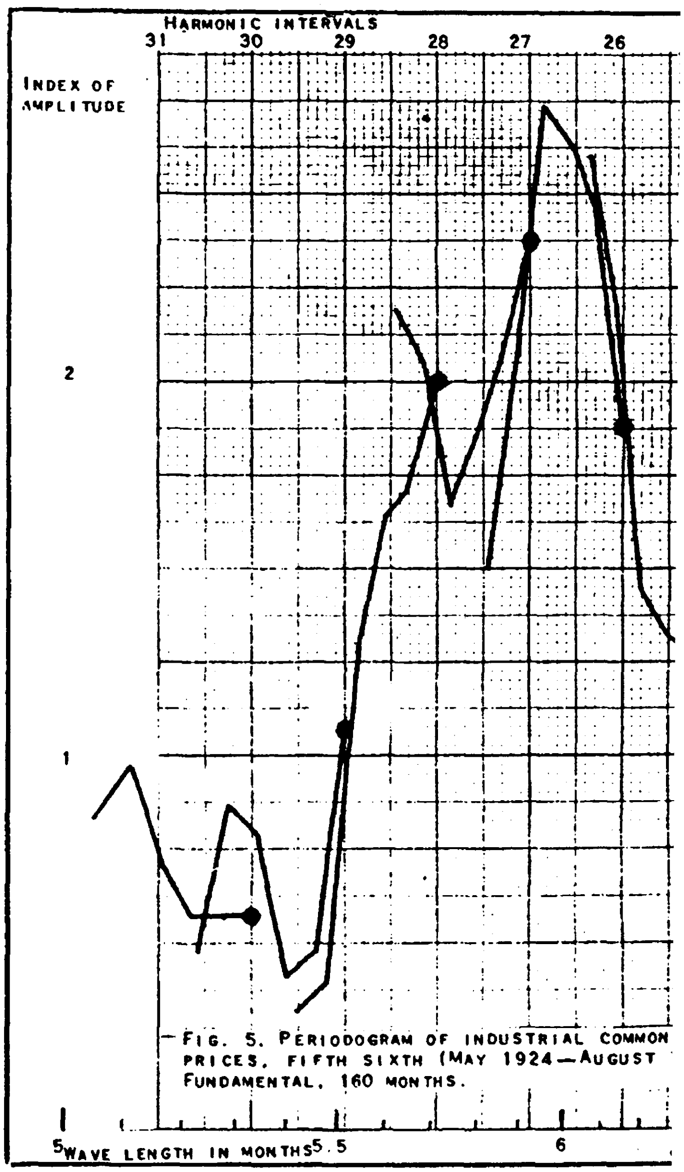
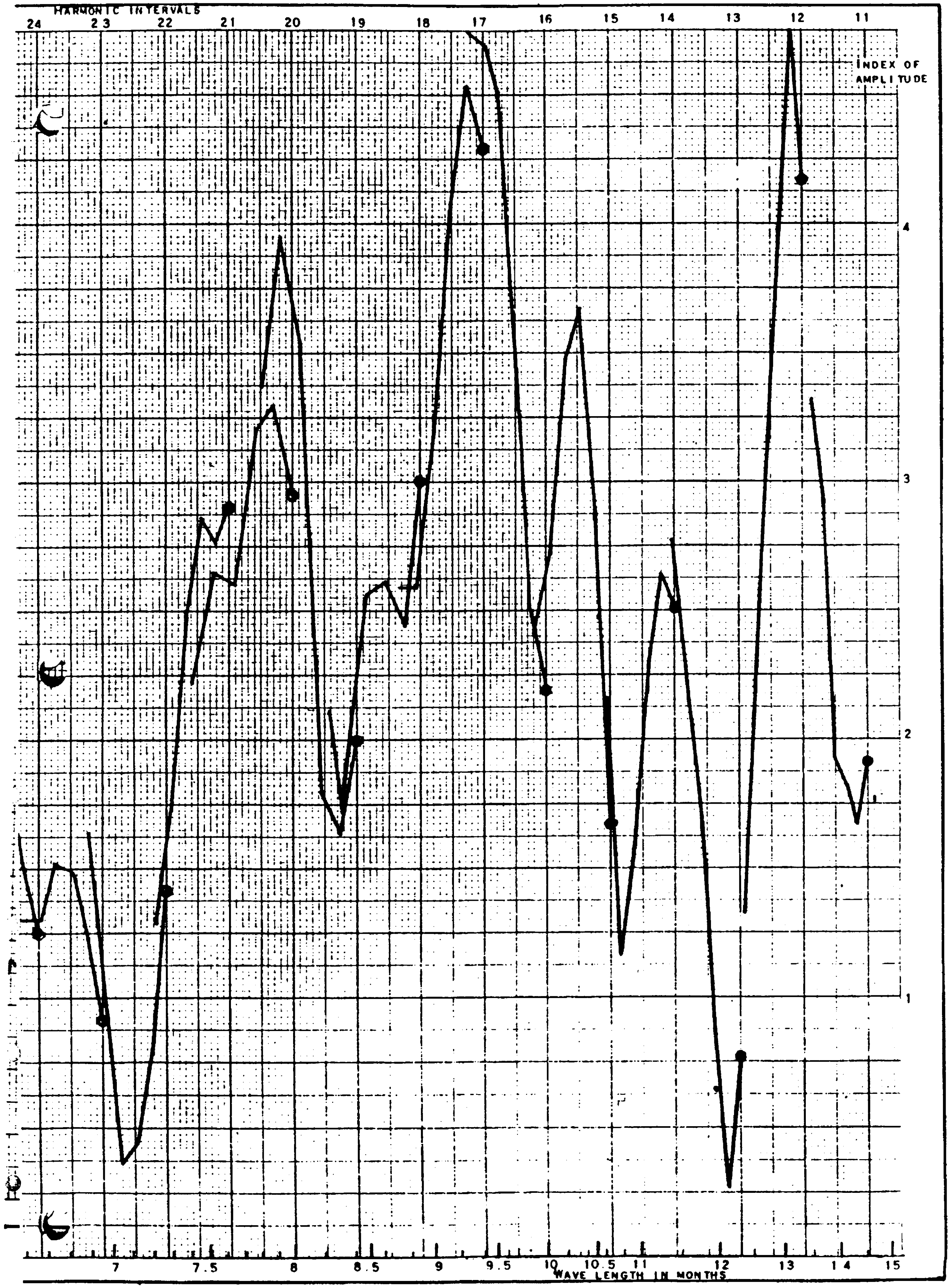


FIG. 5. PERIODOGRAM OF INDUSTRIAL COMMON PRICES, FIFTH SIXTH (MAY 1924—AUGUST FUNDAMENTAL, 160 MONTHS.


HARMONIC INTERVALS
24
23
22
21
20
19
18
17
16
15
14
13
12
11
INDEX OF AMPLITUDE
4
3
2
1
7
7.5
8
8.5
9
9.5
10
10.5
11
12
13
14
15
WAVE LENGTH IN MONTHS

section will just exactly equal the 20th harmonic of the entire series. That is easy to compute. The 20th harmonic of 80 years is 4 years ($\frac{80}{20}$). The 19th harmonic of 76 years is 4 years. The two points will coincide. The 12th harmonic of 80 years is $\frac{80}{12}$ or 6.6, and the 11th harmonic of 76 years is only 6.9 years. We will, therefore, have no coverage between 6.6 years and 6.9 years.

If we chop more off the original 80 year series we can get the 11th harmonic to come out right, and equal the 12th harmonic of 80 years, but then the 19th harmonic of the shorter length will overlap not only the 20th harmonic but all values almost to the 21st harmonic as well.

This being the nature of harmonics there is nothing you can do about it except to choose some middle ground. This works out pretty well except for the first few harmonics, say from the 1st to the 6th. In such cases, we scanted the shorter harmonics because the longer cycles involved have no chance to repeat themselves many times anyway, and therefore, although they may be perfectly real, one can have no good internal evidence of that fact.

Why Do the Different Sections of the Periodogram Show Different Results?

This brings us to the final point of this article. Why do you not find the same cycles in each of the different shorter sections as in the entire series? Why do you not find the same cycles in one short section as in another?

There are many reasons why it could be so. Perhaps in some instances it is one reason, perhaps in another instance it is another.

Here are some of the possible explanations:

1. The peak in the periodogram could reveal merely some grouping of large accidental distortions.
2. The cycle might actually be present as a rhythmic cycle in part of the curve, but might be present as a result of random forces just happening to come at rhythmic intervals (see article called "Cycles in Random Numbers," in our January 1952 report).
3. The cycle may have changed its length over the period of time.
4. The periodogram indications may be wrong.
5. We may be dealing with a compound cycle, the two parts of which largely offset each other in the section of the curve where it did not appear.
6. The cycle may be a characteristic of some particular part of the composite which was not present throughout the series. For example, if the 23-month cycle spoken of above was present from 1897 on, but not from 1871 to 1897, it might mean that this cycle is not present in shipbuilding, mining, iron and steel and other sorts of stocks in the index prior to 1897, but is present in textile, fertilizer, tobacco, or other sorts of stocks which were added to the index at the turn of the century.

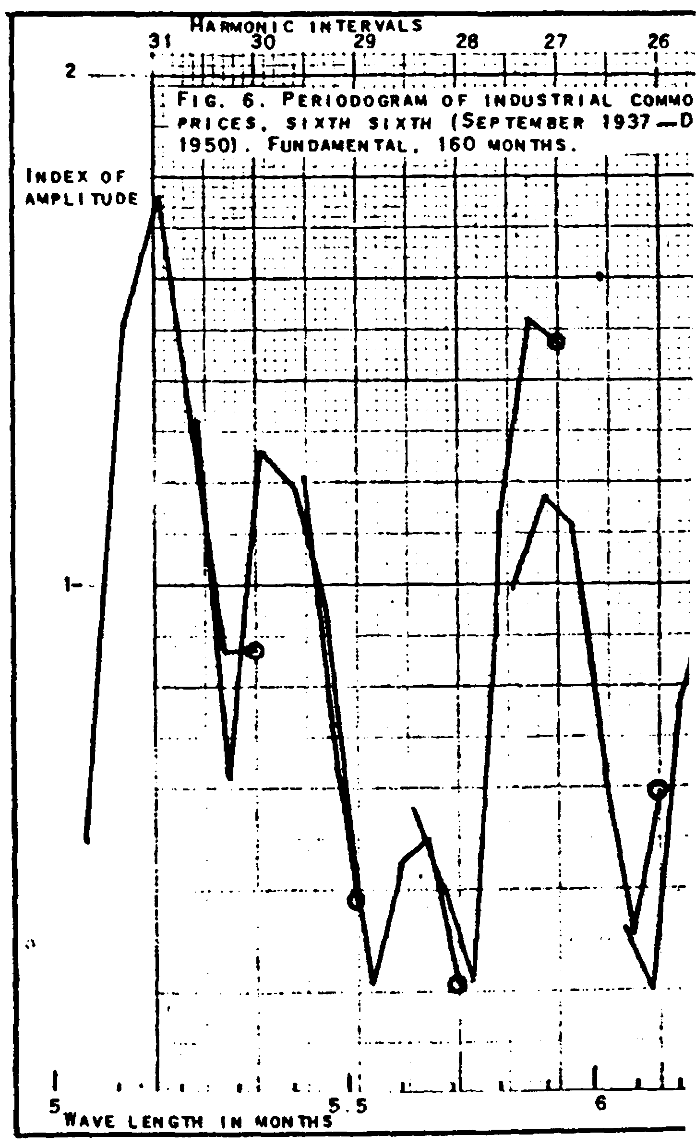
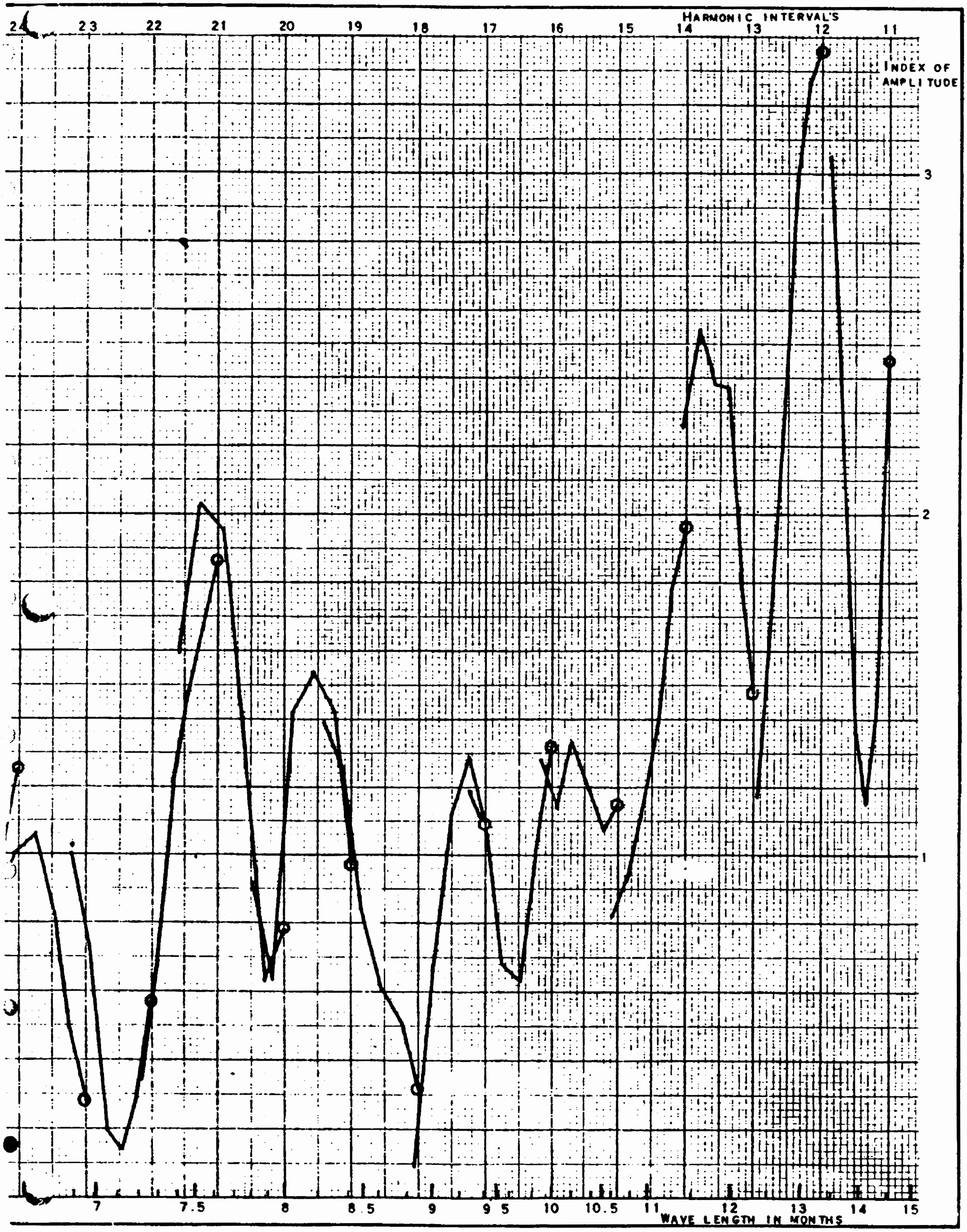


FIG. 6. PERIODOGRAM OF INDUSTRIAL COMMO[DITY] PRICES, SIXTH SIXTH (SEPTEMBER 1937—D[ECEMBER] 1950). FUNDAMENTAL, 160 MONTHS.



HARMONIC INTERVAL'S
24
23
22
21
20
19
18
17
16
15
14
13
12
11
INDEX OF AMPLITUDE
3
2
1
7
7.5
8
8.5
9
9.5
10
10.5
11
12
13
14
15
WAVE LENGTH IN MONTHS

Table 1 Gives Wave Lengths

This brings us now to Table 1 which shows the wave lengths in industrial common stock prices suggested by the six periodograms and, for comparison, the wave lengths suggested by the periodogram of railroad stock prices, 1831-1950, as reported upon in our April report.

Table 1 does not show the strength or amplitude of the various cycles. If you wish this information you can read it from the periodogram concerned. In connection with this matter or strength, remember that if two cycles fall within a harmonic interval of each other they supplement each other's amplitude. Thus, if you have two cycles with amplitude of four with lengths half of a harmonic interval apart they do not both have an amplitude of four. In fact, neither one of them has an amplitude of four. The real amplitude of one adds to part of the real amplitude of the other to give an a p p a r e n t amplitude of four.

I am sorry this article has had to be so long and so technical. Unless you are a serious student I hope you skipped it! When, in future reports, we go on from here to discuss the various cycles suggested and to project them into the future, it will be more interesting.

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TABLE 1.

CYCLE LENGTH SUGGESTED BY PERIODOGRAMS

Industrial Stock Prices: Entire Series Mo.	First Third Mo.	Second Third Mo.	Third Third Mo.	Fifth Sixth Mo.	Sixth Sixth Mo.	Railroad Stock Prices: Entire Series Mo.
				5.1	5.1	
				5.3	5.3	
				5.6	5.6	
					5.8	
				6.0	5.9	
				6.4	6.3	
				6.5		
				6.8	6 7	
				7 5	.7 6	
				7 8		
				8.0	8.1	
				8.5	8.3	
				9.2	9.2	
					9.3	
					9.9	
	10.6		10.5	10.3	10.2	
	10 8	10.9	10 7			
	11.1		11.1	11.2		
	11.5	11.5		11.4	11.6	
	12.0	12.1	11.9		11.9	
	12.5	12.6				
		12.9	12.8			
	13.1	13.2	13.6	13.1	13.3	
	13.7	13.9	13.8			
	14.4	14.5	14.5			
	14.6	14.9	14.7			
	15.3		15.5			
	16 4	16.8				
	17.3		17.8			
	18 3	18.7	18.7			
	19.6	19.6				
	21.6					
		22.2	22.7			
	23.7	23.2	23.7			
			24.3			
		24.9	24 9			
	25.8					
		27.8				
	28.6		29.1			
		30.5	30.2			
	31 2					
		34.2	34.2			

Industrial Stock Prices: Entire Series Mo.	First Third Mo.	Second Third Mo.	Third Third Mo.	Fifth Sixth Mo.	Sixth Sixth Mo.	Railroad Stock Prices: Entire Series Mo.
	35.6		36.2			
			39.0			
	41.6	41.6				
			43.0			
46 6			46.7			47.0
48.6						49.0
Yrs.	Yrs.	Yrs	Yrs.	Yrs.	Yrs.	Yrs.
4.2	4.2		4.2			4.2
4.3						
4.4						4.4
4.5						4.5
						4.6
4.8		4.8				4.8
4.9						5.0
						5.1
5.3						5.3
5.4						5.4
						5.5
			5.7			5.6
						5 9
6 0		6 1				6 1
						6 5
						6.6
7.0						7.0
						7 2
						7.8
7.9						7 9
	8 5	8.2	8.5			8.3
9.3						9.2
10.8			10 7			10 3
11 3						11 3
	12.7					
13.1						13.3
13.6						
						15.8
17.8						
19.1						19.3
		24 2	24.2			24.0
25.8						26 1
						31.6
						54.6
						100.0

POSSIBLE CYCLES IN PIG IRON PRICES, 1784–1951

Summary

A multiple harmonic analysis of pig iron prices, 1784—1951, suggests the length and amplitude of each of a number of cycles which may be present in these figures. Pig iron prices are important from the standpoint of cycle study, not only because pig iron is an important industrial commodity, but because of the success made by Samuel Benner in 1874 in forecasting prices of pig iron by means of cycles. Imaginary purchases and sales, and short sales, made on the basis of Benner's forecast showed a gain-loss ratio of 31 to 1 over a period of 65 years. The multiple harmonic analysis shown below is the first step toward another projection which it is hoped will be as good.

IN previous issues I have told you of the results of multiple harmonic analyses of railroad stock prices, 1831-1950 and industrial stock prices, 1871-1950. These accounts appeared in the April 1952 and May 1952 issues respectively. In this issue we present the results of a multiple harmonic analysis of pig iron prices, 1784-1951, made at the same time.

All these analyses were made possible by money supplied by Stockham Valves & Fittings of Birmingham, Alabama, a machine supplied by the Mico Instrument Company of Cambridge, Massachusetts, and the zeal of Alexander Malinowski of the Foundation staff, who worked 114 hours a week for three weeks in order to achieve the results in the time available.

It seems needless at this point to describe again the techniques and significance of multiple harmonic analysis except perhaps to state that peaks on the periodogram indicate (on the lower scale) the length of rhythmic repetitive cycles which *may* be present in the series of figures being analyzed.

If you want further information about multiple harmonic analysis and periodograms refer to our May 1952 report.

We have indicated for you by means of arrows the positions at which the periodogram curve seems to crest. These lengths, and the relative strength of the cycle indicated by each, are shown in the following table.

TABLE 1

CYCLE LENGTH AND AMPLITUDE SUGGESTED BY PERIODOGRAMS.

LENGTH IN YEARS	AMPLITUDE	LENGTH IN	AMPLITUDE	LENGTH IN YEARS	AMPLITUDE
4.1	1.8	5.8	3.1	7.9	4.8
4.2	1.6	5.9	3.1	9.1	11.0
4.3	1.8	6.1	3.9	9.2	11.1
4.5	2.8	6.3	1.9	10.4	5.8
4.6	1.5	6.4	2.3	11.1	2.2
4.7	2.0	6.5	2.2	12.2	2.5
4.8	2.0	6.6	2.1	13.8	9.6
5.0	2.7	6.8	4.6	15.2	8.5
5.1	1.2	6.9	4.2	17.8	16.4
5.3	4.4	7.2	2.7	27.8	11.8
5.4	4.3	7.4	2.8	52.2	21.4
5.5	3.4	7.6	2.8	151.8	41.2

These lengths must be thought of merely as *hints* which must be explored by more intensive methods of research. They have, however, the virtue that they give a continuous spectrum of possibilities, and that the machine had no preconceived notion of cycles which "ought" to be present in this series of figures.

When these various hints have been explored it may be possible to duplicate the feat of Samuel Benner. In 1874, using merely a knowledge of cycles, Benner made a forecast of pig iron prices which is one of the outstanding achievements of cycle research. If you had bought and sold, and sold short and covered, except during World War I, on the basis of this forecast you would have made 288.5% as against losses of 9.3% for the period 1874 to World War II. These figures work out to a gain-loss ratio of 31 to 1. If you had used the formula right through World War I, your gains would have been 425.8%, making a gain-loss ratio of 44.8 to 1. Benner's forecast was discussed in detail in our reports for February and March 1952.

With the aid of the multiple harmonic analysis presented herewith, it may be possible to duplicate Benner's success.

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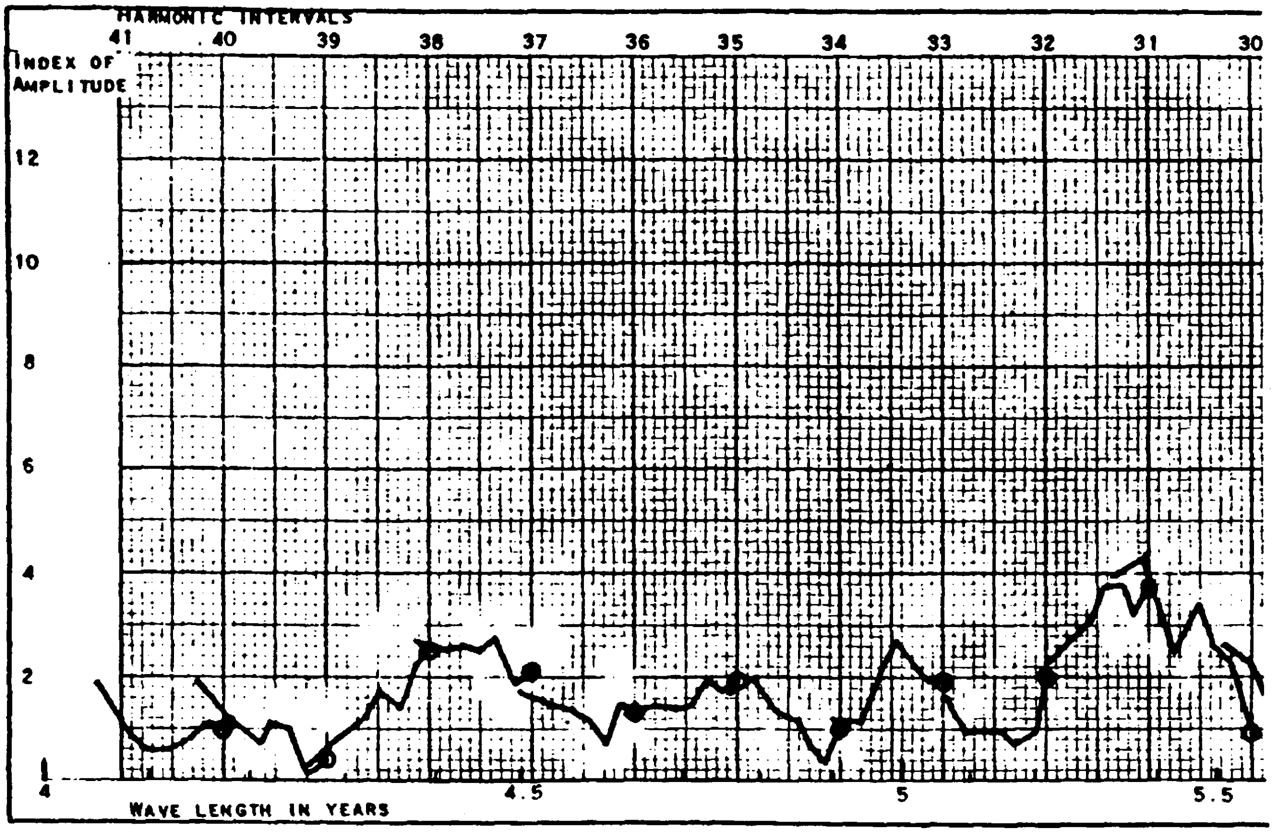
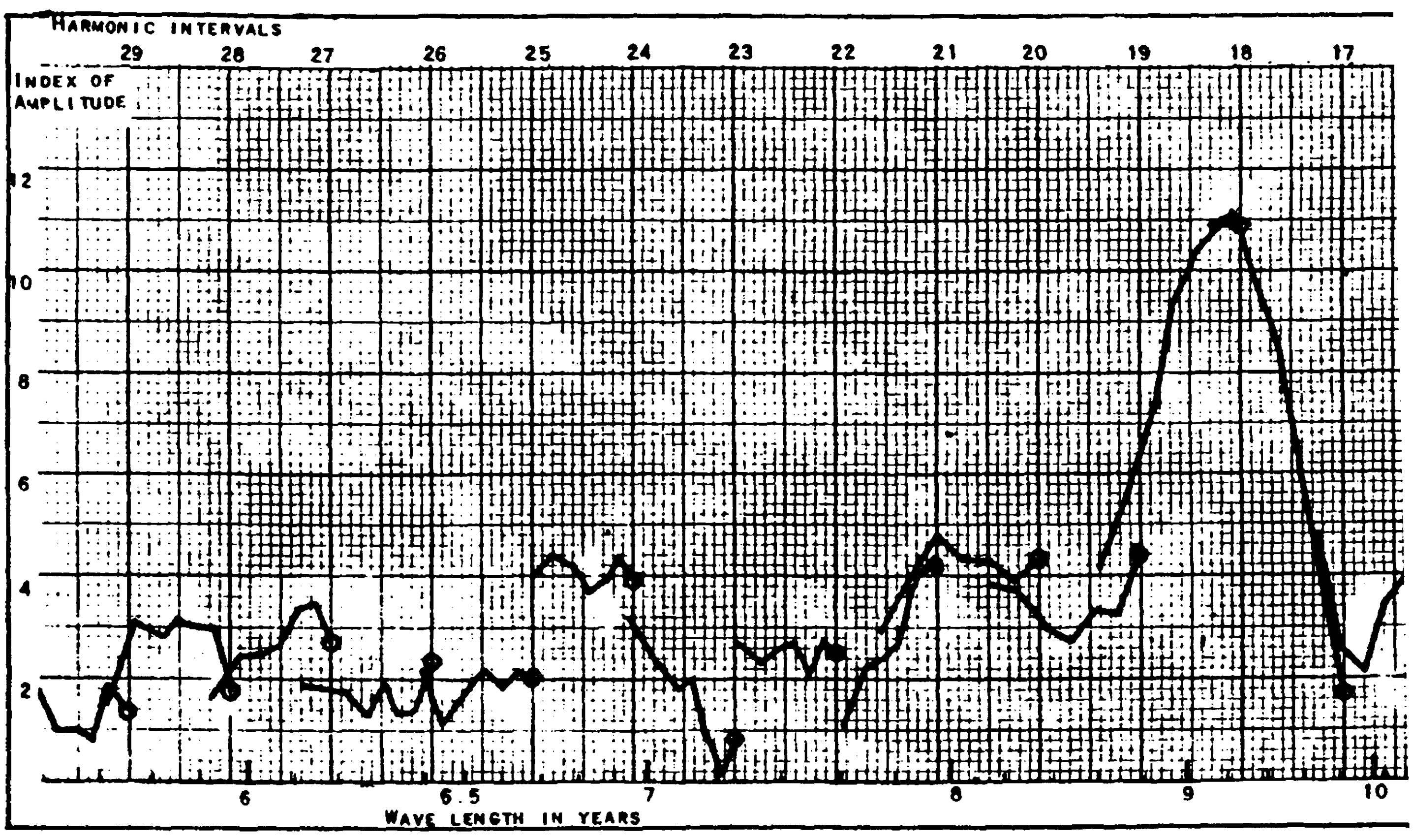
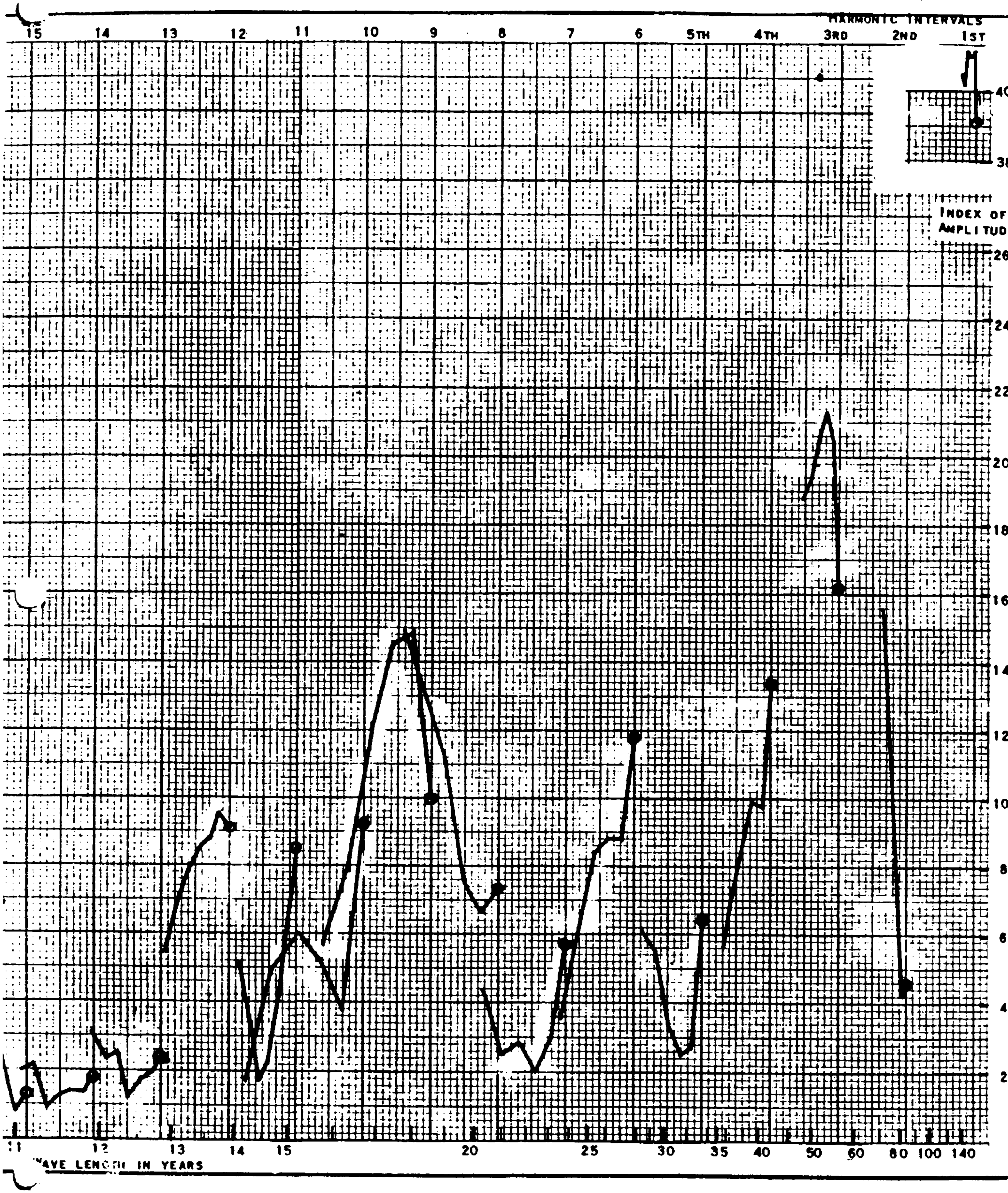

HARMONIC INTERVALS
41 40 39 38 37 36 35 34 33 32 31 30
INDEX OF AMPLITUDE
12
10
8
6
4
2
4 4.5 5 5.5
WAVE LENGTH IN YEARS


HARMONIC INTERVALS
29 28 27 26 25 24 23 22 21 20 19 18 17
INDEX OF AMPLITUDE
12
10
8
6
4
2
6 6.5 7 8 9 10
WAVE LENGTH IN YEARS


HARMONIC INTERVALS
15
14
13
12
11
10
9
8
7
6
5TH
4TH
3RD
2ND
1ST
INDEX OF AMPLITUDE
40
38
26
24
22
20
18
16
14
12
10
8
6
4
2
11
12
13
14
15
20
25
30
35
40
50
60
80
100
140
WAVE LENGTH IN YEARS

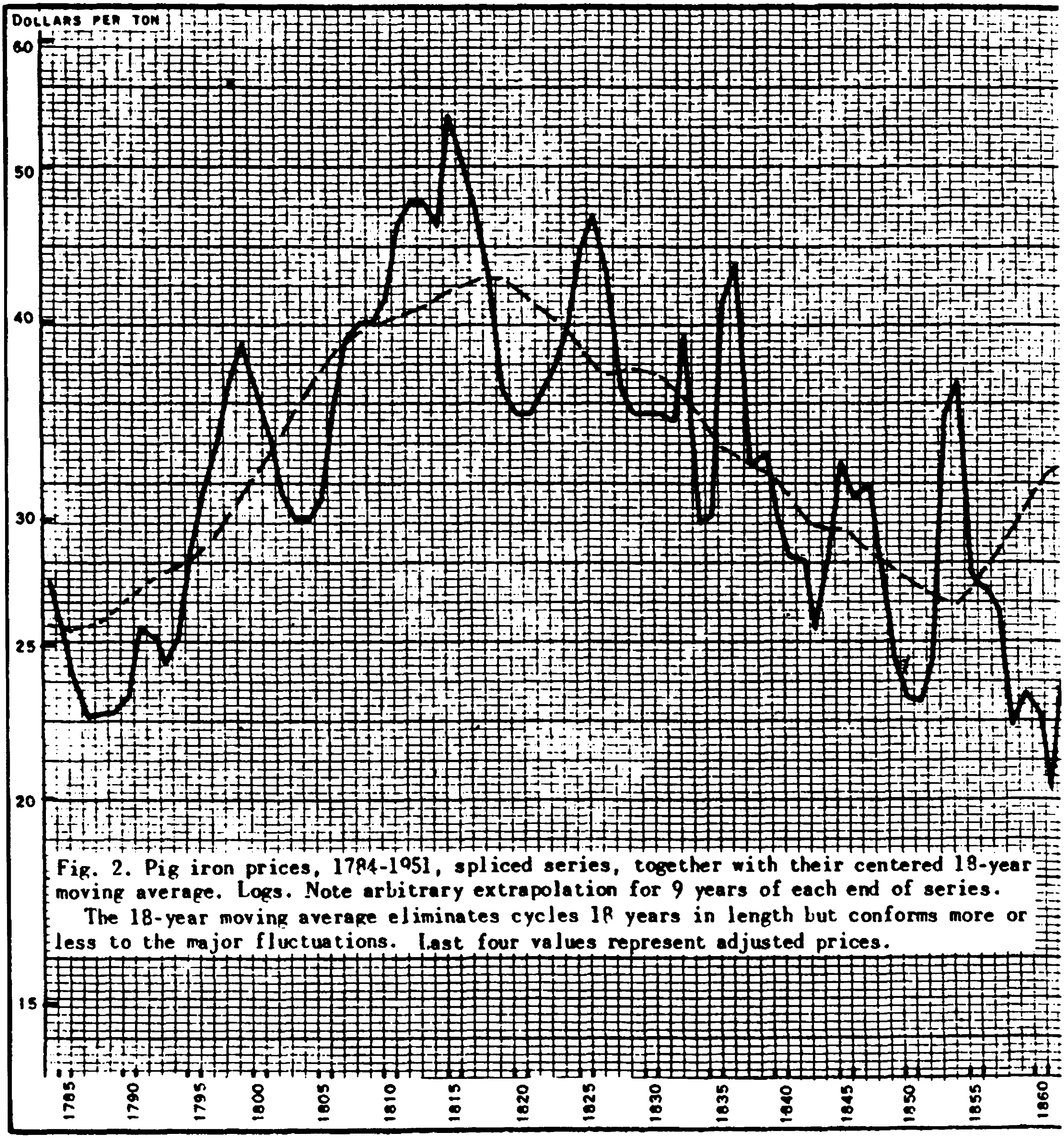
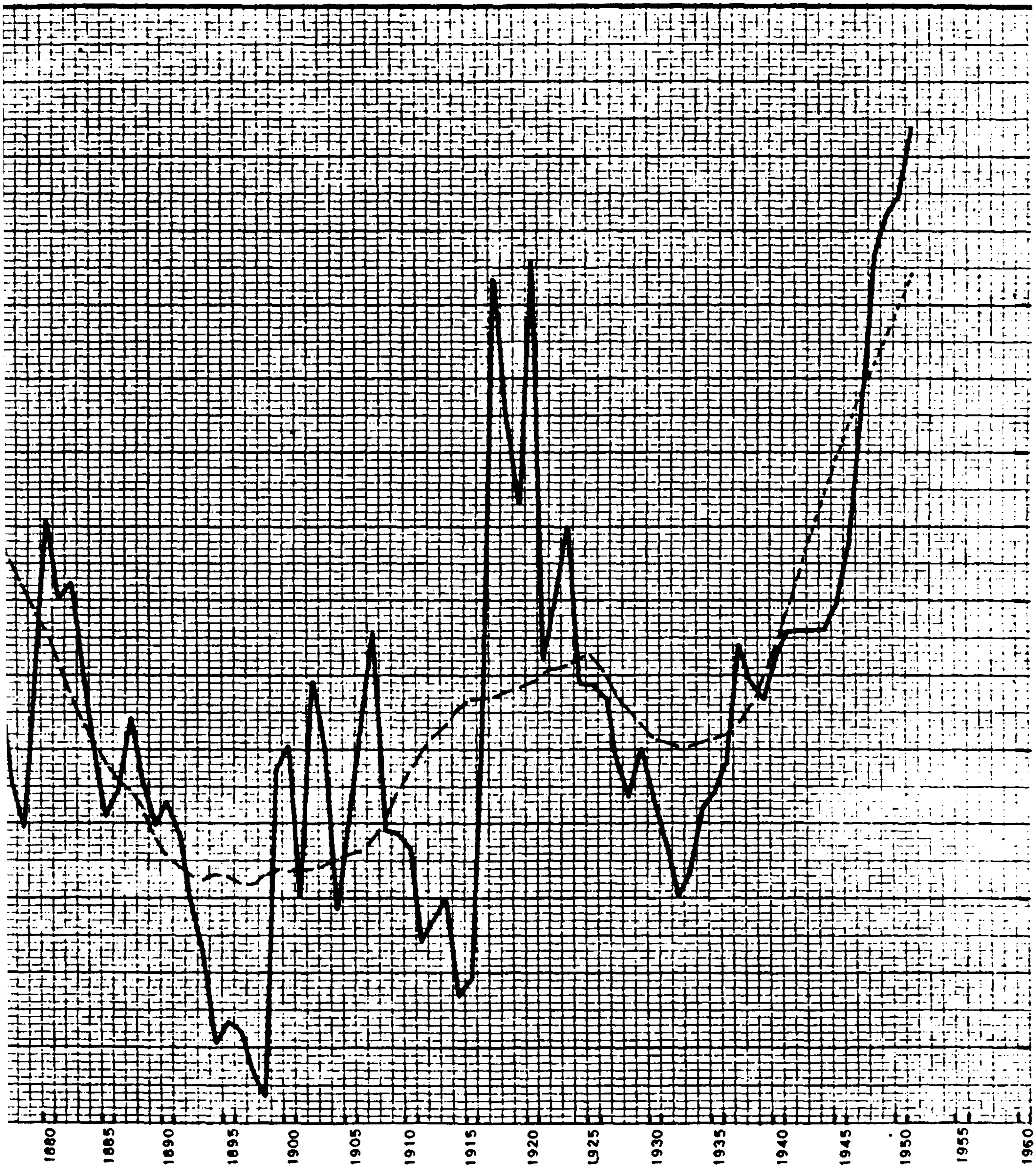


Fig. 2. Pig iron prices, 1784-1951, spliced series, together with their centered 18-year moving average. Logs. Note arbitrary extrapolation for 9 years of each end of series.

The 18-year moving average eliminates cycles 18 years in length but conforms more or less to the major fluctuations. Last four values represent adjusted prices.


1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960

LESSON XVII

MOVING PERCENTAGES, MOVING RATIOS, AND MOVING DIFFERENCES

When the various methods of getting hints of cycles as explained in Lesson VI prove unavailing, or when you want to separate cycles so that each can be seen more clearly, undistorted by the other, you must resort to filtering.

One of the best methods of filtering is the method of moving percentages (and its variants, moving ratios and moving differences).

Moving percentages are the simplest things in the world. Every day you read things like, "Business last month was 10% above the same month a year ago." A month later you will read a similar comparison. A series of such percentages constitutes what is known as a moving percentage.

The interval of time does not have to be a year. It can be anything, but for any given moving percentage it remains constant.

The term moving percentage, like the term moving average, is always qualified by a time interval. Thus we speak of a 12-month moving percentage, an 18-month moving percentage, etc.

The computation of a moving percentage is quite simple. Consider the table at the top of the following page in Column B of which I have computed 12-month moving percentages of variety store sales, in millions of dollars, by months a year and a half.

There are no 12-month moving percentages recorded for 1953 because, in the table, there are no data 12 months previous with which to compare them.

The first of the 12-month moving percentages, the one for month No. 637, is 94.6% because the sales for that month are 94.6% of the value for month No. 625, 12 months earlier.

In column C I have computed 13-month moving percentages in the same way.

To compute moving ratios you merely move the decimal point two positions to the left.

To compute moving differences you merely subtract instead of divide.

In dealing with logs you merely subtract (to accomplish division) and add 2 to your answer so as to avoid negative logs and so as to keep your answer in logs of percentages instead of ratios. See Columns D & E.

If you will take the trouble to look up the anti-logs of Column E you will see that they are the percentages recorded in Column B, except for errors of rounding.

I can't think of anything more to tell you about the mechanics of moving

TABLE TO ILLUSTRATE THE COMPUTATION OF
MOVING PERCENTAGES AND MOVING DIFFERENCES

Month Number	Month Name	A Data	B 12-month moving percentages	C 13-month moving percentages	D Logs of Data	E 12-month moving Differences of Logs of Data (+2.000)
625	Jan.'53	186	--	--	2.2695	--
626	Feb.	193	--	--	2.2856	--
627	Mar.	232	--	--	2.3655	--
628	Apr.	245	--	--	2.3892	--
629	May	235	--	--	2.3711	--
630	June	241	--	--	2.3820	--
631	July	233	--	--	2.3674	--
632	Aug.	242	--	--	2.3838	--
633	Sept.	240	--	--	2.3802	--
634	Oct.	264	--	--	2.4216	--
635	Nov.	257	--	--	2.4099	--
636	Dec.	526	--	--	2.7210	--
637	Jan.'54	176	94.6	--	2.2455	1.9760
638	Feb.	188	97.4	101.1	2.2742	1.9886
639	Mar.	198	85.3	102.6	2.2967	1.9312
640	Apr.	249	101.6	107.3	2.3962	2.0070
641	May	222	94.5	90.6	2.3464	1.9753
642	June	231	95.9	98.3	2.3636	1.9816

percentages, moving ratios, and moving differences.

From the standpoint of cycle analysis moving percentages are important because they enable you to minimize certain elements of a time series and to magnify others. By use of them you can often discover cycles which would otherwise escape you, and discover the characteristics of cycles which might otherwise be vague or uncertain.

I could catalog all the qualities of moving percentages for you, and I would be glad to do it if I did not feel that you would gain a greater mastery of this technique if you make your own discoveries. Therefore, instead of giving you all the answers, I am going to give you a carefully prepared series of problems so that you can get the answers for yourself.

Lesson XVII - Problems

Problem 1. Here is a trend which increases at the constant rate of 2% a year. Compute 3-year and 10-year moving percentages. Also compute 3-year and 10-year moving differences. Space is allowed for the result.

Year	Trend	Moving Percentages 3-Year	Moving Percentages 10-Year	Moving Differences 3-Year	Moving Differences 10-Year
1	100.00				
2	102.00				
3	104.04				
4	106.12	106.12			
5	108.24	106.12			
6	110.41				
7	112.62				
8	114.85				
9	117.15				
10	119.49				
11	121.88		121.55		
12	124.32		121.55		
13	126.80				
14	129.34	106.12			
15	131.92				
16	134.56				
17	137.25				
18	140.00				
19	142.80				
20	145.66		121.9		

Problem 2. Following is a trend that increases at the constant rate of 5% a year. Compute 3-year and 10-year moving percentages and 3-year and 10-year moving differences.

Year	Trend	Moving Percentages 3-Year	Moving Percentages 10-Year	Moving Differences 3-Year	Moving Differences 10-Year
1	100.00				
2	105.00				
3	110.25				
4	115.76	115.76			
5	121.55				
6	127.63				
7	134.01				
8	140.71				
9	147.74				
10	155.13				
11	162.89		162.89		
12	171.03				
13	179.58				
14	188.56				
15	197.99				
16	207.89				
17	218.30				
18	229.21				
19	240.67				
20	252.71				

Problem 3. The following trend increases by the constant amount of 2.5 a year. Compute 3-year and 10-year moving percentages and moving differences.

Year	Trend	Moving Percentages 3-Year	Moving Percentages 10-Year	Moving Differences 3-Year	Moving Differences 10-Year
1	100.0				
2	102.5				
3	105.0				
4	107.5	107.5			
5	110.0				
6	112.5				
7	115.0				
8	117.5				
9	120.0				
10	122.5				
11	125.0		125.0		
12	127.5				
13	130.0				
14	132.5				
15	135.0				
16	137.5				
17	140.0				
18	142.5				
19	145.0				
20	147.5				

Problem 4. The following trend increases by the constant amount of 4.0 a year. Compute 3-year and 10-year moving percentages and moving differences.

Year	Trend	Moving Percentages 3-Year	Moving Percentages 10-Year	Moving Differences 3-Year	Moving Differences 10-Year
1	60				
2	64				
3	68				
4	72	120.			
5	76				
6	80				
7	84				
8	88				
9	92				
10	96				
11	100		166.7		
12	104				
13	108				
14	112				
15	116				
16	120				
17	124				
18	128				
19	132				
20	136				

Note: The charts for Problems 1, 2, 3 and 4 are on Pages 6 and 7 of Supplement 2 of Lesson XVII.

Problem 5. There are two fundamental rules about the effect of moving percentages upon cycles which Problem 5 and Problem 6 should reveal to you.

Compute 20-month moving percentages (or differences) of the following cycles.

Month	20-Month Cycle	20-Month Moving %	10-Month Cycle	20-Month Moving %	5-Month Cycle	20-Month Moving %
1	1		1		1	
2	2		2		2	
3	3		3		3	
4	4		4		3	
5	5		5		2	
6	6		6		1	
7	7		5		2	
8	8		4		3	
9	9		3		3	
10	10		2		2	
11	11		1		1	
12	10		2		2	
13	9		3		3	
14	8		4		3	
15	7		5		2	
16	6		6		1	
17	5		5		2	
18	4		4		3	
19	3		3		3	
20	2		2		2	
21	1	1.0	1	1.0	1	1.0
22	2	1.0	2	1.0	2	1.0
23	3	1.0	3	1.0	3	1.0
24	4		4		3	
25	5		5		2	
26	6		6		1	
27	7		5		2	
28	8		4		3	
29	9		3		3	
30	10	1.0	2	1.0	2	1.0

The cycles are continued below:

Month	4-Month Cycle	20-Month Moving %	2-Month Cycle	20-Month Moving %
1	1		1	
2	2		2	
3	3		1	
4	2		2	
5	1		1	
6	2		2	
7	3		1	
8	2		2	
9	1		1	
10	2		2	

Problem 5 continued.

Month	4-Month Cycle	20-Month Moving %	2-Month Cycle	20-Month Moving %
11	3		1	
12	2		2	
13	1		1	
14	2		2	
15	3		1	
16	2		2	
17	1		1	
18	2		2	
19	3		1	
20	2		2	
21	1	1.0	1	1.0
22	2	1.0	2	1.0
23	3	1.0	1	1.0
24	2		2	
25	1		1	
26	2		2	
27	3		1	
28	2		2	
29	1		1	
30	2	1.0	2	1.0

From the relationship between the length of the moving percentage and the length of the waves, what conclusion do you reach.

Problem 6. For each of the given cycles compute a 30-month moving percentage (or difference).

÷ 1 into 2

Month	60-Mo. Cycle	30-Mo. Mvg. %	20-Mo. Cycle	30-Mo. Mvg. %	12-Mo. Cycle	30-Mo. Mvg. %	4-Mo. Cycle	30-Mo. Mvg. %
1	1		1		1		1	
2	2		2		2		2	
3	3		3		3		3	
4	4		4		4		2	
5	5		5		5		1	
6	6		6		6		2	
7	7		7		7		3	
8	8		8		6		2	
9	9		9		5		1	
10	10		10		4		2	
11	11		11		3		3	

Problem 6 continued.

Month	60-Mo. Cycle	30-Mo. Avg. %	20-Mo. Cycle	30-Mo. Avg. %	12-Mo. Cycle	30-Mo. Avg. %	4-Mo. Cycle	30-Mo. Avg. %
12	12		10		2		2	
13	13		9		1		1	
14	14		8		2		2	
15	15		7		3		3	
16	16		6		4		2	
17	17		5		5		1	
18	18		4		6		2	
19	19		3		7		3	
20	20		2		6		2	
21	21		1		5		1	
22	22		2		4		2	
23	23		3		3		3	
24	24		4		2		2	
25	25		5		1		1	
26	26		6		2		2	
27	27		7		3		3	
28	28		8		4		2	
29	29		9		5		1	
30	30		10		6		2	
31	31	31.0	11	11	7	7	3	3.
32	30	15.0	10	5	6	3	2	1.
33	29	9.7	9	3	5	1.6	1	.3
34	28	7.0	8	2	4	1.	2	1.
35	27	5.4	7	1.4	3	.6	3	3.
36	26	4.3	6	1.	2	.3	2	1.
37	25	3.6	5	.7	1	.1	1	.3
38	24	3.0	4	.5	2	.3	2	1.
39	23	2.7	3	.3	3	.6	3	3.
40	22	2.2	2	.2	4	1.	2	1.
41	21	1.9	1	.1	5	1.6	1	.3
42	20	1.8	2	.2	6	3.	2	1.
43	19	1.5	3	.3	7	7	3	3.
44	18	1.3	4	.5	6	3.	2	1.
45	17	1.1	5	.7	5	1.6	1	.3
46	16	1.0	6	1.	4	1.	2	1.
47	15	.9	7	1.4	3	.6	3	3.
48	14	.8	8	2.	2	.3	2	1.
49	13	.7	9	3	1	.1	1	.3
50	12	.6	10	5.	2	.3	2	1.
51	11	.5	11	11.	3	.6	3	3.
52	10	.45	10	5.	4	1.	2	1.
53	9	.4	9	3.	5	1.6	1	.3
54	8	.3	8	2.	6	3.	2	1.
55	7	.28	7	1.4	7	7.	3	3.
56	6	.23	6	1.0	6	3.	2	1.
57	5	.19	5	.7	5	1.6	1	.3
58	4	.14	4	.5	4	1.	2	1.
59	3	.10	3	.3	3	.6	3	3.
60	2	.07	2	.2	2	.3	2	1.

Here you will observe that the same effect applies to a cycle twice the length of the moving percentage as to a cycle 2/3, 2/5, and 2/15 of the length of the moving percentage. The cycle length used in this problem was selected because each is a whole number. The same effect would pertain to a cycle 2/7 of the length of the moving percentage (or 8 - 4/7 months), or 2/9 of the length (6-2/3 months)

<u>Problem 7.</u> If you are looking for a 9 month cycle what moving percentage would you use? Name 3 possibilities.

<u>Problem 8.</u> Suppose that in another case you are dealing with figures in which you suspect a 24-month wave which you want to confirm, There is also present a strong seasonal. What moving percentage would you use?

<u>Problem 9.</u> If you are checking different series for the presence of a 40-month cycle, what moving percentage would you use?

<u>Problem 10.</u> If a 6-year cycle is present in the S & P C A data, there is a moving percentage which can be computed to eliminate the 9-year cycle and at the same time emphasize any 6. What is it?

LESSON XVII

Supplement 1

MOVING DIFFERENCES

I am reprinting for you a three page article which appeared in the September 1952 issue of Cycles (pages 233-35) called "The Three-Year Cycle in Industrial Common Stock Prices." I am reprinting this article because it illustrates the use of moving differences.

A 27-quarter (81-month) moving difference was used because a moving difference of this length would, for all practical purposes, eliminate 39- to 42-month cycles and 25- to 29-month cycles, and would leave 36-month cycles relatively unaffected.

Problem 1. Set up a regular symmetrical 12-quarter (36-month) cycle, repeated several times. Compute the 27-quarter moving difference. Plot the regular cycle. Under it plot the moving difference.

Problem 2. Prepare an amplitude chart to show for cycles of various lengths the percentage of the original amplitude remaining when the cycles are subjected to a 27-quarter moving difference.

In making your amplitude chart represent the various cycle lengths (from zero up) from left to right along the X axis. Represent the amplitudes remaining (which will range from zero to 200%) along the Y axis.

LESSON XVII

Supplement 2

MOVING PERCENTAGES, MOVING RATIOS, AND MOVING DIFFERENCES—CONTINUED

As I said in the lesson, I would prefer to have you learn about moving percentages, moving ratios, and moving differences, by working out the examples and making your own generalizations. However, some students would prefer to have me do the work; others will want a statement of principles for future reference. Hence this supplement. I ask you not to read it until after you have worked Lesson XVII problems.

The Difference Between Moving Percentages, Moving Ratios, and Moving Differences

(a) Moving Percentages vs Moving Ratios.

Of course there is no difference, numerically, between a moving percentage and a moving ratio. A curve of one looks the same as a curve of the other. The only difference between the two is the location of the decimal point. For my own part I prefer moving percentages. They are more meaningful to me. That is, the meaning of 108.75% is easier for me to grasp than the meaning of the ratio 1.0875, even though the one means the same as the other. This being so, and also for the sake of simplicity, I shall drop the term moving ratio from here on and use the term moving percentage alone.

(b) Moving Percentages vs Moving Differences.

Ordinarily you will do better to use moving percentages instead of moving differences, but there are four exceptions, as follows:

(I) You must use moving differences if your series contains negative values, because a number divided by a negative number gives you a number infinitely large.

(II) You must use moving differences if your series contains zero values, because a number divided by zero gives you a number infinitely large.

(III) You will ordinarily use moving differences if your series contains numbers that are very small in relation to the other numbers. If you do not, your derived series will be greatly distorted. For example, note the two month moving percentage of the following values:

	Month						
	1	2	3	4	5	6	7
Values:	10	12	10	9	.1	9	8
Two month moving percentages:			100	75	1.	100	8,000

The value of 1. for the 5th month is bad enough, but note that $\frac{8.}{.1}$, expressed as a percent, is 8,000!

(IV) And finally, in dealing with logs you <u>must</u> use moving differences of the logs if you wish moving percentages of your original values. If you use moving percentages of the logs you will get a moving root of the original value. You will not ordinarily want this.

<u>The Effect of Moving Percentages and Moving Differences</u>

(a) <u>The Effect Upon Trend</u>.

Moving percentages and moving differences minimize or eliminate trend. Let me illustrate:

<u>Trends which increase at constant rates</u>

Let us first consider a trend which increases at a constant <u>rate</u>. See Fig. 1, in which Curve A increases at 2% per interval, Curve B increases at 5% per interval.

The moving <u>percentage</u> will be a horizontal line above the 100% line. See Fig. 1, Curves C, D, E, and F.

The longer the interval of the moving <u>percentage</u>, the higher above the 100% line will be the moving percentage line. Compare Fig. 1-E with Fig. 1-C; Fig. 1-F with Fig. 1-D.

The moving <u>difference</u> will be nearly horizontal, increasing slightly. See Fig. 1, Curves G, H, I, and J.

The longer the interval of the moving <u>difference</u>, the higher above the 100% line will be the moving difference line. Compare Fig. 1-I with Fig. 1-G; Fig. 1-J with Fig. 1-H.

The greater the rate of growth of the trend (Fig. 1-B versus Fig. 1-A), the higher above the 100% line will be the moving <u>percent</u> lines <u>and</u> the moving <u>difference</u> lines. Compare Fig. 1, Curves D, F, H, and J with their counterparts, Curves C, E, G, and I.

<u>Trends which increase by constant amounts</u>.

Let us now consider a trend which increases at a constant <u>amount</u>. See Fig. 2 in which Curve A increases by 2 points per interval, Curve B increases by 5 points per interval.

The moving <u>percentage</u> will slope down gently toward the right. See Fig. 2, Curves C, D, E, and F.

The longer the interval of the moving <u>percentage</u> the higher above the 100% line will be the moving percentage line. Compare Fig. 2-E with Fig. 2-C; Fig. 2-F with Fig. 2-D.

The moving difference will be horizontal. See Fig. 2, Curves G, H, I and J.

The longer the interval of the moving difference the higher above the 100% line will be the moving difference line. Compare Fig. 2-1 with Fig. 2-G; Fig. 2-J with Fig. 2-H.

The steeper the slope of the trend (Fig. 2-B versus Fig. 2-A) the higher above the 100% line will be the moving percent lines and the moving difference lines. Compare Fig. 2, Curves D, F, H, and J with their counterparts, Curves C, E, G, and I.

* * * * *

If the early values of the trend which increases at a constant amount are very small numbers, the moving percentage will be more distorted from the horizontal than it would be if they were not.

If the trend is increasing at an increasing rate, moving percentages of it will slope upward to the right. Moving differences of such a curve will slope upward to the right compared to what they would if the rate were constant.

If the trend is increasing at an increasing amount, moving differences will slope upward to the right. Moving percentages will increase more than they would if the amount were constant.

The reverse of all this is true for trends which are increasing at a decreasing rate or by decreasing amounts, and for trends which decrease.

However, in most instances, for all practical purposes, moving percentages or moving differences eliminate trend. This is so because the slope of the moving percentages or the moving differences is usually fairly flat in comparison with the slope of the trend from which it was computed.

(b) The Effect Upon Randoms.

Randoms may be thought of as below trend or above trend.

If both randoms are above trend or if both randoms are below trend, a random separated from a previous random by an interval equal to the interval of the moving percentage or the moving difference is lessened by the moving difference calculation, relative to what it was orginally. If one of the two randoms is above trend and the other random is below trend, the second random is increased. As some randoms are lessened and some are increased by moving percentage and moving difference computations, we may say that, for all practical purposes, there is no change in the value of randoms as a whole after such calculations.

There is however an important change in the arrangement of the randoms which I shall now call to your attention.

Consider a series all zeros except for just one random, a low value at position X. A moving percentage or moving difference of such a curve will have a low value at X and a high value at Y, just the interval of the moving difference later. The manipulation has created half of a wave.

Now if there is another low random about twice the interval of the moving percentage or moving difference after the first one, you will have two low values and two high values, each low and each high separated from the other by roughly twice the interval of manipulation. But two lows and two highs gives you the beginning of a rhythmic cycle.

If, later in the series, you have the same thing happening again, more or less in step with the first time, you will have an even stronger indication of a rhythmic cycle. The cycle is there, only, as the manipulation itself has created it, it has no significance.

Of course if your original figures were smoothed by a moving average, the excessively low random we have imagined would have become a low area and the artifacts created a little later would appear as a high area. The manipulation would give you something that would look even more like one complete wave.

In consequence of this fact it is better to apply moving percentages and moving differences to unsmoothed data. The artifacts can be picked out better if you do. They can then be discarded or ignored.

A method to nullify these artifacts will be given later.

(c) The Effect Upon Cycles

Moving percentages and moving differences completely remove any cycle of exactly their own interval and any exact harmonic (unit fraction) of that interval Thus a 60-month moving percentage or moving difference will completely eliminate any 60-month, 30-month, 20-month, 15-month, 12-month, 10-month, etc. cycles down to 2-month cycles (2 months is 1/30th of 60 months) all at one fell swoop.

A moving percentage or a moving difference will almost remove all cycles which are almost its own length or almost unit fractions of its own length.

All this is so regardless of the shape or symmetry of the cycle, as long as it is truly repetitious.

These facts make it possible to use moving percentages or moving differences to simplify a complicated series.

A moving percentage or a moving difference of a symmetrical cycle (regardless of whether it is zigzag or sine shape) will double the amplitude of any cycle twice its length or 2/3, 2/5, 2/9, etc. of its length.

Thus a 60-month moving percentage or moving difference will not only double the amplitude of any symmetrical 120-month cycle which may be present but will double the amplitude of any 40-month, 24-month, 17 1/7-month, 13 1/3-month, 10 9/11-month cycle, etc.

If the cycle involved is almost symmetrical the amplitude will be almost doubled.

If the cycle involved is almost of the indicated length its amplitude will be almost doubled.

Note that the effects are highly selective. For example, in the example given the 60-month moving percentage or moving difference will, at one and the same time, double a 17 1/7-month cycle, eliminate a 15-month cycle, double a 13 1/3-month cycle, eliminate a 12-month cycle, double a 10 11/12-month cycle, eliminate a 10-month cycle, etc.

The Use of Moving Percentages and Moving Differences in Cycle Analysis.

It should be obvious from the characteristics of moving percentages and moving differences as outlined above that these manipulations are useful for minimizing trend, for separating cycles, and for revealing hidden cycles.

For example, if you have particularly complex series of figures which do not yield their cycle content by any of the simple expedients outlined in Lesson VI, you can subject your figures to moving percentage or moving difference manipulation. Plot the result, and see what you see.

Suppose you have about 100 yearly figures. A 1-year moving percentage would double a 2-year cycle and minimize trend and cycles larger than 4 years, but of course 2-year cycles are pretty short to look for in annual data.

A 2-year moving percentage would double cycles 4 years long, eliminate 2-year cycles and minimize trend and cycles over 8 years long.

And so on.

If you compute 1-year, 2-year, 3-year, 4-year, 5-year, 6-year, 7-year, 9-year, and 10-year moving percentages of your figures I'll bet a cookie some cycles will show up in addition to the artifacts introduced by the randoms. (The cycle you find may not be significant, but that's another story.)

Getting rid of Artifacts.

The artifacts don't need to bother you. For example, a 2-year moving percentage will create artifacts which might occasionally look something like a 4-year cycle. The question is, is the 4-year cycle real, or is it a reflaction of the randoms? You can often tell this by reference to the original series, You can use moving percentages to tell you too: Compute a 6-year moving percentage (1 1/2 four year waves). Such a moving percentage will reveal a true 4-year cycle just as well as a 2-year moving percentage. Such a moving percentage will have no 4-year artifacts (its artifacts, if any, will be 12 years, crest to crest). If the 4-year cycle persists in the 6-year moving percentage it cannot be the result of the technique.

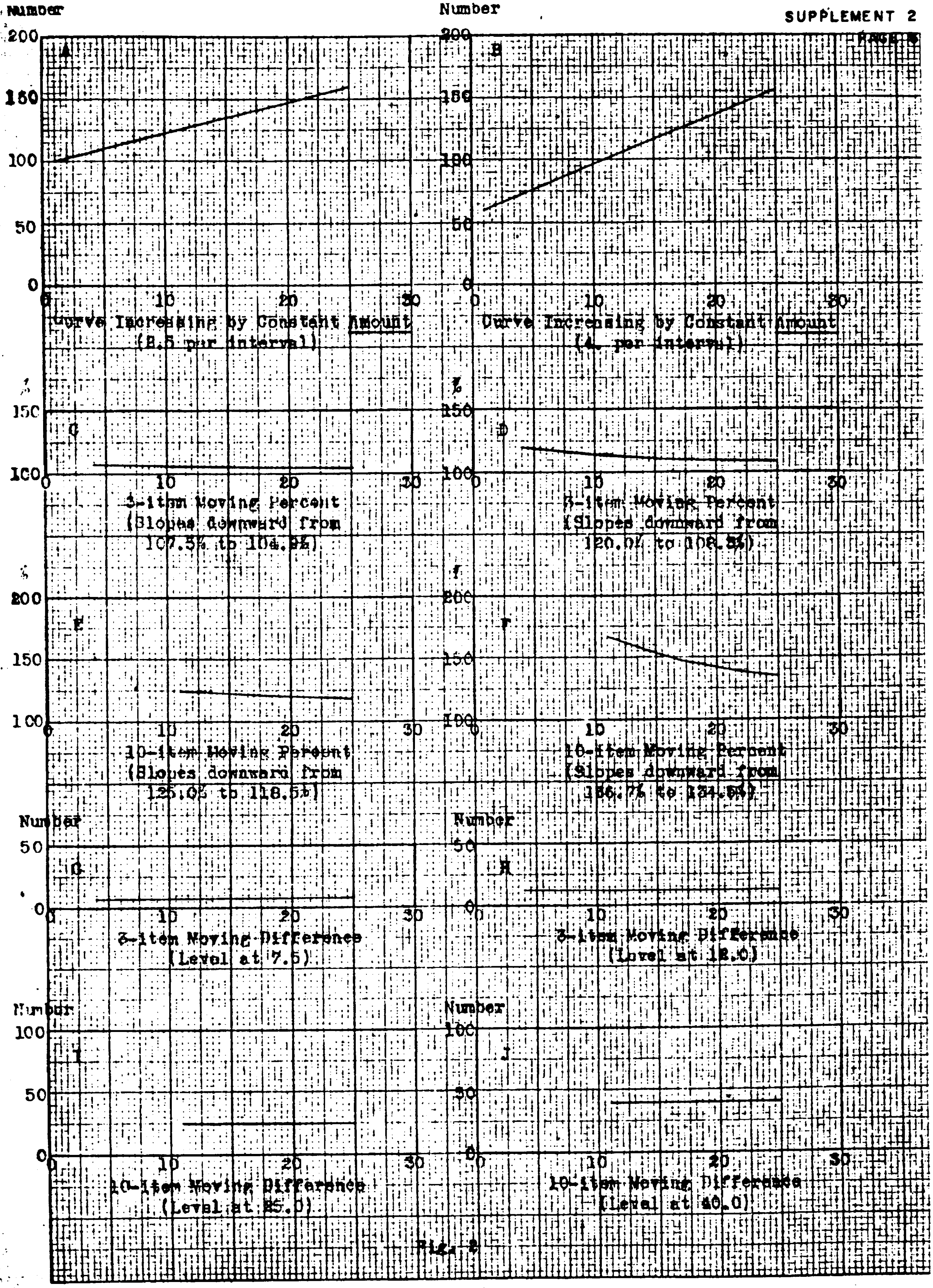

Number
200
150
100
50
0
A
0
10
20
30
Curve Increasing by Constant Amount
(2.5 per interval)
Number
B
Curve Increasing by Constant Amount
(4. per interval)
%
C
3-item Moving Percent
(Slopes downward from
107.5% to 104.9%)
D
3-item Moving Percent
(Slopes downward from
120.0% to 108.3%)
E
10-item Moving Percent
(Slopes downward from
125.0% to 118.5%)
F
10-item Moving Percent
(Slopes downward from
136.7% to 134.6%)
G
3-item Moving Difference
(Level at 7.5)
H
3-item Moving Difference
(Level at 12.0)
I
10-item Moving Difference
(Level at 25.0)
J
10-item Moving Difference
(Level at 40.0)

Fig. 2

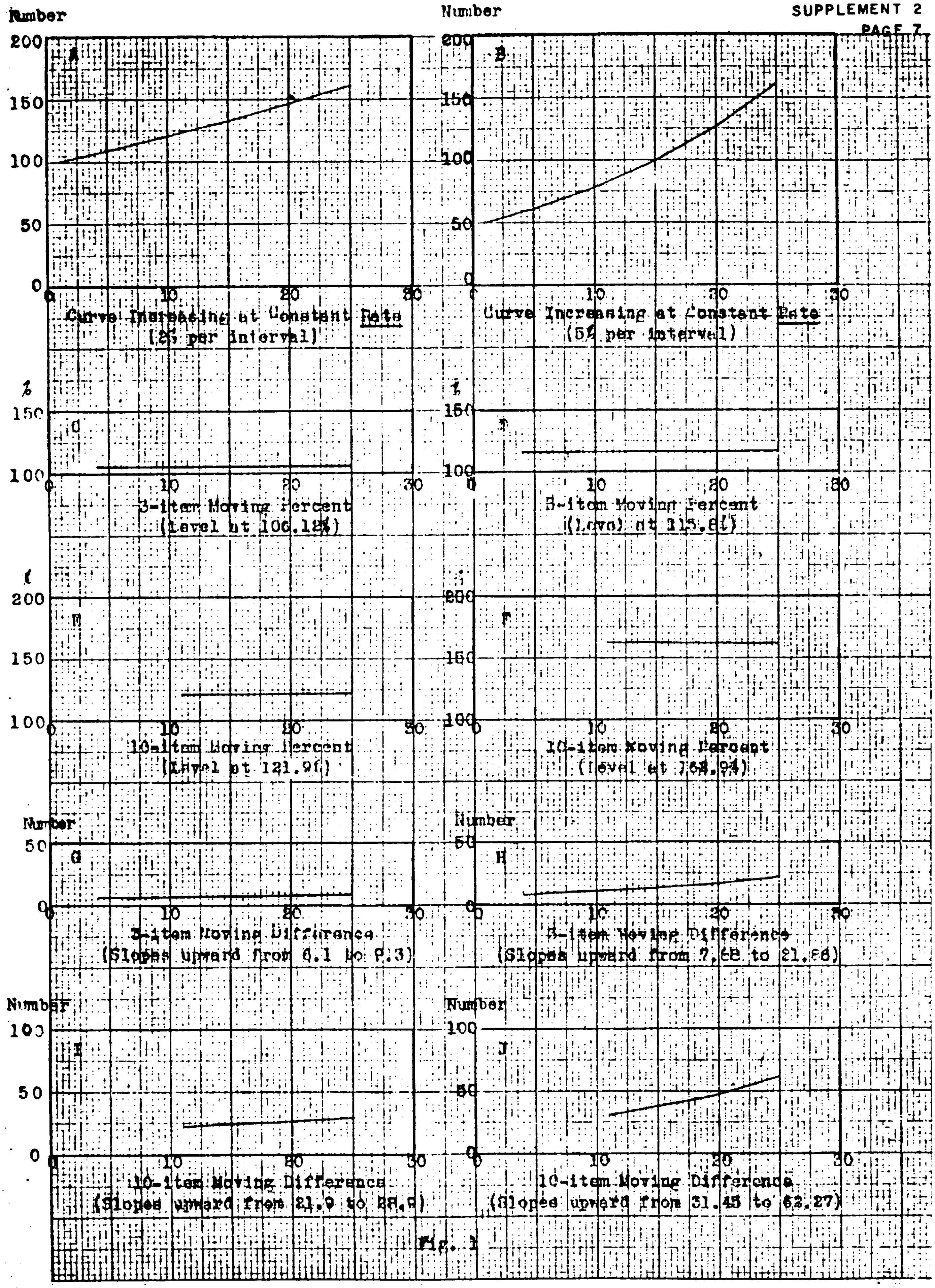

Number
A
Curve Increasing at Constant Rate
(2% per interval)
Number
B
Curve Increasing at Constant Rate
(5% per interval)
%
C
3-item Moving Percent
(Level at 106.12%)
%
D
3-item Moving Percent
(Level at 115.8%)
%
E
10-item Moving Percent
(Level at 121.9%)
F
10-item Moving Percent
(Level at 162.9%)
Number
G
3-item Moving Difference
(Slopes upward from 6.1 to 9.3)
Number
H
3-item Moving Difference
(Slopes upward from 7.88 to 21.86)
Number
I
10-item Moving Difference
(Slopes upward from 21.9 to 28.9)
Number
J
10-item Moving Difference
(Slopes upward from 31.45 to 62.27)

Fig. 1

The Straight Line Trend

When To Use One - How To Compute One

In Lesson III we touched upon the matter of secular (long term) trend.

I told you that for cycle analysis it is generally best to use a moving average trend, with the moving average about as long as the cycle in which you are interested. This is true. A moving average is generally the best trend for cycle analysis. But not always. Sometimes a mathematical curve is better. There are several mathematical trends you can use. Of these, the straight line is simplest. In this lesson I want to tell you (I) When to use a straight line trend and (II) how to fit it.

I. When to Use a Straight Line Trend.

Use a straight line trend only when your data, or the logs of your data, lie pretty much along a straight line.

For example, the curve in Chart 15-2 on page 326 of your text is suitable for a straight line trend. So are the logs of your stock market data (S. & P. C. A. 3).

The curve in Chart 15-7 on page 340 of your text is not suitable for a straight line trend.

However, if your data lie along a straight line (or if their logs do), a straight line trend of the data (or of their logs) is better for cycle analysis in the following three instances:

A. If your series is short relative to the length of the cycles in which you are interested. In this instance a straight line trend is better because deviations from a straight line trend contains an additional wave.

For example, suppose you have a series of figures 20 years long. Suppose further that you want to study the six year cycles as it may appear in these figures. If you compute a six or seven year moving average your trend will be only 14 years long. Deviations from trend will be only 14 years long. You have lost 6/20 or 30% of your data.

On the other hand a straight line trend will be 20 years long. Deviations from it will be 20 years long. This procedure enables you to use all your data.

Even in a series as long as your stock market averages (1854-1953), one hundred years, a 23 year moving average will cost you the use of a quarter of your data (eleven years at each end - 22 years). To study the 19-25 year cycle in these figures a straight line trend is highly desirable.

B. A second reason for using a straight line trend, if it is suitable (i.e. if your figures lie along a straight line) arises when your moving average is too irregular. The irregularity of your moving average may come from excessive randoms or some other cycle, or cycles, or both. In a series which is long relative to the cycle length in which you are interested these irregularities will usually wash out. However, when you have only a few repetitions of the cycle they often cause trouble.

C. Wherever possible use a straight line trend of the data or the logs of the data when making presentations to laymen. People who are uninformed often figure that perhaps the use of a moving average trend <u>creates</u> the cycle.

II. <u>How to Compute a Straight Line Trend</u>.

As explained in your text (pages 322, ff.) there are three methods of computing a straight line trend, (A) graphic method, (B) the method of semi averages, and (C) the method of least squares.

A. <u>Graphic method</u>. Use a ruler applied to a chart of the data or a chart of the logs of the data to fit a straight line by eye. Draw in the straight line trend with a sharp pencil.

To get arithmetic values for your trend select places near the ends of your line where your straight line cuts across a line of your grid. Count intervals from one point to the other, both horizontally and vertically. Find the vertical move per horizontal interval. Add or subtract this amount cumulatively to one of your two points.

For example, suppose your straight line trend cut 1903 at 17, cut 1952 at 48. Your trend has risen 31 points (48-17) in 49 years (1952-1903). It has therefore risen $\frac{31}{49}$ points per year or .632 points per year. This is your <u>increment</u>.

The value for 1904, one year after 1903, would be 17 + .632 or this would round to 17.6. The value for 1905, two years after 1903, would be 17 + .632 + .632. This would round to 18.3. Values of the straight line trend for other years, forward or back, would be computed similarly.

Always run the decimal of your increment out several places; otherwise there will be a cumulative error.

Leave your increment in your machine. Round merely the sum which you record. Do not add your increment to your rounded figure.

See your text, pages 323 and 324, for supplemental information.

B. <u>The method of semi averages</u>. This method is a little more exact, <u>when you have a long series of figures</u>. To use this method, you get the mean of each half of the series and proceed as above.

If your series has an odd number of terms, omit the center term (or count it twice.

If half of your series has an even number of terms your average value will lie between intervals. You must therefore add half an increment to start with.

For example, suppose your data are annual and run from 1907 to 1954 inclusive. This is 48 years of data, 24 in each half.

Suppose the average of the first 24 comes to 36; the average of the second 24 comes to 84. The rise is 48 (84-36) in 24 years (from the center of the first half, half way between 1918 and 1919, to the center of the second half, halfway between 1942 and 1943). A rise of 48 points in 24 years is an increment of 2 points a year. To get the value for 1919 you would add 1 point or 1/2 increment to your average of 36 to get 37. From here on all values to the right would go up by the full increment of 2. The values to the left would go down by 2. Thus the value for 1920 would be 39 (37 + 2). The value for 1918 would be 35 (37 - 2). And so on.

C. <u>The method of least squares</u>.

This method gives the best fit and is the standard method used by statisticians. It is fully described in your text.

This method is really very easy, but somehow all the texts seem to make it sound hard. "It don't tell so good."

Maybe if I put it in simple words you will see how easy it really is.

There are seven easy steps required to get the values of a straight line trend fitted by least squares.

I'll show you these steps by working a simple example. For our problem we will take imports of India rubber by years from 1913 through 1921.

1. List your data in a column headed A (as in Table A below). Add them. In the example the sum amounts to 244. Call this sum A.

2. In column B write down a consecutive series of Numbers, starting with 1. Your last figure will give you the number of terms in your series. In the example given it is 9. (Call this number N.) Add this column. In the example this sum amounts to 45. Let us call this sum B.

3. Multiply each item of your data (Col. A) by the corresponding number in Col. B. Post the products in Col. C.

Add these products (the values in Col. C). In the example this sum comes to 1455. Call this sum C.

4. Average your data. That is divide the sum of your data, A (244) by the number of terms in your series, N (9, the last entry in Col. B). The answer in the problem given is 27.11. Call this average D.

5. Average your consecutive numbers. (Divide the sum, B (45), by the number of terms, N (9). The answer is 5. Call this average E.

6. Substitute the values obtained by the above processes in the following formula:

$$\frac{C - (A \times E)}{\frac{N^3 - N}{12}}$$

In the example given this would work out:

$$\frac{1455 - (244 \times 5)}{\frac{9^3 - 9}{12}} =$$

$$\frac{1455 - 1220}{\frac{720}{12}} = \frac{235}{60} = 3.9166$$

This figure, 3.9166, is the increment by which your straight line trend goes up and down.

7. To get your trend, when you have an odd number of terms in your series, add or subtract your increment cumulatively to the average of your data, D (in the example given, 27.11) posted to the center term. As your series is 9 items long, from 1913 to 1921, the center point is located at the 5th year, or 1947.

Thus your trend is as follows:

1917 27.1 (27.11 rounded)
1918 31.0 (27.11 + 3.9165, the result rounded)
1919 34.9 (27.11 + 3.9165 + 3.9165, the result rounded)

etc.

Or going down

1917 27.1
1916 23.2 (27.11 - 3.9165, the result rounded)
1915 19.3 (27.11 - 3.9165 - 3.9165, the result rounded)

When you have an even number of terms the average of your data falls <u>between</u> years. In this case you must start out by adding half an increment to get the value of the trend for the year following or subtracting half an increment to get the value of the trend for the year before. Then proceed as above.

Note that if A x E is bigger than C your increment is negative. Your trend slopes down from left to right.

Note that D x B = A x E. Therefore D x B may be used instead if you prefer.

Note that N^3 does not need to be computed. It can be looked up in a table of cubes, if you have one.

Note that even the whole expression $\frac{N^3 - N}{12}$ does not have to be computed if the number of terms in your series is less than 100. All you have to do is to read the values from the table at the end of this lesson. Read 10's in the stub, units in the column headings. Thus when N = 9. $\frac{N^3 - N}{12} = 60$.

Of course there are other ways of fitting straight line trends, by least squares, but the method given above is perhaps simplest. All methods come out the same in the end.

TABLE A

Imports of India Rubber each year from 1913 to 1921.
The figures are monthly averages in millions of pounds.

		A	B	C C
Year		Millions of Pounds	Number of Years	AxB
1913		10	1	10
1914		12	2	24
1915		18	3	54
1916		23	4	92
1917		34	5	170
1918		27	6	162
1919		45	7	315
1920		47	8	376
1921		28	9 N	2582
	Total	244 A	45 B	1455 C
	Average	27.11 D	5 E	

Assignment

Review: Spurr, Kellogg, and Smith, pages 322 - 325; 327 - 330.

Study: Spurr, Kellogg, and Smith, pages 325 - 327; 334 - 340 (up to Parabola).

The method given in S, K, & S is slightly different from the method I have given you. You should know it too. It may be a little harder to follow than the method I have given you, but on the other hand it may be a little easier to understand and a little easier to figure when you have a good many terms. Understand both methods. Use either method you prefer.

Values of $\frac{N^3 - N}{12}$ for all values of N from 00 to 99 inclusive

	0	1	2	3	4	5	6	7	8	9
0	-	-	.5	2	5	10	17.5	28	42	60
1	82.5	110	143	182	227.5	280	340	408	484.5	570
2	665	770	885.5	1012	1150	1300	1462.5	1638	1827	2030
3	2247.5	2480	2728	2992	3272.5	3570	3885	4218	4569.5	4940
4	5330	5740	6170.5	6622	7095	7590	8107.5	8648	9212	9800
5	10412.5	11050	11713	12402	13117.5	13860	14630	15428	16254.5	17110
6	17995	18910	19855.5	20832	21840	22880	23952.5	25058	26197	27370
7	28577.5	29820	31098	32412	33762.5	35150	36575	38038	39539.5	41080
8	42660	44280	45940.5	47642	49385	51170	52997.55	54868	56782	58740
9	607425	62790	64883	67022	69207.5	71440	73720	76048	78424.5	80850

Problem

Compute the straight line trend of the Standard and Poor's Combined Annual Index. Use the logs of the data.

Draw a chart of the logs of the data (as on Chart 3) and show the straight-line trend on the same chart.

LESSON XIX

THE PERIODIC TABLE--CONCLUDED

HOW TO USE THE PERIODIC TABLE TO FIND HIDDEN CYCLES

HOW TO USE THE PERIODIC TABLE TO SEPARATE ONE CYCLE FROM ANOTHER

How to Use the Periodic Table to Find Hidden Cycles

You may remember that in Lesson VII, where we first discussed the periodic table, I told you that it had four main uses. I listed these as follows:

1. If you know the length of the cycle, it will reveal to you the typical or average shape, strength, and timing of the cycle, as nearly as the figures will permit.

2. It is one of the ways of revealing to you as exactly as possible the length of any cycle that may be present in the series. That is, it will tell you the length of the cycle in days, months, or years from crest to crest or trough to trough.

3. It will often enable you to separate one cycle from another, when two or more cycles are present concurrently in the same series of figures.

4. It will help you to find hidden and unsuspected cycles.

Previous lessons have discussed the first two uses completely. It remains to discuss, in this lesson, use three. And to complete the discussion of use four.

Of course the fourth use, finding hidden cycles, has already been pretty well covered. When you made seven periodic tables of your stock market figures with lengths that were at harmonic intervals from each other you were scanning your series for hidden cycles. You were doing it in a systematic comprehensive way. You scanned for all possible lengths from the 7th to the 13th harmonics inclusive (more accurately from the $6\frac{1}{2}$th harmonic to the $13\frac{1}{2}$th harmonic, inclusive). No cycle within this range could fail to be reflected in one or more of your tables for at least one-half amplitude (assuming zigzag shape for the cycle).

For example, if you had a cycle with a length $1/7\frac{1}{2}$th of the length of your series (92 years $\div 7\frac{1}{2}$ or 12.26 years), the periodic table with a length 1/7th of the length of your series (92 years $\div$ 7 = 13.14 years) would show it with one-half amplitude; your periodic table with a length 1/8th of the length of your series (92 years $\div$ 8 = 11.50 years) would also show it with one-half amplitude.

One-half of the amplitude of the cycle is the least that would be revealed by your periodic tables because they are harmonic intervals apart, 1/7th, 1/8th, etc., of the length of your series. If the hidden cycles were 1/8th of the length of the series it would be revealed fully by your 1/8th table. If the length of the cycle lay between $1/7\frac{1}{2}$th and 1/8th of the length of the series, your 1/8th (11.50-year) table would show anywhere from one half of the cycle up to all of it, depending upon the length of the cycle. Similarly, if the length of the cycle lay between $1/7\frac{1}{2}$th and 1/7th of the length of your series, your 1/7th (13.14-year) table would show anywhere from one half of the cycle to all of it. Thus, in making this series of periodic tables equally spaced (harmonically), you were in effect scanning the series for hidden cycles.

When you carried your analysis a step further by dividing your tables into halves and/or thirds to get a closer approximation of the length of the cycle you were using the periodic table even more effectively to reveal hidden cycles.

Finally, when (in Lesson XIV) you got to periodograms and (in Lesson XVI) you got to multiple harmonic analysis, you were using periodic tables to the utmost to help you discover hidden cycles. This is so because each bar or point on your periodograms epitomizes a periodic table of that exact length.

What else is there to say on the subject? In principle perhaps nothing. But before concluding the subject of using periodic tables to find hidden cycles I would like to give you one or two suggestions that I have found extremely valuable in practice.

Suppose you wish to investigate cycles about 6 months long in stock market figures 1871 to 1900 inclusive. Suppose scanning, thumbing, and time charts fail to help. Suppose even moving percentages will not help you. You are stuck!

All right, come back to the good old periodic table. Make a six-month table. This will give you six columns and sixty lines. Make it in color, but it will be a hodge podge of red and black, I'm sure.

At this point there are two ways you can proceed.

This is one way:

Divide the table into thirty sections of two lines each, average each section, and post the results into a new six-month Table, being sure to preserve color.

Scan the new table for evidence of pattern. Do the reds and blacks now fall in bands running vertically, or diagonallyto the right or diagonally to the left? Or is it still a complete hodge podge of reds and blacks?

If its still a hodge podge, repeat the process, dividing the thirty line table into fifteen sections of two lines each, average each section, and post the results into a new six-month table of fifteen lines, preserving color. Each line now represents the average of four six-month cycles. Do the colors now fall into bands?

If so, of course you have a hint of a cycle. You can calculate its length in the usual way.

If not, you can repeat the process to get a new six-month table of seven lines, each an averageing of eight six-month cycles, and so on. Or, as fifteen is not exactly divisible by two, you could divide your fifteen line table into five sections of three lines each. There is no magic in the number two.

In practice you will find it easier merely to add the lines and (if you are using logs) to record in the first instance in black all values 4.000 or more, in red all values 3.999 or less. For your second table you would use 8.000 as your dividing line. For your third table you would use 16.000 or 24.000 as the value to separate reds from black, depending upon whether you used two or three lines.

If you are using percentages you could also add, using 200, 400, or 800 or 1200 as the value above which to use blacks, below which to use reds. Always do things the easiest way!

The second way to proceed would be to run moving averages (or moving totals) of various lengths down each of the six columns of your six-month periodic table, posting the moving average (or moving total) values into a new six-month periodic table—and being sure to preserve color. Start with three-item moving averages. If these show nothing, try a five-item moving average, then a seven-item moving average or a nine-item one.

These moving averages may be plotted for better study.

You may ask, if you are eventually going to make nine-item moving averages, why not start with them?

The answer is that a long moving average won't reveal cycles unless they are very close to the length of your periodic table. In the first instances you want your search to be as wide as possible. A five-item moving average might pick up cycles from 5.5 to 6.5 months in length, whereas a 9-item moving average would hardly pick up anything outside 5.7 to 6.3 months long.

In series where randoms are very bad you might want to use moving medians instead of using moving means.

When you finish the next lesson you will realize that, in making moving averages of the columns of a periodic table, what you have been doing is constructing a weighted moving average of your series. A three-item moving average of a six-month table will give you a weighted moving average of the series with the weights 1/3, 0, 0, 0, 0, 0, 1/3, 0, 0, 0, 0, 0, 1/3. But let's not get into that now—just think of it as you go over the next lesson.

These two additional suggestions conclude all I can think of to tell you about using periodic tables to get hints of cycles.

How to Use the Periodic Table to Separate One Cycle from Another

Please go back to page 2 of Lesson VIII and look at Tables 29 and 30. You will be reminded that when you cast a cycle of some particular length into a periodic table table that is longer or shorter than the length of the cycle, and average the table, the amplitude of the cycle decreases as it appears in the average of the table. In the examples given the true peak and trough values of 48 and 0 (24 above and below the mean of 24) reduce to 39 and 9 (15 above and 15 below the mean value.)

What I did not go on to point out in Lesson VIII is that the amount by which the amplitude is reduced depends upon the number of lines of the periodic table.

For example, continue Table 29 so that it has a total of 16 lines, add, and average, as below.

TABLE 1

A 16-DAY CYCLE CAST INTO A 15-DAY PERIODIC TABLE

FIFTEEN 16-DAY CYCLES; SIXTEEN 15-DAY CYCLES

Base	Days after base														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12
15	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18
30	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24
45	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30
60	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36
75	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42
90	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48
105	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42
120	48	42	36	30	24	18	12	6	0	6	12	18	24	30	36
135	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30
150	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24
165	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18
180	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12
195	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6
210	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0
225	6	12	18	24	30	36	42	48	42	36	39	24	18	12	6
(Sum)	384	384	384	384	384	384	384	384	384	384	384	384	384	384	384
Average	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24

TABLE 2

A 16-DAY CYCLE CAST INTO A 17-DAY PERIODIC TABLE

SIXTEEN 17-DAY CYCLES; SEVENTEEN 16-DAY CYCLES

Base	Days after base																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0
17	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6
34	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12
51	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18
68	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24
85	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30
102	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30	36
119	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42
136	48	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48
153	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42
170	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36
187	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30
204	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24
221	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18
238	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12
255	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6
(Sum)	384	384	384	384	384	384	384	384	384	384	384	384	384	384	384	384	384
Av.	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24

As you can see, in both instances the 16-day cycle vanishes. This is so because each column has an equal number of high and low values.

If you were to go on and add more lines to either table, the 16-day cycle would reappear (of course as a 15-day or 17-day cycle depending upon which table it was cast in). It would increase in amplitude for 8 more cycles so that in tables of 24 lines it would have 1/6th of the original amplitude in the average of the entire table (See Table 3 below).

The table shows 1/6th amplitude because it has 1/2 amplitude in one third of the table and no amplitude in the other two thirds. One third times one half is one sixth.

TABLE 3

A 16-DAY CYCLE CAST INTO A 15-DAY PERIODIC TABLE

TWENTY-TWO AND ONE HALF 16-DAY CYCLES; TWENTY-FOUR 15-DAY CYCLES

Base	Days after base														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12
15	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18
30	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24
45	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30
60	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36
75	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42
90	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48
105	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42
120	48	42	36	30	24	18	12	6	0	6	12	18	24	30	36
135	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30
150	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24
165	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18
180	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12
195	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6
210	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0
225	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6
240	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12
255	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18
270	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24
285	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30
300	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36
315	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42
330	36	30	24	18	12	6	0	6	12	18	24	30	36	42	48
345	42	36	30	24	18	12	6	0	6	12	18	24	30	36	42
(Sum)	552	516	492	480	480	492	516	552	600	636	600	672	672	660	636
Av.	23.0	21.5	20.5	20.0	20.0	20.5	21.5	23.0	25.0	26.5	27.5	28.0	28.0	27.5	26.5

If you should add more lines, the amplitude of the cycle would <u>de</u>crease until your periodic table had 32 lines; at this point the amplitude of the original cycle would be completely washed out for a second time. And so on indefinitely (at 40 lines it would have 1/10th of the original amplitude, etc).

This fact needs to be taken into account in connection with multiple harmonic analysis. For example, suppose you made a complete multiple harmonic analysis of controlled data containing but a single cycle--say 16 days long. I have told you that by the time the curve of the periodogram had reached a harmonic interval away your cycle would vanish. This is true. But I did not complicate things for you at the time by telling you that the curve of your periodogram would rise again and would show a peak one and a half (and two and a half, three and a half, etc) harmonic intervals away. Of course each of these "shadows," as they are sometimes called, has less and less amplitude.

I waited to mention this matter until we got further into the matter of periodic tables so that you see how and why this is so.

As I mentioned earlier in this lesson, the periodogram of a multiple harmonic analysis is merely a profile to show the amplitude of a great number of periodic tables of closely related wave length. If you understand how and why cycles wash out of periodic tables when the periodic tables have lengths longer and shorter than the lengths of the cycle--and why they reappear at 1/6th strength, 1/10th strength, 1/14th strength etc--you will understand multiple harmonic analysis better.

Getting back to the matter of using the periodic table to separate one cycle from another, you see from Table 1 that a 16-day cycle can be eliminated from tables of other than 16-days by dividing the tables into sections of appropriate length. On the other hand--taking Table 1 as an example--if we had had a 15-day cycle, it would have been present in the average of the 15-day table in full force and effect. Therefore, it follows, if you had had a series of figures in which you had <u>both</u> a 15-day <u>and</u> a 16-day cycle, and if you put this series of figures into a 15-day table and added this table by sections 16 lines long, the average of the table would have the 15-day cycle present with full force and effect and the 16-day cycle completely eliminated.

Conversely, if you had put the same figures in a 16-day table 15 cycles long, the 15-day cycle would have been completely eliminated and the 16-day cycle would have been present in full force and effect.

This fact has an important hearing on the way you add your periodic tables. For example if you have a series of figures a year long (360 days - twenty-four 15-day cycles) which contains both 15- and 16-day cycles, and you wish to isolate the 15-day cycle, you must average your 15-day table by overlapping two thirds of 16 cycles each in order to eliminate the distorting effect of the 16-day cycle. If you don't do this--if, for example, you divide your periodic table into halves--you will have a certain residual of your 16-day cycle mixed in with your 15-day cycle with the result that your 15-day cycle will be made to look longer than it really is, and bigger.

It follows from this that you need to know about your 16-day cycle before you try to make final isolation of your 15-day cycle.

It follows that you need to know all your cycles before you start to isolate any of them!

It follows that you need to finish your cycle analysis before you are in shape to start it! This isn't as much of an Irish bull as it sounds. In fact its so darn true it isn't funny!

<u>Cycles do not always wash out completely</u>

In certain instances, due to the interrelationships of the wave lengths involved, the cycles do not wash out completely.

Suppose we combine a 6-month and an 8-month cycle, as follows:

6-Month Cycle	8-Month Cycle	Synthesis (by addition)
3	4	7
1	2	3
-1	0	-1
-3	-2	-5
-1	-4	-5
1	-2	-1
3	0	3
1	2	3
-1	4	3
-3	2	-1
-1	0	-1
1	-2	-1
3	-4	-1
1	-2	-1
-1	0	-1
-3	2	-1
-1	4	3
1	2	3
3	0	3
1	-2	-1
-1	-4	-5
-3	-2	-5
-1	0	-1
1	2	3
& repeat	& repeat	& repeat

Now put these values into a 6-month table of 4 lines:

Base	Months after base 1	2	3	4	5	6
0	7	3	-1	-5	-5	-1
6	3	3	3	-1	-1	-1
12	-1	-1	-1	-1	3	3
18	3	-1	-5	-5	-1	3
(Sum)	12	4	-4	-12	-4	4
Average	3	1	-1	-3	-1	1

As you can see, the 8-month cycle vanishes and the 6-month cycle remains.

Now put these same values into an 8-month table. As you can see the 6-month cycle vanishes, and the 8-month cycle remains, but distorted by errors of plus and minus 1/3.

Base	Months after base							
	1	2	3	4	5	6	7	8
0	7	3	-1	-5	-5	-1	3	3
8	3	-1	-1	-1	-1	-1	-1	-1
16	3	3	3	-1	-5	-5	-1	3
(Sum)	13	5	1	-7	-11	-7	1	5
Average	4 1/3	1 2/3	1/3	-2 1/3	- 3 2/3	-2 1/3	1/3	1 2/3
Error	1/3	- 1/3	1/3	- 1/3	1/3	- 1/3	1/3	- 1/3

This error of 1/3 is an artifact--a two month cycle introduced by the fact that a 6-year cycle doesn't wipe out smoothly when the values are thrown into a eight month table.

You can see this by putting the values of the 6-month cycle by themselves into an 8-month table, as follows:

Base	Months after base							
	1	2	3	4	5	6	7	8
0	3	1	-1	-3	-1	1	3	1
8	-1	-3	-1	1	3	1	-1	-3
16	-1	1	3	1	-1	-3	-1	1
(Sum)	1	-1	1	-1	1	-1	1	-1
Average	1/3	- 1/3	1/3	- 1/3	1/3	- 1/3	1/3	- 1/3

There are two or three other facts about periodic tables to which I should call your attention.

A periodic table just half the length of a symmetrical zigzag cycle will eliminate the cycle, when averaged in sections that are multiples of 2. For example, suppose you have an 8-year cycle and throw these values into a 4-year periodic table. The average of any two lines of this table, or any other number of lines that is a multiple of two, will eliminate the 8-year cycle. Theoretically the elimination is complete, but just as we ran into with moving averages, and with periodic tables, too, there may sometimes be minior differences in actual practice.

If your periodic table is one fourth the length of the cycle, the cycle will be eliminated provided the sections of your periodic table are four lines long, or some multiple of four. Thus a two year periodic table of a series containing a symmetrical zigzag eight-year cycle will wash out the eight-year cycle if averaged in groups of four lines or any multiple of four lines.

And so on.

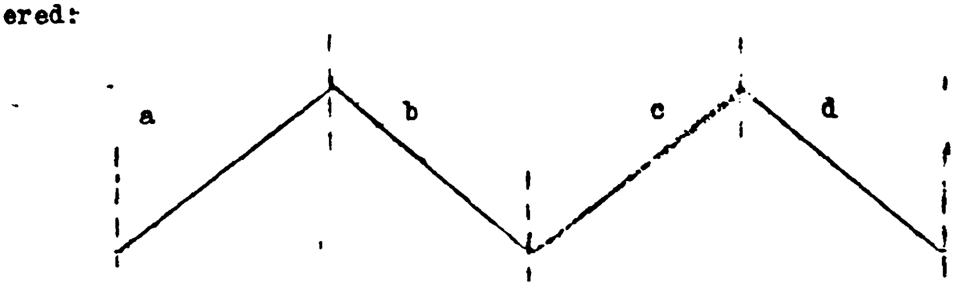
You can generalize by saying that a periodic table will eliminate all symmetrical zigzag cycles that are even multiples of its length, provided the table is averaged in groups that have as many lines as the cycle length is times the periodic table length.

Thus a 4-year periodic table will wash out symmetrical 8-year and 16-year and 24-year cycles, if the table is 12 lines long. It will almost wash out all these cycles if it is almost 12 lines long.

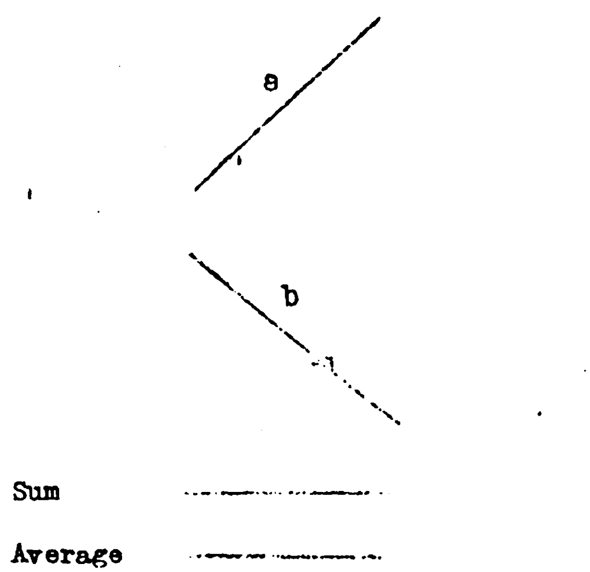
The situation is quite the opposite with symmetrical zigzag cycles that are an odd multiple of the length of the periodic table.

Suppose your periodic table is 3 years long and your series has a symmetrical zigzag cycle 9 years long. Three lines of such a table (or any multiple of three lines) will, when averaged, retain 1/9 (1/3 of 1/3) of the 9-year cycle. Suppose your cycle is 5 times as long as the table, or 15 years. An average of five lines of your table will retain 1/25 (1/5 x 1/5) of the 15-year cycle.

It is easy to see why these things are so if you make some little diagrams. For example here is a cycle cut into segments half the length of the cycle, and lettered:



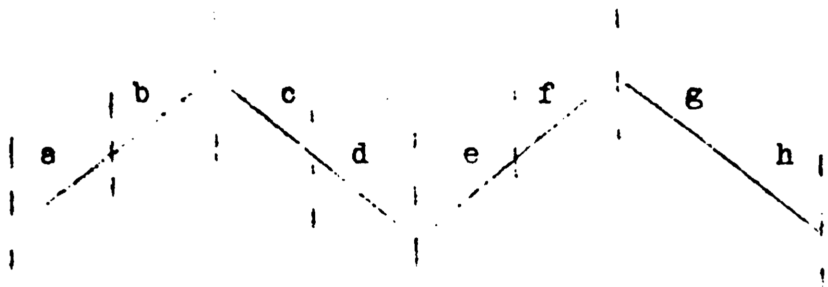
Here are these segments put, graphically, into a periodic table:



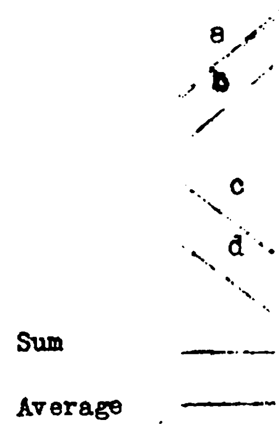
When added, averaged, a offsets b. The two slopes wipe each other out.

This is true no matter how many pairs of segments you average.

Now cut your cycle into four segments.

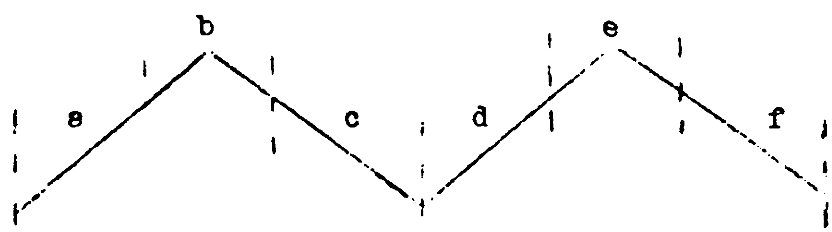


and throw the segments into a periodic table.

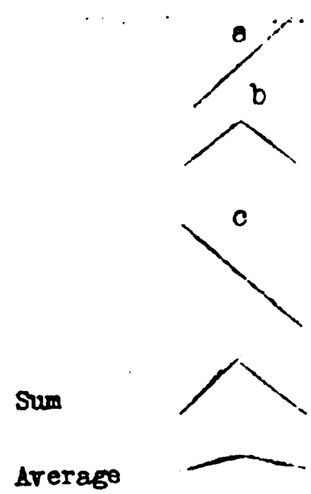


When added and averaged, a offsets c, b offsets d, and so on for each group of four.

Here is a cycle cut into segments 1/3 of its length.



Here are these segments put, graphically, into a periodic table.



Segment a offsets segment c and cancels it out, but segment b remains. Segment b has 1/3 of the amplitude of the original cycle. The sum of the three segments therefore has 1/3 of the amplitude of the original cycle. The average of the three segments has 1/3 of this third, or one ninth of the original amplitude.

And so on.

What do you do if you have two cycles in the same series where one has a length that is an exact or an almost exact multiple of the other?

Well, in pinning down the longer cycle, you can get rid of the shorter one (a) by smoothing it out by a moving average, or (b) by isolating it and adjusting for it, or (c) removing it by a moving percentage, or (d) removing it by a weighted moving average.

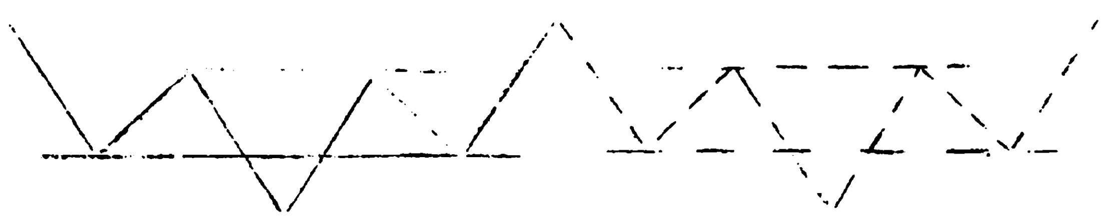
In pinning down the shorter cycle you can't use a moving percentage or a moving average to eliminate the longer one, as it will take out the shorter one also, but you can--and usually should--express the values as a percentage of a short moving average. This will remove the major part of the longer cycle. But you will still have a vestige of the longer cycle left.

If the longer cycle has a length that is an even multiple of the length of the shorter cycle, your periodic table of the shorter cycle will wash it out.

However, suppose the length of the longer cycle is an odd multiple of the length of the shorter cycle? Specifically, suppose you have three-month and nine-month cycles present concurrently in the same series.

Well, deviations from a three-month moving average will get rid of 2/3 of your 9-month cycle, but 1/3 will still be left. Your 3-month periodic table will get rid of 8/9 of this third, but 1/9 of the 1/3 or 1/27 of your original 9-month cycle will still be left. If your 9-month is strong and your 3-month is weak this may still be enough to be troublesome.

In such a case you can make a 9-month periodic table of your deviations from the three-month moving average. Such a periodic table, when averaged, will show you three three-month cycles, one after the other; only, due to the nine month influence, one will be stronger and one weaker than the other, especially if times of peak coincide, thus:



By this expedient it is often easy to pick on the one cycle from the other. In the example given, assuming that the variations are due to a nine month cycle and not to randoms, the overall amplitude of the three-month cycle is the distance between the two horizontal lines.

(Lesson XIX has no problems)

LESSON XX

WEIGHTED MOVING AVERAGES

It will help you to understand weighted moving averages if you realize that the ordinary moving averages you have been dealing with are also weighted moving averages--only the weights are all 1.

To illustrate this fact consider the following:

Suppose you have a series, the terms of which are represented by letters, a, b, c, d, etc. The first term of a 3 item moving average would be $\frac{a + b + c}{3}$. The second term would be $\frac{b + c + d}{3}$. And so on.

If you stop a moment to think you will realize that the first term is really $\frac{1a + 1b + 1c}{3}$. That is, to a taken once you add b taken once and c taken once. The number of times you take each term is once. That is, all the "weights" are 1.

However the weights might be 1, 2, and 1. If so, the first term of the moving average with these weights would be $\frac{1a + 2b + 1c}{4}$.

Note that in this instance the denominator is 4 because there are really 4 items in the numerator, viz: a, b, b, and c.

Note that a moving average of a moving average is a weighted moving average and can be computed directly. For example.

Col. A Series	Col. B 3-item M. A. of Col. A	Col. C 3-item M. A. of Col. B
a		
b	$\frac{a + b + c}{3}$ or $1/3a + 1/3b + 1/3c$	
c	$\frac{b + c + d}{3}$ or $1/3b + 1/3c + 1/3d$	$\frac{1/3a + 2/3b + 3/3c + 2/3d + 1/3c}{3}$
d	$\frac{c + d + e}{3}$ or $1/3c + 1/3d + 1/3c$	etc.
e	etc.	
f		

$\frac{1/3a + 2/3b + 3/3c + 2/3d + 1/3c}{3}$ is the same as

$1/9a + 2/9b + 3/9c + 2/9d + 1/9c$

You note that the fractional weights of the terms add to unity.

Negative Weights and Zero Weights

There is no reason why some of the terms of a weighted moving average cannot be negative or zero.

The Use of Weighted Moving Averages

The traditional use of weighted moving averages is in connection with smoothing series of figures. The standard book on this subject is by Frederick R. Macaulay. It is called The Smoothing of Time Series. It was published in 1931 by the National Bureau of Economic Research. It is now out of print, but it may be consulted at any large library. Sometimes you can find a copy at a second hand book store. This book is quite simple and easy to read.

The cycle analyst can usually get along with a simple moving average, or at most with a moving average of a moving average. I, therefore, see no need to get into the subject of smoothing at this point.

The other use of the weighted moving average is to help you to separate one cycle from another, and/or from randoms and/or from trend. This will be the subject matter of the rest of this lesson.

The Use of Weighted Moving Averages in Cycle Analysis

Let us start our investigation of the effect of weighted moving averages by considering some controlled data. Here for example is our old friend the regular 8-month cycle with values 8, 4, 0, -4, -8, -4, 0, 4 and repeat.

Let us compute various weighted moving averages of this cycle.

Let us start by computing a nine-term moving average with the following weights: 1/4, 0, 0, 0, 1/2, 0, 0, 0, 1/4. The first term of the weighted moving average would be:

$$(1/4 \times 8) + (0 \times 4) + (0 \times 0) + (0 \times (-4)) + (1/2 \times (-8)) + (0 \times (-4)) + (0 \times 0) + (0 \times (+4)) + (1/4 \times 8)$$

This would become

$$2 + 0 + 0 + 0 + (-4) + 0 + 0 + 0 + 2$$

or $2 - 4 + 2$

or 0

All the rest of the terms would become 0 too. The cycle would vanish.

Suppose you now use the weights -1/4, 0, 0, 0, 1/2, 0, 0, 0, -1/4. You would get

$$((-1/4) \times 8) + 0 + 0 + 0 + ((1/2)(-8)) + 0 + 0 + 0 + ((-1/4) \times 8) = (-2) + (-4) + (-2) = -8$$

The next term would be -4, the next 0, the next 4, etc. The original cycle would reappear without distortion.

Now try the weights -1/2, 0, 0, 0, 1, 0, 0, 0, -1/2. You would get $((-1/2) \times 8) + 0 + 0 + 0 + (1 \times (-8)) + 0 + 0 + 0 + ((-1/2) \times 8) = (-4) + (-8) + (-4) = -16$

If you proceed you would get -8 for the next term, then 0, then + 8, then + 16. In other words you would get your 8 month cycle back with double amplitude.

Here then is a technique which will enable you (a) to eliminate a cycle, (b) to reproduce it, (c) to magnify it, at will.

What effect will these techniques have upon randoms, upon trend, and upon other cycles? Let us see.

Effect upon Trend

Consider a trend increasing smoothly as follows: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc. Apply the first formula (1/4, 0, 0, 0, 1/2, 0, 0, 0, 1/4):

$$(1/4 \times 1) + 0 + 0 + 0 + (1/2 \times 5) + 0 + 0 + 0 + (1/4 \times 9) = \frac{1}{4} + \frac{10}{4} + \frac{9}{4} = \frac{20}{4} = 5$$

Applied to the next term we would get.

$$2/4 + 12/4 + 10/4 = 24/4 = 6.$$

Similarly we would get 7, 8, 9, 10, etc. Thus, as you can see there would be no effect whatever on trend. If cycle and trend were combined and the formula applied, cycle would vanish, trend would remain.

Now let's try the second formula:

(-1/4, 0, 0, 0, 1/2, 0, 0, 0, -1/4) on the same trend:

$$((-1/4) \times 1) + 0 + 0 + 0 + (1/2 \times 5) + 0 + 0 + 0 + ((-1/4) \times 9) = -1/4 + 10/4 - 9/4 = 0.$$

Each of the other terms would be zero. Trend would vanish. (The cycle would remain.)

Now let's try the third formula (-1/2, 0, 0, 0, 1, 0, 0, 0, -1/2) on the same trend.

$$((-1/2) \times 1) + 0 + 0 + 0 + (1 \times 5) + 0 + 0 + 0 + ((-1/2) \times 9) = -1/2 + 10/2 - 9/2 = 0.$$

Each of the other terms would also be zero. Again trend would vanish, even though the cycle doubled.

Effect upon Randoms

How about the effect on randoms?

Let us use a few of the actual random numbers used in Lesson V Table 1. These are shown in Col. A of the table below. The results of applying the three weighted moving averages are shown in Col. B, C, and D.

Table 1
Effect of weighted moving averages upon Randoms

Position	Col. A Random	Col. B. Wt. m.a. of Col. A. wts. [illegible], 0,0,0, ½,0,0,0,[illegible]	Col. C. Wt. m.a. of Col. A. wts.-[illegible],0,0,0, ½,0,0,0,-¼	Col. D. Wt. m.a. of Col. A wts.-½,0,0,0, 1, 0,0,0,-½
1	1			
2	9			
3	15			
4	-18			
5	-20	-9 1/4	-10 3/4	-21 1/2
6	8	4 1/4	3 3/4	7 1/2
7	-24	2 3/4	-1 1/4	-2 1/2
8	- 3	1/4	-3 1/4	-6 1/2
9	2	12 1/2	-10 1/2	-21
10	-8	0	-8	-16
11	-58	-39 1/4	-18 3/4	-37 1/2
12	25	13 1/4	11 3/4	23 1/2
13	66	37 3/4	28 1/4	56 1/2
14	8	-2 3/4	10 3/4	21 1/2
15	-17			
16	6			
17	17			
18	-19			

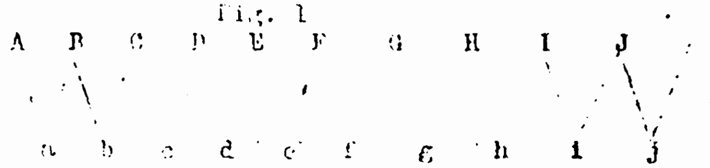
As you can see, the first two weighted moving averages reduce the randoms importantly. For example the sum of the actual randoms from the 5th to the 14th inclusive, regardless of sign, is 222. The first moving average reduced this total to 142; the second one reduces it to 107. The values of the third moving average are seen to be just twice the values of the second moving average. It is thus seen that Column D. offers no advantage as far as randoms are concerned. It doubles the amplitude of the cycle but it doubles the randoms as well, so it offers no relative advantage.

Other Formula

The number of different weighted moving average formulae is literally limitless. Let us discuss a few more.

Refer to Fig. 1 which diagrams several repetitions of an 8 item cycle. The various turning points are lettered.

Fig. 1



Suppose we illustrate our formulae for averages centering on E.

In [illegible] purposes of cycle analysis, we use formulae with weights for the turning points in which we are interested and zeros for the intervening points. As a short cut therefore we can refer merely to the turning points. For example weights of 1/4, 0, 0, 0, 1/2, 0, 0, 0, 1/4 for the section of the curve centering on E might be described in shorthand as 1/4 d + 1/2 E + 1/4 c.

Perhaps the diagram will help you to see why this formula wipes out the cycle. Both d and e are negative values as far below the axis as E is above it. The value of 1/2 E is the same as 2/4 E. That is, you have two quarter values on the plus side offsetting two quarter values on the minus side, and so of course they cancel out.

With the -1/4, 0, 0, 0, 1/2, 0, 0, 0, -1/4 formula (-1/4d, + 1/2E - 1/4c), your values of d and e which are negative to start with are taken negatively which makes them positive. The result is four quarter values piled together at E to make an amount equal to the E you started with.

Here are some other formulae which will eliminate the cycle:

d + 2E + e
d - e
1/2d - 1/2e
1/10d - 1/10e
100d - 100e
b + c + d - e - f - g
c - f
-c + f
1/4A + 1/4b + 1/4g + 1/4 I

and so on to your hearts content. All lower case letters are negative and have the same value. All upper case letters are positive and have the same value. Any upper case letter will offset any lower case letter, any upper case letter taken negatively will offset any other upper case letter taken positively. Any lower case letter taken negatively will offset any other lower case letter taken positively. All letters may be multiplied by the same constant.

Let us now construct a few formula that can be applied so as to leave the cycle unaffected.

-1/4 d + 1/2 E - 1/4 e
-1/8 c - 1/8 d + 1/2 E - 1/8 e - 1/8 f
-1/4 a + 1/2 E - 1/4 h
-1/4 a + 1/4 C + 1/4 G - 1/4 h
1/16 (A + B + C + D + F + G + H + I) - 1/16 (a + b + c + d + e + f + g + h)

etc. etc.

I might remark at this point that although Fig. 1 was set up to diagram an 8 item cycle it really diagrams any cycle with an even number of terms. (Later we will consider cycles with an odd number of terms.)

Let us see where this leads us.

You now know how to devise weighted moving average formulae to leave in a cycle or remove it at will. It should not be hard to devise a formula that will leave in one cycle and remove another. With this in mind, consider Fig. 2 in which 6 and 8 year cycles are diagrammed.

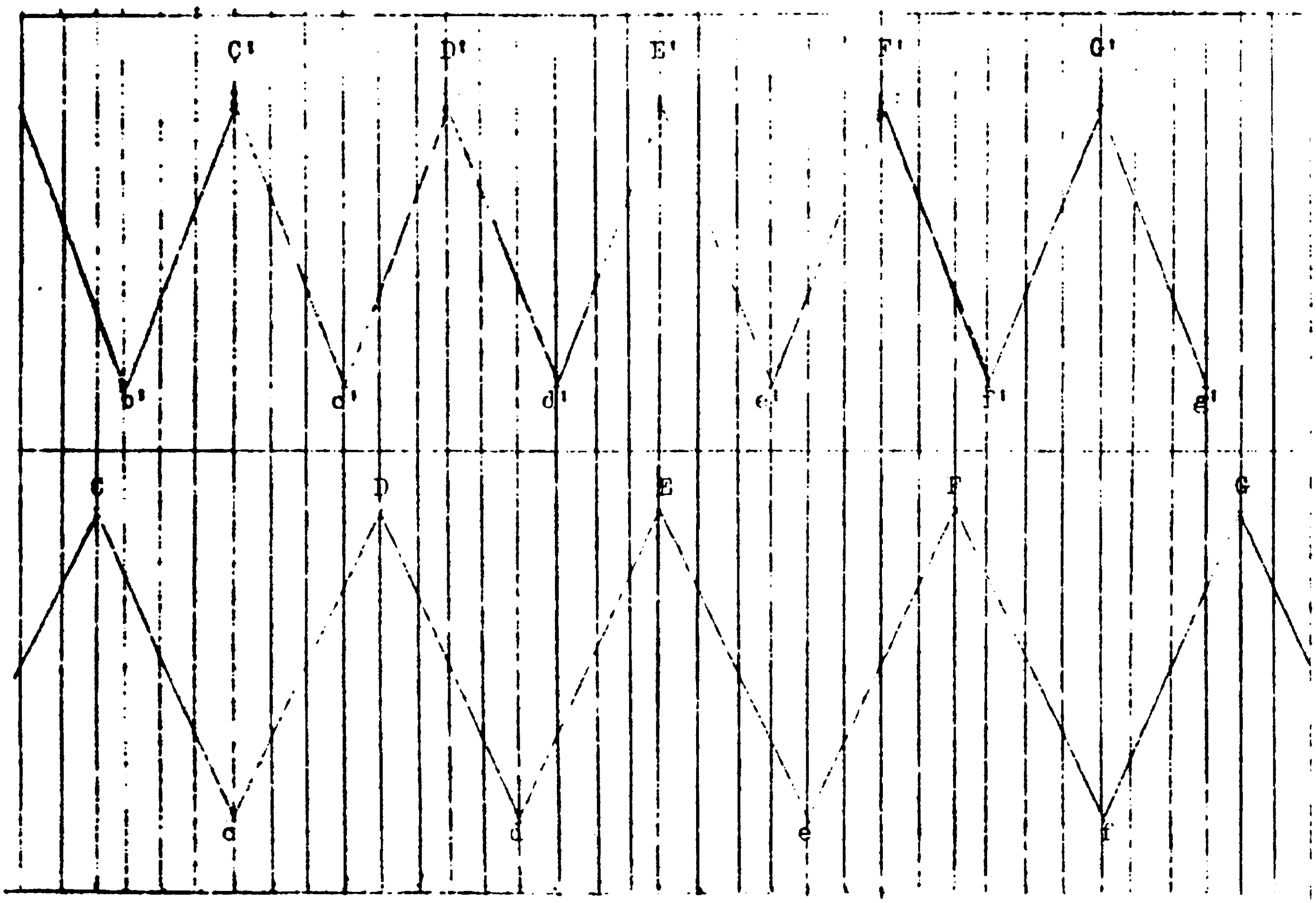


Fig. 2 Upper curve a 6-year cycle. Lower curve an 8-year cycle.

Suppose you have a series of figures in which both these cycles have been combined. You wish to devise a formula that will preserve the 8-year cycle, eliminate the 6-year cycle. Easy. Find some points where the 6-year cycle is down where the 8-year cycle is up, or vice versa. You have such points at c and f of the 8 year cycle or C^1 and G^1 of the 6-year cycle. c and C^1 are both 12 years before E; f and G^1 are both 12 years after G. Your formula would therefore be

$$-1/4c + 1/2E - 1/4f$$

using the designation of the 8 year cycle or $-1/4C^1 + 1/2\ E^1 - 1/4\ G^1$ using the designation of the 6 year cycle. In either event the formula would expand to:

$$-1/4,\ 0,0,0,0,0,0,0,0,0,0,0,1/2,0,0,0,0,0,0,0,0,0,0,0,-1/4$$

For the 8 year cycle the end negative values would be made positive and would add to the center positive values. For the 6-year cycle the end positive values would be made negative and would therefore offset the center positive value.

To reverse the process and eliminate the 8 and keep the 6 you would merely reverse the beginning and ending sign, thus:

1/4,0,0,0,0,0,0,0,0,0,0,0,0, 1/2, 0,0,0,0,0,0,0,0,0,0,0,0, 1/4

Now that you have the idea, you should be able to go on by yourself, but a few more minor points are worth mentioning.

First, the more significant terms to your moving average the more you reduce your randoms. That is, a five term formula of the c,d,E,e,f type or the C,D, d, E, e, F, f type gets rid of randoms better than a three term formula of the d,E,e, or c, E, f type. (But it may not do as good a job of getting rid of some other particular cycle.

Second. In the illustrations given the formulae have been set up so that all values except cycle turning points are zero. It is not essential that you do this. Intermediate values can be given suitable weights too.

Third. In the illustrations given the formulae have been related to a single crest, E. For example 1/2 of E plus one quarter of d plus 1/4 of e all piled up at E will equal E. It perhaps needs to be pointed out again that the formula works equally well for every other point on the curve.

For example, take the formula expressed as -1/4d, 1/2E - 1/4e which builds up to full positive amplitude value for the cycle at point E. When these weights, -1/4, 0,0,0, 1/2, 0,0,0, - 1/4 are applied to center at point e we get - 1/4E + 1/2e - 1/4F.

E and F. are both positive to start with but after you apply the formula they become negative. e, negative to start with, stays negative. Therefore you build up at point e a full negative amplitude for the cycle.

Fourth, with cycles of odd length, either crest or trough will fall between positions. For example consider the 3-year cycle diagrammed in Fig. 3. The lows at d and e fall between years.

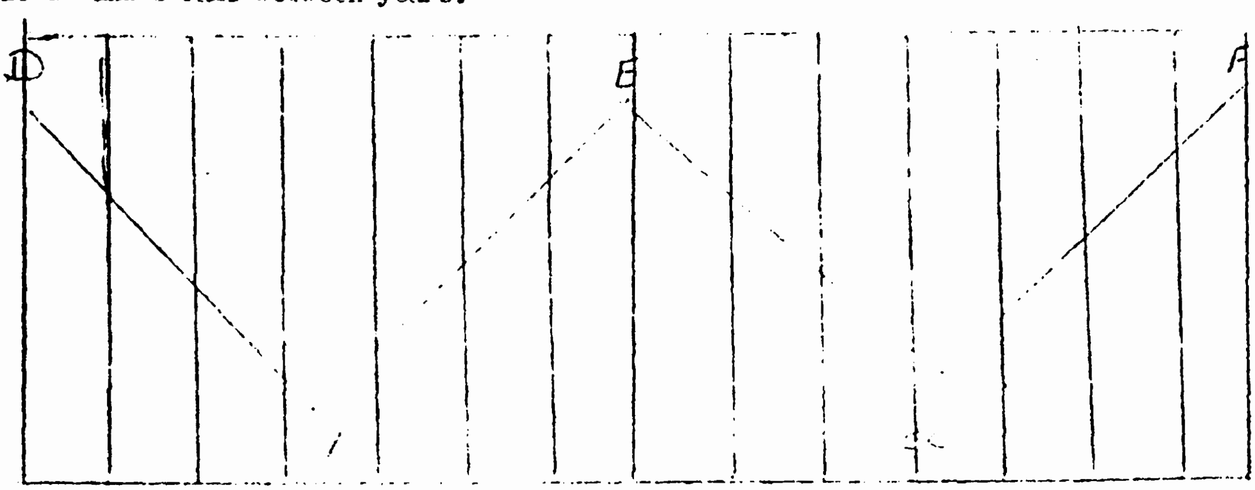


Fig. 3. A 3-year cycle cresting at E. note that troughs d and e fall between years.

You could devise a formula which would give the best results for the value centered at E, but it would not work for the points before and after(d, e, etc.). Under the circumstances you might use a formula that would average the values before and after d and e thus; - 1/8, - 1/8, 0, 0, 1/2, 0,0, - 1/8, - 1/8

Fifth, what I have just said in regard to cycles of odd length applies to cycles of fractional length.

You may not be able to devise a formula that will completely retain one cycle and completely remove another. You will often have to be content with substantial retention and substantial removal.

Sixth, Remember that a formula designed to reveal cycles of a particular length will also reveal cycles of nearby length.

For example, take this formula designed to bring out an eight year cycle:

-1/4, 0,0,0, 1/2, 0,0,0, -1/4

Apply it to a 7 year cycle of value 7,3,-1, -5, -5, -1, 3 and repeat. The results are shown in the following table.

Table 2

Weighted Moving Average
of Formula -1/4, 0, 0, 0, 1/2, 0, 0, 0, -1/4
(suitable for leaving an 8-year cycle unchanged)
applied to a 7-year cycle

Position	Data	Computation of Moving Average		Moving Average
1	7	-		-
2	3	-		-
3	-1	-		-
4	-5	-		-
5	-5	(-1/4x7)+(1/2x-5)+(-1/4 x 3)	=	-5
6	-1	(-1/4x3)+(1/2x-1)+(-1/4x-1)	=	-1
7	3	(-1/4x-1)+(1/2x3)+ (-1/4x-5)	=	3
8	7	(-1/4x-5)+(1/2x7) +(-1/4x-5)	=	6
9	3	(-1/4x-5)+ (1/2x3)+ (-1/4x-1)	=	3
10	-1	(-1/4x-1)+ (1/2x-1)+ (-1/4x3)	=	-1
11	-5	(-1/4x3)+ (1/2x-5) + (-1/4x7)	=	-5
12	-5	(-1/4x7)+ (1/2x-5) + (-1/4x3)	=	-5
13	-1	(-1/4x3) +(1/2x-1) + (-1/4x-1)	=	-1
14	3	-		-
15	7	-		-
16	3	-		-
17	-1	-		-
	etc.			

You will notice that the weighted moving average does not change the length or the timing of the 7 year cycle but that it does change its shape and amplitude slightly.

Seventh, While we are on the subject of computation it may be as well to mention that the method shown in the above table is unduly laborious. I deliberately did it the long way to spell it out for you. In actual practice you would short cut.

Several sorts of short cut are possible. For example instead of using a formula like -1/4, 0,0,0, 1/2, 0,0,0, -1/4 you could use a formula like $\frac{-1,0,0,0,2,0,0,0,-1}{4}$. That is, take the 1st value negatively, take the fifth value twice, take the 9th value negatively, and divide the sum of the three products by four. Actually you would not even need to divide by four. You could chart the sum of the three values and change the scale on the chart after you got through charting, substituting a 1 for a 4, a 2 for an 8, etc.

In instances where you had peculiar weights you could compute each of these weights all at once, record them in separate columns, and assemble as needed. This method makes it practical to divide by use of reciprocals--a great time saver as I have already pointed out.

<u>Eighth.</u> Don't fail to note that weighted moving averages tend to create pseudo cycles out of randoms.

Consider a series all zero except for one random. Subject this series to weighted moving averages of various sorts. Notice how pseudo cycles appear. For some examples see Table 3 and Fig. 4.

Another random a little later in the series will create another pseudo cycle, but this pseudo cycle will not be in phase with the first one, except by accident. However, if it is nearly in phase it will create the illusion of a cycle a little longer or a little shorter than the pseudo cycle.

Table 3

Showing pseudo cycle introduced by application of a moving average formula to a single random number.

Number	Data	Weighted Moving Average $-\frac{1}{4}, 0, 0, \frac{1}{2}, 0, 0, -\frac{1}{4}$	Weighted Moving Average $\frac{1}{4}, 0, 0, \frac{1}{2}, 0, 0, \frac{1}{4}$
1	0		
2	0		
3	0		
4	0	0	0
5	0	0	0
6	0	- 0	0
7	0	-15	15
8	0	0	0
9	0	0	0
10	60	30	30
11	0	0	0
12	0	0	0
13	0	-15	15
14	0	0	0
15	0	0	0
16	0	0	0
17	0	0	0

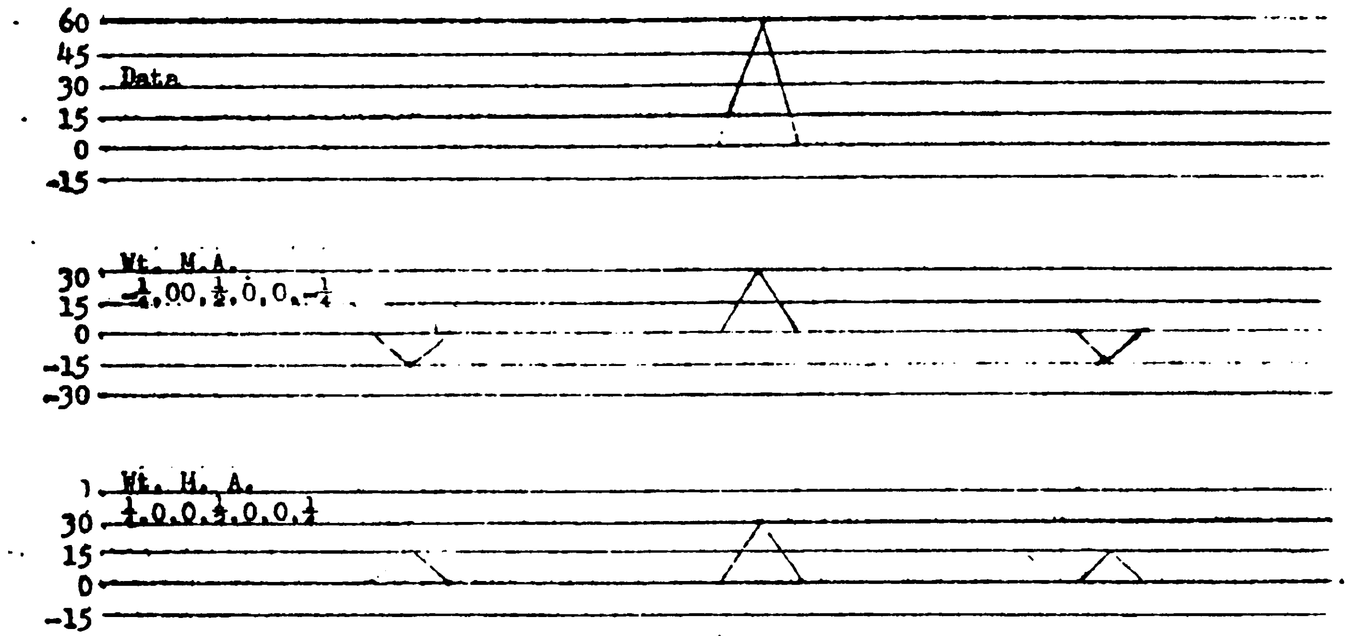


Figure 4

Amplitude Tables and Amplitude Charts

Every time you subject a series of figures to a weighted moving average of any particular sort you effect every cycle that is present in the figures. As in the example given above, a particular weighted moving average may eliminate a 6-year cycle and leave an 8-year cycle unchanged. The same weighted moving average will also have an effect on a 7-year cycle, a 5-year cycle, a 9-year cycle and so on.

Every time you use a weighted moving average you should find out the percentage of amplitude of cycles of various length which remain after the application of the moving average to the series. You have to do this for each cycle length separately. When you have made these computations they can be recorded in what is called an amplitude table and charted in what is called an amplitude chart.

There is no rule I can give you for working these computations. It will depend upon the shape of the cycle, the lenght of the cycle, and the weight of the weighted moving average you are using. Each case has to be computed individually by applying the weighted moving average formula to ideal cycles of the given shape and of the various lenght which interests you.

The Work of Mr. W. C. Yeatman

Mr. W. C. Yeatman of Los Angeles has written a paper about weighted moving averages. This paper is being published in the Journal of Cycle Research for October 1955. A copy of this paper is being sent you herewith. It gives you an example of wave separation by means of weighted moving averages. It also gives you amplitude tables for 54 different moving average formulae each applied to sine waves of 63 different wave lengths—a total of 3,402 values. It gives you examples of amplitude charts. You should consider this paper of Mr. Yeatman's and its tables and charts as part of this lesson.

Weighted Moving Average of Weighted Moving Averages

There is no reason why you cannot compute a weighted moving average of a weighted moving average. One weighted moving average will eliminate or minimize cycles of certain lengths; another weighted moving average will eliminate or minimize cycles of other lengths. By this process you can get the cycle you are interested in revealed more and more clearly--if it is present.

SUMMARY

The weighted moving average is a marvelous tool for "filtering" a series of figures so that certain cycles and randoms can be more or less minimized and other cycles can be left relatively unaffected.

LESSON XX

Supplement 1

W. C. YEATMAN'S PAPER

Weighted moving averages are so important in cycle analysis that I asked Mr. W. C. Yeatman of Los Angeles to allow me to print a paper on this subject which he had written a number of years ago. He consented. The paper has been printed.

The issue of the Journal of Cycle Research containing it (October 1955) is being sent you herewith. It will supplement what I have written in Lesson XX. It will provide you with many useful reference tables. It will give you examples of amplitude charts. I trust you will find it helpful.

LESSON XX

Supplement 2

YEATMAN'S AMPLIFICATION TABLE

Question: Do you know of any interpolation formula which can be used with a table, such as Table 1 of the October '55 Journal which accompanied Lesson XX to interpolate values for any fractional cycle length for constructing amplitude charts and restoring cycle amplitudes?

Since the Yeatman-Lane tables are for sine waves and since we usually consider our cycles to be symmetrical zigzag waves, I am trying to make up my own table for zigzag cycles. There is quite a difference in the amplification factors.

Answer: No, I don't know of any interpolation formula for Yeatman's table, nor do I know of any table for zigzag waves.

I should, however, perhaps call your attention to the fact that you can get interpolated values from the table itself.

For example, a 3-item wave in the 07 Column can be considered as a 6-item wave in the 014 Column or a 12-item wave in the 028 Column or a 24-item wave in the 056 Column and all other values pro-rate.

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A METHOD FOR SEPARATING CYCLES AND ITS USE IN CONJUNCTION WITH OTHER METHODS

by W. C. Yeatman

THE method described is intended to separate a cycle from others which may accompany it, and from the trend line as well. It shows the cycle as a continuous function in its true size. The operation loses points at each end of the series, but if the cycle is shown to be a true one, with regular recurrence, it may be extended to the present time. Where a number of cycles exist in the data, they may be processed to develop each one separately.

Due to the lack of a short descriptive name, this method is listed as 9, with variations 9-A, 9-B, 9-C and 9-D. The method is designed to eliminate, as far as possible, all other cycles than the one being worked on, and to show it as a continuous wave. It might be called a method of moving sums and differences, and it is in effect a mathematical wave filter.

The original 9, (now called Method 9-C), was based on the simple idea of eliminating a wave by subtracting the figure one wave length earlier. This process is commonly used to eliminate a twelve months cycle, or seasonal variation. Many kinds of statistics are given comparing the data for the current month with that for the same month of the previous year, without any thought that this is a mathematical process, as well as a comparison.

It is usual, in a process of this sort, to designate the month being worked on by the reference month number 0 (zero). A month 12 months earlier will hereafter be called 12B, or 12 months before. One 20 months earlier would be 20B. When a month later than that being worked on is used the letter A will follow the number, to show that it is a certain number of months after the reference month.

In carrying out this process it is convenient to obtain cross-section paper in rolls, to cut it to the width needed for the various operations, and of a length sufficient to put the entire data in the first column. The vertical spacing of the figures will be uniform throughout the entire data, and cut-outs can be made of the same paper, with openings, or windows, the desired number of months apart. If the operation being made is 0 - 12B the openings will be 12 months apart, and the figure derived by this subtraction would be put in the second column opposite the reference month 9. It is convenient to use a black pencil for plus figures, and a red one for minus figures.

Using 0 - 12B on a long column of monthly figures would completely eliminate a 12 months cycle, but would leave a complex picture of all other waves present. One difficulty with this process is that the phase of associated waves is shifted. With this operation all waves between 12 and 24 months would be shown late, their tops and bottoms in the derived figures being shown after their true position. A wave 24 months long would be shown in its true position, and of double size. All waves longer than 24 months would be shown earlier than their true position. If a large number of waves is present, the use of this method leaves a distorted jumble that is not a true picture of the remaining rhythms. A further discussion of this method, 9-C, and 9-D, will be given at the end of this paper.

Methods 9-A and 9-B were designed to avoid this phase shift, and to give greater selectivity in wave elimination. They involve the use of points BOTH earlier and later than the reference month. All remaining waves, after repeated operations by this process are left in their true position, there being no phase shift whatever. Cut-outs for these operations would require three windows, the reference month (0) being in the middle.

Method 9-A uses the formula 0 - ½(XA + XB), X being the number of months after and before the reference month for the desired operation. This will eliminate a wave of length X, X/2, X/3, X/4, etc. It will double the size of a wave of the length 2X, 2X/3, 2X/5, 2X/7, etc. The amount of amplification or reduction of all waves which may be present may be found from Table 1.

Method 9-B uses the formula 0 + ½(XA + XB). This will double the size of a wave of length X, X/2, X/3, X/4, etc. It will eliminate a wave of length 2X, 2X/3, 2X/5, 2X/7, etc. Its effect of amplification or reduction on any wave which may be present will be found from the same table used for Method 9-A, except that in this case the figure of the tables is to be subtracted from 2.000. Method 9-B, like 9-A, gives no phase shift to the remaining waves.

These methods may be combined, and repeated several times, to amplify the wave being studied, and reduce the size of others which may be present. Such a multiple operation might be 0 - ½(XA + XB) of 0 + ½(YA + YB) of 0 - ½(ZA + ZB), the distances X, Y and Z being chosen to effect the purpose desired. To obtain the true size of the resultant wave one would multiply the amplification figures given in Table 1, for the three operations in question. An example of the planning for developing a wave is given below. In the case of a series of operations of this sort it makes no difference in the final result which one is done first, though one combination may give a set of starting figures more convenient for the succeeding operations.

A concrete example of the use of this process is given in Table 2, where three sine waves 11, 17 and 24 time periods long are added in column 4. The length of data is 96 time periods long, far too short for separation by periodic table. (For PERFECT separation by periodic table the data would have to be the least common multiple of the three lengths, or 4488 time periods long). Each wave is developed by two operations to remove the other waves. In developing wave 11, the first operation removes wave 24, and the next one wave 17. The amplification for the first operation is 1.841, this being 2.000 minus the figure in column 0-12, for wave 11. The second operation amplifies wave 11 by 1.959, this being taken from the column 0-17, for wave 11. The product of these two figures is 3.606, which is the divisor used to get the true size of the wave. The divisors for waves 17 and 24 are obtained in the same manner.

Another example of this process is shown in Table 3, where waves 20 and 36 months long are added to a trend line. From the sum in column 4 each wave is derived, and the trend line as well. For the 20 months wave the first operation doubles the size of the wave and removes the trend line, (See intersection of column 0-10 with wave 20 in Table 1). The next removes the 36 months wave, and amplifies 20 by 1.809. (See intersection 0-18 column with wave 20, the figure .191 being subtracted from 2.000 for a 9-B operation). The divisors for the other operations are obtained in the same manner. The trend line could be found by subtracting the two waves, but it can also be developed by Method 9-B, removing the waves, and leaving the trend line, as shown in the last column.

The second example shows that waves can readily be separated from a straight trend line. While the same degree of precision cannot be expected where the trend line is a curve, or of an extremely irregular form, the results are surprisingly good in such cases.

These two examples are on synthetic data, where the figures are precise. It is interesting to try the method on observational data, containing observational errors and randoms. A set of tidal measurements, for Morro, Calif., is given in the Coast & Geodetic Survey "Manual of Harmonic Analysis and Prediction of Tides". As Measurements are given only to the nearest tenth of a foot, an observational error might be as large as .05 foot. The effect of winds would cause random errors. Method 9 was used on the tidal data given, and the result is shown in Figure 1.

The solar and lunar tides are so nearly the same length that they are treated as a single wave by this method. A dying away of the diurnal component is shown when the two forces are approaching opposition, the effect of wave interference. This interference is much less in the semi-diurnal wave. The quarter-diurnal wave was computed, and added into the total, but not plotted. The ter-diurnal, sixth-diurnal, and eight-diurnal were so small that they were omitted from the computations. The sum of the three waves derived looks almost the same as the original tidal observations. The sum was then subtracted from the observational figures, and the differences were plotted to double scale. These differences consist of the three small waves omitted, randoms, and the fortnightly tide reaching a low about February, 16th.

The method cannot be used to separate waves of nearly the same length. It will carry them as a pair, or group. Its use finds many cases formerly thought of as a single wave to be compound, either a pair or a group. Many persons have spoken of the action of two waves nearly the same length as if their sum produced a wave of intermediate length. A synthetic investigation of a binary wave is a simple matter, and two graphs are appended (Figures 2 and 3) showing the effect of adding two sine waves 15 and 16 months, and 15 and 18 months long, of different amplitudes.

The first combination cannot be separated by this method, but would be carried through just as shown on Figure 2. The second combination of 15 and 18 months are of enough difference in length to separate the combination into the two components. By the use of periodic tables 15 and 16 could be exactly separated by using the synodic period of 240 months, or multiples of it. For 15 and 18 the synodic period is 90 months, and periodic table separation would require the use of this length, or a multiple of it. In many cases of cyclical data there are so many other waves present that interference with them exists when the synodic period for a pair is used. But if a pair like 15 and 16 are separated from other waves, by Method 9, the resulting figures can be put into a periodic table 240 months long to separate the waves from each other.

One advantage of Methods 9-A and 9-B is that the operations are not tied down to an assumed wave length. A periodic table is based on an assumption of the length of the wave. If the true length is only slightly different from the length used in the periodic table the amplitude will be too small, and the wave out of phase at each end. With Methods 9-A and 9-B there is perfect freedom of length. If the wave comes out a different length from what was expected, the divisor obtained from Table 1 can be changed to fit the length found. If the wave is an economic one, due to some such thing as inventory accumulation and reduction, it may have a varying length. This method will follow the changing length with fidelity, with a slight error in amplitude, due to the fact that the divisor should change with the length. One might say that an economic wave due to inventory changes is not a true rhythm. Even so the method will show how true it is.

One of the most difficult things about the process is planning the operations so that the wave being developed gets as little interference as possible from other waves present. One should assume that a wide range of other waves may be present, and plan to nearly eliminate all of them. To show the planning involved, amplification figures are given for the development of a wave seven weeks long.

The figures in the first column are taken from column 0-3 of Table 1 reading the values opposite the wave lengths shown. Wave length 2 does not appear in the table, but the value is the same as 0-½(6A + 6B) of wave length 4. The figures for the second column are the product of those in

WAVE LENGTH	0 - ½ (3A+3B)	0 - ½ (4A+4B)	0 + ½ (7A+7B)	0 - ½ (18A+18B)	
2	2.000	.000	.000	.000	
3	.000	.000	.000	.000	
4	1.000	.000	.000	.000	To be
5	1.809	1.250	.239	.442	added
6	2.000	3.000	4.500	.000	if nec-
7	1.899	3.602	7.204	13.890	essary
8	1.707	3.414	5.820	5.820	
9	1.500	2.910	3.420	.000	
10	1.309	2.366	1.625	1.122	
11	1.143	1.891	.657	1.087	
12	1.000	1.500	.201	.402	
13	.880	1.193	.035	.061	
14	.777	.951	.000	.000	
15	.691	.764	.017	.012	
16	.617	.617	.047	.003	
18	.500	.413	.097	.000	
20	.412	.285	.117	.022	
22	.347	.203	.119	.078	
24	.293	.146	.108	.108	
26	.252	.109	.096	.130	
28	.218	.082	.082	.133	
30	.191	.063	.070	.137	
32	.169	.050	.060	.115	
34	.151	.039	.050	.099	
40	.109	.021	.031	.060	

the first column and those for the operation of the second column. Those in the third column are the product of the figures in the second column and those for the operation 0 + ½(7A + 7B) of the third column. As column 3 is a 9-B operation the amplification will be 2.000 minus the figures found in the 0-7 column of Table 1. Each column shows the progress of the operations to that point on the list of waves considered. In searching for this 7 week wave the nearest known waves were 4 and 14 weeks long. The band of waves near 7 weeks shown in the third column is rather broad, and if the result of the first three operations shows the presence of another wave near 7 weeks a much higher degree of selectivity can be obtained by the fourth operation.

In many cases a preliminary smoothing operation of the data is desirable by a moving average. Where exact results are desired for the amplitude of the wave the amount of reduction caused by this moving average should be considered. A table of moving average reductions is given in Table 4, which by simple proportion can be used for any length of wave and moving average. If a 3 moving average were used for the operations of the last page, its effect would be calculated for each of the wave lengths shown before that of the 9-A and 9-B operations.

A 2-3-4 moving average is a combination often used for smoothing of erratic data. To get the effect of this on a set of waves like those on the last page, three columns would have to be set up, showing the reduction by each operation on each wave length. The effect of this set of operations on a wave 108 months long would be trivial, only reducing the wave to .996 of true size, but an 8 months wave would be reduced to .45 of its true amplitude.

To supplement the tables of amplification factors (Table 1), Figure 8 is made of a typical operation for each of Methods 9-A and 9-B. This shows that for each of these operations the curve oscillates from 2.000 to zero with greater and greater frequency as the waves become shorter. One way to avoid the interference from short