LESSON VI

Supplement 5

THE TIME CHART PROBLEMS

We are getting now into the realm where there is no longer any "right" answer to things. There is your answer and my answer.

Of course I have probably had more experience at this sort of thing than you, so I suppose my answer is more likely to be right. However, in any given instance, this is not necessarily so. Do not therefore, take what I say as gospel. I could be wrong. (Every man is entitled to his first mistake!) Use the old beano. It's lots of fun to trip the teacher.

Mostly the students got the same answers that I did. More in detail--

Problem 3. (You will remember that this problem asked you to find the most probable cycle in the range 7 to 13 years in the logs of the S. & P. C. A. actuals, expressed as deviations from their 9-year moving average.) My answer is that a cycle a little over 9 years long--perhaps 9.1 or 9.2--is most probable, but that there are hints of a number of other cycles within this range which should be investigated also.

Problem 4. (This problem asked you to find the most probable cycle in the S. & P. C. A. 9-year moving average itself.) My answer is that the most probable cycle is about 20 or 21 years long.

Problem 5. (Here, using the actual S. & P. C. A. logs, you were asked to make a time chart of the length found best in Problem 3.) In working this problem I used a 9 1/7-year length. My conclusion was that this cycle is present in the actual figures as well as in the deviations, but of course not quite so clearly.

Now, although most of you came to the same conclusions that I did, there were frequent differences of detail. Therefore, I think it will be helpful if I work out at least part of this group of problems for you so that you can compare what I did with what you did. I shall do so.

LESSON VI

Supplement 6

SOLUTION TO PROBLEMS

Probably the best way to correct the time charts called for by Lesson VI is to show you how I would have drawn them, and to tell you, for each step of the process, just why I did as I did. You can then compare your work with my work. In some instances our methods and/or our results may differ. In such cases, if you can see the logic of what I did, you can correct your work accordingly. If you think your way better, write me and I will either stand corrected or argue it out with you.

As Problem 5 is simpler of solution than Problem 4, I shall start by giving you my solution of Problem 5. I shall then work out at least a partial solution to Problem 4.

LOG OF PROBLEM 5

Inspection of the 9-year moving average of logs of actuals plotted in S. & P. C. A. #3 (the broken line) shows four and a half clear cut waves.

		Interval
Lows fall:	1858 or earlier	
		17 years, or more
	1875	
		20 years
	1895	
		22 years
	1917	
		19 years
	1936	
Highs fall:	1871	
		14 years
	1885	
		28 years
	1913	
		14 years
	1927	
		22 years, or more
	1949 or later	

The fact (a) that there are two highs 14 years apart, (b) that the 28 years between them is two times 14 years, and (c) that the present wave can hardly end short of 1952, which would make it at least 25 years long (nearly 2 x 14 years), suggests the possibility of a cycle about 14-years in length.

There may indeed be a cycle of about this length present in this series of figures. However, starting at 1885, there is no high at or near 1899; no low at or near 1906, no high at or near 1941, and no low at or near 1948. In other words, there is <u>no</u> consistent 14-year <u>rhythm</u> in the 9-year moving averages of the S. & P. C. A.

By inspection, the only rhythm present in this series of figures is the one of 19½ or more years suggested by the waves. (From the low of 1858 or earlier to 1936 equals 78 years or more. Four cycles in 78 years or more. Four cycles in 78 years or more equals 19½ years or more per cycle.) To try to get a better idea of the length of this cycle let us make a 20-year time chart. This is done. See T. C. No. 1.

We start this time chart with a base one year before the beginning of the series, so as to have room to plot anything we wish back as far as there are any figures at all.

In 1858, we insert a red bar with the symbol ↑ to indicate that the low comes in that year or earlier. Even though there is no clearspan number available we use a full standard bar because the value is so very low. However we use parentheses to emphasize the fact that we do not know that this really is a full standard low.

Similarly for 1949 we use a bar with the symbol ↓ to indicate that the high is in 1949 or later. As a matter of fact the high of the 9-year moving average cannot come before 1952 unless the 1955 value is lower in proportion to 1954 than 1930 was to 1929. The 1952 position is indicated on the time chart by parentheses.

Both the 1858 "low" and the 1949 "high" are of course ignored in trying to determine cycle length. They help us to know that there is something of this general order of magnitude; until we can time them accurately they are of no use in determining cycle length.

Using method of semi-averages we get semi-average points 2 positions (years) down (after base) between the 1873 and the 1893 columns; 3½ positions down between the 1913 and the 1933 columns, 2 columns later. A drop of 1½ positions in 2 columns (two 20-year cycles) suggests $20 + \frac{1\frac{1}{2}}{2}$ or 20.75 as the length.

Similarly for the highs, the first semi-average falls between the 1853 and the 1873 column, 15 positions (years) down; the second semi-average falls between the 1893 and the 1913 columns, 18 positions down. The line of highs is thus $20 + \frac{3}{2} =$ 21.5 years long.

Averaging these two values gives us 21.125 or 21.1 as a rough guess of the probable length of this cycle.

In studying this cycle further, keep in mind the possibility of the 14-year cycle mentioned earlier. Also keep in mind a possible longer cycle of which there is a slight hint from inspection of the 9-year moving average curve as plotted in S. & P. C. A. No. 3.

Just for fun, as a 14-year cycle has been mentioned, let us make a 14-year time chart. See T. C. No. 2.

The "low" in 1858 or before has not been connected, as it probably belongs in the previous (1839) column.

The "high" of 1949 has not been connected for the same reason (it probably belongs in the 1951 column).

The downward step-ladder of the bars suggests that the lows of 1917 and 1936 should be set back a column. However, this would merely give us the 20 or 21-year cycle which we already know about, so we don't do it.

Note that the median position of the highs is 4 positions down and of the lows $10\frac{1}{2}$ positions down, $6\frac{1}{2}$ positions later. As $6\frac{1}{2}$ is close to 7 (half of 14) the 14-year pattern, such as it is, is nearly symmetrical. This fact adds to the credibility of the idea of a possible cycle of this length. It should be kept in mind and sought for later.

Conclusions:

The only rhythm present in the 9-year moving average of S. & P. C. A. is one which yields a T. C. length of about 21-years long.

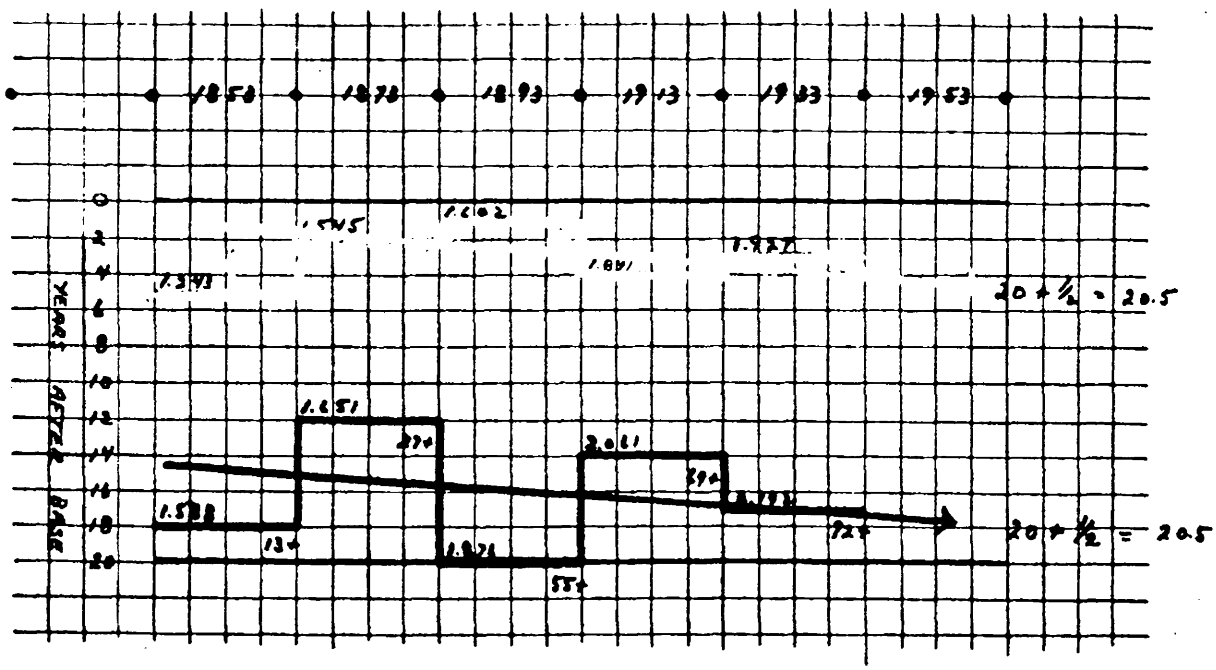
It should be investigated further. Use deviations from a straight line or parabolic trend.

The 14-year behavior is provocative. Clearly however there is no 14-year rhythm. Keep 14-year length in mind in future analysis.

S. & P.C.A.

20-YEAR TIME CHART V_{10} V_3

9-YEAR MOVING AVERAGE OF LOGS OF DATA



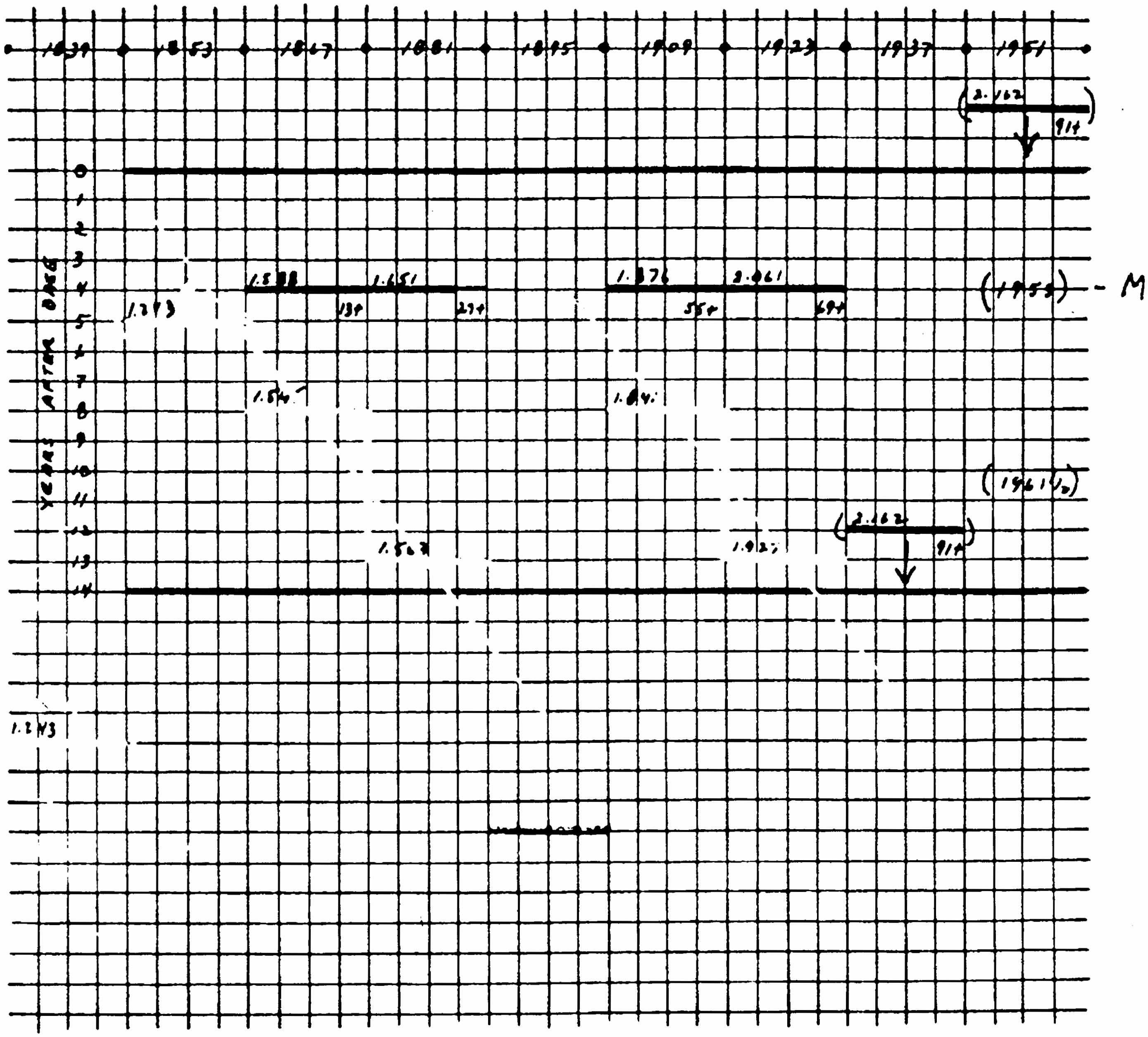
SOURCE: COLUMN F OF T.S.-1 OF S.&P.C.A.

Revised

S. + P. C. A.

14-YEAR TIME CHART V7 V2

9-YEAR MOVING AVERAGE OF LOGS OF DATA



SOURCE: COLUMN F OF T.S.-1 OF S+PCA

LESSON VI

Supplement 7

SOLUTION TO PROBLEM 3

Study of S. & P. C. A. No. 4 (deviations of logs of actuals from their 9-year moving average) shows rather clear cut cycles about 9 years long at each end of the series--a mixed up situation in the middle.

Graduated scale of 9.1 fits fairly well.

Picture clouded by short cycle a little over 3 years long which really should be removed by a 3 year moving average before trying to find longer cycles. (A 3-year moving average will wipe out a 3-year cycle as you will learn in Lesson XI.) See Les. 3 Supl 2 Page 1.

However, let us make a 9-year time chart. See T. C. #3. (The series is not long enough to make a 9.1-year T. C. feasable--only 11 cycles. Hence make 9 year T. C., even though it may droop a little to the right.

We start base numbers in 1853 so as to have room to use any values we wish back to the beginning of the series.

By reference to S. & P. C. A. No. 3 it is clear that no matter how the 9-year moving average goes there will be a crest at or before 1854. This fact is indicated by a bar with an arrow ↑, but as we do not know just when the top really came we do not use this bar in determining length.

Slope of bars for both highs and lows determined by eye (after covering up bars of 1854 and 1954 so they cannot be seen). Lines are drawn in. Convenient points are chosen on the lines and marked with squares. Lines slip about 1 year from square to square (7 columns, or as I prefer to call them, 7 9-year cycles). Hence indicated length is 9 1/7-years.

Semi-average points are computed. They show the same slope for lows, slightly more (9 1/5) for highs. I still prefer the fit by eye.

Note that ideal time for current high, according to line of slope, is midway between 1954 and 1955--i.e. end of 1954.

Tops more regular than bottoms. Suggests possible concurrent cycle of half the 9 1/7 length (4.57 year--just about 55 months, a well known length) coinciding on the crests, opposed to trough. Watch out for it. If not taken into account this half length cycle (if present) may distort the length of the 9 + cycle to make it appear longer or shorter than it really is.

Checked back to S. & P. C. A. #4 with graduated scale. 9 1/7 looks good, but probably something longer or shorter (or both) also present to cause fade out from 1900 to 1925. This length would be approximately the next higher or lower harmonic, i. e. 1/9 or 1/11 of 92, the total number of years in our series of deviations. 1/9 of 92 is 10.22, say 10 1/4; 1/11 is 8.36, say 8 1/3. 8 1/3 looks better. Try 8 1/3 T. C.

Did so. See T. C. #4.

Results not very good. Nowhere nearly as good as T. C. #3.

For what they are worth however results suggest 8 1/6 for highs and 8 2/9 for lows, as per slope lines added.

Note that I have fitted these slope lines by eye. Note also that in measuring slope I have picked points on the line which are in columns of the same length. The 1861 column is 9 years long; so are the 1936 and the 1961 columns. If I had picked a point in the 1928 column for example (a column before a 9-year column), I would have picked the other point in a column prior to another 9 year column.

There could be something a little over 8 years long in this series, but it is quite inconclusive.

One of the chief uses of a time chart is to suggest cycles of a length <u>other</u> than the length of the time chart that may be present. With this in mind note upward stepladder from the red 17 in 1896 to the red 42 in 1921, as indicated by the red dashed line. This slope suggests that, working our way toward the left, we should move the bars of the 1878 column to the previous (1870) column, and so on. Going to the right we would move the bars of the 1928 column into the 1936 column, and so on.

The final result, when we got through, would look like S. & P. C. A. #4-A.

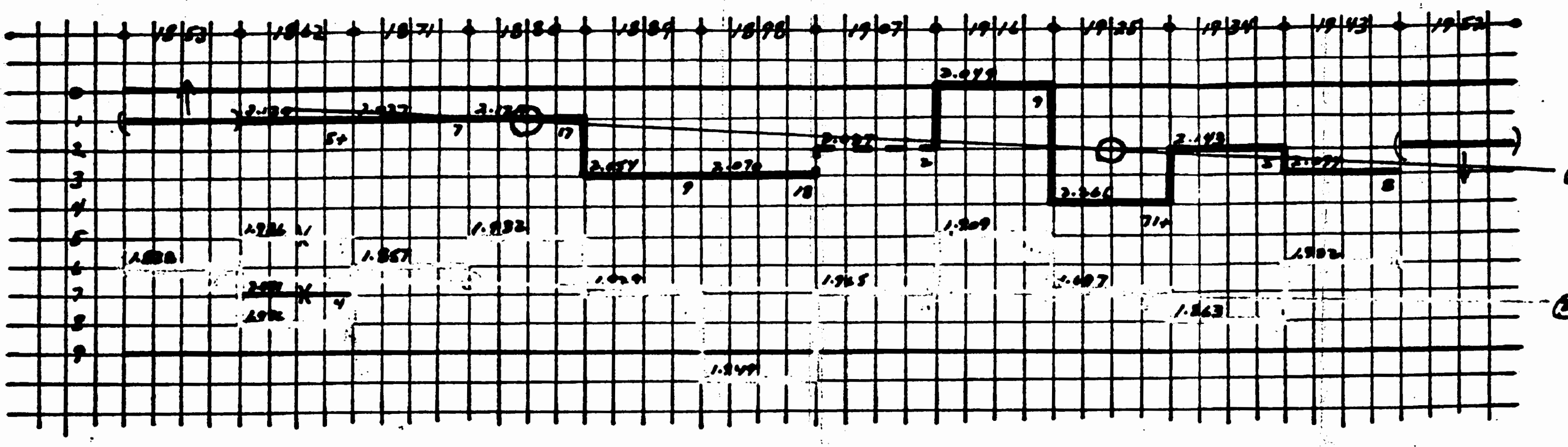
The upward sloping lines measures about 6 years, both red and black.

In T. C. 4-B I have left out the bars as we originally used them in the 8 1/3 year time chart so you can see the 6-year cycle more clearly.

Of course we should construct a 6-year time chart to get the length with greater accuracy. We do, as T. C. #5. Of course in a 6-year T. C. the bars with clearspan of red and black 3 becomes full standard.

It is clear that we must adjust for the 9 1/7 (or whatever) before we can make much progress with whatever it may be that washed the 9 1/7 out in the middle.

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



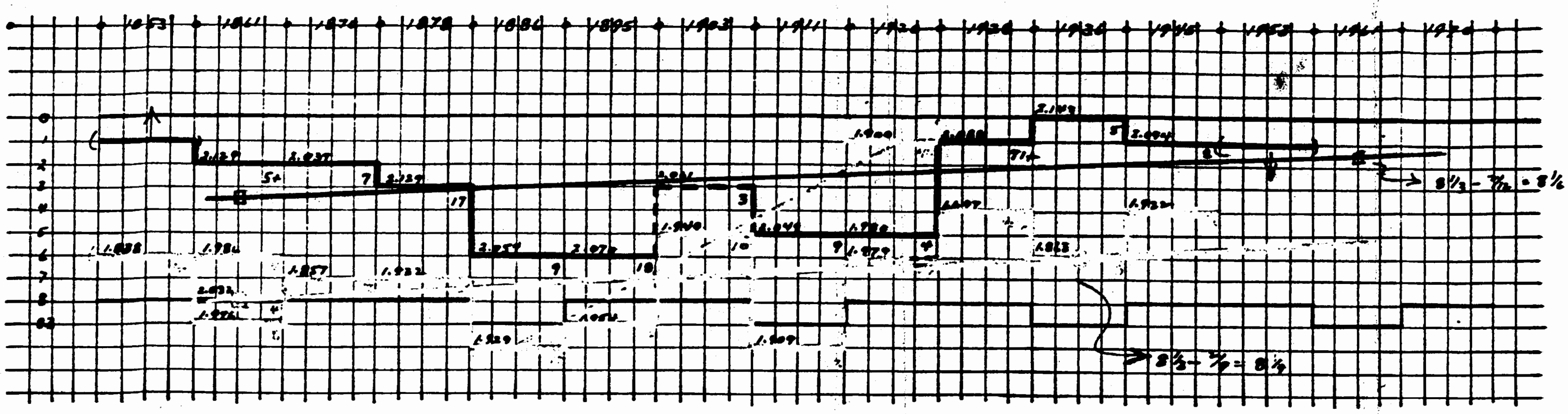
① $9 + \frac{1}{5} = 9.2$ YEARS

② $9 + \frac{.5}{5} = 9.1$ YEARS

INDICATED CYCLE LENGTH

8.3 - YEAR TIME CHART V_4 V_1

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE

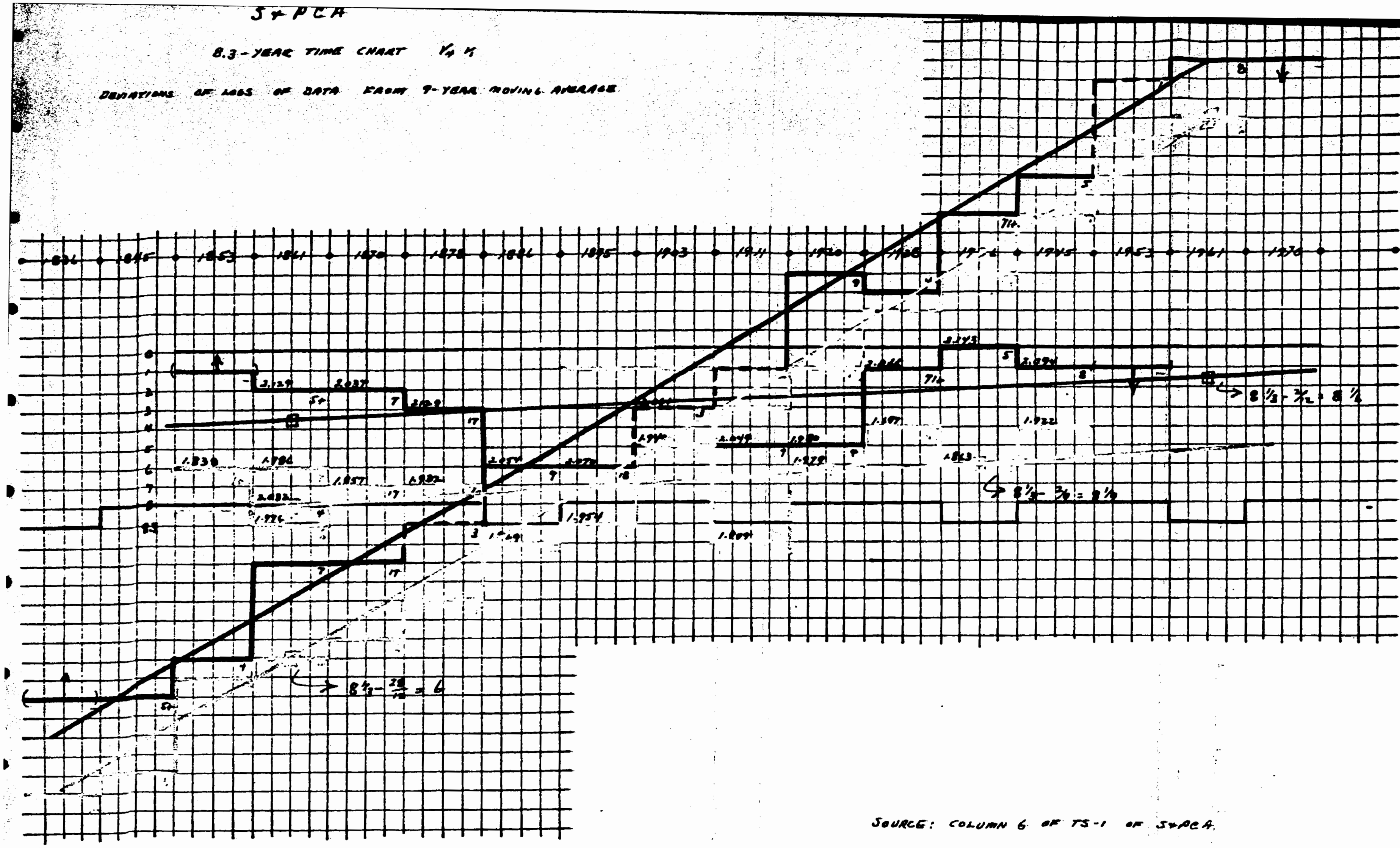


SOURCE: COLUMN 6 OF TS-1 OF ST-FCA

S+PCA

8.3-YEAR TIME CHART 1/4 K

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE

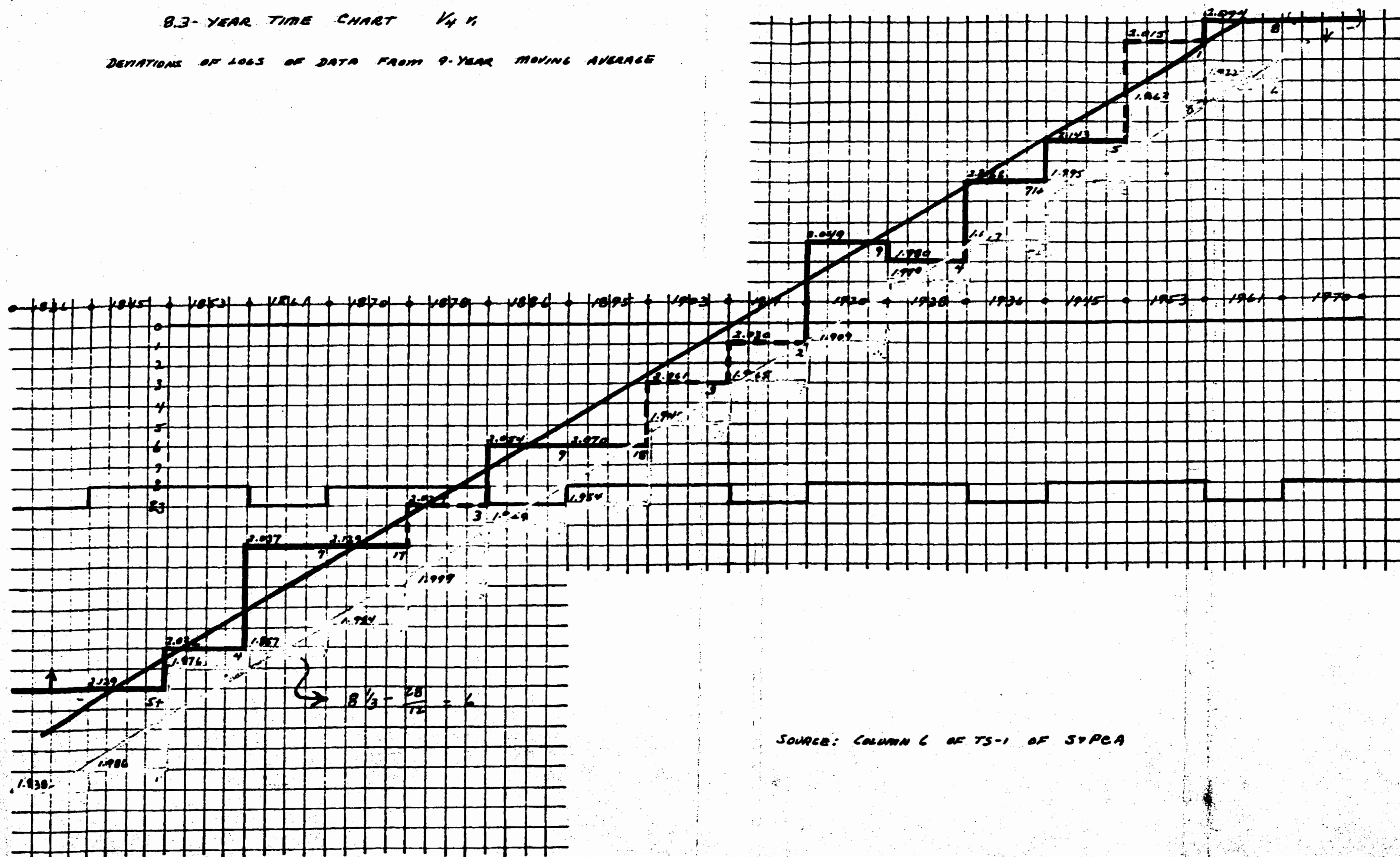


SOURCE: COLUMN 6 OF TS-1 OF S+PCA.

S+PCA

8.3-YEAR TIME CHART 1/4 Y.

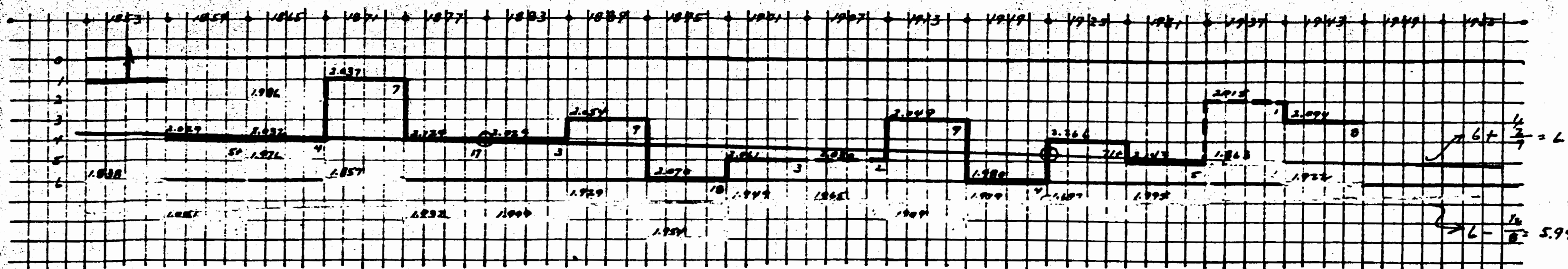
DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



SOURCE: COLUMN 6 OF TS-1 OF S+PCA

6-YEAR TIME CHART V_3 1.

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



CONVERGING BLACK & RED LINES - OBVIOUSLY IMPOSSIBLE

BEST GUESS WOULD BE 6-YEAR CYCLE

SOURCE: COLUMN 6 OF TS-1 OF 3rd PCA

LESSON VI

Supplement 8

SOLUTION TO PROBLEM 3—CONTINUED

In Supplement 7 I showed you how I would start to solve Problem 3. I would get the best possible fit for the most obvious cycle by thumbing, trued up by a graduated scale. Then I would make a time chart of the indicated length. I would do this, first, to get the length of the cycle as exactly as possible, second to get additional evidence as to the validity of the indicated length, and third to see if I could get suggestions of cycles of other lengths which might also be present in the figures.

Having got the first cycle pinned down as best I could by graduated scale and time chart I would then start to look for another cycle.

To get my second cycle I would search again by thumb and graduated scale and, in addition, use my first time chart, to try to get hints of other lengths.

I would proceed in this fashion until the entire range of wave lengths in which I was interested had been scanned.

However, it is not always possible to proceed this way. It is one of the great advantages of the time chart that when inspection, thumbing, and the graduated scale fail, the time chart will often by itself give you the answer you are looking for.

In the solution of Problem 3 of Lesson VI most of you proceeded this latter way in more or less mechanical fashion to make 7-, 8-, 9-, 10-, 11-, 12-, and 13-year time charts. If you could not get hints in other ways this method is all right.

So that you can see what you did right and wrong--at least in comparison with the way I would do it--I enclose herewith 10-, 11-, 12-, and 13-year time charts. With these in front of you, you can correct your own work.

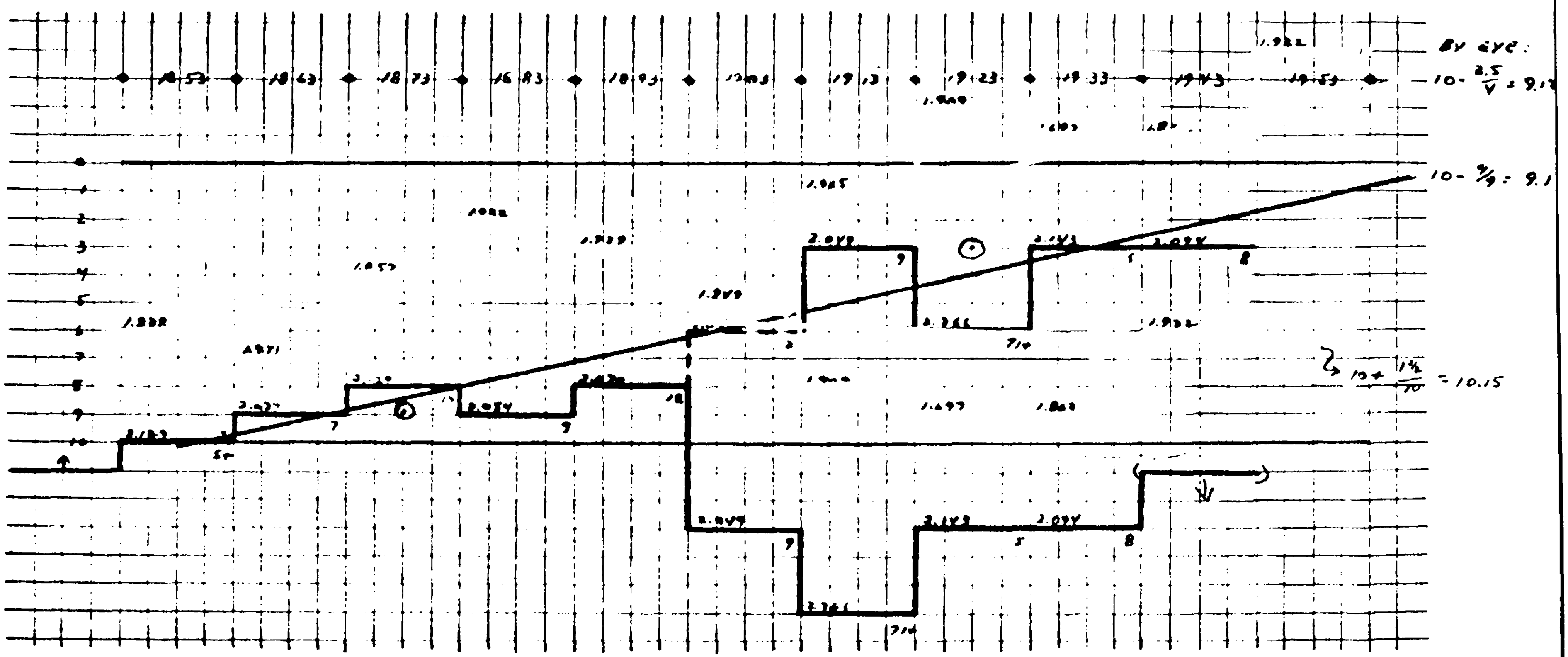
Let me re-emphasize that if you cannot see the logic of the time charts as I have made them, write me (via Miss Shirk).

I am not sending you a 7-year time chart, for two 7-year time charts have already been sent you with Supplement 4.

Also, as 9-year and 8 1/3-year time charts were sent with Supplement 7, I am not sending you 8- and 9-year time charts herewith. You can correct your 8-year time chart from the 8 1/3-year one, just as easily as from an 8-year one.

10-YEAR TIME CHART Y_5 Y_2

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



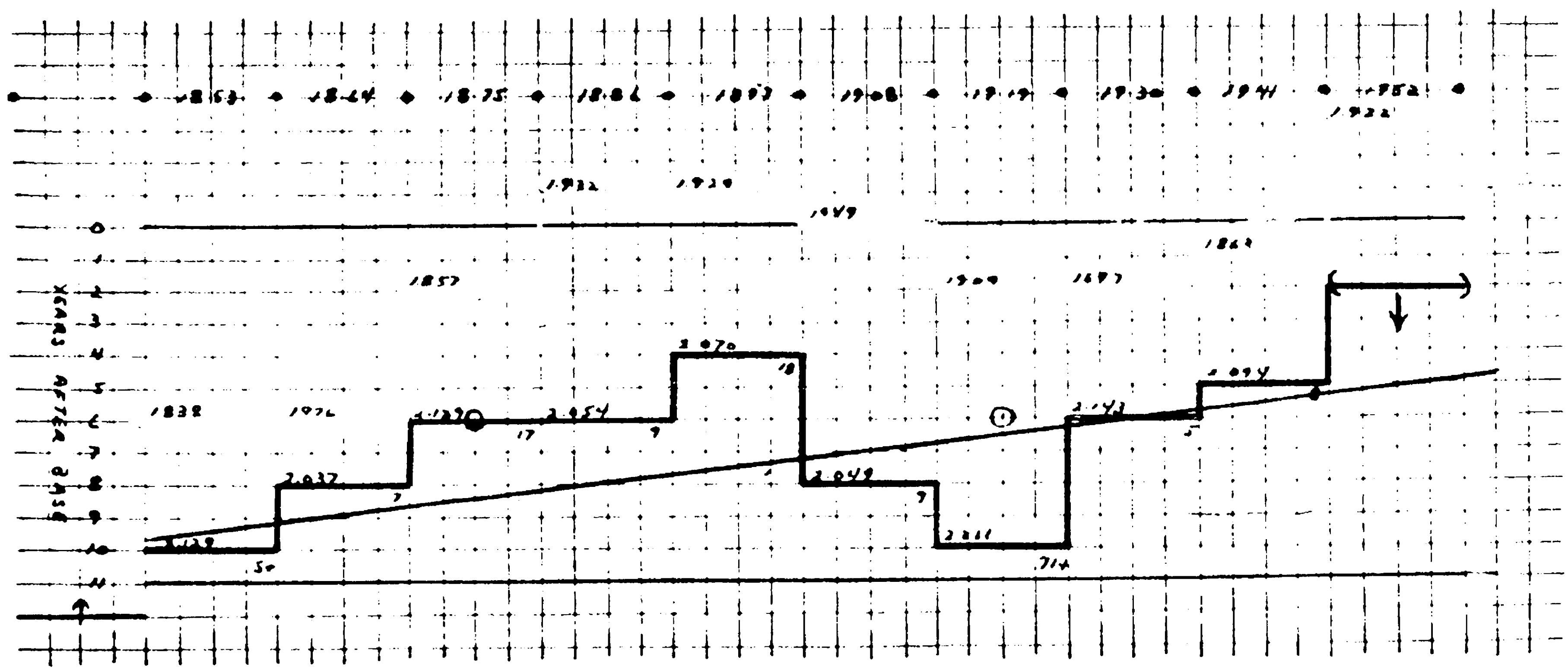
SHOWS ABOUT 9.1-YEAR CYCLE

ALSO A SUGGESTION OF A 10.15 NEARLY AS GOOD - IN FACT, BETTER FROM CLEARSPAN STANDPOINT.

SOURCE: COL. 6. OF T.S.-1 OF S.&P.C.A.

11-YEAR TIME CHART V_5 V_2

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE

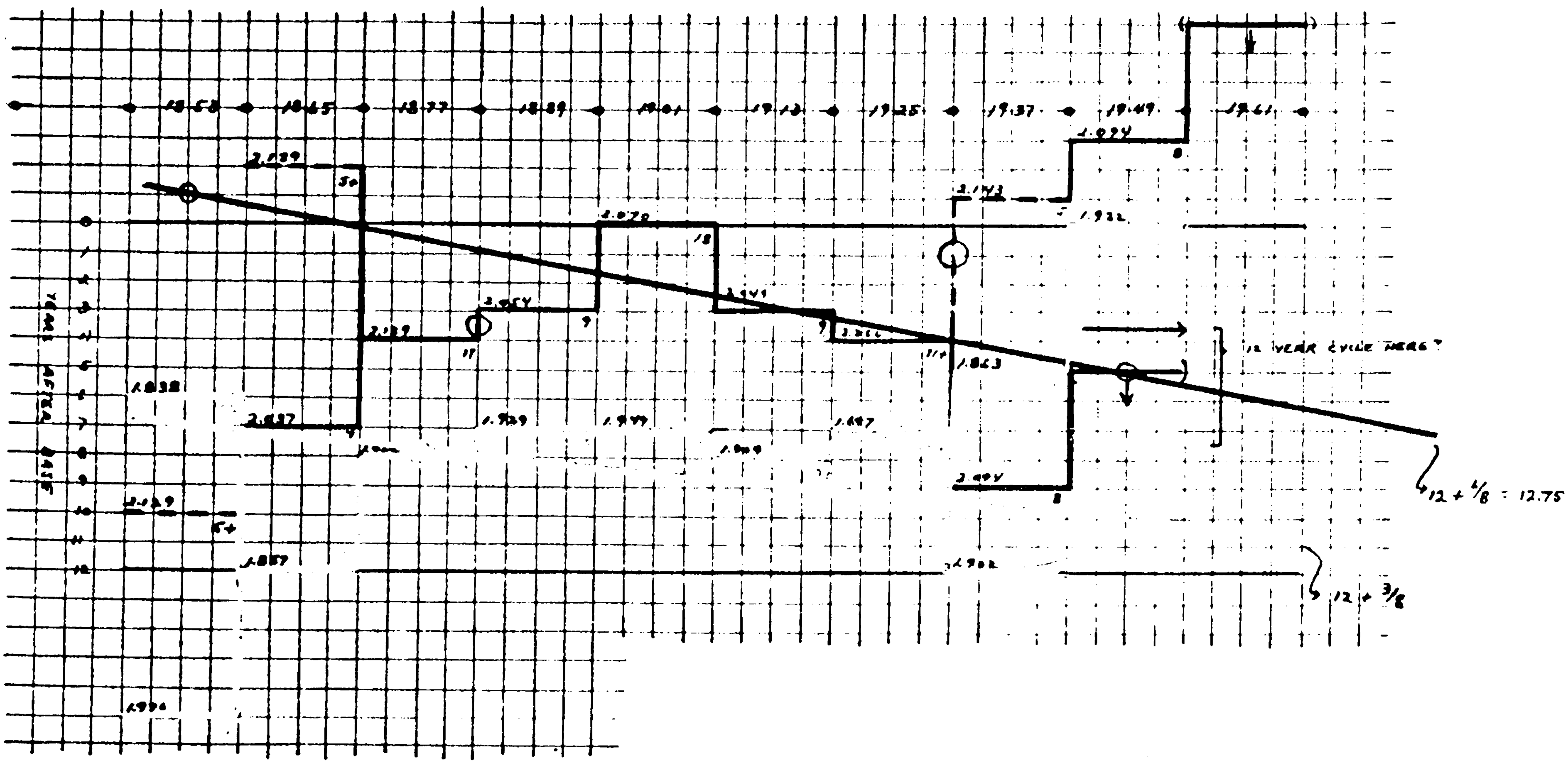


SLOPE BY EYE: $11 - \frac{5}{9} = 10.45$

SOURCE: COLUMN 6 OF TS-1 OF S...

12-YEAR TIME CHART V6 V2

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



IF 12-YEAR CYCLE IS PRESENT, IT IS OBSCURED BY POSSIBLE 10+ CYCLE.

THE 10+ POSSIBLY BETTER, BUT CHANCE OF 12 3/4, OR MORE LIKELY 12 1/2.

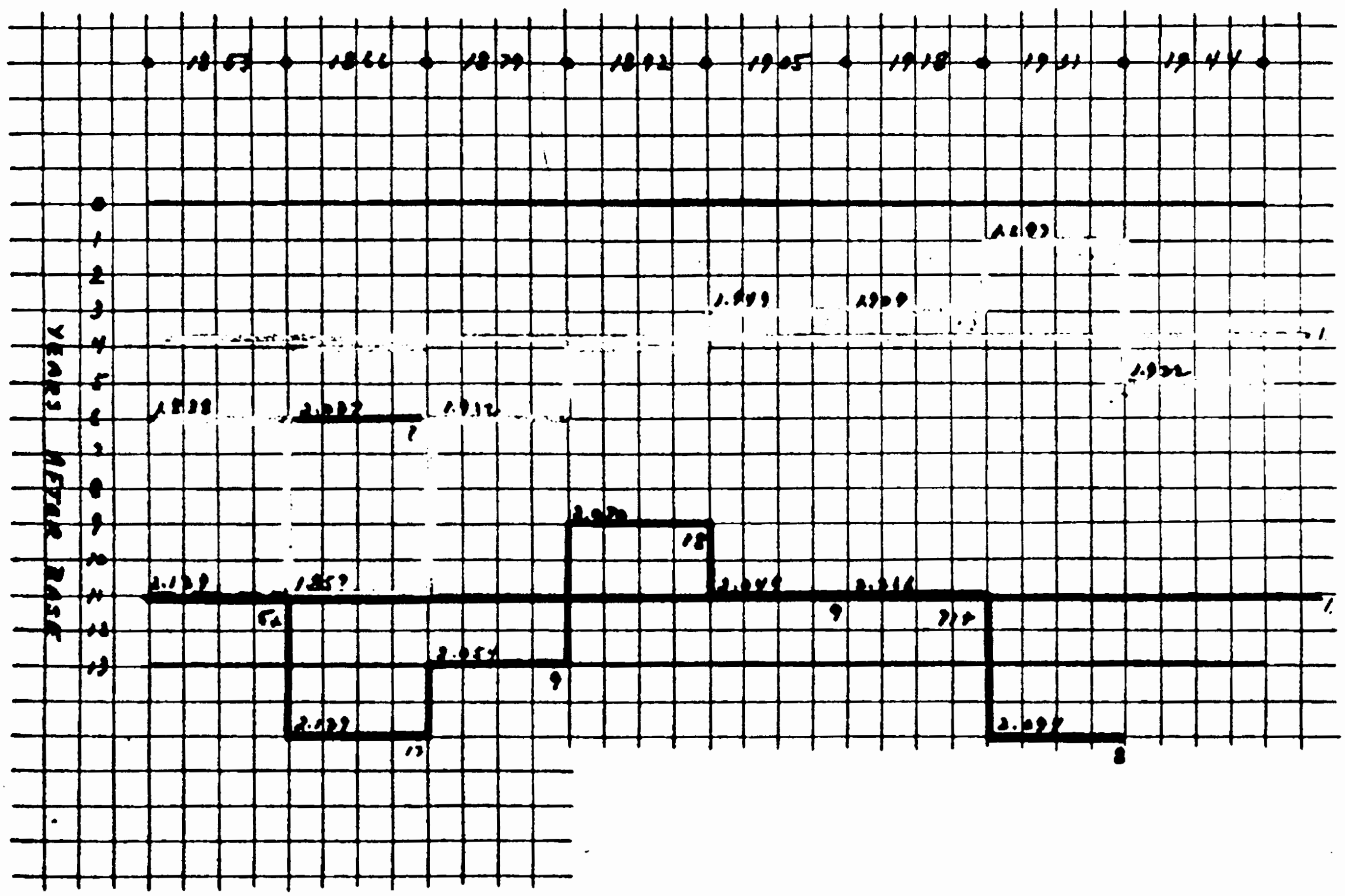
COULD ALSO BE A 12.

SOURCE:

S. & P. C. A.

13 - YEAR TIME CHART V_6 V_2

DEVIATIONS OF LOGS OF DATA FROM 9-YEAR MOVING AVERAGE



1. POSSIBLE 13

SOURCE: COL. 6. OF T.S.-1 OF S.& P.C.A.

LESSON VI

Supplement 9

MAKING TIME CHARTS WORK HARDER

There are many ways in which you can make time chart work harder. Some of them are listed below.

1. Double the bars having double or triple your standard clearspan. Longer cycles are thus often suggested. (See Lesson VI, page 13.)

2. Where you have but a few repetitions of a cycle it often helps to use bars not only for highs and for lows but for upward and downward crossings as well. I use blue and green for the additional bars. When you do this you have four lines across the time chart instead of two. This gives you four slope lines to average. Sometimes this helps.

3. Time charts often enable you to notice reversing cycles. (For a discussion of reversing cycles see Lesson V-A.)

4. Cycles of a particular length plotted on a grid twice or three times as long as the particular length are often illuminating. (See Lesson VI, page 13.)

5. Finally comes the matter of getting from a time chart of one length hints of cycles of other lengths. This subjedt has not perhaps yet been fully covered.

<u>Any</u> tendency for bars to crawl uphill or downhill suggests the possibility of another cycle.

For instance refer to the time chart illustrated by the blue print, page 35 of Lesson VI.

Note first of all the drastic drop off (toward the right) of the reds from the red bar at month 319 to the red bar four columns later 7 lines below the bottom of the grid at position 434.

Connect these two bars with a light pencil line. The pencil line falls off 23 lines in 4 columns, suggesting a length of $23 + \frac{23}{4} = 28\ 3/4$ months.

Now continue the pencil line upward to the left until it cuts the top of the grid (about a quarter of the way into position 289).

Month number 289 at the top of the 7th column is the same as month number 289 at the bottom of the 6th column. Therefore you can continue your line upward and to the left from the bottom of the 6th column, using the same slope (23 lines in 4 columns). Your line will cut the top of the grid about a quarter way into the position representing month 174.

Similarly you can extend your line to the right. It will run from position 404 at the top downward to position 519 at the bottom, and again from position 519 at the top downward to position 634 at the bottom.

Note now that there are fallings off of the red bars from month 158 to month 218 and from month 558 to month 615, but our light pencil line does not go through either of these fallings off. It is too steep.

Now, either by trial and error or by the use of a little arithmetic find the slope of a new line that will take <u>all three</u> of these drastic downsweeps into account.

I shall first proceed by trial and error. To start with, at a guess, I'll run a line from position 558 to position 427. This line has a slope of 16 lines in 5 columns.

Continuing this line shows that it also is not quite right. Perhaps a line that ran through month number 158, down to the bottom of the grid, once across the grid, and down again to the bar at month number 558 would be best. Such a line would have to slip 16 + 23 + 16 lines in 15 columns, of 3 2/3 positions per column. This is 11 positions in 3 columns.

(If you count from the column which contains month number 158 to the column which contains month number 558 you will get 17 columns. However, if you were to connect these two bars by one single line of the sort described you would have to set your month 558 bar back two columns. Hence to get the slope you count two columns less.)

A line drawn in on this basis fits fairly well. It suggests that we try a 26 2/3-month time chart.

Similarly the upward slope of the high from month 231 to month 312 could be investigated. In fact <u>all</u> tendencies of highs and/or lows to run upward or downward offer clues to possible additional cycles. Usually they are all worthy of the few extra minutes required to explore them.

LESSON VI

Supplement 10

ANOTHER TIME CHART

Because time charts are rather scarce--except in the files of the few cycle analysts who know about them--I think you will be interested to see a time chart of the 9.18 month cycle in the ton miles of the Canadian Pacific Railway. This time chart was made by G. Meredith Rountree, Chief Statistician of that company. It is reproduced from Cycles for June, 1951, page 216.

The time chart is of the deviations of actual ton miles, adjusted for seasonal variations, from their 9-month moving average.

You will note that, to simplify the presentation, Mr. Rountree made a 9-month time chart instead of a 9.2- or 9.18-month time chart. He then allowed the bars to fall off to the right and fitted a 9.18-month line to them.

You will also note that, because color was not available for reproduction, some of the usual time chart conventions in regard to color had to be violated.

* * *

To fill up the back of this page I am reproducing for you the Foundation for the Study of Cycles Index of Railroad Stock Prices, 1831-1951, reprinted from Cycles for April 1952.

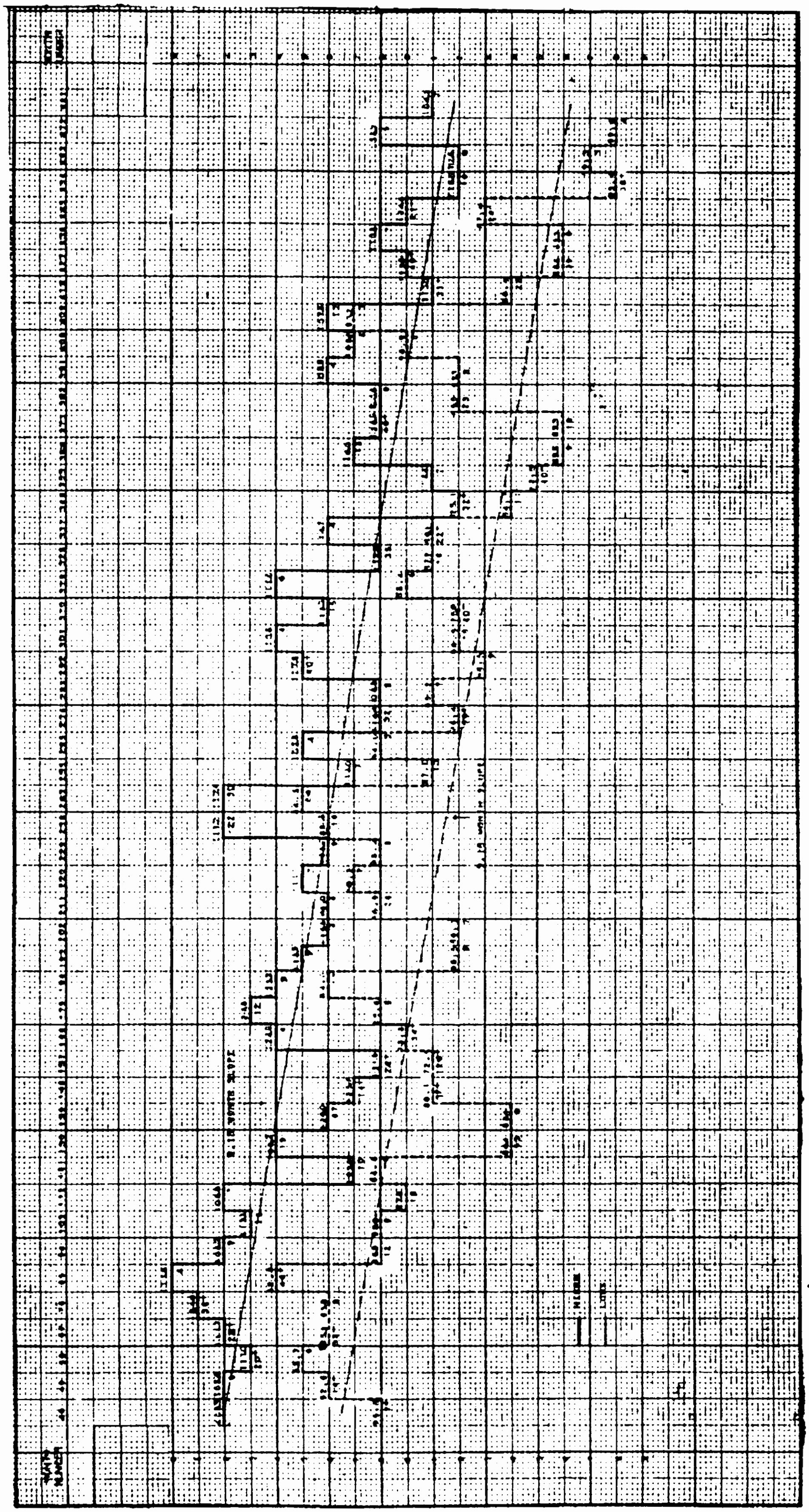


FIG. 6. A 9-MONTH TIME CHART OF THE DEVIATIONS OF THE C.P.R. TON MILES FROM THEIR 9-MONTH MOVING AVERAGE.

rinted from Cycles, June 1951, page 216.

FOUNDATION FOR THE STUDY OF CYCLES'

INDEX OF RAILROAD STOCK PRICES, 1831-1951

To construct an index of railroad common stock prices suitable for cycle analysis we proceeded as follows:

As the basis of this index we used the following source material: (1) For the period 1831-1870 we used the Cleveland Trust Company Index of Rail Stock Prices; (2) for 1871-1938 we used the Standard and Poor's Corporation Index of Railroad Stock Prices as extended backward by Alfred Cowles III and Associates and published by them as Index P-3 in their book **Common Stock Indexes**; and (3) for the period 1939-1951 we used the Standard and Poor's Corporation Railroad Stock Index as published in the Survey of Current Business.

As the Standard and Poor's Corporation Index is now based on 1935-39=100, but was formerly based on 1926=100, and as the Cleveland Trust Company Index differs from the Cowles Index, adjustments were made as follows:

1831-1870, x 1.149
1871-1938, x 2.643
1939-1951, x 1.000

The values of the new index, thus derived, are given below.

You can keep this index up to date from year to year or month to month or even from week to week from the Survey of Current Business.

TABLE 1

THE FOUNDATION FOR THE STUDY OF CYCLES INDEX OF RAILROAD STOCK PRICES, 1831-1951.

Year	Index	Year	Index	Year	Index	Year	Index	Year	Index	Year	Index
1831	55.8	1851	74.3	1871	115.5	1891	123.2	1911	262.7	1931	191.6
32	57.8	52	83.5	72	121.6	92	134.3	12	265.9	32	69.8
33	75.6	53	85.1	73	116.0	93	114.7	13	238.9	33	99.7
34	69.1	54	69.9	74	109.7	94	104.9	14	222.0	34	109.7
35	93.0	55	61.4	75	106.5	95	107.6	15	213.8	35	89.9
36	80.3	56	58.7	76	97.3	96	101.2	16	229.7	36	135.3
37	60.3	57	48.9	77	74.8	97	106.0	17	201.7	37	130.3
38	53.2	58	44.4	78	81.1	98	119.2	18	181.6	38	69.0
39	48.9	59	40.1	79	100.7	99	147.5	19	185.3	39	74.7
1840	42.1	1860	48.1	1880	130.3	1900	150.9	1920	169.2	1940	71.1
41	39.2	61	44.3	81	157.0	01	202.7	21	163.3	41	70.6
42	29.0	62	56.7	82	147.2	02	230.0	22	192.2	42	66.1
43	36.0	63	97.6	83	141.4	03	200.4	23	190.0	43	88.7
44	57.5	64	112.6	84	118.9	04	199.6	24	202.7	44	101.0
45	55.8	65	96.9	85	114.7	05	258.0	25	236.6	45	136.9
46	59.0	66	100.0	86	134.3	06	275.9	26	264.3	46	143.0
47	64.0	67	93.5	87	138.5	07	227.6	27	314.8	47	105.3
48	57.8	68	111.5	88	127.9	08	228.4	28	339.6	48	114.8
49	60.8	69	117.2	89	127.2	09	281.8	29	389.3	49	96.6
1850	67.0	1870	115.3	1890	127.9	1910	266.4	1930	330.1	1950	116.7
										51	148.8

TABLE 2

DEVIATIONS OF THE LOGS OF THE INDEX OF RAILROAD STOCK PRICES (SHOWN IN TABLE 1 ABOVE)

FROM THEIR 19-YEAR MOVING AVERAGE

Year	Logs	Year	Logs	Year	Logs	Year	Logs	Year	Logs	Year	Logs
1831	*2.039	1851	2.123	1871	2.057	1891	1.994	1911	2.065	1931	2.086
32	*2.050	52	2.164	72	2.057	92	2.025	12	2.079	32	1.671
33	*2.161	53	2.162	73	2.026	93	1.945	13	2.033	33	1.849
34	*2.118	54	2.065	74	1.997	94	1.895	14	2.002	34	1.910
35	*2.244	55	1.991	75	1.979	95	1.893	15	1.991	35	1.842
36	*2.175	56	1.950	76	1.937	96	1.851	16	2.026	36	2.035
37	*2.047	57	1.871	77	1.815	97	1.855	17	1.966	37	2.037
38	*1.986	58	1.818	78	1.845	98	1.893	18	1.913	38	1.788
39	*1.946	59	1.761	79	1.937	99	1.973	19	1.918	39	1.849
1840	1.878	1860	1.826	1880	2.047	1900	1.965	1920	1.869	1940	1.857
41	1.843	61	1.779	81	2.125	01	2.075	21	1.849	41	1.865
42	1.708	62	1.879	82	2.097	02	2.115	22	1.927	42	1.819
43	1.799	63	2.108	83	2.076	03	2.035	23	1.951	43	*1.977
44	1.996	64	2.158	84	2.000	04	2.015	24	1.997	44	*2.033
45	1.991	65	2.081	85	1.984	05	2.109	25	2.079	45	*2.165
46	2.021	66	2.079	86	2.051	06	2.122	26	2.149	46	*2.184
47	2.057	67	2.036	87	2.057	07	2.020	27	2.234	47	*2.051
48	2.015	68	2.100	88	2.017	08	2.010	28	2.274	48	*2.088
49	2.039	69	2.106	89	2.010	09	2.097	29	2.356	49	*2.013
1850	2.083	1870	2.081	1890	2.010	1910	2.067	1930	2.304	1950	*2.096
										51	*2.202

* FROM THE ARBITRARY EXTRAPOLATED VALUES PLOTTED IN FIG. 1, PAGES 124/25

LESSON VI

Supplement 11

MORE ABOUT HOW TO GET HINTS OF CYCLES

Several years ago I wrote an article called "How to Get Hints of Cycles." This article was printed in Cycles for September 1950 and October 1950.

It more or less repeats what I have said in Section A of Lesson VI and in other places in the course, however it does have a little new material. And anyway, saying the same thing in other words sometimes helps. I am therefore having it reproduced and am sending it to you herewith. I hope you find it useful.

Many of the methods mentioned will be treated at greater length later in the course. Do not be disturbed if the outline enclosed is too sketchy to be entirely clear. This defect will be corrected as the course progresses.

Technical

CYCLE ANALYSIS

HOW TO GET HINTS OF CYCLES

In Two Parts - Part I

To make a cycle analysis of a series of figures, you must do four things: First you must get a hint of a cycle that may be present. Second, you must find out all you can about this cycle. Third, you must come to an opinion in regard to the significance of the cycle and accept it as a probable reality, or reject it as probably due to random forces. Fourth, you must see if there is a hint of another cycle present, and if so, you must go through the same steps all over again. This process must be continued until all the important cycles present have been discovered, studied, characterized, and evaluated. You have now completed your cycle analysis.

If, following your analysis, you adjust the original figures for the combined effect of all the accepted cycles, the remainder will be a combination of the trend and the sporadic or non-cyclic fluctuations (including in this category the unimportant cycles, if any). To these figures you can fit a trend which you can project with due caution. You can then 'wrap' the discovered and accepted cycles around this trend to show what will happen, except for non-cyclic fluctuations, if the trend and cycles continue as determined.

You will recall that your first step is to get a hint of the cycles that may be present. How do you do this?

At the Institute held at Foundation Headquarters from June 5th to June 10th, we discussed fifteen of the ways in which this can be done. We also discussed types of machines that have been found useful in minimizing computation.

An outline of this part of the work of the Institute follows:

HOW TO GET HINTS OF CYCLES
THAT MAY BE PRESENT
IN A SERIES OF FIGURES

Graphic Methods

1. Simple inspection
2. Quick count
3. Graduated rule
4. Time chart analysis

Computational Methods

5. Limited data
6. Autocorrelation
7. Relatives: (a) simple; (b) complex
8. Moving averages
9. Section moving average: (a) simple; (b) weighted
10. Elimination of trend: (a) by comparison of the data with their moving averages; (b) by other means
11. Comparison of one moving average with another
12. Periodic tables
13. Harmonic analysis: (a) simple, and (b) multiple
14. Adjust for cycles already found
15. Dr. Norbert Wiener's methods

Machines

1. Optical machines
2. Mechanical machines
3. Electrical machines
4. Punch card and tabulating machines

It may prove of value to you if this outline is expanded.

I feel somewhat apologetic about giving you merely a brief description of the different methods, for at the Institute we spent up to two hours apiece in developing and describing each of the nineteen headings listed above, but space limitations prevent an adequate treatment in this letter. As far as I can, I will make up for this deficiency by giving you references, and as soon as possible, I will write Technical Bulletins to cover each of the methods or will get someone else to do so.

GRAPHIC METHODS

1. You should chart the figures (usually on semi-log paper). When this has been done you can often pick out the rhythms by **simple inspection**.

2. You can make a **quick count** of the time intervals between successive crests (or troughs) on the chart thus: Crest; 1, 2 3, 4, 5 crest; 1, 2, 3, 4, 5 crest; 1, 2, 3, 4 crest; 1, 2, 3, 4, 5 crest; etc. This method will often disclose crests (or troughs) coming at more or less regular intervals. Such disclosures can be used as a basis for further study. In the example given you could see if the tendency for crests to come at five unit intervals continued throughout the rest of the series of figures.

3. You can make a series of **graduated rulers** scaled off at various uniform intervals and slip them back and forth upon the chart. These rulers can be made of paper. This method will often give hints of rhythms that escape observation by either of the methods named above. Be sure to look for both highs and lows.

4. You can compute the clearspan numbers and make a **time chart analysis**. This method is explained fully in Technical Bulletin No. 3, issued by the Foundation last year. To my mind the Hoskins Time Chart is the easiest and best way to get hints of rhythms when a chart of the figures does not show the rhythms by simple inspection as suggested by methods 1, 2, and 3 above. It is also a very valuable way to check the regularity of the cycles suggested by other methods.

COMPUTATIONAL METHODS

5. The simplest of the computational methods is one brought to my attention by Boynton Hartz of Detroit which he calls the **limited data method**. This method involves reducing excessive fluctuations of the data on the theory that sporadic fluctuations of great magnitude are necessarily the result of random forces, or of combinations of cycles in addition to the cycle for which one may be seeking.

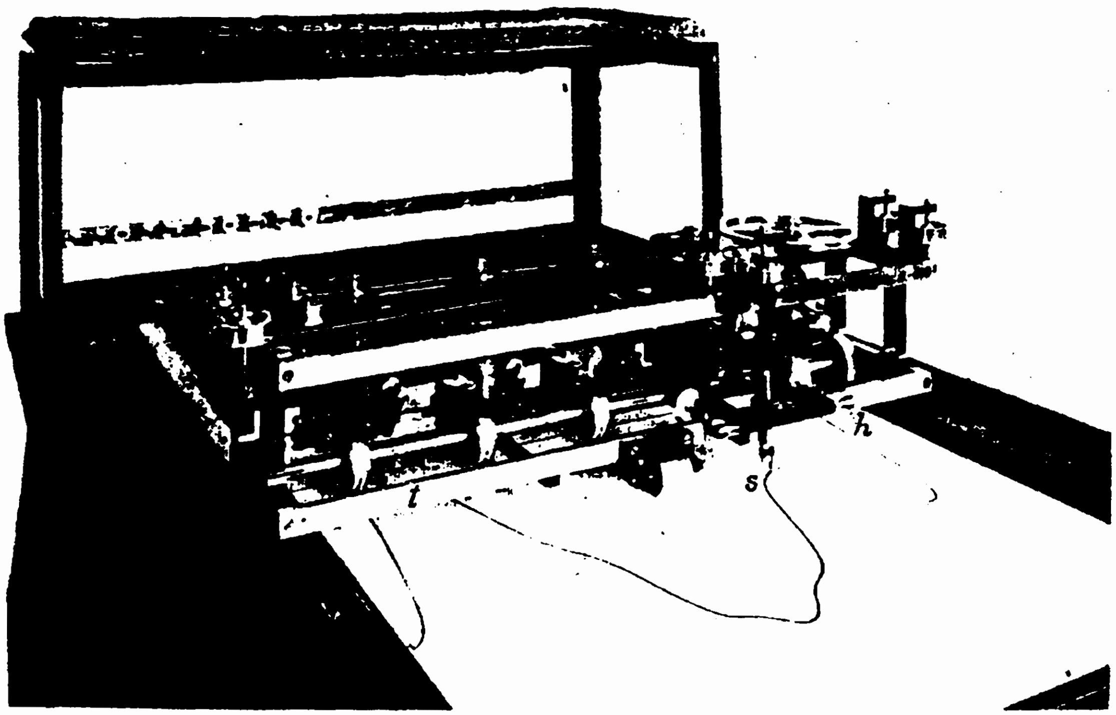
The method involves ranking the moves both up and down and arbitrarily reducing all the extremes in each array to the value of the ninth (or some other) decile.

It can be argued that any such distortion of the series distorts the values and therefore invalidates the conclusions. Of course the method creates distortions, but remember that all we are doing at this point is seeking **hints** of waves. The hints thus obtained must subsequently be verified in other ways. A treatment of these other ways is beyond the scope of this outline.

6. Another method is called **auto-correlation**, which means merely the serial correlation of a curve with itself. You can get a fairly complete description of auto-correlation in **The Analysis of Economic Time Series** by Harold T. Davis, published by the Principia Press of Bloomington, Indiana, in 1941.

7. One way to bring hidden cycles into relief is the computation of **relatives**. There are two kinds of relatives, one called **moving percentages**, the other called **moving differences**. Which to use depends upon the nature of the series being studied.

Relatives always involve a comparison of each point on the curve with another point or points on the curve a fixed distance away. (For example, comparing the sales of January of this year with the sales of January of last year, the sales



HENRICI'S HARMONIC ANALYZER

REPRODUCED FROM THE SCIENCE OF MUSICAL SOUNDS BY DAYTON C. MILLER, © 1916, THE MACMILLAN CO.

of February of this year with the sales of February of last year, and so on.)

To learn more about relatives, you may want to get hold of **Direct Methd of Determining Cyclical Fluctuation of Economic Data**, by M. A. Brumbaugh, Prentice Hall, 1926.

Relatives must always be referred to in such a way as to indicate the time interval of the comparison and whether or not the comparison is made by subtraction or division.

One-item moving differences are ordinarily referred to as 'first differences;' 'second differences' however are not two-item moving differences, but are one-item moving differences of one-item moving differences.

The effect of relatives is to eliminate waves of the length of the relative and of all its aliquot parts (that is, one-half, one-third, one-fourth, etc.), provided of course, that the wave is uniform as to period, shape, and amplitude. Relatives also magnify waves in the general neighborhood of twice their length.

In using relatives you should realize that every excessively low or excessively high value, whether random or not, creates an important reflection in the opposite direction in your relative series at just the length of the relative away. This fact tends to create the appearance of waves just twice the length of the relative, but if the points thus reflected come at random intervals, there will not be a tendency—except by accident—for these points to be in any regular phase relationship with each other.

Complex relatives were invented independently by W. C. Yeatman of Los Angeles and G. T. Lane of Rochester. I told you about them briefly in my letter for June.

8. The simplest way for you to minimize the minor rhythms and the minor random fluctuations is to smooth the data by means of a short term **moving average**. (You will find many details in regard to moving averages in the Foundation's Technical Bulletin No. 4.) When you effect such a smoothing, the longer rhythms, originally obscured by the short term movement, very often stand out clearly.

Using moving averages of different lengths will help you to see rhythms of various lengths. That is, if you smooth by means of a short-term moving average you will be helped to find rhythms of medium length; if you smooth with moving averages of medium length, you will be helped to find long cycles.

Having smoothed the original figures by means of a moving average, you can subject the smoothed figures to inspection or time chart analysis, or any of the other methods for obtaining hints in regard to the length of possible cycles.

At this point I should warn you that cycles can be found even in random numbers. This is particularly true if you compare groups of random numbers, as you do when you look for cycles in smoothed figures. Obviously one group of random numbers is almost certain to be larger than another group. It is no great trick to vary the grouping (by means of taking moving averages of various lengths) in such a way that some of the high and low groups will fall into rhythmic patterns. Of course, such patterns in random numbers are not significant, but while they last they look like cycles and in fact are cycles—only they are cycles that have been created by accident and not by a constant underlying force that can be expected to continue.

It is worth pointing out that it is not the moving average that **created** the cycle. The cycle was already there (in the figures taken as groups); the moving average merely revealed it more clearly.

If you wish a mastery of moving averages as used in cycle analysis, you will be helped if you (a) make up for yourself a simple series of figures that go up and down regularly, such as 8 7 6 5 4 5 6 7 8 7 6 5 4 5 6 7 8.....and take various moving averages of it, charting both the wave and each of the moving averages; (b) make up a number of such rhythmic series of figures and subject each of them to the same moving average, plotting each of the waves and the corresponding moving average, and (c) get hold of or make up different sorts of random series of figures and subject them to different moving averages, plotting both the random series and their moving averages.

9. The **section moving average** as I call it, is also a useful tool for finding hidden periodicities and for separating one cycle from another. It is described briefly in Appendix III of **Cycles--The Science of Prediction** by Mr. E. F. Dakin and myself.

The **weighted section moving average** is an improvement upon the simple section moving average. It was devised by Mr. Lane and was also described briefly in my June letter.

Space limitations prevent consideration of the other methods in this issue of **Cycles—A Monthly Report for September.** The remaining methods and the section on machines will appear next month.

Technical

HOW TO GET HINTS OF CYCLES
THAT MAY BE PRESENT IN A SERIES OF FIGURES
In Two Parts - Part II

In the September report I described for you four graphic methods for obtaining hints of cycles that might be present in a series of figures. I also described five of the computational methods.

In this section of the outline, I want to tell you about six other computational methods that are useful and also to describe briefly some of the machines that have been used to minimize the labor of computation.

Let us then pick up the story where we left it last month and proceed to the consideration of the tenth item on our outline.

10. **Elimination of trend.** There are, of course, a variety of methods of determining trend and, upon occasion, almost any of these will be useful in helping to get hints of cycles that may be present in the data.

For a discussion of many ways of trend determination, refer to Croxton and

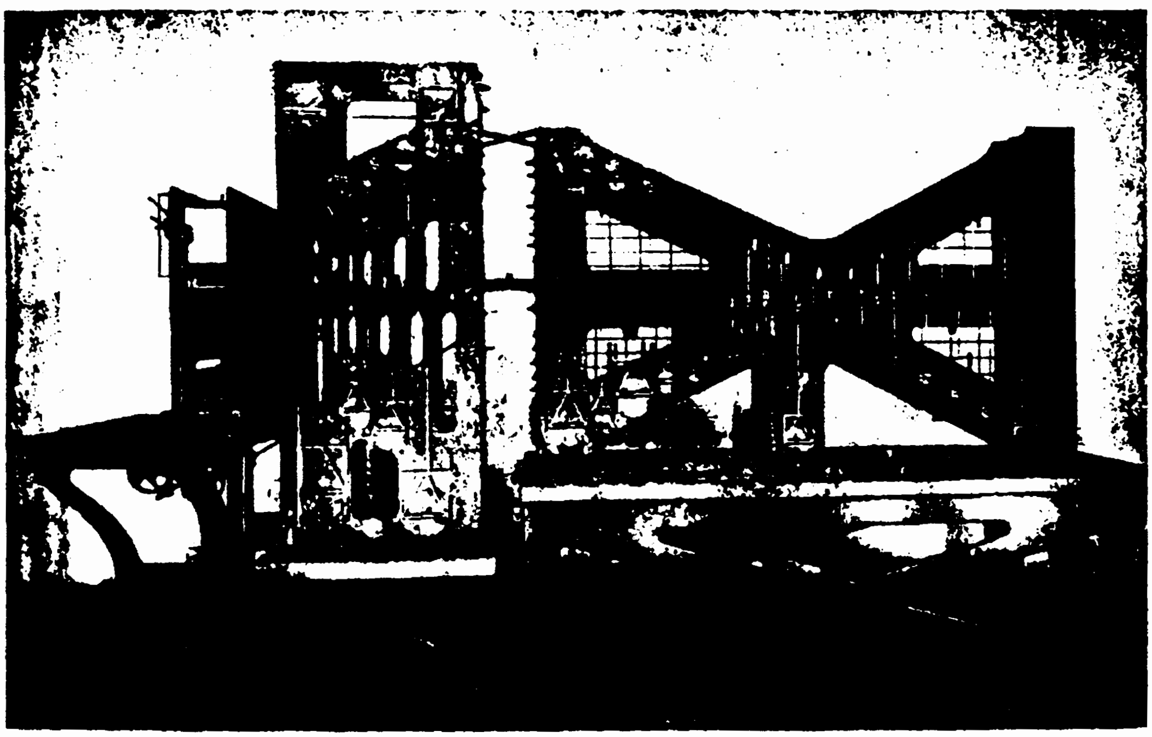


FIG. 102. Tide predictor of the United States Coast and Geodetic Survey.

Cowden's **Applied General Statistics**, pps. 363-463. Having determined the trend, one should remove it by dividing each point on the curve by the corresponding value of the trend.

However, as an aid in finding hints of cycles, probably the best form of trend to use is the moving average even though, from other points of view, the moving aver-

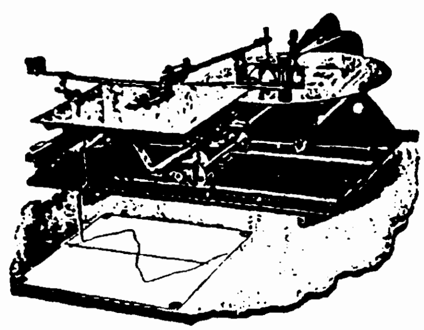


Fig. 104. Rowe's harmonic analyzer.

age may be one of the worst forms of trend to use. Comparison of the actual data with the moving average will emphasize certain rhythms at the expense of others and will often reveal cycles that might be otherwise almost completely hidden. This method has the added advantage that you can know the effect of the manipulation upon all regular cycles that may be present in the series.

For a description of the effect of comparing the actual data with their moving averages, see the Foundation's **Technical Bulletin No. 4.**

11. **Comparing one moving average with another** is another means by which you can manipulate the data to reveal hidden rhythms. But here also, as always, you need to be on your guard in interpreting your results.

12. Getting slightly more complicated, you may now wish to make a series of **periodic tables.**

A Periodic table is merely a table with as many columns as there are items in the cycle being studied and as many lines as there are possible repetitions of the waves. Into this table you post the data consecutively.

For a rudimentary outline of the periodic table, you might refer to Croxton and Cowden's **Applied General Statistics**, pps. 554-562, Prentice Hall, 1941.

For a better description of the periodic table, you could examine a chapter called "Analysis of Non-Harmonic Periodic Functions" in a book by Worthing & Geffner called **Treatment of Experimental Data**, John Wiley & Sons, 1943.

From the amplitude of a sine curve fitted to the averages of the columns one can get some idea of the relative importance of the average wave with a period equal to that of the table.

By dividing the table horizontally into halves, thirds, or other fractions and taking subtotals of the columns, one can get some idea of the continuity of the given average wave throughout the series.

If the crests and troughs of the average wave in each section slip slightly forward or back in relation to the section before, one has a hint that the rhythm, if present, is slightly longer or shorter than the given length.

If the nature of the data permits, numbers above the trend or above 100 can be recorded in black, the others in red. In such a table bands of color running diagonally across the page will often reveal hints of rhythms that may be present of lengths other than the length of the table.

13. **Harmonic analysis** will give you hints of cycles that may be present. If done by a mechanical or electrical analyzer, it is very easy. For an excellent description of the principles involved, see Dayton C. Miller's **The Science of Musical Sounds**, published by The Macmillan Company in 1937. This book also gives an extended description of mechanical harmonic analysis.

Before making a harmonic analysis you must correct for trend. Harmonic analysis gives you the amplitude (height) and phase (position in time) of the waves that may be present **on the average** in the series as a whole, after corrections for trend, and for each of the parts when averaged by halves, thirds, fourths, fifths, sixths, etc. In harmonic analysis the waves are assumed to be sine shaped, but of course they may not actually be of this shape.

Unless the data are truly repetitive, as with a sound wave, cycles obtained by harmonic analysis are merely mathematical building blocks, and have no necessary relations to reality. We merely know that when combined they will reproduce the original wave, but for purposes of forecasting, one might as well project a photostat of past behavior

Although you will often find the hints developed by harmonic analysis of use in finding rhythms that may be present in the figures, you can get much better results by a method that I have christened **Multiple Harmonic Analysis**. As far as I know, it has never been described in print, and so I cannot give you any references to books or articles where you can read about it in detail, but in principle it is very simple. You merely make successively a number of harmonic analyses (perhaps 8 or 10) of shorter and shorter sections of the original data. For example, if your series consisted of 200 figures, you might make one complete harmonic analysis of them (as far down the scale as your machine would go), then another complete harmonic analysis of the first 198 figures, then a third one of the first 196 figures, and so on, choosing lengths according to your particular needs.

Multiple harmonic analysis will give you many more points on your periodogram—ten times as many points if you make ten analyses. A periodogram of this sort (with ten times as many points) is really useful and will point out possibilities of real cycles instead of merely recording mathematical abstraction.

(A periodogram is merely a chart on which, after due adjustment, you record the amplitude of the average waves at various wave lengths. In other words, it is a spectrum.)

14. One of the best ways to get hints of cycles is to adjust for and eliminate cycles already found, just as you would adjust for a seasonal cycle.

Fig. 103. Michelson's harmonic analyzer and synthesizer for twenty components.

15. No account of methods of finding hints of rhythms would be complete without reference to the methods developed by Dr. Norbert Weiner. These methods were restricted during the war but the restriction has been lifted and the methods are described in a book by Dr. Weiner entitled **Extrapolation, Interpolation, and Smoothing of Stationary Time Series** published jointly

by the Technology Press of the Massachusetts Institute of Technology and John Wiley & Sons, Inc., New York, in 1949.. These methods attempt to extrapolate any regularities present in a series of figures, together with a measure of the probability of the accuracy of the extrapolations.

The description of the method is quite complicated and unless you have a good mathematical background, these methods are probably better read about than attempted.

MACHINES

1. A discussion of **optical machines** is more or less academic, for there are only two such machines that I know about, one in Tucson, Arizona, and the other in Chicago.

The Arizona machine was invented by Andrew E. Douglass in 1914 and is described in "The Cyclograph: Variable Grating Type" in **Reports of the Conferences on Cycles**, Carnegie Institution of Washington, 1929, pps. 34-40.

The Chicago machine was developed independently some thirty years later by L. V. Mitelman. It operates along more or less the same lines.

2. The most important of the **mechanical machines** used to get hints of cycles in a series of figures or curves are the harmonic analyzers. These were touched upon above in the section on harmonic analysis. The best one that I know about is manufactured by the Mico Instrument Company of Cambridge, Massachusetts. The latest model costs $12,500. We would like one, so, if you are feeling generous......!

In addition, I should perhaps mention a machine developed by C. G. Abbot for the mechanical resolution of curves. You will find it described in Smithsonian Miscellaneous Collections, Vol. 87, No. 4, under the title **The Periodometer: An Instrument For Finding and Evaluating Periodicities in Long Series of Observations**. I suspect the machine was not too satisfactory.

3. A fairly large number of **electrical machines** for obtaining hints of waves that may be present in curves have been developed.

One of these has been described by R. L. Wegel and C. R. Moore in an article entitled "An Electrical Frequency Analyzer," published in the April 1924 issue of the **Bell System Technical Journal**.

Another machine has just been developed by H. Bruderlin of Los Angeles. It was first put into use in May of this year.

Samuél F. Bagno and Charles C. Works of New York have also built a machine for electrical resolution. This machine was demonstrated at the June 27th meeting of the Foundation for the Study of Cycles. (Refer to the September report, pages 29 to 32 inclusive, and to pages 28 and 29 of this report.)

Mitelman also has invented an electrical device for analyzing cycles. This machine was patented October 14, 1941, under patent 2,258,859. You can get a description of it from the patent office for twenty-five cents. Mitelman built a rather crude model, to prove that the machine would work, which I saw in Chicago in 1942. I have not heard what he may have done since.

4. The use of **punch card and tabulating machines** for manipulating figures is too well known to require particular comment at this point. Vedder Hughey of Florida writes that he has devised a method for computing simple section moving averages by means of punch cards.

G. T. Lane writes that he has made a 39-term Macaulay weighted moving average of a series of 1100 figures recording water levels in Lake Ontario over a period of years in a total elapsed time of eight hours, including punching the cards and running them through the machines.

All of these machines are of use in minimizing labor, but often they involve more fuss and bother than the time they save is worth. Their use depends chiefly upon the quantity of work of this sort with which one is engaged, and the facilities one has for carrying through on the results of the machine work.

* * * * * * *

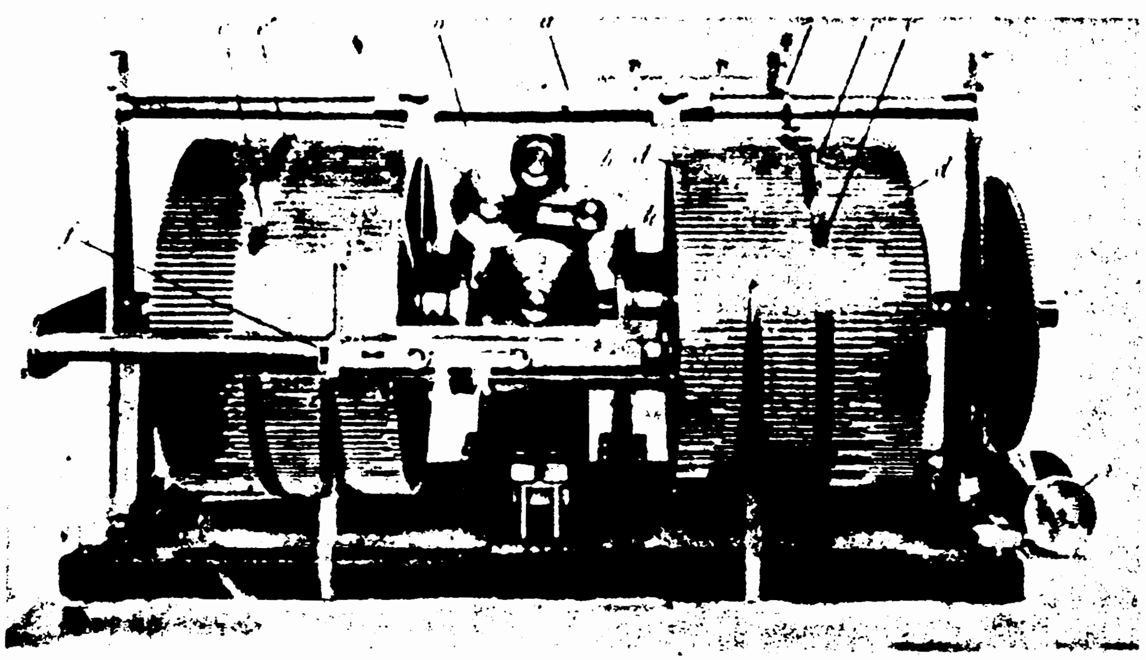
Having used one or a combination of these methods to obtain a hint of the length of a rhythm that may be present in a series of figures or in a curve, your next step, as stated in the beginning, is to find out all you can about the indicated cycle. For example, you must find out the exact length of the cycle, its amplitude, its phase position, and its shape. More important than this, you must find out whether or not it is regularly repetitive, that is, rhythmic, or if it is a mere mathematical abstraction.

If it is rhythmic, you must then go further and come to an opinion in regard to its significance. That is, how many times out of a hundred could a rhythm of this regularity exist in a series of figures as a result of separate random forces. As I have mentioned before, you can easily find rhythms in random figures. but such rhythms will not continue over many repetitions. The problem before you at this point is to decide the probabilities of the observed behavior having arisen from random forces that cannot be expected to continue or from some continuing force that probably will continue.

Both of these two additional steps are beyond the scope of tnis outline but it will be treated in later monthly reports.

Acknowledgment: All of the pictures in this article, except the one immediately below, are taken, with the permission of the Case School of Applied Science from Dayton C. Miller's book **The Science of Musical Sounds**, © The Macmillan Company, 1916. The picture below is taken from one of the Smithsonian Institute publications, as indicated.

SMITHSONIAN MISCELLANEOUS COLLECTIONS VOL. 87 NO. 4, PL. 1



THE PERIODOMETER

LESSON VI

Supplement 12

THE HOSKINS TIME CHART

The time chart was invented by Mr. Chapin Hoskins, a consultant of New York City and Hopkinton, New Hampshire. He made his invention sometime prior to 1936, in which year he revealed this invention to me.

In Lesson VI I did not call the time chart I described to you the "Hoskins Time Chart" because what I described to you is not the time chart exactly as Mr. Hoskins invented it and has since developed it. Mr. Hoskins would not want the time chart I described called the "Hoskins Time Chart." He would, I suppose, call it the "Dewey Time Chart."

The chief differences between the Hoskins Time Chart and the Time Chart I described to you are as follows:

1. Mr. Hoskins uses reds for highs and blacks for lows whereas I use the reverse coloring.

The reason I use the colors I do is because I sometimes use a time chart in conjunction with a periodic table done in color. When time charts are used this way, it helps to have the colors of the bars the same as the colors of the figures in the periodic table.

Also it seems more logical to me to have the lows or negative numbers represented by red in accordance with ordinary bookkeeping conventions.

2. The second difference between the time chart I described to you and the real McCoy is one of terminology. I used the word "clearspan" for the concept which Mr. Hoskins calls "violence."

I finally prevailed upon Mr. Hoskins to issue an authorized description of the Hoskins Time Chart. This description was written by Paul Davis and was printed in the Journal of Cycle Research for January 1956. A copy of this magazine is being sent you herewith. I trust that Mr. Davis' exposition of the Hoskins Time Chart will add something to your knowledge of time chart technique.

Note that the printer got the colors on page 10 reversed. For that page read red for black and black for red.

Lesson VI, Supplement 13

The Time Chart

If you are performing cycle analysis manually (rather than by computer) another useful device you can use to inspect data for evidence of cycles is the time chart. Time charts may reveal possible periods (lengths). In addition, the time chart will give an indication of phase, but it will not verify the validity of the cycle, nor reveal its precise amplitude or shape. If a possible cycle is indicated, additional analysis must follow.

The time chart was invented by Mr. Chapin Hoskins of the Institute of Trend Research of Hopkinton, New Hampshire.

The time chart is a diagram of highs and lows set up in such a way as to give you hints of rhythms that may be present in your time series. It is a most important adjunct for cycle analysis. It depends for its usefulness upon an objective method for determining the "highs" and "lows" for any given wave length being sought. The time chart itself is then merely a method of diagramming these highs and lows in such a way as to throw light upon the length and consistency of the rhythm or rhythms that may be present in the figures.

The efficacy of the time chart technique depends upon the concept of clearspan; it is necessary to understand clearspan before attempting time charts.

First, the definition: clearspan is a number giving the distance measured in time units from a point in time going back to the prior point where, for rising values, there is a value ***equal to or higher than*** the value at the starting point; for falling values, clearspan is the distance (in time units) back to the point where there occurs a value ***as low as or lower than*** the value at the starting point.

Note that clearspan is a number that counts units of time. If a data point is above the immediately prior data point, clearspan answers the question: How long has it been since the data were that high? You count back so many time units until you bump into a value as high as or higher than the one you started with. What is counted is the time units, and the answer is that it has been so many weeks, or months, or whatever, since there occurred a value as high (or higher than) the one being examined.

Consider the following series of data points.

Time Period Number	Data (i.e.: value)
1	102
2	90
3	98
4	91
5	96
6	98
7	101
8	93

The clearspan number for the value at position 6 is 2. The value at position 6 is 98, and the value is up from the previous one. You count back two places until you reach the 98 at the third position. With a rising value the clearspan is a black value, and is obtained by counting back until you reach a value ***as high as or higher than*** the value at the starting point. A black clearspan means a higher value. The clearspan for the 101 value at position 7 is a black 5, there being five time periods between the 101 at position 7 and the 102 at position 1.

When the value of the data point is down from the previous one, the clearspan is red or minus. A red or minus clearspan means a lower value. Thus, the clearspan for the value of 93 at position 8 is red or minus 3, there being three time periods between the 93 at position 8 and the 91 at position 4.

Clearspan is counted and noted beside the data being studied; the time chart is then plotted from the clearspan numbers.

The time chart can also be plotted directly from a chart of the figures being studied. The technique, first counting and noting the clearspan numbers of a series of figures, then making a time chart (or making the time chart from the plotting of the figures) can be used at any stage in the analysis. The technique can be applied to the original data, or to detrended data, for example.

The time chart is a grid constructed for recording the dates of selected highs and lows, the highs and lows being selected according to the full standard criterion established by the period of the time chart, and the clearspan being determined by the movement and levels of the data being studied.

Only highs and lows turning points are noted on the time chart, and it is the *date* of their occurrence which is noted. The rigid schedule of the time chart is determined by the period being checked. To be entered on the time chart, a high or low must have a *clearspan of at least half the length of the period being checked* (or the next lower number if the period is an odd number). Such a clearspan number is "full standard."

Thus, to check the possibility of a 12-month cycle you would construct a 12-month time chart. The clearspan necessary for a high or low to be noted on the time chart is 6 months. A high must be the highest high with full standard clearspan and must be preceded and followed by lows of full standard clearspan. A low must be the lowest low with full standard clearspan and it must be preceded and followed by highs of full standard clearspan. The time chart records the time of occurrence of high, low, high, low, etc., according to its schedule.

These are the main rules with which the game is started. A rigid format is set to check a certain period. If the data show evidence of that period the time chart will reveal it as the dates of the turning points with full standard clearspan are marked. If the period is not the same length as the time chart, but is near to the length, the time chart will enable you to compute an estimate of the probable length.

As the work on the time chart proceeds, the rules become a little more involved, but we will start the demonstration with the simplest possible example. Assume a cycle six terms long: 1, 2, 3, 4, 3, 2, and repeat. The clearspan would be as follows:

Time Period	Data Value	Clearspan
1	1	
2	2	1+
3	3	2+
4	4	3+
5	3	-1
6	2	-3
7	1	-5
8	2	1
9	3	3
10	4	5
11	3	-1
12	2	-3
(Etc.)	(Etc.)	

+ means it could be more

- means a red figure; the curve is going down.

Clearspan is computed from the data value. Consider the 4 at position 10. The data are rising. You will go back 5 places before you bump into a value as high as 4. The clearspan is a black 5.

Here is another example:

Time Period	Data Value	Clearspan
1	100	
2	400	1+
3	800	2+
4	700	-1
5	600	-2
6	500	-3
7	600	1
8	400	-5
9	300	-7
10	200	-8
11	400	2
12	900	11+
13	1,000	12+
14	750	-2
15	700	-3
16	900	2
17	750	-1
18	650	-6

Highs and lows are turning points terminating rises and declines. If a cycle is contained in the figures, it should contribute some of the turns seen—not all of them—but at least the turning points on *its* schedule, if its amplitude is sufficient. Therefore, only the highs and lows on the schedule of the period being checked are valid to consider. Using a schedule based on an assumed period is an objective method for checking the presence of the assumed period.

The six-term cycle used above to illustrate clearspan is plotted below. The cycle, being an ideal situation, repeats precisely and continuously. (Such a rhythm is technically labelled a periodicity.) We'll use five repetitions of the cycle here to make a time chart.

Note that in the repetition of the cycle, the last term of each repetition is the base (or zero term) of the next repetition.

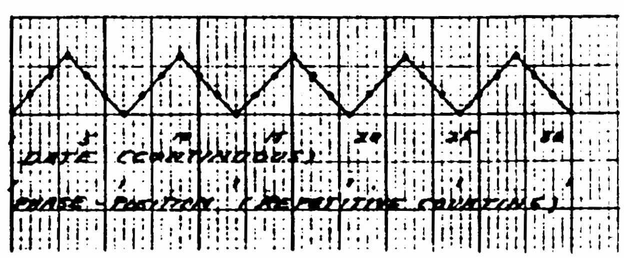


Fig. 1: A 6-TERM CYCLE REPEATED FIVE TIMES

To check if there is a 6-term cycle, a 6-term time chart is constructed. There are six vertical positions. The first position in each column is zero to that column and the positions going down are 1, 2, 3, 4, 5, or 6 places after. Going down the fourth column, the one headed by 18, 18 is the date of the zero position or base for that column. The first position below 18 marks the 19th time slot; the sixth position below marks the 24th time slot.

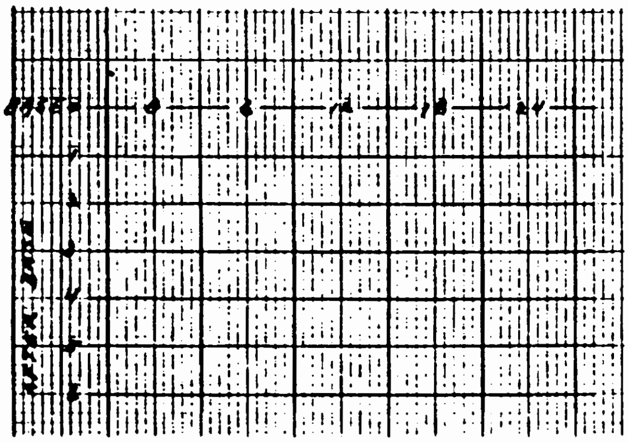


Fig. 2: A 6-TERM TIME CHART GRID

The first high (crest) in the 6-term cycle occurs at time position 4.

On a 6-term time chart, a high or low must have a clearspan of at least 3 to be entered. The first high occurs at time position 4, and has a clearspan of 3. It is followed by a full standard clearspan low, and can therefore be entered on the time chart at position 4, four places after 0 in the first column. The second high also qualifies under the rules—a clearspan of at least 3, and lows of full standard clearspan before and after; it can be entered at position 10, four places after 6 in the second column.

The crests would all be entered as shown below.

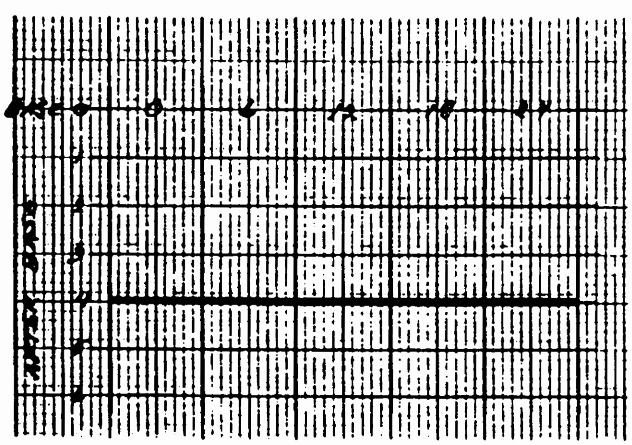


Fig. 3: HIGHS OF A 6-TERM CYCLE ENTERED ON A 6-TERM TIME CHART

However, crests and troughs are more correctly entered successively; the complete chart follows. Crests are entered as black lines, troughs as red lines where color is possible. Here a broken line is used to denote lows.

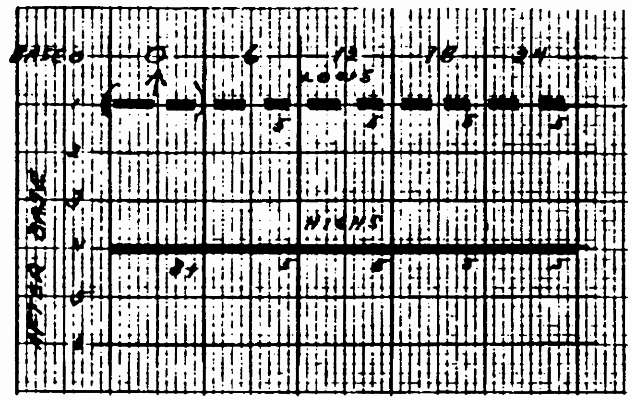


Fig. 4: A 6-TERM TIME CHART OF A 6-TERM CYCLE

The first entry on the chart is a low at position one. Because you cannot be sure it is a low—no data occurring before that point—parenthesis are placed around it on the chart and an arrow noted.

As mentioned above, highs and lows are entered successively. Although the result here is two horizontal lines, the lines eventuated only as each separate entry was made; the separate entries just happen to connect.

The horizontal line on the 6-term time chart indicates the possible presence of a 6-term cycle; further analysis is necessary to verify the significance of the cycle, and to obtain a description of its characteristics. It is obvious, however, that you could reach a conclusion about the timing (phase) of the cycle from the above time chart.

Note that the clearspan is written to the right under each entry as the entry is made. In a complicated situation this notation can be helpful.

The time chart can be plotted from either the clearspan numbers, or the chart of the data. When the chart is used, clearspan is checked visually as shown in the following enlarged section of our 6-term cycle. (See figure 5.)

Our example is as regular a cycle as it is possible to find. Unless you are unusually perceptive or are making a 12-month time chart of a monthly weather series, or department store sales, or agricultural prices, it would of course be unusual to draw a time chart with the bars going horizontally with such precision.

ANOTHER EXAMPLE

The following tabulation records the warmest month of the year at San Francisco for the years 1931–1940. The month numbers are also noted, as the time chart is set up

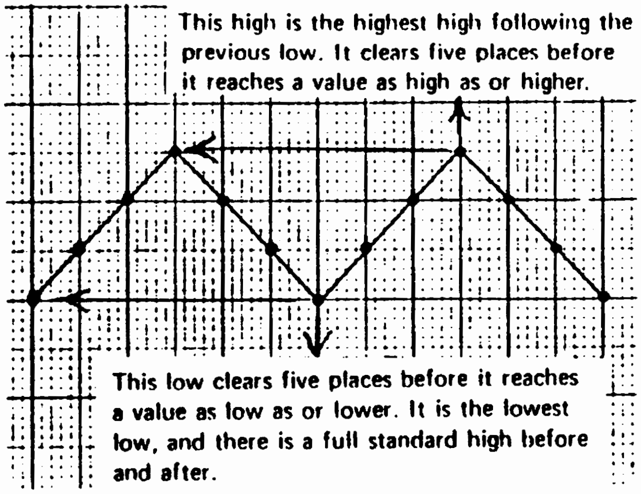


Fig. 5: CHART SHOWING VISUAL INSPECTION FOR FULL STANDARD CLEARSPAN

using continuous month numbers.

Year	Month	Number	Year	Month	Number
1931	Sep	369	1936	Sep	429
1932	Sep	381	1937	Oct	442
1933	Oct	394	1938	Oct	454
1934	Sep	405	1939	Sep	465
1935	Jun	414	1940	Sep	477

The problem is to set up a 12-month time chart. No clearspan is given--the data are selected correctly, and all that is necessary is to enter these highs on a 12-month chart. Try it!

APPENDICES

The appendicies this month include the BASIC program for computing clearspan, and a tabulation of standard month numbers. The standard month numbers should be part of your library if you are performing cycle analyses.

Appendix 4: Program for Computing Clearspan Numbers

```
10 DEF FNA(X)=INT(100*(X-INT(X)+.001))
20 DEF FNB(X)= INT(1000*(X-INT(X)+.0001))
30 DIM G$[70],H$[70],E[500],C[500],D[500]
40 FILES LN752
50 GOSUB 3000
60 LET P=6
70 LET L=0
80 FOR I=2 TO N-1
90 IF (E[I] > = E[I-1] AND E[I]> E[I+1]) THEN 120
100 IF (E[I] < = E[I-1] AND E[I] < E[I+1]) THEN 190
110 GOTO 310
120 FOR J=2 TO I-1
130 IF E[I] < = E[I-J] THEN 150
140 NEXT J
150 LET J=J-1
160 IF L=0 THEN 260
170 IF C[L] < 0 THEN 260
180 GOTO 310
190 FOR J=2 TO I-1
200 IF E[I] > = E[I-J] THEN 220
210 NEXT J
220 LET J=J-1
230 LET J=-J
240 IF L=0 THEN 260
250 IF C[L] <0 THEN 310
260 LET L=L+1
270 IF J<0 THEN 300
280 LET C[L] =J+I/1000
290 GOTO 310
300 LET C[L] =J-I/1000
310 NEXT I
320 LET X=INT(P/2)
330 LET J=0
340 FOR I=1 TO L
350 IF ABS(C[I])<X THEN 380
360 LET J=J+1
370 LET D[J] =C[I]
380 NEXT I
390 PRINT TAB(25);"CLEARSPANS"
400 PRINT "POINT","PEAK","POINT","TROUGH"
410 PRINT "-------------------------"
420 PRINT
430 FOR I=1 TO J
440 IF D[I]< 0 THEN 530
450 IF I=J THEN 570
460 IF D[I+1]>0 THEN 570
470 LET Y=-D[I+1]
480 LET V=INT(Y)
490 LET W=INT(D[I])
500 PRINT FNB(D[I]),W,FNB(Y),V
510 LET I=I+1
520 GOTO 580
530 LET Y=-D[I]
540 LET V=INT(Y)
550 PRINT TAB(30);FNB(Y),V
560 GOTO 580
570 PRINT INT(1000*(D[I]-INT(D[I])+.0001)), INT(D[
580 NEXT I
590 STOP
```

Appendix 5: Standard Month Numbers

A numbering system for months is almost indispensable for cycle analysis of monthly data. Month numbers are necessary for computing clearspan, for arranging arrays, and for various other manipulations. These numbers start with December of 1650 as 0. December of 1700 is thus month number 600. (Only the numbers for 1750 forward are shown here.) December 1900 is month number 3000, according to this system. If you are working with recent monthly data, you drop the 3000 and use December 1900 as month number 0.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1750	1,189	1,190	1,191	1,192	1,193	1,194	1,195	1,196	1,197	1,198	1,199	1,200
1751	1,201	1,202	1,203	1,204	1,205	1,206	1,207	1,208	1,209	1,210	1,211	1,212
1752	1,213	1,214	1,215	1,216	1,217	1,218	1,219	1,220	1,221	1,222	1,223	1,224
1753	1,225	1,226	1,227	1,228	1,229	1,230	1,231	1,232	1,233	1,234	1,235	1,236
1754	1,237	1,238	1,239	1,240	1,241	1,242	1,243	1,244	1,245	1,246	1,247	1,248
1755	1,249	1,250	1,251	1,252	1,253	1,254	1,255	1,256	1,257	1,258	1,259	1,260
1756	1,261	1,262	1,263	1,264	1,265	1,266	1,267	1,268	1,269	1,270	1,271	1,272
1757	1,273	1,274	1,275	1,276	1,277	1,278	1,279	1,280	1,281	1,282	1,283	1,284
1758	1,285	1,286	1,287	1,288	1,289	1,290	1,291	1,292	1,293	1,294	1,295	1,296
1759	1,297	1,298	1,299	1,300	1,301	1,302	1,303	1,304	1,305	1,306	1,307	1,308
1760	1,309	1,310	1,311	1,312	1,313	1,314	1,315	1,316	1,317	1,318	1,319	1,320
1761	1,321	1,322	1,323	1,324	1,325	1,326	1,327	1,328	1,329	1,330	1,331	1,332
1762	1,333	1,334	1,335	1,336	1,337	1,338	1,339	1,340	1,341	1,342	1,343	1,344
1763	1,345	1,346	1,347	1,348	1,349	1,350	1,351	1,352	1,353	1,354	1,355	1,356
1764	1,357	1,358	1,359	1,360	1,361	1,362	1,363	1,364	1,365	1,366	1,367	1,368
1765	1,369	1,370	1,371	1,372	1,373	1,374	1,375	1,376	1,377	1,378	1,379	1,380
1766	1,381	1,382	1,383	1,384	1,385	1,386	1,387	1,388	1,389	1,390	1,391	1,392
1767	1,393	1,394	1,395	1,396	1,397	1,398	1,399	1,400	1,401	1,402	1,403	1,404
1768	1,405	1,406	1,407	1,408	1,409	1,410	1,411	1,412	1,413	1,414	1,415	1,416
1769	1,417	1,418	1,419	1,420	1,421	1,422	1,423	1,424	1,425	1,426	1,427	1,428
1770	1,429	1,430	1,431	1,432	1,433	1,434	1,435	1,436	1,437	1,438	1,439	1,440
1771	1,441	1,442	1,443	1,444	1,445	1,446	1,447	1,448	1,449	1,450	1,451	1,452
1772	1,453	1,454	1,455	1,456	1,457	1,458	1,459	1,460	1,461	1,462	1,463	1,464
1773	1,465	1,466	1,467	1,468	1,469	1,470	1,471	1,472	1,473	1,474	1,475	1,476
1774	1,477	1,478	1,479	1,480	1,481	1,482	1,483	1,484	1,485	1,486	1,487	1,488
1775	1,489	1,490	1,491	1,492	1,493	1,494	1,495	1,496	1,497	1,498	1,499	1,500
1776	1,501	1,502	1,503	1,504	1,505	1,506	1,507	1,508	1,509	1,510	1,511	1,512
1777	1,513	1,514	1,515	1,516	1,517	1,518	1,519	1,520	1,521	1,522	1,523	1,524
1778	1,525	1,526	1,527	1,528	1,529	1,530	1,531	1,532	1,533	1,534	1,535	1,536
1779	1,537	1,538	1,539	1,540	1,541	1,542	1,543	1,544	1,545	1,546	1,547	1,548
1780	1,549	1,550	1,551	1,552	1,553	1,554	1,555	1,556	1,557	1,558	1,559	1,560
1781	1,561	1,562	1,563	1,564	1,565	1,566	1,567	1,568	1,569	1,570	1,571	1,572
1782	1,573	1,574	1,575	1,576	1,577	1,578	1,579	1,580	1,581	1,582	1,583	1,584
1783	1,585	1,586	1,587	1,588	1,589	1,590	1,591	1,592	1,593	1,594	1,595	1,596
1784	1,597	1,598	1,599	1,600	1,601	1,602	1,603	1,604	1,605	1,606	1,607	1,608
1785	1,609	1,610	1,611	1,612	1,613	1,614	1,615	1,616	1,617	1,618	1,619	1,620
1786	1,621	1,622	1,623	1,624	1,625	1,626	1,627	1,628	1,629	1,630	1,631	1,632
1787	1,633	1,634	1,635	1,636	1,637	1,638	1,639	1,640	1,641	1,642	1,643	1,644
1788	1,645	1,646	1,647	1,648	1,649	1,650	1,651	1,652	1,653	1,654	1,655	1,656
1789	1,657	1,658	1,659	1,660	1,661	1,662	1,663	1,664	1,665	1,666	1,667	1,668
1790	1,669	1,670	1,671	1,672	1,673	1,674	1,675	1,676	1,677	1,678	1,679	1,680
1791	1,681	1,682	1,683	1,684	1,685	1,686	1,687	1,688	1,689	1,690	1,691	1,692
1792	1,693	1,694	1,695	1,696	1,697	1,698	1,699	1,700	1,701	1,702	1,703	1,704
1793	1,705	1,706	1,707	1,708	1,709	1,710	1,711	1,712	1,713	1,714	1,715	1,716
1794	1,717	1,718	1,719	1,720	1,721	1,722	1,723	1,724	1,725	1,726	1,727	1,728
1795	1,729	1,730	1,731	1,732	1,733	1,734	1,735	1,736	1,737	1,738	1,739	1,740
1796	1,741	1,742	1,743	1,744	1,745	1,746	1,747	1,748	1,749	1,750	1,751	1,752
1797	1,753	1,754	1,755	1,756	1,757	1,758	1,759	1,760	1,761	1,762	1,763	1,764
1798	1,765	1,766	1,767	1,768	1,769	1,770	1,771	1,772	1,773	1,774	1,775	1,776
1799	1,777	1,778	1,779	1,780	1,781	1,782	1,783	1,784	1,785	1,786	1,787	1,788
1800	1,789	1,790	1,791	1,792	1,793	1,794	1,795	1,796	1,797	1,798	1,799	1,800
1801	1,801	1,802	1,803	1,804	1,805	1,806	1,807	1,808	1,809	1,810	1,811	1,812
1802	1,813	1,814	1,815	1,816	1,817	1,818	1,819	1,820	1,821	1,822	1,823	1,824
1803	1,825	1,826	1,827	1,828	1,829	1,830	1,831	1,832	1,833	1,834	1,835	1,836
1804	1,837	1,838	1,839	1,840	1,841	1,842	1,843	1,844	1,845	1,846	1,847	1,848

STANDARD MONTH NUMBERS (Continued)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1805	1,849	1,850	1,851	1,852	1,853	1,854	1,855	1,856	1,857	1,858	1,859	1,860
1806	1,861	1,862	1,863	1,864	1,865	1,866	1,867	1,868	1,869	1,870	1,871	1,872
1807	1,873	1,874	1,875	1,876	1,877	1,878	1,879	1,880	1,881	1,882	1,883	1,884
1808	1,885	1,886	1,887	1,888	1,889	1,890	1,891	1,892	1,893	1,894	1,895	1,896
1809	1,897	1,898	1,899	1,900	1,901	1,902	1,903	1,904	1,905	1,906	1,907	1,908
1810	1,909	1,910	1,911	1,912	1,913	1,914	1,915	1,916	1,917	1,918	1,919	1,920
1811	1,921	1,922	1,923	1,924	1,925	1,926	1,927	1,928	1,929	1,930	1,931	1,932
1812	1,933	1,934	1,935	1,936	1,937	1,938	1,939	1,940	1,941	1,942	1,943	1,944
1813	1,945	1,946	1,947	1,948	1,949	1,950	1,951	1,952	1,953	1,954	1,955	1,956
1814	1,957	1,958	1,959	1,960	1,961	1,962	1,963	1,964	1,965	1,966	1,967	1,968
1815	1,969	1,970	1,971	1,972	1,973	1,974	1,975	1,976	1,977	1,978	1,979	1,980
1816	1,981	1,982	1,983	1,984	1,985	1,986	1,987	1,988	1,989	1,990	1,991	1,992
1817	1,993	1,994	1,995	1,996	1,997	1,998	1,999	2,000	2,001	2,002	2,003	2,004
1818	2,005	2,006	2,007	2,008	2,009	2,010	2,011	2,012	2,013	2,014	2,015	2,016
1819	2,017	2,018	2,019	2,020	2,021	2,022	2,023	2,024	2,025	2,026	2,027	2,028
1820	2,029	2,030	2,031	2,032	2,033	2,034	2,035	2,036	2,037	2,038	2,039	2,040
1821	2,041	2,042	2,043	2,044	2,045	2,046	2,047	2,048	2,049	2,050	2,051	2,052
1822	2,053	2,054	2,055	2,056	2,057	2,058	2,059	2,060	2,061	2,062	2,063	2,064
1823	2,065	2,066	2,067	2,068	2,069	2,070	2,071	2,072	2,073	2,074	2,075	2,076
1824	2,077	2,078	2,079	2,080	2,081	2,082	2,083	2,084	2,085	2,086	2,087	2,088
1825	2,089	2,090	2,091	2,092	2,093	2,094	2,095	2,096	2,097	2,098	2,099	2,100
1826	2,101	2,102	2,103	2,104	2,105	2,106	2,107	2,108	2,109	2,110	2,111	2,112
1827	2,113	2,114	2,115	2,116	2,117	2,118	2,119	2,120	2,121	2,122	2,123	2,124
1828	2,125	2,126	2,127	2,128	2,129	2,130	2,131	2,132	2,133	2,134	2,135	2,136
1829	2,137	2,138	2,139	2,140	2,141	2,142	2,143	2,144	2,145	2,146	2,147	2,148
1830	2,149	2,150	2,151	2,152	2,153	2,154	2,155	2,156	2,157	2,158	2,159	2,160
1831	2,161	2,162	2,163	2,164	2,165	2,166	2,167	2,168	2,169	2,170	2,171	2,172
1832	2,173	2,174	2,175	2,176	2,177	2,178	2,179	2,180	2,181	2,182	2,183	2,184
1833	2,185	2,186	2,187	2,188	2,189	2,190	2,191	2,192	2,193	2,194	2,195	2,196
1834	2,197	2,198	2,199	2,200	2,201	2,202	2,203	2,204	2,205	2,206	2,207	2,208
1835	2,209	2,210	2,211	2,212	2,213	2,214	2,215	2,216	2,217	2,218	2,219	2,220
1836	2,221	2,222	2,223	2,224	2,225	2,226	2,227	2,228	2,229	2,230	2,231	2,232
1837	2,233	2,234	2,235	2,236	2,237	2,238	2,239	2,240	2,241	2,242	2,243	2,244
1838	2,245	2,246	2,247	2,248	2,249	2,250	2,251	2,252	2,253	2,254	2,255	2,256
1839	2,257	2,258	2,259	2,260	2,261	2,262	2,263	2,264	2,265	2,266	2,267	2,268
1840	2,269	2,270	2,271	2,272	2,273	2,274	2,275	2,276	2,277	2,278	2,279	2,280
1841	2,281	2,282	2,283	2,284	2,285	2,286	2,287	2,288	2,289	2,290	2,291	2,292
1842	2,293	2,294	2,295	2,296	2,297	2,298	2,299	2,300	2,301	2,302	2,303	2,304
1843	2,305	2,306	2,307	2,308	2,309	2,310	2,311	2,312	2,313	2,314	2,315	2,316
1844	2,317	2,318	2,319	2,320	2,321	2,322	2,323	2,324	2,325	2,326	2,327	2,328
1845	2,329	2,330	2,331	2,332	2,333	2,334	2,335	2,336	2,337	2,338	2,339	2,340
1846	2,341	2,342	2,343	2,344	2,345	2,346	2,347	2,348	2,349	2,350	2,351	2,352
1847	2,353	2,354	2,355	2,356	2,357	2,358	2,359	2,360	2,361	2,362	2,363	2,364
1848	2,365	2,366	2,367	2,368	2,369	2,370	2,371	2,372	2,373	2,374	2,375	2,376
1849	2,377	2,378	2,379	2,380	2,381	2,382	2,383	2,384	2,385	2,386	2,387	2,388
1850	2,389	2,390	2,391	2,392	2,393	2,394	2,395	2,396	2,397	2,398	2,399	2,400
1851	2,401	2,402	2,403	2,404	2,405	2,406	2,407	2,408	2,409	2,410	2,411	2,412
1852	2,413	2,414	2,415	2,416	2,417	2,418	2,419	2,420	2,421	2,422	2,423	2,424
1853	2,425	2,426	2,427	2,428	2,429	2,430	2,431	2,432	2,433	2,434	2,435	2,436
1854	2,437	2,438	2,439	2,440	2,441	2,442	2,443	2,444	2,445	2,446	2,447	2,448
1855	2,449	2,450	2,451	2,452	2,453	2,454	2,455	2,456	2,457	2,458	2,459	2,460
1856	2,461	2,462	2,463	2,464	2,465	2,466	2,467	2,468	2,469	2,470	2,471	2,472
1857	2,473	2,474	2,475	2,476	2,477	2,478	2,479	2,480	2,481	2,482	2,483	2,484
1858	2,485	2,486	2,487	2,488	2,489	2,490	2,491	2,492	2,493	2,494	2,495	2,496
1859	2,497	2,498	2,499	2,500	2,501	2,502	2,503	2,504	2,505	2,506	2,507	2,508
1860	2,509	2,510	2,511	2,512	2,513	2,514	2,515	2,516	2,517	2,518	2,519	2,520
1861	2,521	2,522	2,523	2,524	2,525	2,526	2,527	2,528	2,529	2,530	2,531	2,532
1862	2,533	2,534	2,535	2,536	2,537	2,538	2,539	2,540	2,541	2,542	2,543	2,544
1863	2,545	2,546	2,547	2,548	2,549	2,550	2,551	2,552	2,553	2,554	2,555	2,556
1864	2,557	2,558	2,559	2,560	2,561	2,562	2,563	2,564	2,565	2,566	2,567	2,568

STANDARD MONTH NUMBERS (Continued)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1865	2,569	2,570	2,571	2,572	2,573	2,574	2,575	2,576	2,577	2,578	2,579	2,580
1866	2,581	2,582	2,583	2,584	2,585	2,586	2,587	2,588	2,589	2,590	2,591	2,592
1867	2,593	2,594	2,595	2,596	2,597	2,598	2,599	2,600	2,601	2,602	2,603	2,604
1868	2,605	2,606	2,607	2,608	2,609	2,610	2,611	2,612	2,613	2,614	2,615	2,616
1869	2,617	2,618	2,619	2,620	2,621	2,622	2,623	2,624	2,625	2,626	2,627	2,628
1870	2,629	2,630	2,631	2,632	2,633	2,634	2,635	2,636	2,637	2,638	2,639	2,640
1871	2,641	2,642	2,643	2,644	2,645	2,646	2,647	2,648	2,649	2,650	2,651	2,652
1872	2,653	2,654	2,655	2,656	2,657	2,658	2,659	2,660	2,661	2,662	2,663	2,664
1873	2,665	2,666	2,667	2,668	2,669	2,670	2,671	2,672	2,673	2,674	2,675	2,676
1874	2,677	2,678	2,679	2,680	2,681	2,682	2,683	2,684	2,685	2,686	2,687	2,688
1875	2,689	2,690	2,691	2,692	2,693	2,694	2,695	2,696	2,697	2,698	2,699	2,700
1876	2,701	2,702	2,703	2,704	2,705	2,706	2,707	2,708	2,709	2,710	2,711	2,712
1877	2,713	2,714	2,715	2,716	2,717	2,718	2,719	2,720	2,721	2,722	2,723	2,724
1878	2,725	2,726	2,727	2,728	2,729	2,730	2,731	2,732	2,733	2,734	2,735	2,736
1879	2,737	2,738	2,739	2,740	2,741	2,742	2,743	2,744	2,745	2,746	2,747	2,748
1880	2,749	2,750	2,751	2,752	2,753	2,754	2,755	2,756	2,757	2,758	2,759	2,760
1881	2,761	2,762	2,763	2,764	2,765	2,766	2,767	2,768	2,769	2,770	2,771	2,772
1882	2,773	2,774	2,775	2,776	2,777	2,778	2,779	2,780	2,781	2,782	2,783	2,784
1883	2,785	2,786	2,787	2,788	2,789	2,790	2,791	2,792	2,793	2,794	2,795	2,796
1884	2,797	2,798	2,799	2,800	2,801	2,802	2,803	2,804	2,805	2,806	2,807	2,808
1885	2,809	2,810	2,811	2,812	2,813	2,814	2,815	2,816	2,817	2,818	2,819	2,820
1886	2,821	2,822	2,823	2,824	2,825	2,826	2,827	2,828	2,829	2,830	2,831	2,832
1887	2,833	2,834	2,835	2,836	2,837	2,838	2,839	2,840	2,841	2,842	2,843	2,844
1888	2,845	2,846	2,847	2,848	2,849	2,850	2,851	2,852	2,853	2,854	2,855	2,856
1889	2,857	2,858	2,859	2,860	2,861	2,862	2,863	2,864	2,865	2,866	2,867	2,868
1890	2,869	2,870	2,871	2,872	2,873	2,874	2,875	2,876	2,877	2,878	2,879	2,880
1891	2,881	2,882	2,883	2,884	2,885	2,886	2,887	2,888	2,889	2,890	2,891	2,892
1892	2,893	2,894	2,895	2,896	2,897	2,898	2,899	2,900	2,901	2,902	2,903	2,904
1893	2,905	2,906	2,907	2,908	2,909	2,910	2,911	2,912	2,913	2,914	2,915	2,916
1894	2,917	2,918	2,919	2,920	2,921	2,922	2,923	2,924	2,925	2,926	2,927	2,928
1895	2,929	2,930	2,931	2,932	2,933	2,934	2,935	2,936	2,937	2,938	2,939	2,940
1896	2,941	2,942	2,943	2,944	2,945	2,946	2,947	2,948	2,949	2,950	2,951	2,952
1897	2,953	2,954	2,955	2,956	2,957	2,958	2,959	2,960	2,961	2,962	2,963	2,964
1898	2,965	2,966	2,967	2,968	2,969	2,970	2,971	2,972	2,973	2,974	2,975	2,976
1899	2,977	2,978	2,979	2,980	2,981	2,982	2,983	2,984	2,985	2,986	2,987	2,988
1900	2,989	2,990	2,991	2,992	2,993	2,994	2,995	2,996	2,997	2,998	2,999	3,000
1901	3,001	3,002	3,003	3,004	3,005	3,006	3,007	3,008	3,009	3,010	3,011	3,012
1902	3,013	3,014	3,015	3,016	3,017	3,018	3,019	3,020	3,021	3,022	3,023	3,024
1903	3,025	3,026	3,027	3,028	3,029	3,030	3,031	3,032	3,033	3,034	3,035	3,036
1904	3,037	3,038	3,039	3,040	3,041	3,042	3,043	3,044	3,045	3,046	3,047	3,048
1905	3,049	3,050	3,051	3,052	3,053	3,054	3,055	3,056	3,057	3,058	3,059	3,060
1906	3,061	3,062	3,063	3,064	3,065	3,066	3,067	3,068	3,069	3,070	3,071	3,072
1907	3,073	3,074	3,075	3,076	3,077	3,078	3,079	3,080	3,081	3,082	3,083	3,084
1908	3,085	3,086	3,087	3,088	3,089	3,090	3,091	3,092	3,093	3,094	3,095	3,096
1909	3,097	3,098	3,099	3,100	3,101	3,102	3,103	3,104	3,105	3,106	3,107	3,108
1910	3,109	3,110	3,111	3,112	3,113	3,114	3,115	3,116	3,117	3,118	3,119	3,120
1911	3,121	3,122	3,123	3,124	3,125	3,126	3,127	3,128	3,129	3,130	3,131	3,132
1912	3,133	3,134	3,135	3,136	3,137	3,138	3,139	3,140	3,141	3,142	3,143	3,144
1913	3,145	3,146	3,147	3,148	3,149	3,150	3,151	3,152	3,153	3,154	3,155	3,156
1914	3,157	3,158	3,159	3,160	3,161	3,162	3,163	3,164	3,165	3,166	3,167	3,168
1915	3,169	3,170	3,171	3,172	3,173	3,174	3,175	3,176	3,177	3,178	3,179	3,180
1916	3,181	3,182	3,183	3,184	3,185	3,186	3,187	3,188	3,189	3,190	3,191	3,192
1917	3,193	3,194	3,195	3,196	3,197	3,198	3,199	3,200	3,201	3,202	3,203	3,204
1918	3,205	3,206	3,207	3,208	3,209	3,210	3,211	3,212	3,213	3,214	3,215	3,216
1919	3,217	3,218	3,219	3,220	3,221	3,222	3,223	3,224	3,225	3,226	3,227	3,228
1920	3,229	3,230	3,231	3,232	3,233	3,234	3,235	3,236	3,237	3,238	3,239	3,240
1921	3,241	3,242	3,243	3,244	3,245	3,246	3,247	3,248	3,249	3,250	3,251	3,252
1922	3,253	3,254	3,255	3,256	3,257	3,258	3,259	3,260	3,261	3,262	3,263	3,264
1923	3,265	3,266	3,267	3,268	3,269	3,270	3,271	3,272	3,273	3,274	3,275	3,276
1924	3,277	3,278	3,279	3,280	3,281	3,282	3,283	3,284	3,285	3,286	3,287	3,288

STANDARD MONTH NUMBERS (Continued)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1925	3,289	3,290	3,291	3,292	3,293	3,294	3,295	3,296	3,297	3,298	3,299	3,300
1926	3,301	3,302	3,303	3,304	3,305	3,306	3,307	3,308	3,309	3,310	3,311	3,312
1927	3,313	3,314	3,315	3,316	3,317	3,318	3,319	3,320	3,321	3,322	3,323	3,324
1928	3,325	3,326	3,327	3,328	3,329	3,330	3,331	3,332	3,333	3,334	3,335	3,336
1929	3,337	3,338	3,339	3,340	3,341	3,342	3,343	3,344	3,345	3,346	3,347	3,348
1930	3,349	3,350	3,351	3,352	3,353	3,354	3,355	3,356	3,357	3,358	3,359	3,360
1931	3,361	3,362	3,363	3,364	3,365	3,366	3,367	3,368	3,369	3,370	3,371	3,372
1932	3,373	3,374	3,375	3,376	3,377	3,378	3,379	3,380	3,381	3,382	3,383	3,384
1933	3,385	3,386	3,387	3,388	3,389	3,390	3,391	3,392	3,393	3,394	3,395	3,396
1934	3,397	3,398	3,399	3,400	3,401	3,402	3,403	3,404	3,405	3,406	3,407	3,408
1935	3,409	3,410	3,411	3,412	3,413	3,414	3,415	3,416	3,417	3,418	3,419	3,420
1936	3,421	3,422	3,423	3,424	3,425	3,426	3,427	3,428	3,429	3,430	3,431	3,432
1937	3,433	3,434	3,435	3,436	3,437	3,438	3,439	3,440	3,441	3,442	3,443	3,444
1938	3,445	3,446	3,447	3,448	3,449	3,450	3,451	3,452	3,453	3,454	3,455	3,456
1939	3,457	3,458	3,459	3,460	3,461	3,462	3,463	3,464	3,465	3,466	3,467	3,468
1940	3,469	3,470	3,471	3,472	3,473	3,474	3,475	3,476	3,477	3,478	3,479	3,480
1941	3,481	3,482	3,483	3,484	3,485	3,486	3,487	3,488	3,489	3,490	3,491	3,492
1942	3,493	3,494	3,495	3,496	3,497	3,498	3,499	3,500	3,501	3,502	3,503	3,504
1943	3,505	3,506	3,507	3,508	3,509	3,510	3,511	3,512	3,513	3,514	3,515	3,516
1944	3,517	3,518	3,519	3,520	3,521	3,522	3,523	3,524	3,525	3,526	3,527	3,528
1945	3,529	3,530	3,531	3,532	3,533	3,534	3,535	3,536	3,537	3,538	3,539	3,540
1946	3,541	3,542	3,543	3,544	3,545	3,546	3,547	3,548	3,549	3,550	3,551	3,552
1947	3,553	3,554	3,555	3,556	3,557	3,558	3,559	3,560	3,561	3,562	3,563	3,564
1948	3,565	3,566	3,567	3,568	3,569	3,570	3,571	3,572	3,573	3,574	3,575	3,576
1949	3,577	3,578	3,579	3,580	3,581	3,582	3,583	3,584	3,585	3,586	3,587	3,588
1950	3,589	3,590	3,591	3,592	3,593	3,594	3,595	3,596	3,597	3,598	3,599	3,600
1951	3,601	3,602	3,603	3,604	3,605	3,606	3,607	3,608	3,609	3,610	3,611	3,612
1952	3,613	3,614	3,615	3,616	3,617	3,618	3,619	3,620	3,621	3,622	3,623	3,624
1953	3,625	3,626	3,627	3,628	3,629	3,630	3,631	3,632	3,633	3,634	3,635	3,636
1954	3,637	3,638	3,639	3,640	3,641	3,642	3,643	3,644	3,645	3,646	3,647	3,648
1955	3.649	3.650	3.651	3.652	3.653	3.654	3.655	3.656	3.657	3,658	3.659	3,660
1956	3.661	3.662	3.663	3.664	3.665	3.666	3.667	3.668	3.669	3,670	3,671	3,672
1957	3.673	3.674	3.675	3.676	3.677	3.678	3.679	3.680	3,681	3,682	3.683	3,684
1958	3.685	3.686	3.687	3.688	3.689	3.690	3.691	3.692	3.693	3,694	3,695	3,696
1959	3.697	3.698	3.699	3.700	3.701	3.702	3.703	3.704	3,705	3,706	3,707	3,708
1960	3.709	3.710	3.711	3.712	3.713	3.714	3.715	3.716	3.717	3,718	3.719	3,720
1961	3.721	3.722	3.723	3.724	3.725	3.726	3.727	3.728	3.729	3,730	3.731	3.732
1962	3.733	3.734	3.735	3.736	3.737	3.738	3.739	3.740	3.741	3,742	3.743	3.744
1963	3.745	3.746	3.747	3.748	3.749	3.750	3.751	3.752	3.753	3,754	3.755	3.756
1964	3.757	3.758	3.759	3.760	3.761	3.762	3.763	3.764	3.765	3,766	3.767	3,768
1965	3.769	3.770	3.771	3.772	3.773	3.774	3.775	3.776	3.777	3,778	3.779	3.780
1966	3.781	3.782	3.783	3.784	3.785	3.786	3.787	3.788	3.789	3,790	3.791	3.792
1967	3.793	3.794	3.795	3.796	3.797	3.798	3.799	3.800	3,801	3,802	3.803	3.804
1968	3.805	3.806	3.807	3.808	3.809	3.810	3.811	3.812	3.813	3,814	3.815	3.816
1969	3.817	3.818	3.819	3.820	3.821	3.822	3.823	3.824	3.825	3,826	3,827	3.828
1970	3.829	3.830	3.831	3.832	3.833	3.834	3.835	3.836	3,837	3,838	3.839	3.840
1971	3.841	3.842	3.843	3.844	3.845	3.846	3.847	3.848	3,849	3,850	3.851	3,852
1972	3.853	3.854	3.855	3.856	3.857	3.858	3.859	3.860	3.861	3,862	3.863	3.864
1973	3.865	3.866	3.867	3.868	3.869	3.870	3.871	3.872	3.873	3,874	3.875	3.876
1974	3.877	3.878	3.879	3.880	3.881	3.882	3.883	3.884	3,885	3,886	3.887	3.888
1975	3.889	3.890	3.891	3.892	3.893	3.894	3.895	3.896	3.897	3,898	3.899	3.900
1976	3.901	3.902	3.903	3.904	3.905	3.906	3.907	3.908	3.909	3,910	3,911	3.912
1977	3.913	3.914	3.915	3.916	3.917	3.918	3.919	3,920	3.921	3,922	3,923	3.924
1978	3.925	3.926	3.927	3.928	3.929	3.930	3.931	3.932	3.933	3,934	3.935	3.936
1979	3.937	3.938	3,939	3.940	3.941	3.942	3.943	3.944	3.945	3,946	3,947	3,948
1980	3.940	3.950	3.951	3.952	3.953	3.954	3.955	3.956	3.957	3,958	3.959	3.960
1981	3.961	3.962	3.963	3.964	3.965	3.966	3.967	3.968	3.969	3,970	3.971	3.972
1982	3.973	3.974	3.975	3.976	3.977	3.978	3.979	3.980	3.981	3,982	3,983	3.984
1983	3.985	3.986	3.987	3.988	3.989	3.990	3.991	3.992	3.993	3,994	3.995	3.996
1984	3.997	3.998	3.999	4.000	4.001	4.002	4.003	4,004	4,005	4,006	4,007	4,008

CYCLE ANALYSIS:

A DESCRIPTION OF THE HOSKINS TIME CHART

By

Edward R. Dewey

Technical Bulletin No. 3

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1949

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A DESCRIPTION OF THE HOSKINS TIME CHART

By Edward R. Dewey
Director, Foundation for the Study of Cycles

In 1935 Mr. Chapin Hoskins, a business consultant of New York City and a pioneer in rhythm analysis, devised a useful method for getting an indication of the length and importance of the rhythmic fluctuations that may be present in a series of figures. He calls the method a "Time Chart."

The Time Chart depends for its usefulness upon an objective method for determining the "highs" and "lows" for any given wave length being sought. The Time Chart itself is then merely a method of diagramming those highs and lows in such a way as to throw light upon the length and amplitude of the rhythm or rhythms that may be present in the figures.

Mr. Hoskins' objective method of determining the "highs" and "lows" can best be explained by an illustration. But first we must introduce a concept that will be new to most readers. Consider the curve in Fig. 1. As everyone knows, each point on the curve has a value, depending upon its height above the base line. Thus the point for the year 1920 has a value of 850. But the point for the year 1920 has another characteristic. It is higher than any of the points of the three preceding years. That is, a horizontal line projected backward from the point at 1920 until it intersects the curve between years 1916 and 1917, as indicated by the fine dotted line, would skim over three points. Similarly, a horizontal line projected backward from the point at year 1922 would be lower than the ten preceding points--but not lower than the eleventh. Those numbers--the whole years or time units backward on a horizontal line that are in the clear before the curve is intersected, or the end of the series reached--three and ten in the examples given, are called by Mr. Hoskins the <u>violence</u> of the point.* If the given point has a value greater than that of the point next preceding, the horizontal line would skim above preceding points and the violence would be indicated by a black number. If the given point is

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*"Violence" is an arbitrary word used by Mr. Hoskins for this concept, to imply that the rise or decline, the results of which are being measured, come about through motion. To the author the word <u>clearspan</u> would seem preferable. Certainly it is more descriptive.

below the point next preceding, the horizontal line would skim below the preceding points and the violence would be denoted by a red number. When, as with the years 1902 and 1903, the values are higher than at any previous time, this fact is indicated by a + sign after the violence, the plus to be read "or more." Thus the clearspan or violence for 1902 would be 1+, one or more. When the value of a point is exactly equal to the value of the point next preceding, the violence is 0, usually written in the opposite color from the violence next preceding.

The value and violence of each point on the curve in Fig. 1 are given in the following table.

Year	Value	Violence (Clearspan)	Year	Value	Violence (Clearspan)
1900	—	—	1917	750	1
1901	100	—	1918	650	6
1902	400	1+	1919	700	1
1903	800	2+	1920	850	3
1904	700	1	1921	850	0
1905	600	2	1922	500	10
1906	500	1	1923	550	1
1907	600	1	1924	600	2
1908	400	5	1925	900	8
1909	300	7	1926	800	1
1910	200	8	1927	700	2
1911	400	2	1928	50	27+
1912	900	11+	1929	200	1
1913	1,000	12+	1930	250	2
1914	750	2	1931	200	1
1915	700	3	1932	100	3
1916	900	2			

The violence can be computed graphically by counting the points over or under which an imaginary backward horizontal projection would skim until it intersects the curve, or passes beyond the first point of the curve.

The violence can be computed more easily from the numerical data. This is done as follows: First notice if the value of the point for which the violence is being computed is over or under the value of the next preceding point. If it is above, the violence will be recorded in black. If below, in red. The numerical value of the violence will be the number of points back to a value as high or higher, less one, or as low or lower, less one, as the case may be. Thus the violence of the point at year 1920 is found by counting the points backward until one finds the first number with a value as great as or greater than 850, and subtracting 1. In this case, one must go back four years to 1916 to get a value as high or higher than 850. The number of the violence is therefore 4 less 1, or three, recorded in black. Again, the clearspan or violence of the point at year 1922 is 11 less 1, or 10, recorded in red.

If the points are numbered consecutively, as they are when the series of figures are annual ones, one does not need to count,—one can subtract. Thus, to get the violence of 1922, one scans the value column (without counting) until one reaches a value as low or lower than 500, because the value for 1922 is below the value for 1921. Having found the value sought, one notices that it is at year 1911. Subtract 1911 from 1922; the difference is 11. Subtract 1 to get the clearspan or violence of 10; record it in red. (In practice, of course, one subtracts the year <u>after</u> 1911, namely 1912, from 1922, and obtains 10 directly.)

If the data used are monthly, in order to make these computations practicable it is necessary to give consecutive numbers to all the months. Any consecutive numbers will do, but Mr. Hoskins chose December 1900 as month zero. This scheme makes December of 1901 month number 12, December of 1902 month number 24, etc., December of any year being 12 times the number of years after 1900. For longer series, December 1900 can be taken as month number 1,000 or 3,000. Hoskins month numbers from January 1901 to December 1960 are given in the table on Page 13. They are standard for all who use this method of analysis.*

We are now ready to proceed to a description of Mr. Hoskins' objective method of determining highs and lows.

"Highs" and "lows" can be defined as terminal points of cyclical rise and cyclical decline. To meet "full standard" for a high or a low, a point must have a clearspan or violence equal to at least one half of the length of the wave being sought, or the next larger odd number. Thus, for either 6- or 7-year waves, the violence of full standard highs or lows must be 3 or more, black for the highs, red for the lows.

The high is the highest point of the required violence between two lows of the required violence. Thus the high for 6- or 7-year

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*For the analysis of short term rhythms where weekly data are used, Mr. Hoskins has set the week ending July 2, 1932, as week number zero. With this numbering plan the week ending December 31, 1949 would be week number 4913. For pre-1932 calculation, he calls the week ending July 2, 1932, week 4000. For work with daily figures a daily number in which Saturday is seven times the week number could be used.

In passing, it should be noted that in all rhythm analysis using daily figures, Saturdays, Sundays, and Holidays must be included, either with real or interpolated values.

waves is the highest point with a violence of black 3 or more between two lows of red 3 or more; a low, the lowest point between two highs of black 3 or more.

For example, in the curve given in Fig. 1, if we are seeking 6- or 7-year waves, we could start at the beginning and go forward until we get a violence of 3 or more, either red or black. We find such a point at 1906. This year, with a violence of red 3, could mark (as far as violence is concerned) a full standard low for a 6-year wave, and is such a low, unless a lower value is present before the next black 3. We then proceed to look for the next point with a violence of black 3 or more. We find such a point at 1912 with a violence of black 11+ . The year with the lowest value between 1906 and 1912 is 1910 with a value of 200 and a clearspan or violence of red 8. It is the low of the move.

We then go on from year 1912 until we find a point with a red violence of 3 or more, as at the year 1915. The highest value between 1910 and 1915 is at 1913 with a value of 1,000 and a violence of black 12 . This point is the high of the move. Similarly, year 1918 is a low, year 1920 is a high, year 1922 a low, year 1925 is a high, and year 1928 is a low.

Year 1903 is probably a high and year 1901 is possibly a low, and these facts may be indicated, but neither point has a known violence great enough fully to meet standards.

Suppose we were seeking 2- or 3-year waves. We would take as our full standard highs the highest points with a violence of black 1 or more between points with violences of red 1 or more. By this rule, highs would be present at years '03, '07, '13, '16, '20, '25, and '30; lows at years '06, '10, '15, '18, '22, and '28. Lows are also perhaps at 1901 and 1932, but this cannot be known. (In practice one would hardly expect to find a 2-year wave through the use of annual figures, but we are concerned here with principles only.)

Thus, by this method, for any given length of rhythm, is it possible to make an objective determination of highs and lows. It is the only method known to the author whereby this can be done.

Suppose we wish to use the Time Chart to scan the curve in Fig. 1 (the values of which are recorded in the table on Page 2) for all possible wave lengths that would be revealed by this method. The longest wave that could repeat itself completely within the length of this series of figures would be half of 32 or 16 years in length. Full standard highs and lows for such a wave would have a violence or clearspan of black 8 or more and red 8 or more, and would be determined by the methods already described. From Fig. 1 or from the table on Page 2 we see that highs of these standards occurred at 1913, and 1925, and that full standard lows came in 1910, 1922, and 1928. There is a possibility that 1903, exceeded as it is by only six values in the entire 32 years, is also a high, but this fact cannot be known without earlier figures.

These facts could be diagrammed on the grid shown in Fig. 2 using a solid line ——— for highs, a wavy line ～～～ for lows.* High or low values that come so near the beginning of the series that we do not know their true violence could have this fact indicated by a parenthesis. In Fig. 3 the base year, 1900, is written in at the top and the lines are numbered to represent years after base. Fig. 3 is little more than the grid of Fig. 1 and Fig. 2 turned from sideways to up and down and marked to indicate highs and lows.

Another way to represent the time position of these highs and lows would be to cut the grid in two, as in Fig. 4, and place the second section beside the first. The numbers at the top of the column still represent the base years, in this case 1900 and 1916; the numbers down the side represent the number of years after base. The lines in the first column represent the first sixteen years of the series, the lines in the second column represent years 16 to 32 inclusive. The last year represented in the first column is the base year of the second column.

The first low (wavy line) comes at year 1910; the first high (solid line) at year 1913, but there is a probable high (solid line in parentheses) at 1903; the second low (wavy line) comes at year 1922, or position 6 after base year of 1916; the next high (solid line) comes at year 1925, or 1916 plus 9; the last low (wavy line) at year 1928, or 1916 plus 12.

Now, if there were a 16-year rhythm in this series of figures there would <u>tend</u> to be one high and one low in each column of 16 years; the highs would be about 16 years from each other and would therefore fall in about the same section of the grid. In other words, the solid lines would tend to make more or less of a horizontal line. The lows would tend to do likewise. In Fig. 4 they do not do this. It is therefore obvious that in this series of figures we do not have a 16-year rhythm.

But our Time Chart can be made to tell us more. As well as telling us of lengths that are absent it will tell us the approximate length of the rhythms that are present (if there are any).

Consider Fig. 5. Here we have extended the diagram of Fig. 4 by adding two additional 16-year sections based on 1884 and 1932 respectively. In the 1884 section, let us now repeat three years

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*On work sheets or in drawings, one uses <u>color</u>; but the expense of color printing makes it necessary at this point to use the symbolism of straight lines for highs and wavy lines for lows. Any two colors would do, but Mr. Hoskins uses red for highs to indicate danger, and black for lows. The author prefers to use black for highs and red for lows as this scheme fits better into other methods which have been found to be indispensible supplements to the Time Chart, and which will be described in later bulletins.

after its end, the first solid line, "A", of 1903. Similarly, let us repeat the wavy line "B" of 1928 in the 1932 section, five years before its beginning. We can do this because any position in time can be plotted in any column if we extend it far enough one way or the other. We now have no more than one pair of highs and lows in each column, with all the lows on the same side (in this case above) all the highs next following. Let us connect highs and lows with vertical lines of their own sort--solid or wavy (in actual use, according to the author's standards, black or red).

We see that the highs and the lows thus represented do line up after a sort, but not along a horizontal line as they would if the wave were 16 years long. To represent how they really fall, let us connect the midpoint of the first and last of the highs and the first and last of the lows by dotted lines. We note that for the wavy lines or lows, the dotted line slopes upward by 14 years in two cycles. This suggests a length of 16 years less 7 years (14/2) or 9 years. The dotted line connecting the highs slopes upward 10 years in two cycles or five years (10/2) per cycle. This suggests a length of 11 years (16 years less 5 years). Averaging the two indicated lengths (9 and 11 years) we get a length of 10 years as a first approximation of the length of the rhythm.

Let us now make a 10-year Time Chart of the values (using a violence or clearspan of five (V_5)). This is done in Fig. 6. It happens that V_5 gives us no additional highs and lows in this instance. As before, wavy line "A" and "A" both represent an identical low point, in this case the low at 1922, (position 1910 plus 12 or position 1920 plus 2). Fitting a straight line to these positions will give us the closest length that we can get by the time chart method. For so few repetitions it will not be too accurate, but if the series is long enough the time chart will indicate the length within 1% or less.

Note that on the Time Chart there is a number above each of the lines and a number below each of the lines. The numbers above the lines give the value of the curve at the time indicated--i.e. the height above the base line; the numbers below the lines give the violence or clearspan for the particular turning point at this position--bold face for black, italics surrounded by circles for red.

This is how one would interpret such a chart: There has been a tendency in the data plotted in Fig. 1 and recorded in the table on Page 2 for highs to come at about 10-year intervals and lows likewise. It is true that there have been but three repetitions of this wave, but there could not have been more repetitions of a wave of this length in a series as short as this. For the significance of such behavior we must resort to additional tests, but for what it may be worth we can observe directly from the Time Chart that no high has ever had a value of less than 800 and that the most distorted high came only two years after median position. Also, that insofar as we have the figures, no high has failed to be higher than all values for at least the preceding 8 years. The median high position is 3 years after base, and _if this behavior continues_, we may expect highs in the general neighborhood (perhaps two years one way or the other) of 1933, 1943, and 1953, etc.

As for the lows, no low has failed to have a value lower than any value in at least the 8 preceding years; no low has ever had a value in excess of 500, and the median behavior indicates a value of 200 as being closer to probabilities. No low has come more than 2 years before median timing, nor more than two years after such timing. (In longer series it is helpful to speak of the range within which the majority of the highs or lows have fallen.) _If this behavior continues_, lows may be expected 2 years one way or the other from 1940, 1950, and 1960.

Other methods can be used to refine these observations, to evaluate them, and to determine their significance.

Originally we started to look for whatever rhythm might be present at about half of our overall length of 32 years. We came up with a rhythm of about a third of our overall length. We might next look for a rhythm of a quarter of 32 years, or 8 years, to see what we would find, then to a fifth of 32 years, a sixth of 32 years, and so on for as far as we wished.

On the other hand, we might start by simple observation and use a Time Chart to see the regularity of some particular rhythm we thought might be present. Or we could start from a preconceived idea and explore the extent to which a 9-year or a 3½-year or a 6-year wave might or might not be present. In any event, the Time Chart will be useful and will throw light on our problem.

Occasionally one finds an extra pair of highs or lows within a cycle, as in Fig. 7. The Time Chart for such behavior would be like the one in Fig. 8. The extra pairs should be diagrammed and crossed out.

Often one fails to find highs and lows of full standard clear-span or violence. In such cases one is compelled to fill in with sub-standard highs or lows. When this is done, this fact should be indicated by broken lines. If the violence of the point used for a high or low is less than full standard but over one-third of the proper violence, the point is called a secondary high; it is indicated by a dashed line. If the violence is less than one-third of the proper violence, but greater than one ninth, the high is called a tertiary high and it is indicated by a dotted line.

Sometimes one finds a cycle with no highs or lows at all. In such a case the cycle without high or low is left entirely blank and no effort is made to connect to each other the highs and lows on either side of it, or they are connected with curved lines, thus .

The Time Chart can be made to tell much more. For instance, hints of longer cycles may be revealed by double or triple underlining of all highs and lows which have double or triple the required violence. Also, it is often helpful to use a grid twice or three times as long as the length of the wave being sought. Thus, in exploring for a 6-year rhythm, one might use a grid 12 or 18 years long and repeat the wave two or three times per column. This procedure often reveals unsuspected facts in regard to the behavior of the series. Such a Time Chart would be called "Twelve-over-two" (12/2-year) Time Chart V_3v_1 or "Eighteen-over-three" (18/3-year) Time Charts, V_3v_1 to indicate the length of the grid and the violence used for full standard and secondary standard highs and lows.

The above outline describes the Time Chart only in its most elementary form. There is much more that could be said in regard to it, its use, and its interpretation. In fact, so much could be said that, if he could find the time, Mr. Hoskins could undoubtedly write a small monograph on the subject. It is the hope of the author of this description that someday he will.

The value of the Time Chart method lies first in the concept of "violence" or "clearspan," second in the objective method of determining highs and lows, and third, in the fact that the individual cycles are not lost sight of through any sort of mathematical averaging.

Finally, a note of warning: Simple as its elements are, the Time Chart must be used with care, or unjustified conclusions will be drawn. It must be remembered that after all, the Time Chart is merely a tool for obtaining hints of rhythmic behavior. It does not reveal whether the indicated rhythms, if any, are "real" or are merely the result of random forces.

CHARTS AND TABLES

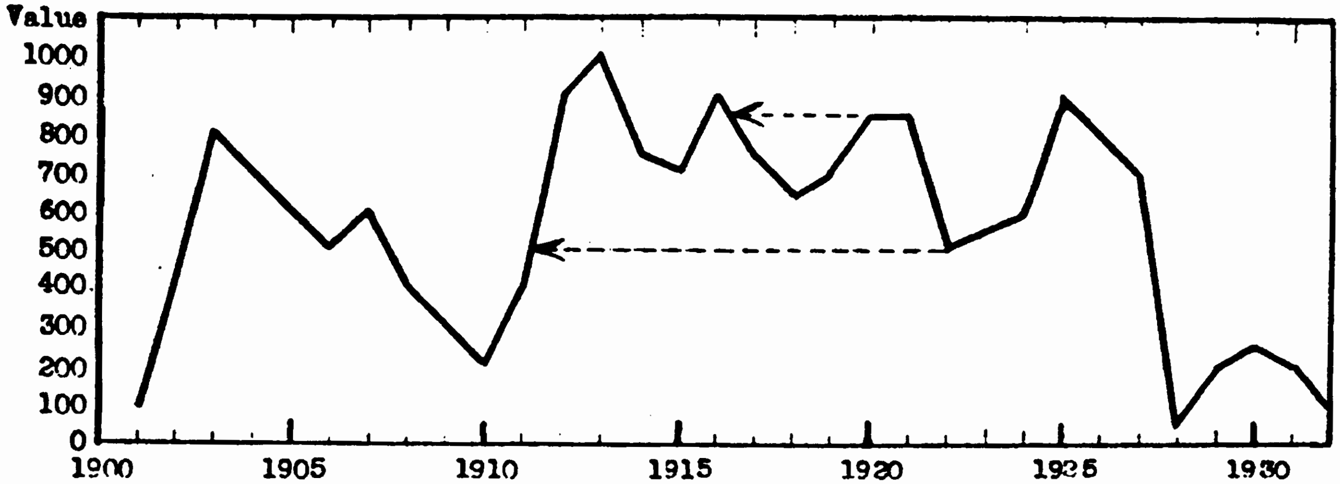


Fig. 1. This chart shows a hypothetical curve from 1901 to 1932 inclusive. The fine dotted lines illustrate the concept of clearspan or "violence".

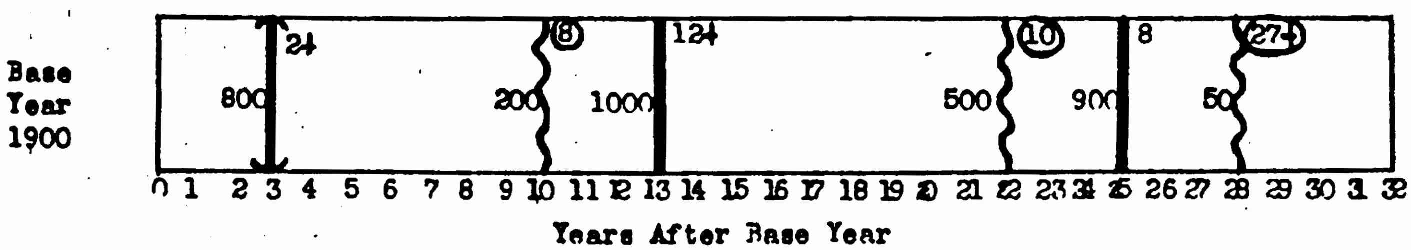


Fig. 2. This chart represents time from zero years after1900 to 32 years after 1900; in other words, 1900 to 1932. Heavy solid vertical lines indicate highs; wavy vertical lines indicate lows. (In actual practice, one would use black and red.) The parentheses that surround the first high indicate that we do not know for certain that it is a high. The numbers to the left of the line refer to the value of the curve at the time indicated. The numbers to the right of the line indicate "violence" or "clear-span. Red violence is indicated by a circle, but in actual practice would be written in red. This chart is merely a short-hand method of indicating the highs and lows with a "violence" of 8 or more, as found in Fig. 1 above.

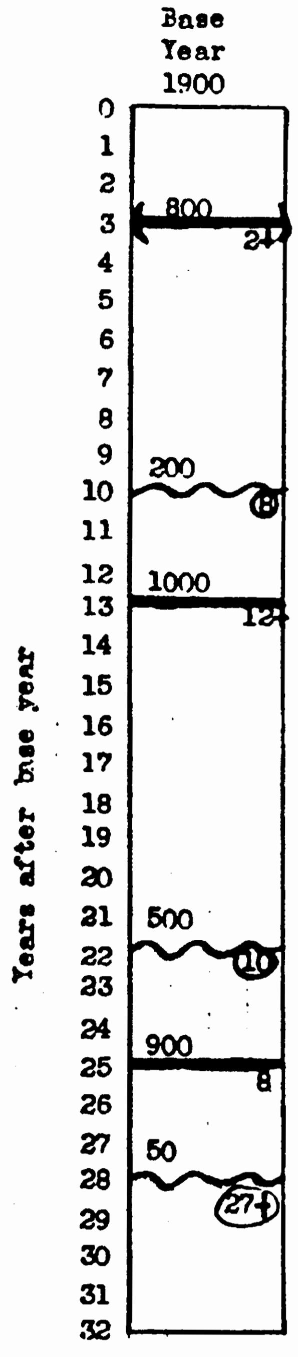


Fig. 3. This diagram is the same as Fig. 2 except that it is represented in a vertical instead of a horizontal position.

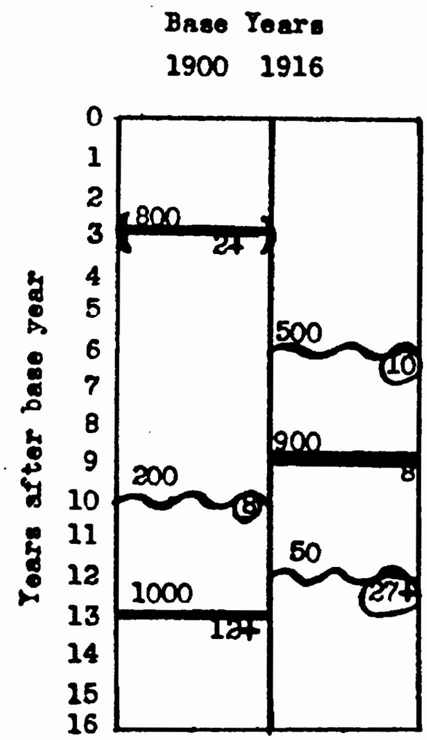


Fig. 4. This diagram shows the first step in the making of a 16-year Time Chart of the data plotted in Fig. 1. The "violence" standard is V_8. The highs and lows are not yet connected. Fig. 4 is the same as Fig. 3 except the second half of Fig. 3 has been moved over and up, thus showing two consecutive 16-year sections. The first low in the second column comes 6 years after base year 1916, or 1922.

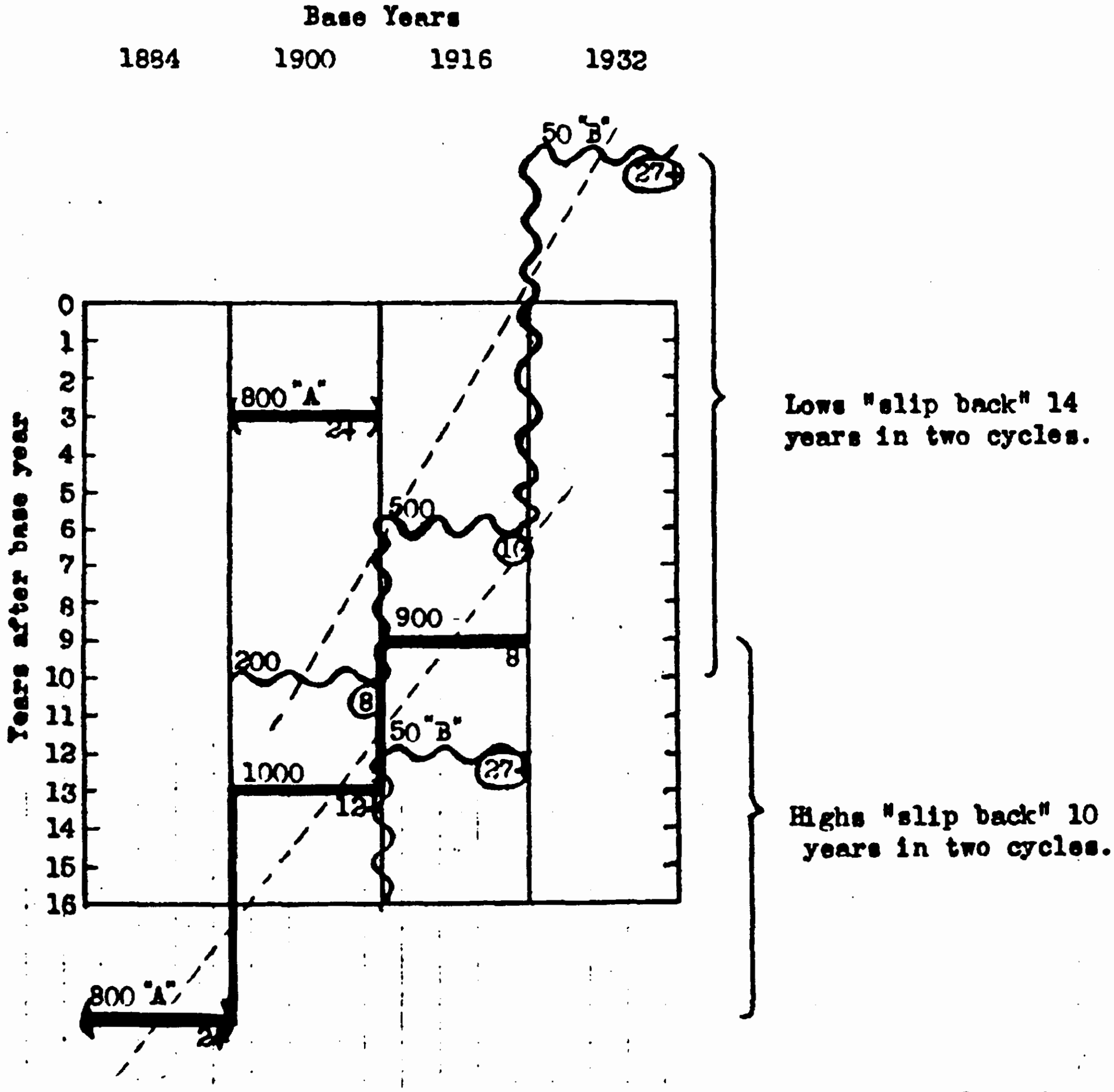


Fig. 5. This is a completed 16-year Time Chart of the data plotted in Fig. 1. Violence standard V_8. Two more cycles have been added to the grid and the first high, "A", is then repeated in the new section based on 1884. The last low, "B", is then repeated in the new section based on 1932. You will note that both the first high and the last low fall outside of the grid. The highs and lows were then connected. Because the highs and lows do not fall opposite each other in a horizontal line, it is obvious that there is no 16-year rhythm in the given curve.

From the upward slope of the line one sees that there is a rhythm of something less than 16 years. As the lows drop back 14 years in 2 cycles, we see the indicated length is 16 minus 14 over 2 or 7 years. As the highs slip back 10 years we see the indicated length is 16 minus 10 over 2 or 5 years. It is obvious that the length of the rhythm is approximately 6 years (7+5 over 2) less than 16 years, or 10 years.

This diagram would be clearer if red and black lines were used instead of wavy and straight lines.

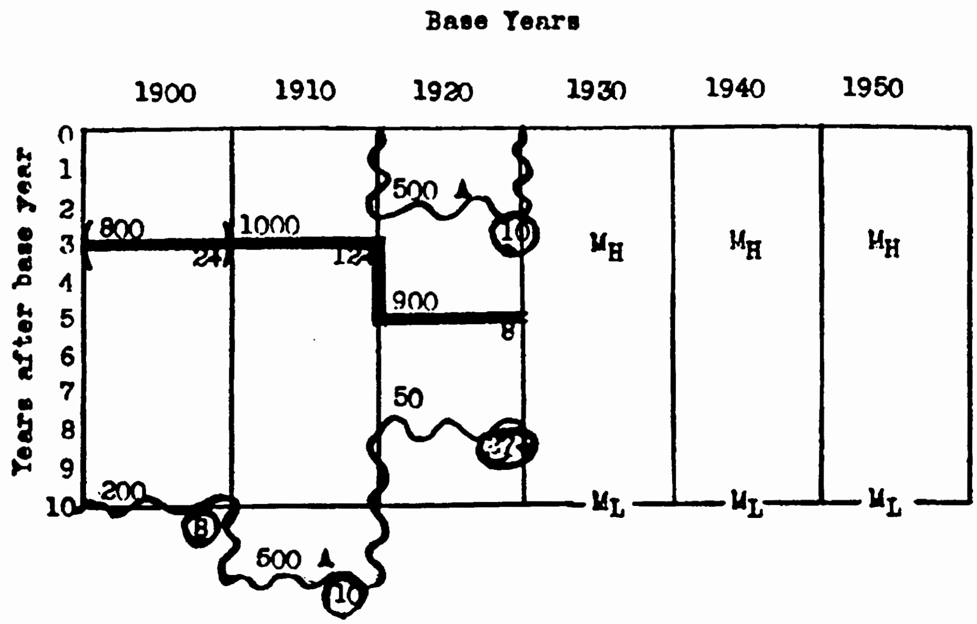


Fig. 6. A 10-year Time Chart of the data plotted in Fig. 1. Violence standard V_5. Highs and lows connected. M_H signifies predicted median position of highs. M_L signifies predicted median position of lows.

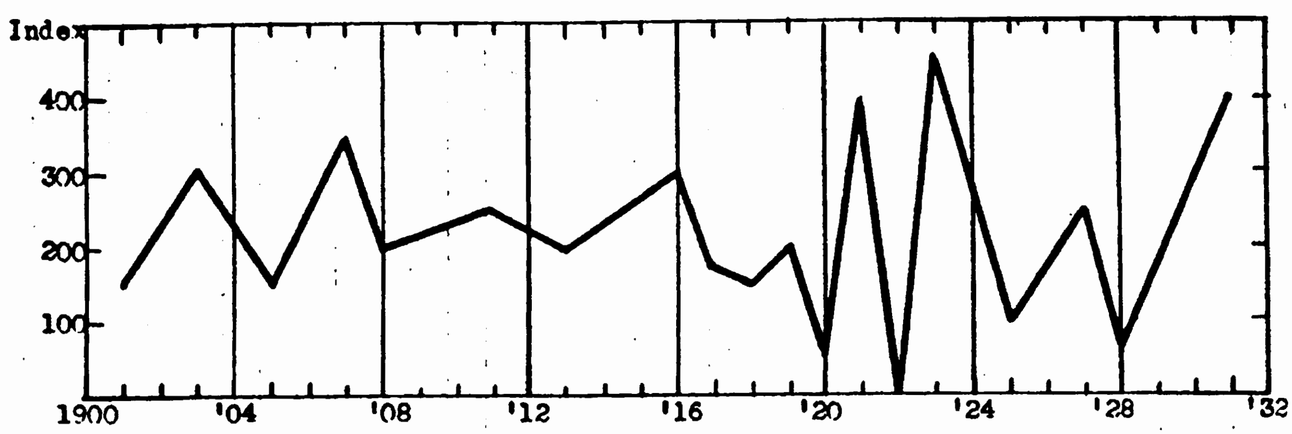


Fig. 7. This chart shows another hypothetical series of data, 1901-1931.

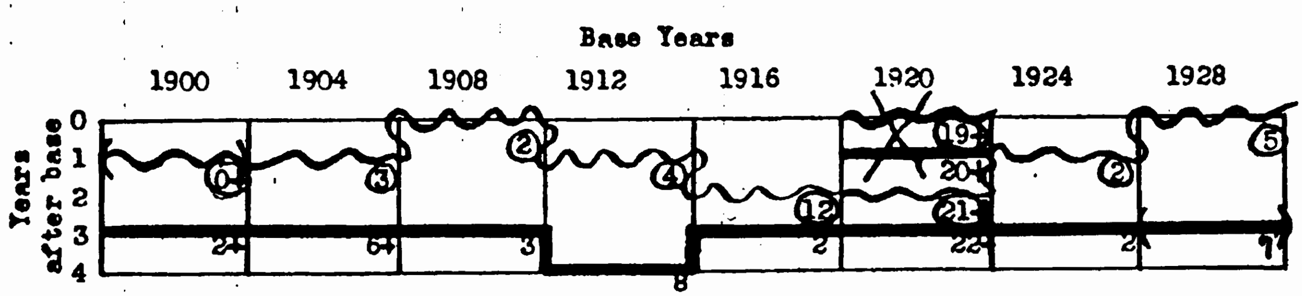


Fig. 8. A 4-year Time Chart of the data shown in Fig. 6.

HOSKINS STANDARD MONTH NUMBERS

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1901	1	2	3	4	5	6	7	8	9	10	11	12
1902	13	14	15	16	17	18	19	20	21	22	23	24
1903	25	26	27	28	29	30	31	32	33	34	35	36
1904	37	38	39	40	41	42	43	44	45	46	47	48
1905	49	50	51	52	53	54	55	56	57	58	59	60
1906	61	62	63	64	65	66	67	68	69	70	71	72
1907	73	74	75	76	77	78	79	80	81	82	83	84
1908	85	86	87	88	89	90	91	92	93	94	95	96
1909	97	98	99	100	101	102	103	104	105	106	107	108
1910	109	110	111	112	113	114	115	116	117	118	119	120
1911	121	122	123	124	125	126	127	128	129	130	131	132
1912	133	134	135	136	137	138	139	140	141	142	143	144
1913	145	146	147	148	149	150	151	152	153	154	155	156
1914	157	158	159	160	161	162	163	164	165	166	167	168
1915	169	170	171	172	173	174	175	176	177	178	179	180
1916	181	182	183	184	185	186	187	188	189	190	191	192
1917	193	194	195	196	197	198	199	200	201	202	203	204
1918	205	206	207	208	209	210	211	212	213	214	215	216
1919	217	218	219	220	221	222	223	224	225	226	227	228
1920	229	230	231	232	233	234	235	236	237	238	239	240
1921	241	242	243	244	245	246	247	248	249	250	251	252
1922	253	254	255	256	257	258	259	260	261	262	263	264
1923	265	266	267	268	269	270	271	272	273	274	275	276
1924	277	278	279	280	281	282	283	284	285	286	287	288
1925	289	290	291	292	293	294	295	296	297	298	299	300
1926	301	302	303	304	305	306	307	308	309	310	311	312
1927	313	314	315	316	317	318	319	320	321	322	323	324
1928	325	326	327	328	329	330	331	332	333	334	335	336
1929	337	338	339	340	341	342	343	344	345	346	347	348
1930	349	350	351	352	353	354	355	356	357	358	359	360
1931	361	362	363	364	365	366	367	368	369	370	371	372
1932	373	374	375	376	377	378	379	380	381	382	383	384
1933	385	386	387	388	389	390	391	392	393	394	395	396
1934	397	398	399	400	401	402	403	404	405	406	407	408
1935	409	410	411	412	413	414	415	416	417	418	419	420
1936	421	422	423	424	425	426	427	428	429	430	431	432
1937	433	434	435	436	437	438	439	440	441	442	443	444
1938	445	446	447	448	449	450	451	452	453	454	455	456
1939	457	458	459	460	461	462	463	464	465	466	467	468
1940	469	470	471	472	473	474	475	476	477	478	479	480
1941	481	482	483	484	485	486	487	488	489	490	491	492
1942	493	494	495	496	497	498	499	500	501	502	503	504
1943	505	506	507	508	509	510	511	512	513	514	515	516
1944	517	518	519	520	521	522	523	524	525	526	527	528
1945	529	530	531	532	533	534	535	536	537	538	539	540
1946	541	542	543	544	545	546	547	548	549	550	551	552
1947	553	554	555	556	557	558	559	560	561	562	563	564
1948	565	566	567	568	569	570	571	572	573	574	575	576
1949	577	578	579	580	581	582	583	584	585	586	587	588
1950	589	590	591	592	593	594	595	596	597	598	599	600
1951	601	602	603	604	605	606	607	608	609	610	611	612
1952	613	614	615	616	617	618	619	620	621	622	623	624
1953	625	626	627	628	629	630	631	632	633	634	635	636
1954	637	638	639	640	641	642	643	644	645	646	647	648
1955	649	650	651	652	653	654	655	656	657	658	659	660

LESSON VI-A

HOW TO GET HINTS OF CYCLES —CONTINUED

My object in giving this course is to make a trained cycle analyst out of you. I want to teach you enough so that I could hire you and you could get to work without further instructions. Even more, I want to teach you enough so that you can make cycle analyses on your own and come up with the right answer.

The guts of the course is Lesson VI, How to Find Hints of Cycles. You cannot possibly learn how to find hints of cycles in one lesson. It requires practice. It's something of an art.

It is almost as if a professor at a medical school were to say to his students, "And now this morning, gentlemen, we will cover the subject of diagnosis."

Yet in Lesson VI, I said about all I could think of to say on the subject. And I gave you about all the problems I thought wise for one time.

The answer to the dilemma would seem to be to go back to the subject (as many times as may prove necessary) with additional problems interspersed among the other lessons.

Therefore, for this week's assignment, instead of going on to a new subject, please review Lesson VI and work the additional problems given on the attached sheet.

Lesson VI-A — Problems

Problem 1. Now I want you to take a series of figures of your own choosing and carry it through the procedures called for up to and including Lesson VI.

I don't care what series of figures you choose. Take some daily, or weekly, or monthly stock market figures, or some monthly or annual lake level figures, or the hourly variation of terrestrial magnetism, or daily spot oat prices, or consecutive random numbers, or the monthly or annual sales of your own company.

More particularly, having chosen your figures—

1. Fill up gaps in your data, if any.
2. Convert your data to logs.
3. Chart.
4. Determine if trend should be removed.
5. If so, scan for cycles, compute the appropriate moving average trend, express the data as deviations from the moving average trend, and chart.
6. Scan deviations for cycles.
7. Smooth deviations, if necessary, and scan again.
8. Time chart the smoothed and unsmoothed deviations.

In doing all this I want you to observe the Standard Practice Instructions so far given.

Finally, and this is most important, I want a copious log to tell me for each step of the analysis (a) exactly what you did, and (b) why you did it, and (c) what you concluded as a result of your labors.

How much time did this problem take you?

LESSON VI-A

Supplement 1

Dear Fellow Student of Cycles:

I gave you Lesson VI-A as kind of a test.

First, I wanted to make sure you would not run into any problems in connection with your own particular time series which had not been covered in the lesson material so far presented.

Second, I wanted to see how well you had mastered the techniques which have been presented so far.

Third, I wanted to get some sort of an idea of how good a job I, myself, had done in presenting the material.

As I write you this letter only one student, Mr. S, has sent back Lesson VI-A.

It may, however, help him and help you if I comment in some detail upon his procedure.

The series of figures in which Mr. S is interested consisted of about a year of daily market prices which fluctuated from about 66 to about 76.

Mr. S proceeded as follows:

First, he interpolated values for Saturdays, Sundays, and holidays.

Second, he plotted his data and his interpolations on arithmetic scale.

Third, he converted to 4 place logarithms.

Fourth, he smoothed these logarithms by means of a three-day moving average.

Fifth, he computed a 9-day moving average of the logarithms and expressed the smoothed data as deviations from this 9-day moving average.

Sixth, he plotted these deviations on arithmetic scale.

Seventh, he studied this plotting by means of dividers to try to find highs and lows that were equidistant.

Eighth, he computed clearspan numbers.

Ninth, he made three time charts of the deviations of the 3-day moving average from the 9-day moving average. One time chart was 8 days long, one was 15 days long, and one was 16 days long.

Now let me review what was right and what was wrong with what Mr. S did.

__First__, he was right in interpolating values for Saturdays, Sundays, and holidays.

__Second__, he was right (my instructions to the contrary notwithstanding) in plotting on arithmetic scale. When you have so small a range as 66 to 76 there is no need to plot on logarithmic scale. In fact arithmetic scale is much better because you can exaggerate the vertical scale so as to see the cycles much more clearly. You cannot do this when you use ordinary semi-log paper (ratio scale).

You __do__ need logarithmic scale for initial plotting when there is a considerable amount of trend and/or when the variations are considerable, but neither of these conditions prevailed for Mr. S's series.

__Third__, the conversion to logarithms was wrong--at this stage of the game. As no computations are required, and as the variation was so small, the use of logarithms--here--is totally unnecessary labor.

However, Mr. S was right in thinking that 4 place logarithms were probably called for, if and when logarithms were used, because with a series as flat as his, the cycles would be bound to be of inconsequential amplitude.

Mr. S could have saved some work by omitting the characteristics of his logarithms. As all of the figures with which he had to deal had two digits to the left of the decimal, the characteristics in all cases were one. Therefore, he could just as easily have omitted them. Omitting his characteristics would have been the same as making them zero. This is the same, in effect, as dividing his data by 10 so that there would have been only one digit to the left of the decimal. Multiplying or dividing your data by a constant does no harm from the standpoint of cycle analysis.

__Fourth__, as all of Lesson VI-A is merely reconnaissance, there was no need to smooth the series. Smoothing is often necessary but it comes later in the game when you know what you want to smooth out.

__Fifth__, as there was no trend to get rid of it was wrong to express the data as deviations from trend (the 9-day moving average).

In the stock market study you __had__ to use deviations from a trend line because the figures ran all the way from 12 up to 200. Figures that vary as much as the stock market figures __must__ have the trend removed before you can deal with them but with figures running from 66 to 76 there was no such necessity.

Moreover even with figures varying like the stock market figure there is a preliminary step before you can decide upon the trend to use. In order to know what kind of a trend to use you have to have a preliminary knowledge of the cycles. Therefore, your __initial__ reconnaissance must __always__ be with the raw figures. In Mr. S's case he should have stayed with the raw figures.

__Sixth__, Mr. S was right in plotting the deviations (once he had them) on arithmetic scale. This is so because they were in lows. But even if they had been in percentages he should have used arithmetic scale so that he could get suitable amplitude for his cycles.

The scale chosen was correct but the plotting was a waste of time because his original plotting would have served him as well or better. In fact, the charting of the deviations was a hindrance to the discovery of any cycles except cycles in the general neighborhood of 9 days (6 days or so to 12 days or so).

__Seventh__, as a rule it is not a good plan to study a chart by means of dividers. If this were the way to look for cycles I would have told you so.

The trouble with looking for cycles with dividers is that the dividers tend to pick out equidistant high __points__ rather than equidistant high __areas__ (or low points and areas as the case may be).

The thumb and pencil, being crude, enables you to concentrate on __areas__ more easily. The graduated scale likewise enables you to concentrate on __areas__ of high and areas of low. Also, with a graduated scale, you can place the scale along the horizontal axis of the chart, covering up either the upper or lower portion of the cycles at will. Finally, the graduated scale has the advantage that it enables you to see __all__ the highs (or lows) simultaneously.

Incidentally, Mr. S should have sent in his graduated scales with his other papers. Miss Shirk and I need them to evaluate his conclusions.

__Eighth__, the clearspan numbers were computed correctly.

__Ninth__, Mr. S's time charts were __not__ made correctly.

To help keep you from making the same mistakes I shall tell you the things Mr. S did wrong.

(a) He did not follow standard practice instructions. For example, he did not put on the time chart the clearspans that he had used for standard. He will have to spend time to figure them out if, some 3 months from now, he should wish to review this work. Also, I need them to evaluate the charts.

(b) Mr. S made his time chart of highs only, leaving out the lows. As a rule no one can interpret a time chart adequately without lows as well as highs.

(c) Mr. S left out his clearspan numbers. Clearspan numbers are indispensable. (Miss Shirk and I will not even bother to correct time charts without clearspan numbers on them.)

(d) Mr. S failed to understand that in putting highs in a time chart you must use __the__ high of the move, not just __a__ high that happens to fall here you would like to have high. That is, in looking for an 8-month cycle with a clearspan standard of V_4 you might have a red 7, then a very nice black 6 just where you wanted it, followed by a red 3, followed by a black 23, followed by a red 4. Under these circumstances the black 6 would not be the high because we do not get a low until the red 4, and the highest value between the red 7 and the red 4 is black 23. Under these circumstances the black 23 must be used. (Similarly of course for lows.)

(e) Finally the nature of Mr. S's manipulation, namely, deviations from a 9-day moving average would eliminate at least 1/3 of any 15- or 16-day cycle that might have been present in the original figures. In most instances it would be unfruitful to bother with time charts of these lengths (15 or 16 days) in deviations of this sort.

Summary. Mr. S's trouble was that he tried to hard. He tried to do too good a job. He did not realize that we are, at this stage of things, engaged merely in reconnaissance. All he has been taught so far is how to find hints of cycles.

If he had spent one fifth of the time (i) studying the curve of the data, (ii) thumbing, (iii) refining his thumbing by means of graduated scales, and (iv) refining his graduated scale conclusions by making suitable time charts--and making them correctly--he would have got much farther.

Live and learn!

LESSON VI-A

Supplement 2

As I write this supplement, quite a number of answers to Lesson VI-A have come in.

The most common mistake is the use of <u>daily</u> instead of <u>weekly</u> or <u>monthly</u> figures.

For preliminary reconnaissance:

(a) Use daily figures for cycles of 3 or 4 days up to perhaps 3 or 4 weeks.

(b) Use weekly figures for cycles 3 or 4 weeks up to 3 or 4 months.

(c) Use monthly figures for cycles 3 or 4 months up to three or four quarters.

(d) Use quarterly figures for cycles 3 or 4 quarters up to three or four half-years.

(e) Use semi annual figures for cycles 3 or 4 half years up to 3 or 4 years.

(f) Use annual figures for cycles 3 or 4 years and up.

Going at it this way cuts down the work and keeps you from becoming bogged down in detail. Or, for the same work it enables you to get more repetitions of your cycle.

Then, <u>after</u> you have your cycles pretty well definitized, you can, with a minimum of effort, drop back to finer subdivisions of data.

For example, suppose, using monthly figures, you have explored a dozen possibilities between 6 and 7 months long. Suppose, you have decided upon 6.41 months as best. Now, with a <u>minimum of effort</u>, you can use weekly data and test for that <u>one particular length</u>. If your work with weekly data does refine the length, you can, if you wish, go still farther and use daily data. But to use daily or weekly data to start with will only bog you down in a morass of detail from which you will never be able to extricate yourself.

LESSON VI-A

Supplement 3

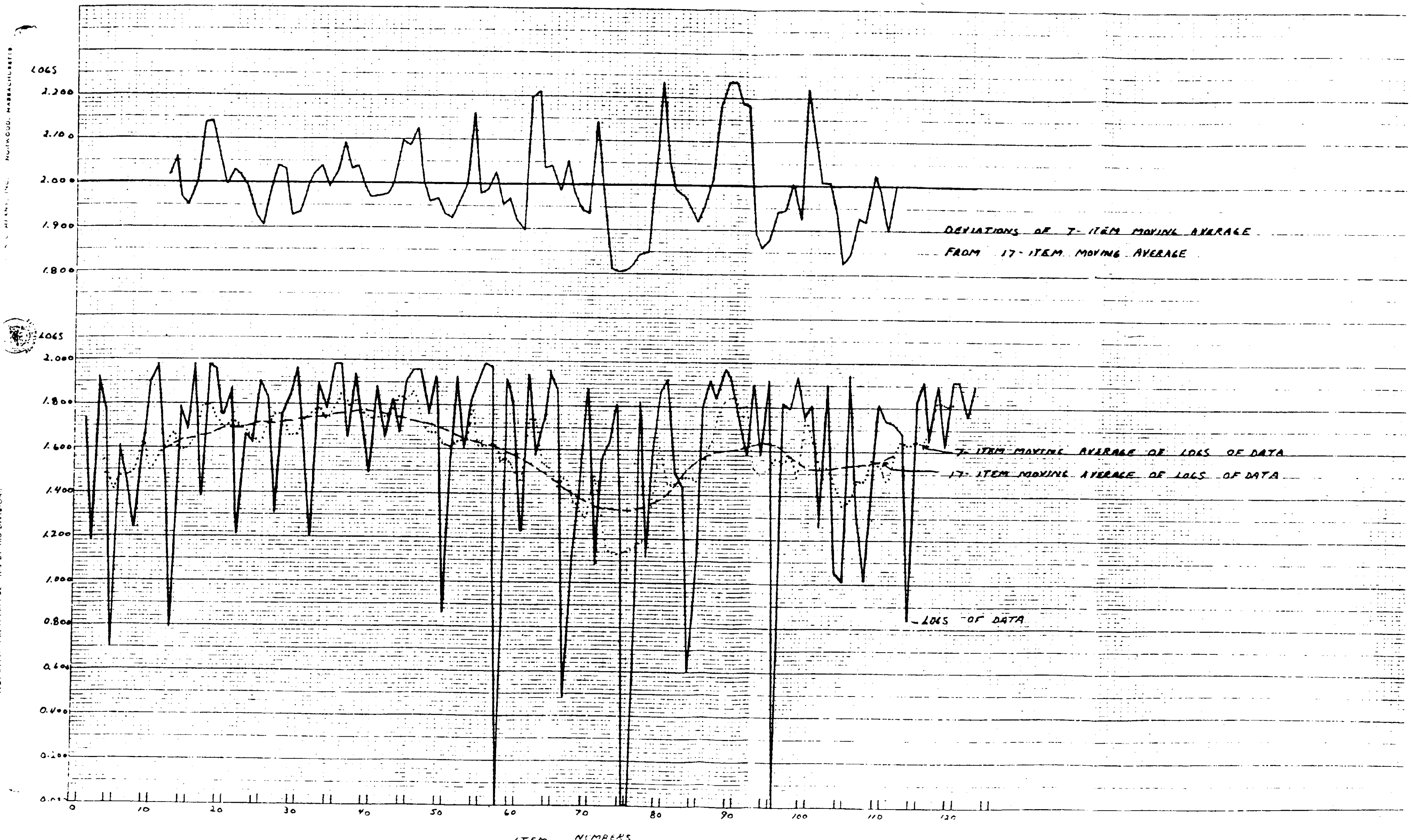
Random Numbers

As I have told you before, random numbers evidence cycles too. However, the cycles found in random numbers are not significant and will not continue.

Two students have already chosen random numbers for the problem called for in Lesson VI-A. One of these series, chosen by Gardner B. Miller, shows such a nice cycle that I have asked Mr. Miller to reproduce his original work chart. A blue-print of it is attached hereto.

You should become familiar with the kinds of cycles you can find in random numbers to help you to distinguish between these evanescent cycles and cycles which are significant and can be expected to continue.

A SERIES OF RANDOM NUMBERS



LESSON VII

HOW TO MAKE AND USE A PERIODIC TABLE—Part I

You will find the Periodic Table an important tool in cycle analysis.

It has four main uses, viz.:

1. If you know the length of the cycle, it will reveal to you the typical or average shape, strength, and timing of the cycle, as nearly as the figures will permit.

2. It is one of the ways of revealing to you as exactly as possible the length of any cycle that may be present in the series. That is, it will tell you the length of the cycle in days, months, or years from crest to crest or trough to trough.

3. It will often enable you to separate one cycle from another, when two or more cycles are present concurrently in the same series of figures.

4. It will help you to find hidden and unsuspected cycles.

On the other hand there are certain limitations to the use of the periodic table. These will be explained as we go along.

This lesson will be limited to a discussion of use 1 only.

Let us start by seeing what a periodic table is.

HOW TO MAKE A SIMPLE PERIODIC TABLE

In principle at least, you will find the making of a simple periodic table the easiest thing you ever did.

For example, suppose the average annual prices of widgets, for a 12-year period, are as follows:

1939	12¢	1945	13¢
1940	10¢	1946	11¢
1941	10¢	1947	15¢
1942	11¢	1948	16¢
1943	8¢	1949	15¢
1944	10¢	1950	13¢

Let us now make a 3-year periodic table of these data. It is done like this:

First of all you construct a grid onto which you can post all of the figures. In a column headed "Base Year" you put <u>the number of the year before your series starts</u>, and every third year thereafter.

Then across the page, under the caption "Years After Base Year" you will write at the head of each of three columns the numbers 1, 2, 3. Thus:

TABLE 10

The Form of a 3-Year Periodic Table Data 1939-1950

Base Year	Years After Base Year		
	1	2	3
1938			
1941			
1944			
1947			

We are going to post the prices of widgets into the appropriate blanks, add the columns, and average. But before we do let us make doubly sure that you understand the grid.

Here is the grid again, and into each space we have posted the number of the year represented.

TABLE 11

The Form of a 3-Year Periodic Table Data 1939-1950
With the Various Positions Filled in With Year Numbers

Base Year	Years After Base Year		
	1	2	3
1938	1939	1940	1941
1941	1942	1943	1944
1944	1945	1946	1947
1947	1948	1949	1950

Note that the base year for any line is always the same as the last year in the line above.

Now let us make the actual periodic table.

Into the space opposite the first base year and in the first column you will post the price of widgets for one year after base year, i.e. the price for 1939. The price is 12¢.

On the same line, into the second column, you will put the price for two years after base year, i.e. the price for 1940. (10¢)

On the same line, into the third column, you will post the price for the third year after base year, or the price for 1941. (10¢)

You will then proceed in the same way to fill the spaces in the second line, and the third line, and the fourth.

When you get through you will add each column and divide by four to get the average, and your periodic table will be completed. Thus:

TABLE 12

A 3-Year Periodic Table of the Price of Widgets 1939-1950

Base Year		Years After Base Year		
		1	2	3
1938		12	10	10
1941		11	8	10
1944		13	11	15
1947		16	15	13
	Sums	52	44	48
	Averages	13	11	12

That is all there is to it! The job is done.

In other words, you "chopped" the series of figures with which you started into sections each of which in this case is three years long. Because there are twelve years in your series you came up with four sections.

You then arranged the figures of the first section in a horizontal line across the page, and put the figures of the second section underneath it. Similarly for the third and fourth sections.

You then add and average. Now you know how to make a periodic table. Simple, isn't it?

INTERPRETATION

Now that you have done all this, what does it mean? It means that IF there is a 3-year cycle in the figures you know its average phase (position in time), shape, and amplitude (height) (subject however to distortion due to random elements, other cycles, and trend which may also be present). And this is all it means.

By itself the presence of this average wave does not mean that the wave has any significance whatever, or that it is rhythmic or repetitive. You will always get an average wave of the exact length of your periodic table, unless all the values of your data are precisely the same or the irregularities completely cancel out.

However, you do see that IF there is a 3-year cycle the ideal time for the crests is one year after the base years, for that is the year when the average value is highest. Thus, a crest is in 1939, which is one year after 1938, and every three years thereafter. Similarly, IF there is a 3-year cycle, the ideal time for the trough will come two years after the base years. A trough is in 1940, two years after 1938, and every three years thereafter.

Now it happens that in these figures for widgets there is a 3-year wave, just because I put one in when I read the figures from the ceiling. Therefore, the average in this case does have significance. It gives you the average phase, shape, and amplitude of a cycle that is really there, as nearly as we can determine it from the figures.

However, there is no 2-year cycle in the figures and no 4-year rhythm in the figures (because I put them together in such a way that there would not be). Nevertheless, a 4-year periodic table and a 2-year periodic table will show average 4-year and 2-year cycles respectively.

Suppose, just to test this out, we make a 4-year periodic table of the same figures:

TABLE 13

A 4-year Periodic Table of the Price of Widgets 1939-1950

Base Year		Years After Base Year			
		1	2	3	4
1938		12	10	10	11
1942		8	10	13	11
1946		15	16	15	13
	Sums	35	36	38	35
	Averages	11 2/3	12	12 2/3	11 2/3

As before, the line called averages means merely that if there is a 4-year rhythm in the figures, it has the shape and height and position in time indicated, but as we already know that there is no such wave, we know that the line is meaningless.

Let us make a 2-year periodic table of the same figures:

TABLE 14

A 2-Year Periodic Table of the Price of Widgets 1939-1950

Base Year		Year After Base Year	
		1	2
1938		12	10
1940		10	11
1942		8	10
1944		13	11
1946		15	16
1948		15	13
	Sums	73	71
	Averages	12 1/6	11 5/6

From these examples it should be clear that you cannot rely on a periodic table for evidence of rhythm, but nonetheless it is a very useful tool, as we shall see.

Use of the Periodic Table to Give Us the Average Phase, Shape, and Amplitude of a Cycle When the Length of the Cycle is Known

It should follow immediately from what has been said above that if, from some other source of information you can know that there really is, in any series of figures, a rhythm of any certain length, the periodic table will give you very valuable information as to the average phase (timing), shape, and amplitude (height) of the typical or average wave.

Therefore, knowing this, and knowing that widgets do have a 3-year cycle, we can go back to the 3-year table and get some additional valuable facts.

You will remember that the averages of all four 3-year cycles were as follows: (Table 12): 13, 11, 12, and that the base of the table was 1938 and every three years thereafter.

By averaging the above figures we get the axis about which the cycle fluctuates:

$$\frac{13 + 11 + 12}{3} = 12$$

Therefore we not only see that this particular 3-year cycle has its crest at one year after the base years, as mentioned before (namely at 1939, 1942, 1945, and 1948, 1951, etc.), but also we see that this crest averages one cent over the axis of the cycle. Similarly, the trough comes two years after base years (or at 1940, 1943, 1946, and 1949) and has a value one cent below the axis.

The entire wave, expressed as differences from the axis, has values as follows: 13-12; 11-12; 12-12 or +1, -1, 0.

You have now found a real use for the periodic table, namely to show you the average phase, shape, and amplitude of a wave where you know the length.

HOW TO MAKE MORE COMPLICATED PERIODIC TABLES

A. Tables of Fractional Length

Before going on to consider other uses of the periodic table, you should learn how to make periodic tables of fractional as distinct from integral length.

Suppose, for example, that in a series of figures you know that the length of the wave is not an integral number of time units. Instead of 2 years or 3 years or 4 years suppose the cycle is $2\frac{1}{2}$ years long.

Let us draw such a cycle, as in Fig. 31 and read consecutive values from the chart. These values are: 8, 4, 6, 6, 4, and repeat.

For such a cycle you might make a periodic table as follows:

TABLE 15

A $2\frac{1}{2}$-Year Periodic Table of Controlled Data (an Ideal $2\frac{1}{2}$-year Cycle)

Base Year		Years After Base Year		
		1	2	$2\frac{1}{2}$
1938		8	4	-
1940		6	6	4
1943		8	4	-
1945		6	6	4
1948		8	4	-
1950		6	6	4
	Sums	42	30	12
	Averages	7	5	4

As you will have noticed, a 2½-year periodic table is made up of 2-year and 3-year lines alternately. In this way we get an average length of 2½ years. The 3rd column is headed 2½ to put us on notice that we do not have a 3-year periodic table.

In this connection there are several things to note:

First of all you will observe, if you plot the values 7, 5, 4 in position 1, 2, and 2½ after base, and repeat, as in Fig. 32, that they do not exactly reproduce the original wave with which we started.

Next you will note that if, instead of having set up your 2½-year periodic table so that the first line was 2 and the second line 3 years long, as we did, you had set it up on a basis, such that the first line was three years long and the second line two years long, as shown below, you would get a still different shaped wave, as shown in Fig. 33.

TABLE 16

Another 2½-year Periodic Table of Controlled Date (A 2½-Year Cycle)

Base Year		Years After Base		
		1	2	2½
1938		8	4	6
1941		6	4	-
	Sums	14	8	6
	Averages	7	4	6

Finally, you can easily see, by making trial, that is, instead of taking 2, 4, 6 or some other even number of cycles so that you would come around to a repetition of your wave in exact phase, you had taken an odd number of cycles so that you would not have an equal number of long and short lines, you would have got a still different shaped wave, as illustrated in the two 2½-year periodic tables given below.

The first of the two tables is set up with an odd number of lines on a basis--such that figures for 2 years were entered on the first line and 3 years on the next. The second table was set up in reverse, that is, figures for three years are entered on the first line and figures for two years on the second.

TABLE 17

Another 2½-year Periodic Table of Controlled Data (A 2½-year cycle)

Base Year		Years After Base		
		1	2	2½
1938		8	4	-
1940		6	6	4
1943		8	4	-
	Sums	22	14	4
	Averages	7 1/3	4 2/3	4

TABLE 18

Another 2½-year Periodic Table of Controlled Data (A 2½-year cycle)

Base Year		Years After Base		
		1	2	2½
1938		8	4	6
1941		6	4	-
1943		8	4	6
	Sums	22	12	12
	Averages	7 1/3	4	6

To summarize, there are four different average waves that can be got from 2½-year periodic table of data containing nothing but a perfectly regular 2½-year wave, and none of them is right!

Of course the differences between the average wave as derived from the periodic table get less as the number of terms in the cycle increase. Thus, if we had a perfectly regular 24½-month wave, and set up a 24½-month periodic table (that is, a 24-month, 25-month table), the differences between the average wave derived from such a table and the actual wave would be negligible. However, the shorter cycle (fewer terms) used in our example illustrates a fact that you should not forget, namely that where the cycle is not an integral number of units long, the average wave as revealed by the periodic table is distorted.

Also, it may be noted in passing, that if you have a long series of figures with many repetitions of the cycle it is less important that you stop the periodic table at a place such that the pattern has come around again to the place of beginning. One odd line out of twenty is much less distorting than one odd line out of three.

Before we go on there is just one other minor point you should notice. That is, in those two 2½-year periodic tables where we started with three figures on the first line instead of two, the third column, headed 2½ really represents positions that are <u>three</u> years after the base year. That is, the figure 6 in the first line is not the value for 2½ years after 1938, it is the value for three years after base.

As the column headed "2½" really contains in all instances values that are three years after the base year of the line involved, you may wonder why we do not head it "3". We could, if we wished, but it saves confusing a 2½-year table with a 3-year table if we can look at the top of the last column and see immediately what the true length of the table is.

As long as the last column represents three years after base, we could just as well insert all values that are three years after base. Let us do so, putting parentheses around them to remind us that the same numbers will appear on the next line also.

TABLE 19

Another 2½-year Periodic Table of Controlled Data (A 2½-year cycle)

Base Year		Years After Base		
		1	2	2½
1938		8	4	(6)
1940		6	6	4
1943		8	4	(6)
1945		6	6	4
	Sums	28	20	20
	Averages	7	5	5

The value of the average of the third column should be plotted 3 years after base. Constructing your periodic table this way gives you still another set of values for your average wave--also wrong.

How can you overcome the difficulties presented, and get an average wave that will be <u>identical</u> with the wave in the original data?

Very easy when the cycle is ½ a time unit more than an integral length. Make a periodic table of the original values, 2 wave lengths long, like the 5-year one below. (Two times 2½ is 5. Five is an integer (a whole number).

TABLE 20

A 5-Year Periodic Table of Controlled Data (A 2½-year cycle)

Base Year		Years After Base Year				
		1	2	3	4	5
1938		8	4	6	6	4
1943		8	4	6	6	4
1948		8	4	6	6	4
	Sums	24	12	18	18	12
	Averages	8	4	6	6	4

This gives us <u>exactly</u> the wave we started with. You can see a chart of it in Fig. 34.

If we chart this wave it will not <u>look</u> quite the same as the wave we started with, because we are merely connecting the points and not interpolating the correct values between points, but this is not the fault of the periodic table. It is because the data taken from the original curve were not numerous enough.

Sometimes the series is so short that it is not feasible to make a periodic table long enough to handle the matter as above. In that event there is another way of getting a closer approximation of the original wave than in a periodic table of the original data, viz:

In the example given above, interpolate values between years, and, instead of a 2½-year periodic table, make a 5-half-year periodic table thus.

TABLE 21

The Interpolation of Half-Yearly Values

Actual Values	Interpolated Values (Half-way between Actuals)	
8	6	(8 + 4) ÷ 2
4	5	(4 + 6) ÷ 2
6	6	(6 + 6) ÷ 2
6	5	(6 + 4) ÷ 2
4	6	(4 + 8) ÷ 2
and repeat	and repeat	

TABLE 22

A 5-Half-Year Periodic Table of Controlled Data (a $2\frac{1}{2}$-year cycle with interpolated values)

Base Year		Half Years After Base Year				
		1	2	3	4	5
1938			8	6	4	5
1940		6	6	6	5	4
1943		6	(and repeat)			
	Sums	12	14	12	9	9
	Averages	6	7	6	$4\frac{1}{2}$	$4\frac{1}{2}$

The results are plotted in Fig. 35.

We have been considering periodic tables where the fractional length is $\frac{1}{2}$. How about periodic tables of other fractional length? As an example let us construct a periodic table 4.222 months long, data starting with month number 49.

To make such a table you put your base number (the number of the time unit <u>before</u> the beginning of your data) on the first line, and in your machine. Then put 4.222 in your machine. For your second base number, you add 4.222 once and post a rounded value to your table. For your third base number, you again add 4.222 (to the unrounded number) and read off a round number. And so on, thus:

TABLE 23

Form of a 4.222-month Periodic Table, Base Month No. 48

Base	Work Column	Months after base				
		1	2	3	4	4.222
48		x	x	x	x	--
52	(48 + 4.222)	x	x	x	x	--
56	(48 + (2 x 4.222))	x	x	x	x	x
61	(48 + (3 x 4.222))	x	x	x	x	--
	etc.			etc.		

In actual practice the work column is omitted. I used it merely to show you how the successive base numbers are obtained.

When it comes to adding and averaging periodic tables of fractional length, if the fraction is less than .5, you ordinarily ignore the last column. If the fraction is .5 or over, the table is ordinarily filled in and averaged, as in Table 19.

B. To Make Very Long Periodic Tables

When there are a great many terms to the cycle being investigated and only a few repetitions of the cycle it is often wise to interchange columns and lines, and to set up your periodic table the way you would set up a time chart.

TABLE 24

Form of Periodic Table for Long Cycles
50 month cycle, Base month 300

Base	300	350	400	450	500	Sum	Average
1							
2							
3							
4							
5							
6							
etc.							
to							
50							

C. To Make Periodic Tables of Data Which Include Trend

So far we have been considering the simplest possible cases.

Let us now introduce some complications.

Suppose we make a 4-year periodic table of a perfectly regular 4-year cycle, as plotted in Fig. 36 thus:

TABLE 25

A 4-year Periodic Table of Controlled Data (a 4-year cycle)

	Years After Base Year			
Base Year	1	2	3	4
1938	8	6	4	6
1942	8	6	4	6
1944	8	6	4	6
Average	88	6	4	6
Axis of the Cycle $\frac{8+6+4+6}{4}$	6	6	6	6
Cycle as Deviation from axis	2	0	-2	0

Obviously, the average is the wave we started with.

Let us now combine this 4-year cycle with a trend line, which goes up one unit every year. The composite is plotted in Fig. 37. Let us now make a periodic table of these values:

TABLE 26

A 4-year Periodic Table of Controlled Data (a 4-year Cycle and Trend)

	Years After Base Year			
Base Year	1	2	3	4
1938	8	7	6	9
1942	12	11	10	13
1944	16	15	14	17
Sum	36	33	30	39
Average	12	11	10	13
Axis of the cycle $\frac{12+11+10+13}{4}$	$11\frac{1}{2}$	$11\frac{1}{2}$	$11\frac{1}{2}$	$11\frac{1}{2}$
Cycle as deviation from axis	$+\frac{1}{2}$	$-\frac{1}{2}$	$-1\frac{1}{2}$	$+1\frac{1}{2}$

Plotting this average wave across the page shows a totally distorted pattern, as you would expect. See Fig. 38.

From this example you can easily see that the values you record in a periodic table must be corrected for trend. There are various ways that this correction can be made, as I explained in Lesson III and Lesson IV. Probably the simplest way is to express the data as percentages of a moving average trend.

Another way in which you can remove the effect of trend from the average of a periodic table is to <u>rotate</u> these averages.

First of all, you have to make some estimate of the trend, end to end. As you are dealing with controlled data you can extend the series one additional term. It is 20 (8 + 12). As, <u>at the same position in the cycle</u>, trend has gone up 12 points in 3 cycles (20, the value of the 13th term, less 8, the value of the 1st term, equals 12), it is clear that trend has gone up 4 points per cycle (12 ÷ 3), 1 point per year (4 ÷ 4).

In rotating the cycle we want to offset this distortion. We do it as follows:

TABLE 27

A 4-year Periodic Table of Controlled Data
(a 4-year Cycle and Trend), with Averages
Rotated to remove Effect of Trend

	Years after Base Year			
Base Year	1	2	3	4
1938	8	7	6	9
1942	12	11	10	13
1944	16	15	14	17
Sum	36	33	30	39
Average	12	11	10	13
Correction for trend	+1½	+½	-½	-1½
Sum	13½	11½	9½	11½
Axis of the Cycle $\left(\frac{13\frac{1}{2}+11\frac{1}{2}+9\frac{1}{2}+11\frac{1}{2}}{4}\right)$	11½	11½	11½	11½
Cycle as Deviation from axis	2	0	-2	0

In connection with this manipulation (Table 27) there are several things to note.

Note first that the rotation is effected by lowering the values of the average in Column 3 and 4 and by raising the values of the average in Column 1 and 2. (If the trend had been downward we would have rotated the averages the other way.)

Note that, because there are an even number of terms to be rotated, the axis around which the rotation takes place, the zero, is <u>between</u> the two center terms. (If their had been an odd number of terms the axis would have been <u>at</u> the center term. It would have remained unaltered.)

Note that the total amount of trend to be compensated for, 4, is divided by 4, because there are four terms to the cycle. We thus have a correction factor of $\frac{4}{4}$ or 1 per position. As the 2nd and 3rd year after base are ½ position from center we add and subtract ½ respectively. As the 1st and 4th year after base are 1½ positions from center, we add and subtract 1½ respectively.

Note that the sum of the maximum negative rotation, -1½ and the maximum positive rotation, +1½, does <u>not</u> equal 4, the amount of correction we wish, but 3. This is because the cycle really extends from ½ of a position before 1 year after base to ½ of a position after 4 years after base. You always make your correction in such a way as to rotate your average wave, first term to last term inclusive, by an amount that is 1/x less than full correction, when x equals the number of terms in the cycle.

How do you estimate trend when you are not dealing with controlled data?

Well, sometimes you have a few values in the cycle after the last full cycle. In this case you can compute the move from position 1 in the 1st cycle to position 1 in the cycle after the last one you are interested in.

Or, you can find the move from 1st cycle to last full cycle and prorate to allow for the last cycle.

In "wild" series you may want to average several initial and several terminal values to come up with a representative number.

D. The use of Color in the Periodic Table

When making periodic tables of deviations of numbers that are plus (+) and minus (-) around an axis you always post negative numbers, percentages less than 100%, and logs of less than 2.000 in red.

Positive values, or percentage values 100% or more, and logs 2.000 or more are posted in black.

When making periodic table of most raw data or logs of such data, use black only.

The reason for the use of color will become clear in a subsequent lesson, but see if you cannot figure it out for yourself in the meantime.

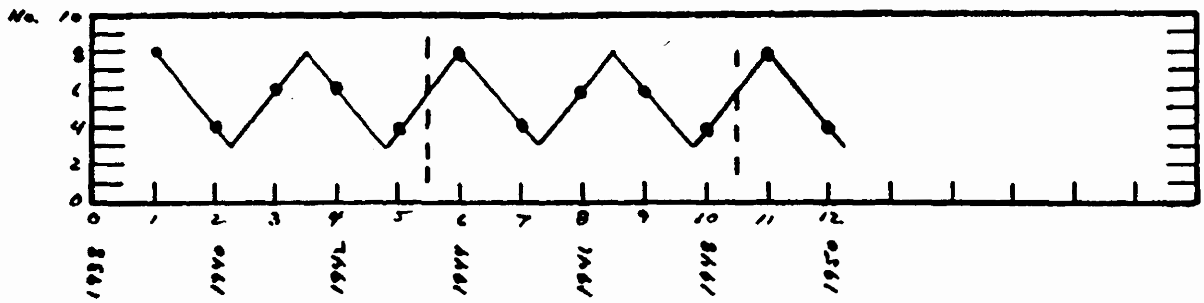


Fig. 31. An Ideal 2-1/2 Year Cycle

Source: Page 6, Lesson VII

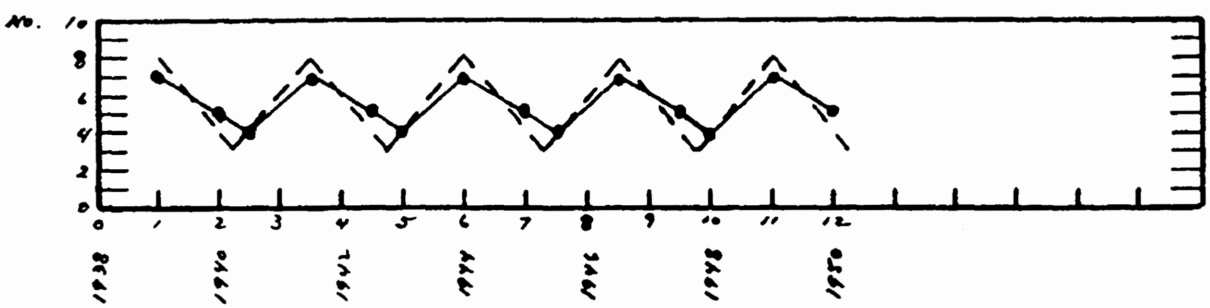


Fig. 32. 2-1/2 Year Cycle Derived from Periodic Table (Table 15) Compared to Original Ideal Cycle (from Fig. 31)

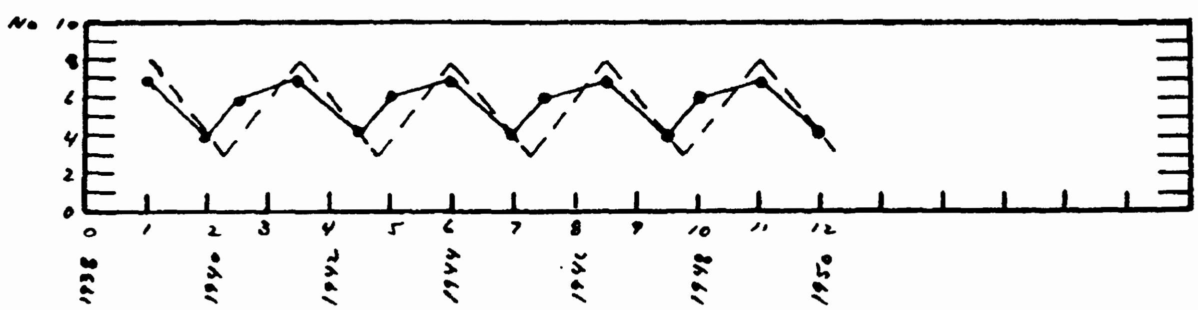


Fig. 33. 2-1/2 Year Cycle Derived from Periodic Table (Table 16) Compared to Original Ideal Cycle (from Fig. 31)

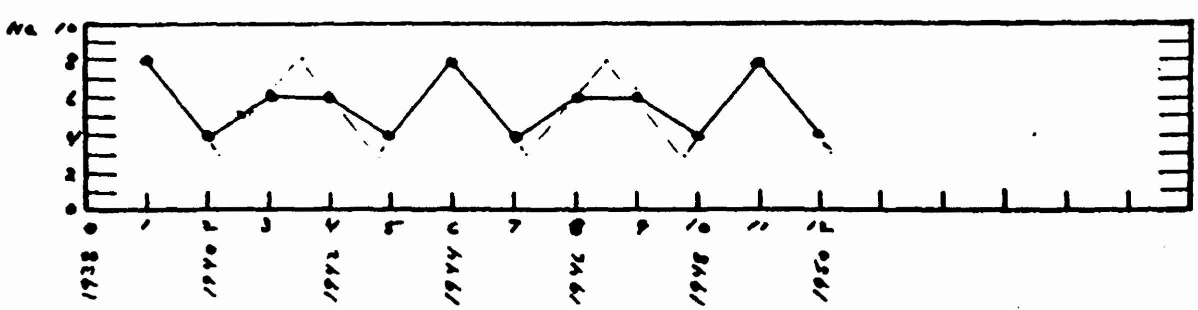


Fig. 34. 2-1/2 Year Cycle Derived from 5-Year Periodic Table (Table 20)

Page 16 is continued on Page 16-A

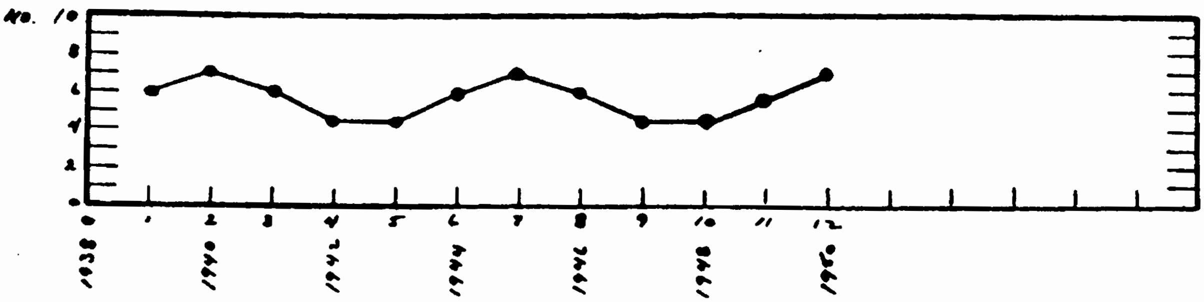


Fig. 35. 2-1/2 Year Cycle (with Interpolated Values) Derived from 5 Year Periodic Table

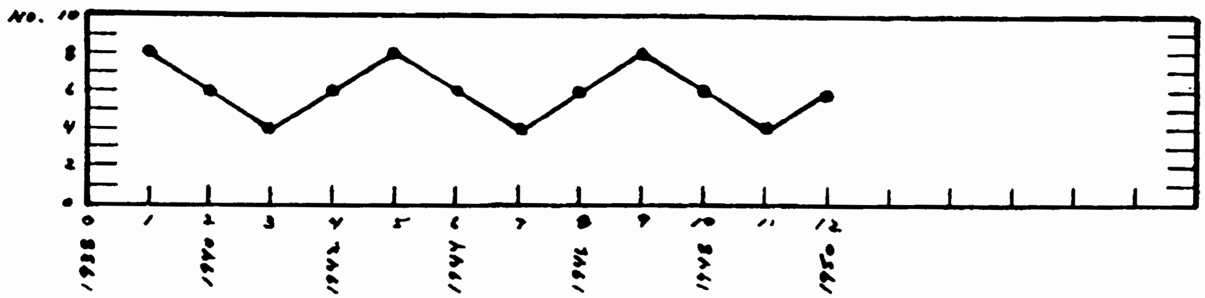


Fig. 36. Ideal 4 Year Cycle (Table 25)

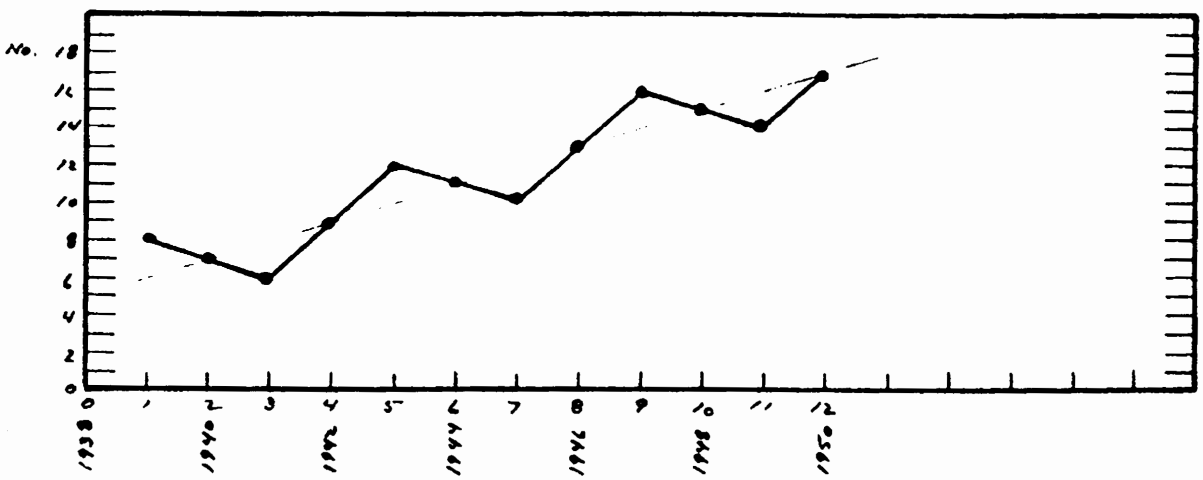


Fig. 37. Ideal 4 Year Cycle Combined with Trend (Table 26)

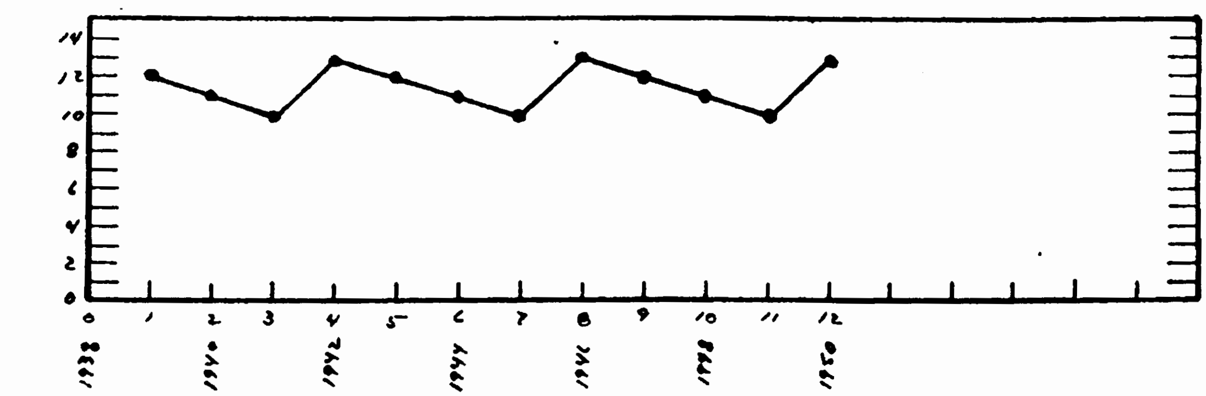


Fig. 38. 4 Year Cycle Derived from Periodic Table of Controlled Data Containing Cycle and Trend (Table 26)

Lesson VII-Problems

Problem 1. Make periodic tables of your stock market figures, using deviations of logs of data from their 9-year moving average. Make tables for harmonic intervals from 7th to 13th inclusive (7 tables). Average and plot the averages.

Harmonics are unit fractions. Unit fractions are the reciprocals of whole numbers, i.e. 1 over intergers.

You have 92 items in your series. The 7th harmonic of 92 is 92 ÷ 7 or 13.14. The 8th harmonic of 92 is 92 ÷ 8 or 11.50.

Compute harmonic lengths to three decimal places. Round to two places. (Always compute everything to one more place than you use.)

Be sure to use color. Red for values 1.999 and less, black for values 2.000 or more.

Problem 2. Make three periodic tables of the logs of your data (Col. B of S. & P. C.A.,T.S.-1) omitting however, for purposes of comparability, the first four and the last four terms.

Let the length of the table be the 9th, 10th and 11th harmonics of the number of data used, i.e. 92.

Rotate to compensate for trend.

STANDARD PRACTICE INSTRUCTIONS

8. Periodic Tables

1. Place your initial and date in the upper left hand corner.

2. Place the title of the periodic table at the top of the sheet. After naming series, always start the sub-title with the length of the periodic table. Be accurate and complete. For example:

Cotton Production

8.33-year periodic table of dev. of 3-year m.a. of logs of data from 9-year m.a. of logs of data.

3. Enter length of periodic table in Index of Time Charts and Periodic Tables in consecutive numerical (and hence in chronological) order.

4. Place number of periodic table in upper right hand corner of periodic table. Get numbers from Index of Time Charts and Periodic Tables. Place page number (Page 1) under periodic table number.

5. Next, set up your table and enter your base numbers. Enter (in parentheses) the decimal, if any, appertaining to one of the last base numbers so that one can continue the table without recalculating all the base numbers from the beginning. Example: 625 (.5).

6. Record in the lower right hand corner the source of the data used in the periodic table. Mark "compared," with the date and your initial.

7. When the periodic table has been averaged, check and mark "ckd.", date, and initial.

8. Now take a sheet of 5 to the inch quadrille paper. Head it the same title as the periodic table. Give it the same chart number, but call it Page 2. Be sure to post the base number of the periodic table as part of the heading.

9. Onto Page 2 of your periodic table, chart the averages of the various columns, repeated at least one and a half times and preferably twice.

That is, if you have a four day periodic table plot the averages for day 1, day 2, day 3, day 4, day 1, day 2, day 3, day 4. This will give you your average wave repeated twice so that you can look at it from crest to crest or trough to trough at your option.

If you have a 4½ day table you will have a value for the 5th column. Plot it in position 5. Then plot the value for day 1 one half a time interval later so that it will be 1½ time intervals after the plotting of day 4.

Etc.

10. If a cycle is idealized do this on Page 2 of the periodic table, or set

up a separate Page 3. Page 2 or Page 3 is also the place to record a list of dates of highs and lows of the indicated cycle.

11. Be sure to record in log what you learn from the periodic table. State what led you to make the table, what you found out, what is indicated as the next move. Record notes also on periodic table itself. Example: "Try 7.65"

12. Fold periodic table so that number in upper right hand corner is clearly visible.

13. File, intermingled with time charts, by lengths of periodic table (not by length of the indicated rhythm), shortest length on top.

14. In large jobs, set up cross index sheets for length of cycle, if different from length of periodic table.

LESSON VII

SUPPLEMENT I

Question from a student:

(Lesson VII, Page 17, Problems 1 & 2)

I do not understand the significance of "Harmonics" as applied to a series of a length not necessarily identified as the common multiple of several cycles and integral numbers of such cycles.

If the 7th harmonic of 92 is 13.14 does this mean that the cycle length is 13.14? Obviously not, so what is the significance of the harmonics of a random number? The number 92 is random. A year later it will be 93.

Answer:

The reasons I chose harmonic lengths for the various periodic tables will become clear when we get a bit further into the course. The results of this work will fit into the lessons on Harmonic Analysis, Multiple Harmonic Analysis, and the Periodogram.

In the meantime it may be said that--

(a) Periodic tables of all harmonic lengths will completely scan your series and will pick up at least half of any cycles present. They are good lengths to start with for reconnaissance. They provide systematic scanning.

(b) Periodic tables which are harmonic lengths of your series of figures will always come out even. There will be no numbers left over. This simplifies things.

(c) A cycle which is as long as the next higher or lower harmonic is completely washed out of a periodic table of the length of the harmonic. For example, if there is a cycle 9.2-years long in 92 years of data, it will appear full force in the averages of the 9.2-year periodic table, but will be completely washed out of the averages of a periodic table as long as the 9th harmonic and as long as the 11th harmonic of 92 years (10.22 and 8.37 years respectively).

(d) It's a good idea to get used to thinking in terms of harmonic intervals.

(e) I wanted to give you fractional length periodic tables and thought I might as well give you logical and systematic fractional lengths, as random ones.

LESSON VII

Supplement 2

THE USE OF COLOR IN THE PERIODIC TABLE

I am enclosing a reprint of a little article I published in Cycles for May 1951, pages 179-80, called "The Use of Color in the Periodic Table."

This article also gives you an example of a periodic table.

The reference to throwing out high and low values in each column of the periodic table anticipates part of Lesson X, "Randoms: Three Ways to Minimize Them."

THE USE OF COLOR IN THE PERIODIC TABLE

ONE neat little trick you should know about is the use of color in the periodic table. If, when making a periodic table of deviations, you record all the numbers 100.00% or over in black and all the numbers 99.99% or less in red, you will often find the result to be quite revealing. Of course if you are using logs you would put all values 2.0000 or over in black, all values 1.9999 or less in red

In preparing such tables I use a two-ended Autopoint automatic pencil with red lead at one end and black lead at the other.

As we do not have color printing available I cannot show you here just how such a table would look in red and black, but I will try to give you an idea by setting up such a table in light face and bold face type, as below.

For the example I have chosen twelve lines from a 9.20-month periodic table of industrial common stock price deviations. The deviations, in logs, are deviations from a 9-month moving average of the data. The deviations have been adjusted for the effect of an 8.18-month cycle.

In this table the numbers in the left hand column, called Base Month, are the Hoskins' Standard Month Numbers, as printed in our report for September 1950. Month number 465 is September 1939. Therefore 2.0243, the first figure in the table, is the logarithm of the corrected percentage deviation for October 1939, the first month after base. The last base month number given, 566, is the number for February 1948.

It is easy for the eye to see a tendency for strength (black) in the center of the table. (It is easier to see on the work sheets where the light face type numbers are in red.) This tendency for strength at about 9.20-month intervals is confirmed by averaging the figures in each column. These averages gives us the average shape of the 9.20-month wave, if there is a cycle of this length in these figures.

I might say in passing, as an aside, that in averaging columns of a periodic table of deviations (or any series where there is no trend) I usually find it more helpful to omit the highest and lowest values in each column, or the two highest and the two lowest values, or sometimes the three highest and three lowest values, to obtain the average median of the column. Such a procedure has no effect upon any perfectly regular wave of the same length as the length of the periodic table, but it does tend to eliminate major accidental variations where the columns of the table are not long enough for these variations to be offset by the laws of chance.

But getting back to the matter of color, the presence of color in our example will tell you several things.

First, by the fact that the black items and red items more or less tend to fall in vertical lines, it confirms the fact that you may be on the right track in searching for a wave of about 9.20 months in these figures.

Second, the relative uniformity of each color tells you that the indicated wave is rhythmic. That is, it shows you that the average weakness at position 1 and average strength at position 6 has also been strong, weak, strong, weak, strong, weak, time after time in each of the lines.

Of course there are better ways of determining rhythm, but this is an easy way achieved by a twirl of your fingers as you write.

Third, color will often show you whether the indicated average strength is or is not the result of a wave which is three (or five) times the length of the periodic table. In the former instances the black (and red) areas will show up more strongly on every third (or fifth) line.

Fourth, color will often indicate the concurrent presence of another wave. In the example given it is clear that the black (and red) figures tend to slip off to the right as well as to group themselves in vertical columns.

For example, draw a line from the figure 2.0070 in the 1st line 4th column (Month No. 469 (465 + 4), to the 6th line 10th column (Month No. 521 (511 + 10). Here we have an indicated length of 10.4 months.(Five cycles in 52 months (521 - 469 = 52), average 10.4 months per cycle.

You can draw a continuation of this line from the 7th line, 1st column to the 12th line 7th column and you find the behavior seeming to continue.

This slippage shows the possible presence of another wave in these figures about 10.4 months in length. After the exact length of the wave about 9.20 months has been determined you can make a 10.4-month periodic table as a first step toward seeing if the 10.4-month wave is real. Thus color has given you a clue which might otherwise have escaped you.

Always make your periodic tables in two colors when you can.

9.20-MONTH PERIODIC TABLE

INDUSTRIAL COMMON STOCK PRICES, STANDARD & POOR'S CORPORATION INDEX

DEVIATIONS OF THE DATA, IN LOGS, FROM THEIR 9-MONTH MOVING AVERAGE

(DEVIATIONS ADJUSTED FOR THE EFFECT OF AN 8.18-MONTH WAVE)

BASE MONTH NUMBER	MONTHS AFTER BASE MONTH									
	1	2	3	4	5	6	7	8	9	10
465	2.0243	2.0181	2.0087	2.0070	2.0124	2.0203	2.0319	1.9800	1.9492	
474	1.9708	1.9879	2.0129	2.0154	2.0158	2.0028	2.0084	1.9872	1.9930	1.9866
484	1.9867	1.9968	2.0171	2.0172	2.0187	2.0081	2.0080	1.9964	2.0170	
493	2.0029	1.9809	1.9654	1.9612	1.9876	2.0038	2.0005	2.0007	2.0098	
502	1.9949	1.9835	1.9835	2.0004	2.0071	2.0158	2.0258	2.0220	2.0212	
511	1.9944	1.9935	1.9952	1.9842	1.9942	2.0108	1.9999	2.0008	1.9824	
520	1.9941	2.0057	2.0177	2.0089	1.9986	2.0039	1.9902	1.9894	1.9948	2.0000
530	1.9947	2.0123	2.0123	2.0138	1.9889	1.9737	1.9900	1.9965	2.0085	
539	2.0082	2.0181	2.0087	1.9923	2.0152	2.0230	2.0203	2.0235	2.0256	
548	1.9725	1.9764	1.9740	1.9972	2.0021	2.0125	1.9990	1.9857	1.9761	
557	1.9984	2.0200	2.0095	1.9973	2.0013	2.0045	2.0082	1.9979	1.9737	
566(.4)	1.9753	2.0000	2.0138	2.0238	2.0143	1.9996	1.9981	2.0178	1.9984	1.9956
AVERAGE EX 1 H, 1 L*	1.9913	1.9983	2.0034	2.0084	2.0048	2.0082	2.0055	1.9994	1.9973	

*AVERAGE OF ITEMS EXCEPT FOR THE HIGHEST AND LOWEST VALUES IN EACH COLUMN.

LESSON VII

Supplement 3

ROTATING TO COMPENSATE FOR TREND

Rotating is a problem you will rarely meet because it is not often that the original data are put into a periodic table. And if original data are used they will not necessarily contain a pronounced trend.

Of course when you are using deviations from a trend the trend is horizontal at 100 or 0 and there is no problem.

But if you are using the data, and they show a pronounced trend, and you make a periodic table as a quick check on a prominent wave you may see by a first inspection of the data, you may then reach the point where you want to take the trend out of the averages of the table.

However, there is a trend in the data, and of course when you split the data into a table, a certain amount of the trend is present in each line of the table, and an average amount is present in the averages of the table.

For instance, suppose on inspecting S & P C A Chart 3, which shows the logs of the data, you decide there is a wave about 9 years long. You have not yet worked a moving average so you put the logs of the data into a periodic table, and average the columns.

As an example here, we will use the 9.2-Year Periodic Table of the Logs of the S & P C A Data because Problem 2 of Lesson VII was set up that way. The original averages of this table follow:

Column	Average
1	1.655
2	1.657
3	1.694
4	1.720
5	1.777
6	1.822
7	1.833
8	1.831
9	1.795

A chart of these averages is plotted in Figure 1.

As you can see, the drop between the ninth position and the second plotting of the first position is partly cyclic, and partly trend.

There are numerous ways to remove the trend, and I will start with the quickest.

When the table was set up, the first value entered was 1.161, and the last value entered was 2.084. This is a growth of about 0.923 over the entire table. Since there are ten lines in the table, this is an average growth of 0.092 per line. Also, the average of the table from position 1 through position 9 will have a growth of 0.092 in it.

Because there are 9 positions in the average wave, this would be 0.010 per position. The middle or zero position in the rotation is column 5, and the adjustment becomes:

(If you are working with a table with a fractional column which you are using, for instance a 5.8 table, then in computing the movement by position, you would divide the average growth by 6.)

0 in column 5
+.010 in column 4 and -.010 in column 6
+.020 in column 3 and -.020 in column 7
+.030 in column 2 and -.030 in column 8
+.040 in column 1 and -.040 in column 9

(If the table you are working with has an even number of columns the middle position comes halfway between the middle two columns and the first move is plus or minus one-half a full move. For example if we had an 8-year cycle and an average growth of .080, the middle position would be $4\frac{1}{2}$ and the corrections would be +.035, +.025, +.015, +.005, -.005, -.015, -.025, -.035.)

(If the trend were downward you would lower the left hand half of the table and raise the right hand half. However downward trends are unusual.)

This adjustment applied to the averages given on page 1 would work like this:

Column	Average	Trend Adjustment	Adjusted Average
1	1.655	+.040	1.695
2	1.657	+.030	1.687
3	1.694	+.020	1.714
4	1.720	+.010	1.730
5	1.777	0	1.777
6	1.822	-.010	1.812
7	1.833	-.020	1.813
8	1.831	-.030	1.801
9	1.795	-.040	1.755

The corrected average for the 9.2-year table is drawn in Figure 2.

This method is the quickest, and serves here just to illustrate how to rotate, that is, when you divide, and subtract, and add, etc.

However, this method is also the least reliable, because it is so very crude. For instance, in the table used the next value that would have been entered is 2.166. This of course would materially change the results.

Variations which serve to refine the trend correction used require slightly different arithmetic, but once you understand what was done above, it is relatively easy to change the method to match the circumstances.

There are two things wrong with the method given above. (1) Growth is distorted slightly because you are not measuring between points which are in the same part of the cycle. (2) Growth is distorted to the extent that the positions you choose for measuring growth are distorted by randoms (or other cycles).

The first distortion can be avoided by figuring growth from the value of the 1st position 1 to the value of the last position 1. (instead of, as in the example given, to the last position 9.).

The second distortion can be minimized by finding the growth between groups of points instead of single points. For instance, in the example given, you could find the growth between the average of the first three values in the first line and the first three values in the last line (always keep measurments between the same part of the cycle).

Better yet, average the entire first cycle and the entire last cycle and get the difference between these averages.

Note that in both methods just given to get the average growth per cycle you divide by the number of cycles between the groups of points taken. In our example you would divide by 9 instead of by 10, because it's 9 cycles from the 1st cycle to the 10th cycle.

Of course ideally you would determine growth by fitting a growth curve to the data, but that is getting a little ahead of ourselves at this stage of the course.

Once you understand the basic idea and process of rotating it can be handled in what for each particular instance seems to be the most logical way.

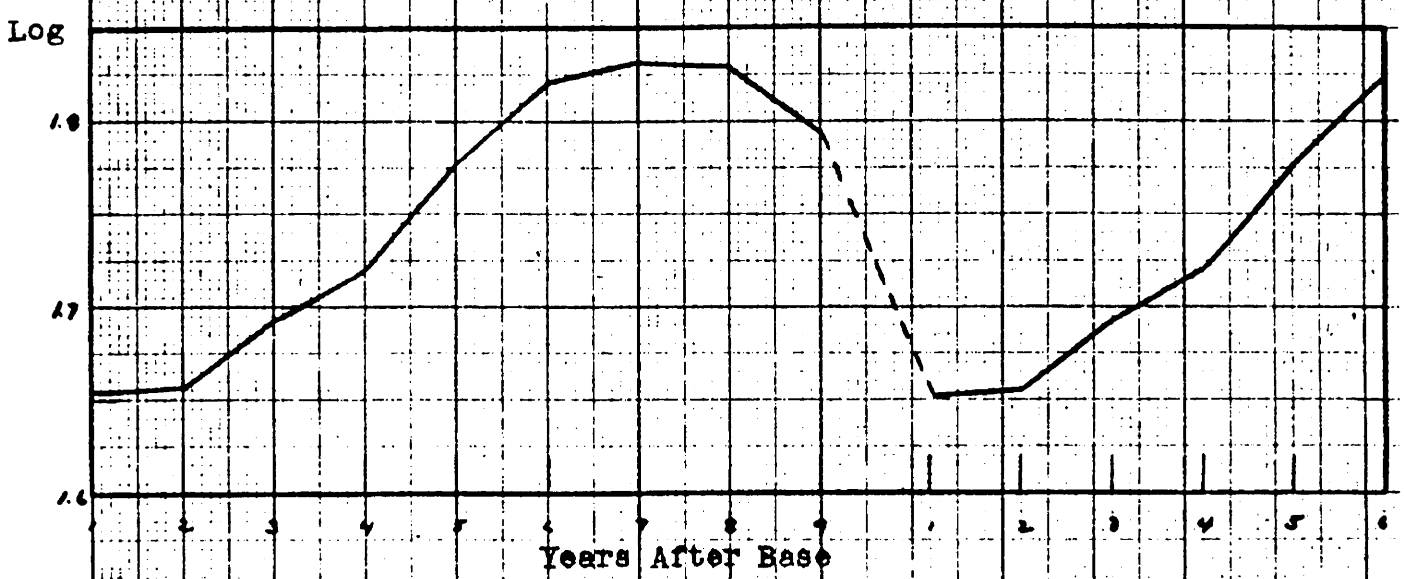


Fig. 1. Standard & Poor's Combined Index - Diagram of Average of 9.2-Year Periodic Table of Logs of Data - Before Trend Adjustment

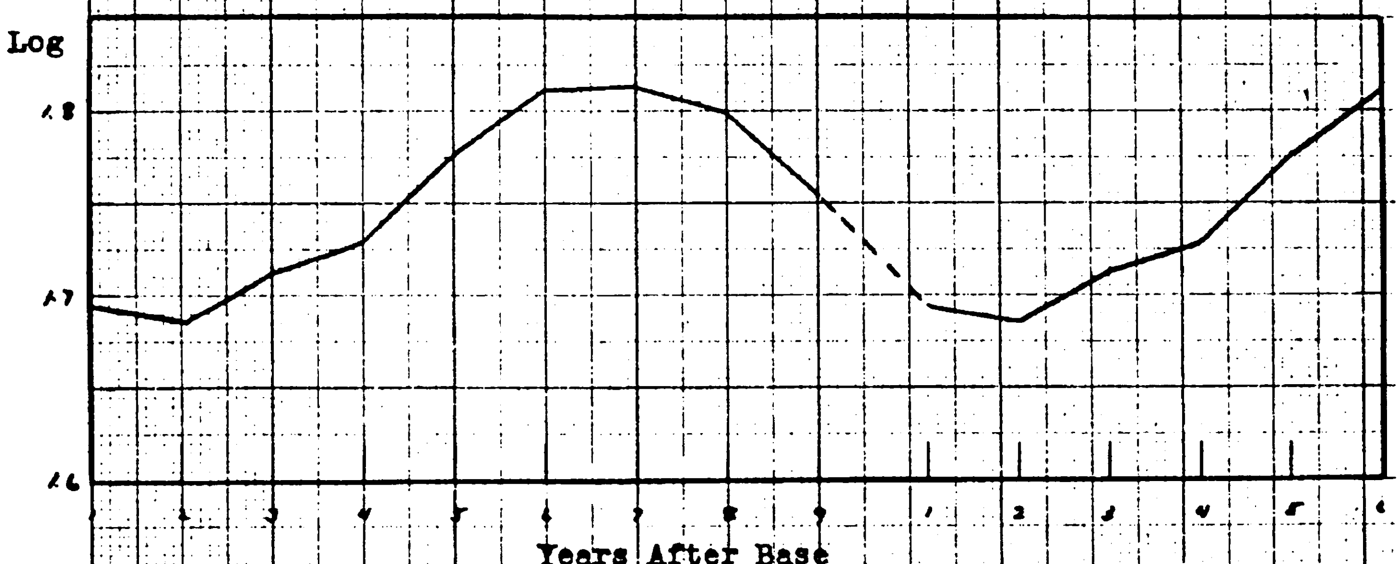


Fig. 2. Standard & Poor's Combined Index - Diagram of Average of 9.2-Year Periodic Table of Logs of Data - After Trend Adjustment

Source: Lesson VII, Supplement 3, Pages 1 and 2

Calendars for 201 Years, 1776-1976

Index of Years

Year	Calendar Number	Year	Calendar Number	Year	Calendar Number	Year	Calendar Number	Year	Calendar Number	Year	Calendar Number
1776	1	1809	10	1844	1	1876	5	1909	4	1944	5
1777	2	1810	6	1845	2	1877	6	1910	9	1945	6
1778	3	1811	7	1846	3	1878	7	1911	10	1946	7
1779	4	1812	14	1847	4	1879	2	1912	1	1947	2
1780	5	1813	4	1848	5	1880	8	1913	2	1948	8
1781	6	1814	9	1849	6	1881	9	1914	3	1949	9
1782	7	1815	10	1850	7	1882	10	1915	4	1950	10
1783	2	1816	1	1851	2	1883	6	1916	5	1951	6
1784	8	1817	2	1852	8	1884	11	1917	6	1952	11
1785	9	1818	3	1853	9	1885	3	1918	7	1953	3
1786	10	1819	4	1854	10	1886	4	1919	2	1954	4
1787	6	1820	5	1855	6	1887	9	1920	8	1955	9
1788	11	1821	6	1856	11	1888	12	1921	9	1956	12
1789	3	1822	7	1857	3	1889	7	1922	10	1957	7
1790	4	1823	2	1858	4	1890	2	1923	6	1958	2
1791	9	1824	8	1859	9	1891	3	1924	11	1959	3
1792	12	1825	9	1860	12	1892	13	1925	3	1960	13
1793	7	1826	10	1861	7	1893	10	1926	4	1961	10
1794	2	1827	6	1862	2	1894	6	1927	9	1962	6
1795	3	1828	11	1863	3	1895	7	1928	12	1963	7
1796	13	1829	3	1864	13	1896	14	1929	7	1964	14
1797	10	1830	4	1865	10	1897	4	1930	2	1965	4
1798	6	1831	9	1866	6	1898	9	1931	3	1966	9
1799	7	1832	12	1867	7	1899	10	1932	13	1967	10
1800	2	1833	7	1868	14	1900	6	1933	10	1968	1
1801	3	1834	2	1869	4	1901	7	1934	6	1969	2
1802	4	1835	3	1870	9	1902	2	1935	7	1970	3
1803	9	1836	13	1871	10	1903	3	1936	14	1971	4
1804	12	1837	10	1872	1	1904	13	1937	4	1972	5
1805	7	1838	6	1873	2	1905	10	1938	9	1973	6
1806	2	1839	7	1874	3	1906	6	1939	10	1974	7
1807	3	1840	14	1875	4	1907	7	1940	1	1975	2
1808	13	1841	4	1876	5	1908	14	1941	2	1976	8
		1842	9					1942	3		
		1843	10					1943	4		

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Calendar No. 1

LEAP YEARS

1776 1844 1940
1816 1872 1968
1912

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29		

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

MAY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

JULY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Calendar No. 2

COMMON YEARS

1777 1800 1823 1851 1879 1913
1783 1806 1834 1862 1890 1919
1794 1817 1845 1873 1902 1930
1941 1947 1958 1969 1975

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

MAY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

JULY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

CALENDAR NO. 3

COMMON YEARS

1778 1801 1835 1874 1914 1953
1789 1807 1846 1885 1925 1959
1795 1818 1857 1891 1931 1970
1829 1863 1903 1942

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

MAY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

JULY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

CALENDAR NO. 4

COMMON YEARS

1779 1813 1841 1869 1897 1926
1790 1819 1847 1875 1909 1937
1802 1830 1858 1886 1915 1943
1954 1965 1971

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28						

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

MAY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

JULY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

CALENDAR NO. 5

LEAP YEARS

1780 1848 1944
1820 1876 1972
1916

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29				

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

MAY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

JULY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

CALENDAR NO. 6

COMMON YEARS

1781 1810 1849 1883 1917 1951
1787 1821 1855 1894 1923 1962
1798 1827 1866 1900 1934 1973
1838 1877 1906 1945

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28			

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

MAY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

JULY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

Calendar No. 7

COMMON YEARS

1782 1805 1833 1861 1889 1907
1793 1811 1839 1867 1895 1918
1799 1822 1850 1878 1901 1929
1935 1946 1957 1963 1974

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28		

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

MAY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

JULY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Calendar No. 8

LEAP YEARS

1784 1852 1948
1824 1880 1976
1920

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29						

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

MAY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

JULY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Calendar No. 9

COMMON YEARS

1785 1814 1842 1881 1921 1949
1791 1825 1853 1887 1927 [illegible]
1803 1831 1859 1898 1938 1966
1870 1910

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28					

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

MAY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

JULY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Calendar No. 10

COMMON YEARS

1786 1815 1843 1871 1899 1922
1797 1826 1854 1882 1905 1933
1809 1837 1865 1893 1911 1939
1950 1961 1967

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

MAY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

JULY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

Calendar No. 11

LEAP YEARS

1788 1856 1924
1828 1884 1952

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

MAY

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

JULY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Calendar No. 12

LEAP YEARS

1792 1832 1928
1804 1860 1956
1888

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29			

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

MAY

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

JULY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24	25	26	27	28	29

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

Calendar No. 13

LEAP YEARS

1796 1836 1892
1808 1864 1904
1932 1960

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29					

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

MAY

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

JULY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Calendar No. 14

LEAP YEARS

1812 1868 1936
1840 1896 1964
1908

JANUARY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

FEBRUARY

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

MARCH

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

APRIL

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

MAY

Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24/31	25	26	27	28	29	30

JUNE

Su	Mo	Tu	We	Th	Fr	Sa
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

JULY

Su	Mo	Tu	We	Th	Fr	Sa
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

AUGUST

Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23/30	24/31	25	26	27	28	29

SEPTEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

OCTOBER

Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

NOVEMBER

Su	Mo	Tu	We	Th	Fr	Sa
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

DECEMBER

Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

DAY NUMBER AND DECIMAL EQUIVALENT

(NON-LEAP-YEARS ONLY)

JANUARY

1	1	.003
2	2	.005
3	3	.008
4	4	.011
5	5	.014
6	6	.016
7	7	.019
8	8	.022
9	9	.025
10	10	.027
11	11	.030
12	12	.033
13	13	.036
14	14	.038
15	15	.041
16	16	.044
17	17	.047
18	18	.049
19	19	.052
20	20	.055
21	21	.058
22	22	.063
23	23	.063
24	24	.066
25	25	.068
26	26	.071
27	27	.074
28	28	.077
29	29	.079
30	30	.082
31	31	.085

FEBRUARY

1	32	.088
2	33	.090
3	34	.093
4	35	.096
5	36	.099
6	37	.101
7	38	.104
8	39	.107
9	40	.110
10	41	.112
11	42	.115
12	43	.118
13	44	.121
14	45	.123
15	46	.126
16	47	.129
17	48	.132
18	49	.134
19	50	.137
20	51	.140
21	52	.143
22	53	.145
23	54	.148
24	55	.151
25	56	.153
26	57	.156
27	58	.159
28	59	.162

MARCH

1	60	.164
2	61	.167
3	62	.170
4	63	.173
5	64	.175
6	65	.178
7	66	.181
8	67	.184
9	68	.186
10	69	.189
11	70	.192
12	71	.195
13	72	.197
14	73	.200
15	74	.203
16	75	.205
17	76	.208
18	77	.211
19	78	.214
20	79	.216
21	80	.219
22	81	.222
23	82	.225
24	83	.227
25	84	.230
26	85	.233
27	86	.236
28	87	.238
29	88	.241
30	89	.244
31	90	.247

APRIL

1	91	.249
2	92	.252
3	93	.255
4	94	.258
5	95	.260
6	96	.263
7	97	.266
8	98	.268
9	99	.271
10	100	.274
11	101	.277
12	102	.279
13	103	.282
14	104	.285
15	105	.288
16	106	.290
17	107	.293
18	108	.296
19	109	.299
20	110	.301
21	111	.304
22	112	.307
23	113	.310
24	114	.312
25	115	.315
26	116	.318
27	117	.321
28	118	.323
29	119	.326
30	120	.329

MAY

1	121	.332
2	122	.334
3	123	.337
4	124	.340
5	125	.342
6	126	.345
7	127	.348
8	128	.351
9	129	.353
10	130	.356
11	131	.359
12	132	.362
13	133	.364
14	134	.367
15	135	.370
16	136	.373
17	137	.375
18	138	.378
19	139	.381
20	140	.384
21	141	.386
22	142	.389
23	143	.392
24	144	.395
25	145	.397
26	146	.400
27	147	.403
28	148	.405
29	149	.408
30	150	.411
31	151	.414

JUNE

1	152	.416
2	153	.419
3	154	.422
4	155	.425
5	156	.427
6	157	.430
7	158	.433
8	159	.436
9	160	.438
10	161	.441
11	162	.444
12	163	.447
13	164	.449
14	165	.452
15	166	.455
16	167	.458
17	168	.460
18	169	.463
19	170	.466
20	171	.468
21	172	.471
22	173	.474
23	174	.477
24	175	.479
25	176	.482
26	177	.485
27	178	.488
28	179	.490
29	180	.493
30	181	.496

JULY

1	182	.499
2	183	.501
3	184	.504
4	185	.507
5	186	.510
6	187	.512
7	188	.515
8	189	.518
9	190	.521
10	191	.523
11	192	.526
12	193	.529
13	194	.532
14	195	.534
15	196	.537
16	197	.540
17	198	.542
18	199	.545
19	200	.548
20	201	.551
21	202	.553
22	203	.556
23	204	.559
24	205	.562
25	206	.564
26	207	.567
27	208	.570
28	209	.573
29	210	.575
30	211	.578
31	212	.581

AUGUST

1	213	.584
2	214	.586
3	215	.589
4	216	.592
5	217	.595
6	218	.598
7	219	.600
8	220	.603
9	221	.605
10	222	.608
11	223	.611
12	224	.614
13	225	.616
14	226	.619
15	227	.622
16	228	.625
17	229	.627
18	230	.630
19	231	.633
20	232	.636
21	233	.638
22	234	.641
23	235	.644
24	236	.647
25	237	.649
26	238	.652
27	239	.655
28	240	.658
29	241	.660
30	242	.663
31	243	.666

SEPTEMBER

1	244	.668
2	245	.671
3	246	.674
4	247	.677
5	248	.679
6	249	.682
7	250	.685
8	251	.688
9	252	.690
10	253	.693
11	254	.696
12	255	.699
13	256	.701
14	257	.704
15	258	.707
16	259	.710
17	260	.712
18	261	.715
19	262	.718
20	263	.721
21	264	.723
22	265	.726
23	266	.729
24	267	.732
25	268	.734
26	269	.737
27	270	.740
28	271	.742
29	272	.745
30	273	.748

OCTOBER

1	274	.751
2	275	.753
3	276	.756
4	277	.759
5	278	.762
6	279	.764
7	280	.767
8	281	.770
9	282	.773
10	283	.775
11	284	.778
12	285	.781
13	286	.784
14	287	.786
15	288	.789
16	289	.792
17	290	.795
18	291	.797
19	292	.800
20	293	.803
21	294	.805
22	295	.808
23	296	.811
24	297	.814
25	298	.816
26	299	.819
27	300	.822
28	301	.825
29	302	.827
30	303	.830
31	304	.833

NOVEMBER

1	305	.836
2	306	.838
3	307	.841
4	308	.844
5	309	.847
6	310	.849
7	311	.852
8	312	.855
9	313	.858
10	314	.860
11	315	.863
12	316	.866
13	317	.868
14	318	.871
15	319	.874
16	320	.877
17	321	.879
18	322	.882
19	323	.885
20	324	.888
21	325	.890
22	326	.893
23	327	.896
24	328	.899
25	329	.901
26	330	.904
27	331	.907
28	332	.910
29	333	.912
30	334	.915

DECEMBER

1	335	.918
2	336	.921
3	337	.923
4	338	.926
5	339	.929
6	340	.932
7	341	.934
8	342	.937
9	343	.940
10	344	.942
11	345	.945
12	346	.948
13	347	.951
14	348	.953
15	349	.956
16	350	.959
17	351	.962
18	352	.964
19	353	.967
20	354	.970
21	355	.973
22	356	.975
23	357	.978
24	358	.981
25	359	.984
26	360	.986
27	361	.989
28	362	.992
29	363	.995
30	364	.997
31	365	1.000

LESSON VII-A
REVIEW, LESSONS V – VII

You are adding to your bag of tricks.

From Lesson V you learned to combine any collection of trends, randoms, and cycles when the values of each for each time interval are given.

(In this connection remember that where examples were given of combination by addition it was just so you could follow the arithmetic move easily. You almost never combine by addition (unless the values are expressed in logs). You should almost always combine by multiplication.)

Right now you could combine 40 different cycles if I gave them to you.

You also learned how to unscramble any collection of trends, randoms, and cycles when the values of each for each time interval are given.

From Lesson VI you learned all the easy ways known to me to get hints of cycle length, whether trend has been removed or not (only it's easier with trend removed). You learned how to make and how to use time charts and graduated scales. (I almost never have to resort to more complicated methods.)

From Lesson VII you learned how to find the average shape, timing, and amplitude of any cycle when the length of the cycle is known, whether trend has been removed or not.

You need lots more practice in these matters, but I do not see that you need any more instruction. The lessons will, therefore, press on to other matters.

LESSON VIII

HOW TO MAKE AND USE A PERIODIC TABLE - Part II

In Lesson VII, I told you something of the first of the four main uses of the periodic table, namely to tell you the typical or average shape, strength, and timing of the cycle, as nearly as the figures will permit, if you know the length of the cycle.

In this lesson I wish to speak of the second main use of the periodic table, viz: to reveal to you, as exactly as possible, the length of any cycle that may be present in the series. Let us see how the periodic table can be made to do this.

Let us start with controlled data.

Consider a 16-day cycle with values which run smoothly from 0 at trough to 48 at crest, troughs being in position 1 after base. (I have chosen these values so we won't have to bother with fractions.) Enter these values into a 16-day periodic table as follows:

TABLE 28

A 16-day Periodic Table of Controlled Data (a 16-day cycle).

Base	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6
16	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6
32	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6
Average	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6

A chart of these averages repeated twice, is shown in Fig. 39. It is obvious that, as the periodic table has the same length as the cycle, the average of any number of cycles will give you merely the values of one typical wave.

Let us now put these values into 15 and 17 day periodic tables respectively. You know from Lesson VII that when you add and average you will get 15 and 17 day cycles, but let us do it anyway.

TABLE 29

A 15-day Periodic Table of Controlled Data (a 16-day cycle).

Base	Days after base: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12
15	6	0	6	12	18	24	30	36	42	48	42	36	30	24	18
30	12	6	0	6	12	18	24	30	36	42	48	42	36	30	24
45	18	12	6	0	6	12	18	24	30	36	42	48	42	36	30
60	24	18	12	6	0	6	12	18	24	30	36	42	48	42	36
75	30	24	18	12	6	0	6	12	18	24	30	36	42	48	42
(sum)	90	66	54	54	66	90	126	162	198	222	234	234	222	198	162
Average	15	11	9	9	11	15	21	27	33	37	39	39	37	33	27

These averages repeated twice are charted in Fig. 40. You will notice (1) that the cycle you get from the 16-day cycle when cast into a 15-day periodic table is 15 days long, (2) that it has lower amplitude than in the 16-day table, (3) that it has a different timing from in a 16-day table, and (4) that it is not symmetrical.

If now we make a 17-day periodic table of our 16-day cycle you will see that we get the same four results. The 17-day periodic table is given below. Its average is charted in Fig. 41.

TABLE 30

A 17-day Periodic Table of Controlled Data (a 16-day cycle).

Base	Days after base: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0
17	6	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6
34	12	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12
51	18	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18
68	24	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24
85	30	36	42	48	42	36	30	24	18	12	6	0	6	12	18	24	30
	90	126	162	198	222	234	234	222	198	162	126	90	66	54	54	66	90
Average	15	21	27	33	37	39	39	37	33	27	21	15	11	9	9	11	15

Incidentally you will notice that when the cycle is longer than the periodic table, as in Table 29 the crests and the troughs slip to the RIGHT as we go from top to bottom. Note also, that if we had continued the table the crests and troughs would jump a whole cycle when they started in at the left again, two cycles later.

On the other hand, when the cycle is shorter than the periodic table as in Table 30, the crests and troughs slip to the LEFT as we go from top to bottom. Moreover, the crests and troughs, when they start in at the right again double up and start in the same cycle in which they ended.

You will find it necessary to keep these facts in mind as we progress with the work.

Suppose now we divide Table 29 into two halves of 3 cycles each, add, average, and plot as in Fig. 42. The averages are as follows:

TABLE 29 AVERAGED BY HALVES

	Days after Base 0														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Av. 1st half	6	4	6	12	18	24	30	36	42	44	42	36	30	24	18
Av. 2nd half	24	18	12	6	4	6	12	18	24	30	36	42	44	42	36

The averages of the first half of the table summarize or epitomize the first half. They boil it down to a series of values properly represented at the mid-point of the block being averaged. That is, just as when you average the 12 months of a year to get a yearly value you represent it as belonging to the mid point of the year, when you average three cycles you consider the averages as collapsing the three cycles into one--the middle one. (If you had averaged an even number of cycles your average would have fallen between the two center ones.)

Similarly the average of the last three cycles must be thought of as epitomizing the last three cycles. In other words we have boiled Table 29 down so that it looks like this:

TABLE 29 AVERAGED BY HALVES

Base	Days after base 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
15	6	4	6	12	18	24	30	36	42	44	42	36	30	24	18
30	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
45	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
60	24	18	12	6	4	6	12	18	24	30	36	42	44	42	36
75	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

(Incidentally, if we averaged the values given above we would obtain the original averages of Table 29.)

It should be clear from the above that the two averages should be thought of as three cycles apart.

It should also be clear from an inspection of the table, or of Fig. 42, that the crest of the cycle in the second half (and the trough likewise) has moved three spaces to the right of the cycle in the first half.

If the cycle moves three positions in three cycles it must move one position in one cycle. In other words it must be 15 days plus 1 (3 ÷ 3) day long. That is to say, the slippage to the right from one half of the periodic table to the other reveals that the inherent cycle is 1 day longer than the length of the periodic table. As the periodic table is 15 days long, the cycle is 16 days long.

Dealing with controlled data we knew this to start with, so it comes as no surprise. But dealing with actual data, where we do not know the true length, this manipulation will tell us, subject to the distortion caused by random, trend, and other cycles, as nearly as it is possible to know, what the true length really is.

We have averaged our periodic table by halves. Let us now average it by thirds.

TABLE 29 AVERAGED BY THIRDS

1st third (1st two cycles)	3	3	9	15	21	27	33	39	45	45	39	33	27	21	15
2nd third (3rd-4th cycles)	15	9	3	3	9	15	21	27	33	39	45	45	39	33	27
3rd third (5th-6th cycles)	27	21	15	9	3	3	9	15	21	27	33	39	45	45	39

The results are plotted in Fig. 43, but even without the chart we can see the cycle slipping across the table two positions to the right in two cycles--one position per cycle.

Of course in Table 29 you can see the cycle slipping across one position every cycle, but remember that in Table 29 you are dealing with controlled data where there is but one cycle present and where there are no distorting randoms. In real life the cycles are usually so distorted, or the movement per cycle is so small, that you cannot obtain the best possible answer by visual inspection of the periodic table or by any means other than measuring the slippage of groups of cycles in a periodic table.

In principle all this is very easy:

1st, by means of inspection or thumbing of data (usually expressed as deviations from trend), you get a hint of a cycle.

2nd, you subject your hint to the test of the graduated scale or its brother with the college education, the time chart.

3rd, having gone as far as you can with these tools you then make a periodic table of the estimated length of the cycle. (Of course you could start blindly with one or more periodic tables. We will get into this subject later when we consider harmonic analysis and multiple harmonic analysis.)

4th, by positioning the cycle in different parts of the periodic table, noting the slippage from the cycle in one part to the cycle in another part, and the number of cycles between parts, you can compute the length of the cycle as exactly as the irregularities and the nature of your data will allow.

How do you "position" a cycle? That is, how do you determine just when the crests and troughs come? With the controlled data given above the answer is easy; in real life it is harder. Definitizing cycles will be the subject of the next lesson.

How do you determine how many cycles to average into a group? That also will be the subject of another lesson.

However as part of this lesson let us be sure you know how to locate the distance between groups of cycles.

Suppose your periodic table is 10 cycles long. The average of the first five cycles would fall at cycle 3. The average of the last five cycles would fall at cycle 8. The two averages would be 5 cycles apart. The slippage between the first group and the second group would be divided by 5 to get the average slippage per cycle.

Suppose now you averaged the first six cycles and the last four cycles. The average of the first six would be at cycle $3\frac{1}{2}$; of the last four would be at $8\frac{1}{2}$. The two groups would still be five cycles apart.

Supposed you averaged the first seven and (overlapping at the middle) the last seven. The average of the first group would fall at cycle 4, the average of the last seven would be at 7. The groups would be three cycles apart.

Suppose you averaged the first seven and the last six. The average of the first seven would fall at cycle four. The average of the last six would fall at cycle seven and a half. There would be $3\frac{1}{2}$ cycles between groups.

From a study of the examples given above you should be able to see how to measure the distance, in cycles, of any group of cycles in a periodic table from any other group of cycles in the same table.

In connection with slippage from one half of a periodic table to another note that if there is any considerable amount of slippage, there is no way of knowing from the averages of the two halves of the table whether the slippage is to the right or to the left.

Suppose for example you have an 8 month periodic table. Suppose the average of the first half of the table crests in position 3, the average of the second half of the table crests at position 6. From these facts you must not assume it slips to the right by 3 places. It could have slipped to the left by 5 places.

Therefore, it is always necessary to check the direction in which the slippage occurred by some other means.

Probably the best means of checking the slippage is to average the table by thirds as well as by halves so that you can see the successive position of the cycle as it marches one way or the other across the periodic table. You should also check by graduated scale and by time chart.