

STANDARD PRACTICE INSTRUCTIONS

4. THE LABORATORY WORKER'S NOTEBOOK OR LOG

You should maintain a log or a laboratory worker's notebook.

If you studied chemistry in school you kept a notebook in which you recorded each step of each experiment so that afterwards you could know what you did and why you did it.

You should do the same for a cycle analysis.

To start off on the right foot I am going to write up the first page of your log for you.

To save you the trouble of copying what I write, I am furnishing you this write-up on a separate page so that you can insert it at the proper place in your binder.

In this instance the log will serve a double purpose. It will provide a guide for future logs. It will also explain to you the reason for choosing a 9-year moving average trend instead of a trend of some other length.

General Instructions Regarding Logs:

1. Each sheet of log should have in the upper right-hand corner the code to indicate the time series to which it pertains.
2. Each entry in the log should be headed with the date upon which the work was done.
3. Use abbreviations wherever possible. The log is merely for your own future reference.

LOG

STANDARD & POOR'S COMBINED ANNUAL, 1854--1953

(Date)

Set up TS--1. Entered data supplied into Column A.
Charted data on S & P C A--1 (arithmetic) and S & P C A--2 (ratio).

(Date)

Looked up logs and entered into TS--1, Column B.

(Date)

Studied Chart 2. Noted pronounced trend and many oscillations.
Major cycle about 20 years long.

Measuring from low to low

1859--1877	18 years
1877--1896	19 years
1896--1921	25 years
1921--1942	21 years
1942-- ?	

Fourth cycle broken all to pieces by the low of 1932.

The series is so short that we have only about 4 1/2 repetitions of the 20-year cycle. Too few to study fruitfully until shorter cycles removed.

Noted crests in 1953 (or later). 1946, 1937, 1929, at about 8-year, intervals. These 8- or 8 1/2 year cycles seem to vanish as we go backward for the next 25 years or so, but seem to reappear with a crest at 1902, 1892, 1881, 1872, 1864, and perhaps 1856.

Admittedly, the troughs between 1863 and 1872, and between 1881 and 1892 are very bad. But we have to start somewhere.

Let us therefore explore for cycles in the neighborhoods of 7, 8, 9, 10, or 11 years.

(Date)

As a first step I definitized trend by computing a 9-year moving average of the logs of the data.

The 9-year moving total was posted into Col. C, 9-year moving average posted into Col. C'.

Logs of data and their 9-year moving average then plotted onto Chart 3.

(Date)

Next I computed deviations, logs of data from their 9-year moving average, as described in Lesson IV. Posted deviations into Col. D, and plotted onto Chart 4.

I studied this chart and noticed:

LESSON III

Supplement 1

Question:

Why did you choose a nine-year moving average for the stock market averages instead of an 11-year or perhaps a 7- or a 5-year moving average?

Answer:

If you study a chart of the data you will see by inspection a cycle about 9 years long. You cannot tell by inspection just how long it is but you can see something about this length very clearly in the figures plotted on log paper or in the logs of the figures plotted on arithmetic paper. (S. & P. C. A. No. 2 and No. 3).

You generally use a moving average trend that is as near as possible to the length of the cycle or the cycles you are going to isolate. Also, you generally use a moving average with an odd number of terms.

Thus, if you had observed a 6-year cycle in these figures, you would have used a 5- or 7-year moving average. If you had observed a 13-year cycle you would have used a 13-year moving average.

You can, of course, use a moving average that differs from the length of the cycle, but if you do you have to make adjustments. Also, as you will learn, if your moving average is shorter than the length of the cycle, you will lose some of the cycle. If the moving average is longer than the length of your cycle you won't do the best possible job of getting rid of randoms and longer cycles.

All of this will become clear as you proceed with the course.

LESSON IV

HOW TO REMOVE TREND

When you have trend values for each year it is a simple matter to express the actual datum for each year as a percentage above or below the trend, as the case may be. (You will remember that datum is the singular of data. It means one of the numerical facts with which you started--one of the values of your original time series.) Such percentages are called percentage deviations from trend. They represent the data adjusted for trend.

Suppose the value of the datum for a particular year is 600. Suppose the trend for that year has a value of 500. The datum is 100 above trend; 100 is 20% of 500. Hence the datum for that year is 20% above trend; i.e., the deviation from trend is plus 20%.

If the datum for that year had had a value of 400, it would have been 100 below trend. One hundred is still 20% of 500, the value of the trend. The relationship of datum to trend--the percentage deviation from trend--would have been minus 20%.

When such computations (percentage deviations above or below trend) have been made, trend of course vanishes. That is, it becomes zero in all cases. Thus, when charted it becomes a horizontal axis at zero, around which the deviations oscillate.

For an illustration of charts of computations of this sort refer to Spurr, Charts 14-1 and 14-4 on pages 296 and 301 respectively.

(Of course what is said here about yearly trend values applies equally to trend values for any other period of time for which the data exist--half years, quarter years, months, weeks, days, hours, etc.)

Another method of proceeding is as follows: Instead of computing the percentages from the trend, compute the percentages that the data are of the corresponding trend values. In the examples given, 600 would be 120% of trend, 400 would be 80% of trend. Trend becomes a horizontal straight line with value 100%.

The curves obtained by the two methods are identical. However, in the second method the scale is 100% larger. Plus 20% becomes 120%, zero % becomes 100%, minus 20% becomes 80%.

For an example of the second method refer to Spurr, page 332.

Strictly speaking, the values obtained by the second method are not deviations. The word deviation (or departure) implies going away from, up or down. However, the word deviation is used loosely to include percentages of trend as well as percentages from trend.

When you are dealing with the data in their original form and when the deviations are small there is little to choose between the two methods. However, (a) when the deviations are large or (b) when you are working with logs of the data, the second method is essential.

When the deviations are large—for rule of thumb, let us say over and under, plus or minus 20%—you will want to plot them on ratio scale. But negative numbers cannot be plotted on ratio scale. You must convert to positive numbers.

Similarly, when you are working with logs you have to use the second method because, as you will remember, negative numbers have no corresponding logarithms.

Let us work out the actual arithmetic:

	Curve Above Trend	Curve Below Trend
A Data	600	400
B Trend	500	500
C Ratios (A divided by B)	$\frac{600}{500} = 1.2$	$\frac{400}{500} = .8$
D Percents (Ratios x 100)	120.	80.

Now, in Logs:--

	Curve Above Trend	Curve Below Trend
A Logs of data	2.7782 (Log of 600)	2.6021 (Log of 400)
B Logs of trend	2.6990 (Log of 500)	2.6990 (Log of 500)
C Logs of ratio (Subtract B from A)	.0792 (Log of 1.2)	-.0969 (Log of .8)
D Logs of percents (add 2.)	2.0792 (Log of 120)	1.9031 (Log of 80)

Of course, in actual practice you put your 2. into the machine along with the log of your datum, all in one operation. If the log of your datum reads

2.7782, you put 4.7782 into the machine, you subtract 2.6990, and obtain 2.0792 directly. Similarly, if your datum is 2.6021 you put 4.6021 into the machine. Then, when you subtract 2.6990 you obtain 1.9031 without having to bother with negative logs and an additional operation.

Note that when dealing with logs your "deviations" are differences of the logs, which is the same as ratios or percentages of the numbers.

All that I have said above applies to trends of any sort--graphic trends, straight line trends, mathematical trend such as parabola, and moving average trends.

NOTE

Lessons III and IV have failed to fulfill a promise I made in the Introduction that you could learn something from them no matter how many courses in statistics you might have taken.

When I got into the actual writing of Lessons III and IV I decided to simplify them so as to limit them to moving average trends. This aspect of moving averages is well understood by statisticians.

The effect of moving averages upon any cycles which may be present in the original curve and in the deviations from a moving average trend will be treated later when we get into the matter of isolating cycles. It is this aspect of moving averages that is not generally understood.

Problems

1. Adjust the logs of your stock market data for the 9-year moving average trend. Post the result in T.S.-1, Col. D.

2. Plot the values of T.S.-1, Col. D onto a sheet of 11" x 17" chart paper (Chart 4) using the same horizontal scale as you used previously, and using a vertical scale of your own choosing.

3. Define the following terms. (If you wish you may write answers on slips of 3" x 5" paper (or cards) which can later be filed alphabetically as part of a glossary.)

a. Reciprocal	y. Stub (of tabulation sheet)	ww. Secular Trend
b. Origin of a chart	z. The base of a ratio	xx. Rate-of-change chart
c. Refined ratio	aa. X scale of a chart	yy. Tabulation sheet
d. Common denominator	bb. Data (datum)	zz. Geometric moving av.
e. Ratio	cc. Average	aaa. Log-log chart
f. Ordinate	dd. Time series	bbb. Geometric mean
g. Quotient	ee. Cycle	ccc. Length of moving av.
h. Ratio chart	ff. Logarithms	ddd. Slope of a curve
i. Array	gg. Chart of graph	eee. Significant figures
j. Statistics	hh. Deviation	fff. Index number
k. Root	ii. Rhythm	ggg. Denominator
l. Grid	jj. Trend fitting	hhh. Series
m. Transposition	kk. Isolate	iii. Decimal fraction
n. Median	ll. Evaluate a cycle	jjj. Improper fraction
o. Factor	mm. Weighted moving average	kkk. Arithmetic mean
p. Common fraction	nn. Semi-logarithmic chart	lll. Parabola
q. Scale	oo. Axis of a chart	mmm. Moving total
r. Moving average	pp. Oscillation	nnn. Dividend (in arith.)
s. Ratio scale	qq. Random fluctuation	ooo. Numerator
t. Rounding off numbers	rr. Arithmetic scale	ppp. Point date value added
u. Term (of a time series)	ss. Cycle Analysis	qqq. Modified mean (date
v. Y scale of a chart	tt. Divisor	rrr. Curve (as plotted on
w. Percent	uu. Line graph	a chart)
x. Trend	vv. Mathematical trend	sss. Abscissa
		ttt. Node

4. Send in your stock market log (if you have added to what I wrote), your stock market tabulation sheet, and all four of your stock market charts.

5. Tell me how long you spent on Lesson IV and on each of the three assigned problems.

6. Tell me if you want (a) more home work, (b) less home work, (c) about what I am giving. Tell me if the lesson material is (d) too hard, (e) too easy, (f) about right.

LESSON IV-A

REVIEW, LESSONS I--IV

We are using the case method of instruction. That is, we are engaged in making an actual cycle analysis of a real series of numbers. The numbers chosen were annual stock market averages for the past 100 years, but any other series of figures would have served.

I kept the problem down to 100 numbers in order to hold the arithmetic to a minimum. On the other hand, I felt it necessary to have at least 100 numbers to work with in order to illustrate the various principles involved.

I told you that the first step in a cycle analysis was to chart the data on ratio scale. To make sure that you understood all about charts and charting, and particularly about charting on ratio scale, we paused to cover this subject sufficiently for the purposes of cycle analysis.

In charting the stock market averages we discovered that they evidence trend. I told you a little about the various ways in which trend can be definitized. In particular I told you that the way to definitize trend for purposes of cycle analysis was to compute a moving average of more or less the same length as the length of the cycles you wish to study. I also told you that the trend, so definitized, must be removed.

As all of this involves arithmetic we paused briefly to review the use of numbers, including logarithms.

I then told you in detail how to compute a moving average trend. Then I told you how to remove it.

Applying all of this to our stock market averages we obtained a series of values, (deviations of the logs of the data from their 9-year moving average trend), suitable for cycle analysis. (S. & P. C. A., T. S.-1, Col. D, and S. & P. C. A., Chart 4).

We are now ready to study these figures and this chart for rhythmic (i.e. repetitive) cycles of 7, 8, 9, 10, or 11 years in length.

These first four lessons have posed a bit of a problem for me, as the persons taking the course have had very different training in statistics. Some are competent statisticians. To such, much of these four lessons must have seemed childishly obvious. Some, however, know nothing of statistics. For them it must have been pretty hard, for we have covered in four lessons what, in a college course in statistics, would have taken about 1/3 of a half-year course. The net of it is that probably no one is satisfied.

From here on, however, it will be new ground for most of you. The lessons will therefore bear upon you much more uniformly.

The subsequent steps of your cycle analysis are, for each cycle present, (1) to find its length, (2) to definitize its shape, (3) to evaluate its significance (i.e., to try to answer the question, how easily could it be the result of random forces), (4) to adjust the data to remove the cycle as a basis for finding other cycles, or finding secular trend, and (5) to project the cycle into the future (either alone or in combination with other cycles and/or with trend).

Now all these steps (except finding cycle length, and evaluating the significance of the cycle) are quite commonplace statistical procedures.

They are discussed in a general sort of way in any good textbook of statistics under the heading of seasonal variation. (Seasonal variation is merely a 12-month cycle.

Assignment

Therefore, as your assignment, please read Spurr, Chapter 16, page 356, down to "Business Cycles" on page 376. This will help you to get ready for a more refined application of the same principles to cycles of other lengths.

Except for incidental references we are now through with Spurr, Kellogg, and Smith.

LESSON IV

Supplement 1

THE GLOSSARY

One student wrote in objecting to the work involved in preparing the glossary.

I don't blame him a bit. I, too, find that sort of thing an awful bore.

However, I had several reasons in mind in asking the questions.

1. I felt that if you really knew your stuff you would write the answers out of your head in short order.

2. I felt that if you had to look the answers up in the text, you needed to do the extra work in order to get the subject matter in hand--or should I say "in head?"

3. I felt that if you had studied the text thoroughly it wouldn't be too hard to find the answers in the book. However, I mixed up the terms to be defined so that if you hadn't studied the text you would have the heck of a time to find the answers--and would absorb quite a bit in the process.

4. I felt that the actual writing of the answers would help fix the meaning of the terms in your mind. It does for me.

5. I felt that if you got any of the answers wrong it would give us a chance to set you straight.

These are the reasons I asked the question. But maybe it wasn't such a bright idea. What do you think?

Lesson IV

Supplement 2

The Log

Question: Is it essential that a log be kept? This is one form of record keeping which I specifically abhor and have consistently found to be of little value, usually serving only to distract attention. But then perhaps I am only rationalizing for the benefit of my laziness.

Answer: I hate keeping logs as much as you do, or more. Just between us, I'll admit I often skip keeping a log; but when I do I always regret it.

A cycle analysis takes so many sheets of paper and involves exploration in so many different directions that it is almost impossible to keep in mind everything you do, and why you did it, and what you concluded from having done it.

When you pick up the work 6 months, or a year, or 5 years from now and see, let us say, a 5.5-year periodic table and a 5.6-year periodic table of the same series of figures you will ask yourself "What lead me to make each of these tables? Which did I make first? Which did I conclude was the better? What I was lead to do next? What other cycles did I get hints of from a study of this table?"

If you do not have a log which explains all of this it sometimes takes hours to discover the answers.

What a blessing it is to look back to old work and to find a well written log that gives the answers to all these and other similar questions! I assure you it is worth the trouble. When I do it I am always glad; when I fail to do it, I am always sorry.

At the very least, note on each time chart or periodic table:

1. The date when the work was done.
2. What lead you to make a table or chart of this particular length.
3. What you saw in the table or time chart requiring further investigation in other directions.
4. What you concluded from the table or chart.
5. The next step indicated by the table or the chart.

This information is, I would say, the bare minimum requirement.

LESSON IV

Supplement 3

COMPUTING PER CENT OF TREND

Question: Please tell me if the following statements are true.

The terms in Column C' - 9-year moving average of "B" - are the trend values of the years.

The terms in Column "B" - Logs of "A" - are the values of the data for the years.

Therefore, to compute the terms in Column D - the per cent of trend - the equation becomes $\frac{100B}{C'}$ equals % of trend.

Example: the term for 1858 equals $\frac{1161 \times 100}{1242}$, or 93.5%

Answer: Your statements are true - up to a point. That is, you do compute the percent of trend, as you have illustrated, IF you are using natural numbers.

But here you are using the logs of the numbers. You will remember that instead of multiplying (as with numbers) you add the logs. And if you want to divide the numbers, but are using logs, then you must subtract.

Using the figures for the year 1858, and using a formula for computing per cent of trend (in logs) you get this computation:

(1.161 plus 2.000) less 1.242 equals 1.919

2.000 is the log of 100. So, instead of multiplying 1161 by 100, you must add 2.000 to 1.161. Because 1.161 is a log. And, instead of dividing by 1242 you must subtract 1.242.

If you want to know what this is in per cent, then you look up the anti-log of 1.919.

Or, to state the matter another way, you compute Column D by adding 2.000 to the figures in Column B, and subtracting the figures in Column C' for each year.

The answers will fluctuate above and below 2.000 - just as they would fluctuate around 100 if expressed in percentages.

PART II - CYCLE ANALYSIS

LESSONS V - XXIII

LESSON V

CYCLE ANALYSIS

A. HOW TO MAKE A CYCLE ANALYSIS OF A SERIES OF NUMBERS--AN OUTLINE

How do you make a cycle analysis of a series of numbers?

The complete answer to this question will take all the rest of this course. However, at this point, it may be helpful to introduce, for purposes of orientation, a brief outline of procedure.

Do not be dismayed if the outline is largely incomprehensible to you at this stage of your study. Before you will understand the outline fully, you will have to cover the rest of the material. At this time get what you can from the outline; then come back to it.

To make a cycle analysis of a series of numbers, you must do five things:

First, you must prepare your figures, chart them, etc.

Second, you must get a hint of a cycle that may be present.

Third, you must find out all you can about this cycle.

Fourth, you must come to an opinion in regard to the significance of this cycle. You must accept it as a probable reality, or reject it as probably due to random forces.

Fifth, you must see if there is a hint of another cycle present; if so, you must go through the same steps all over again.

This process must be continued until all the important cycles present have been discovered, studied, characterized, and evaluated. You have now completed your cycle analysis.

If, following your analysis, you adjust the original figures for the combined effect of all the accepted cycles, the remainder will be a combination of the trend and the sporadic or non-cyclic fluctuations (including in this category the undiscovered cycles, if any, and the introduced artifacts or errors). To these figures you can fit a trend which you can project with due caution. You can then wrap a projection of the discovered and accepted cycles around this trend to show what will happen, except for non-cyclic fluctuations, if the trend and cycles continue as determined.

B. A DETAILED OUTLINE

Now, let me expand the above summary into a more detailed outline of eighteen steps:

1. Be sure your data are accurate and are as homogeneous as possible.
2. For serious work you should next convert your data to logarithms. (For purposes of preliminary reconnaissance you may omit the conversion to logarithms.)
3. Next, you should chart the logarithms on arithmetic scale. (If you have decided to proceed without converting to logarithms, start your analysis by plotting your data on ration scale.)
4. Inspection of your chart will tell you what to do about adjustment for the growth factor and will indicate whether you should start your analysis with the investigation of long cycles, short cycles, or cycles of medium length.

The terms long and short refer to the length of the series being analysed. When dealing with a thousand years of tree ring widths, the long cycles might be those over one hundred years in length; dealing with a year of daily stock market figures, the long cycles might be those longer than a month or so.

Inspection of your chart, or one of the single adjuncts to the inspection which will be described, in Lesson VI, may, even at this stage, give you a hint of one or more rhythmic cycles.

5. If your data are characterized by growth, your next step will be to establish tentative values for the growth factor; that is, to find "trend."

Trend is a very flexible concept. There are many trends. The trend that you choose will be determined by the length of the cycles you wish to investigate first. For short cycles or cycles of medium length, you will probably use a moving average trend, or you may decide upon one of the so-called mathematical trends to be discussed later.

It should be emphasized here that the trend used for cycle determination is probably not the trend that will ultimately be used for purposes of projection.

6. You next express your data as differences from trend (if you have converted to logs) or as percentages of trend (if you are using raw data) in order to obtain what are called departures or deviations.

This process gives you data adjusted for growth. It eliminates, or at least minimizes, the distortions due to growth. (This is the place we are now in the analysis of stock market figures.)

This step may be a necessary prerequisite for obtaining hints of rhythm. Or, if you have already obtained a hint of a rhythm (Step 4), this step may be a stage in the process of isolation and evaluation.

7. If you do not already have a hint of a rhythm, you should now, in the data adjusted for growth, attempt to find one. This is almost always done either by inspection or by one of several simple supplemental methods.

8. If no rhythms are suggested by inspection or by the simple aids to inspection which will be described, you should manipulate the data in such a way as to minimize certain rhythms and the random elements and to leave certain other rhythms relatively unaffected. This process is called filtering.

9. When you have a hint of a rhythm, you should then determine, as exactly as possible, the typical (a) length, (b) timing, (c) shape, and (d) amplitude (strength) of the cycle. That is, you idealize the rhythmic cycle into a true periodicity (a curve which repeats with perfect regularity). This process is called isolating the cycle.

10. You should then attempt to determine the significance of the periodicity which you have isolated.

11. You then adjust the original data or the deviations, or both, for the effect of the isolated cycle.

12. You now proceed to discover, isolate, and evaluate a second cycle. For this purpose you use (a) the original numbers and/or (b) the original numbers adjusted for the cycle already discovered, and/or (c) the original deviations, and/or (d) the deviations adjusted for the cycle already discovered.

13. You continue this process as many times as may be necessary in order to find all the significant cycles in your series of numbers.

14. You then project into the future each isolated periodicity.

15. You then compute the combined effect of all the cycles which you have accepted as probably significant.

16. You then adjust your original data for all these cycles.

What is left is (a) true growth trend, (b) any random fluctuations, that may be present in the original data, (c) any cycles not yet isolated, and (d) any distortions you may have introduced by erroneous adjustments.

17. To the residual obtained by Step 16, you now fit the nearest possible approximation to the truth growth trend, You then project it cautiously a short distance into the future.

18. You now synthesize (combine) the growth factor obtained by Step 17 with all the cycles which you have accepted as probably significant to obtain a composite curve (a) to compare with the actual figures with which you started and (b) to project into the future.

The light that this projection will throw upon actual future behavior will depend upon the extent to which your isolated periodicities represent the effect of real periodic forces which will continue, and the extent to which such forces govern.

C. SOME BASIC CONCEPTS

At this point, at the risk of a little duplication, I would like to use this section to explain some basic concepts which you will need to keep in mind when you come to look for cycles.

The present section will cover the following ground:

1. The Purpose of Cycle Analysis.
2. The Concept of Time.
3. The Forces Creating Time Series.
4. Synthesis--The Combination of Growth, Periodic, and Random Components.
5. Analysis--The Separation of Growth, Periodic, and Random Components.
6. The Concept of Rate of Change.

1. The Purpose of Cycle Analysis

As I said in the introduction, it is the purpose of cycle analysis to detect, isolate, and determine the significance of the cycles (rhythms) in a series of numbers--usually a time series. The significant cycles so detected and isolated can be projected to throw light on the probabilities of the future.

As you doubtless remember, a series is an arrangement of numbers one after another, according to some fixed plan. When the numbers are arranged according to time occurrence, the resulting series is called a time series. The numbers that make up a series are called the terms of the series. They are also called items or data.

Cycle analysis (rhythm analysis) is concerned almost wholly with time series.

2. The Concept of Time

Time flows evenly. However, the figures with which you deal do not necessarily represent this fact.

For example, in dealing with daily price quotations you will often find that no values are available for Saturdays, Sundays, and holidays, or for days when the exchange is closed. Values for such days must be estimated and interpolated. (Interpolate means to insert between.) Thus, if the closing price for Saturday is 10 and the closing price for Monday is 12, you will have to estimate the closing price for Sunday. It would be 11. (Ten, the price for Saturday plus 1, half the difference between Saturday and Monday prices, equals 11. We use half the difference because there is a two-day interval involved.)

If the close for Friday is 12, and there is no quotation for Saturday or Sunday, and if the price for Monday is 10, the estimated price for Saturday would be $11 \frac{1}{3}$, for Sunday $10 \frac{2}{3}$. (12 minus $\frac{2}{3}$ for Saturday, 12 minus 2 times $\frac{2}{3}$ for Sunday.) If the exchange is closed for a week and a day all eight values must be estimated and interpolated.

Your data must always represent points of time that are approximately equidistant. Missing hours, days, weeks, months, or years must always be interpolated.

I used the word approximately in the above paragraphs because exactly equidistant time intervals are not always available or necessary. For example, a Saturday noon close would be only 21 hours after a Friday close, instead of 24 hours as we would ideally like to have it, but it is the best we have and it is good enough for all practical purposes. The average price for January represents a point in time a little more than $1/24$ of a year after the beginning of the year because January is a little longer than an average month), but the error is small and unimportant.

On the other hand, monthly production figures are often distorted enough by the different numbers of days in a month so that, for the discovery of short term cycles, suitable adjustments have to be made. One way to do this is to divide the total production for each month by the number of working days in that month in order to obtain an average production per day. Such figures would be comparable with each other. Another way to adjust is to prorate the monthly production figures up or down to compensate for variations in the length of the work month. Examples of each method follow:

For the first three months of 1953 a plant produced 12,000 tons in January, 11,000 tons in February, and 11,000 tons in March. If the plant works $5\frac{1}{2}$ days a week it will work $23\frac{1}{2}$ days in January, 22 days in February, and 24 days in March. The average daily production for these three months will be 510.6, 500.0, and 458.3 respectively. These figures are obtained by dividing 12,000 by $23\frac{1}{2}$ for January, dividing 11,000 by 22 for February, and dividing 11,000 by 24 for March.

As an example of the second method in prorating the monthly tonnages consider the following circumstances: a company works every day in 1953 except the five standard holidays, Sundays, and half-time Saturdays. This gives a total work year of 282 days. The typical work month should therefore be $23\frac{1}{2}$ work days long. If the company produced 12,000 tons in January, 11,000 tons in February, and 11,000 tons in March you would get the adjusted production by dividing the tonnage by the actual number of work days and multiply the daily average thus obtained by $23\frac{1}{2}$. We would therefore obtain the following adjusted monthly tonnages. January 12,000 ($12,000 \div 23\frac{1}{2} \times 23\frac{1}{2}$); February 11,750 ($11,000 \div 22 \times 23\frac{1}{2}$); March 10,534 ($11,000 \div 24 \times 23\frac{1}{2}$). These adjustments change the monthly values with which you started into truly comparable twelfth-year values.

The same principle applies in theory to sales figures, but I have found that in practice it does not work out that way. In many companies there is a tendency to force more bills into short months. In such instances connecting for the length of the month on the above basis may do more harm than good.

In connection with this matter of time, it should be apparent by now that the time units with which we deal are relatively unimportant. That is, looking for a 6-month cycle in monthly figures involves exactly the same procedures as looking for a 6-year cycle in annual figures, or a 6-day cycle in daily figures.

It would be awkward throughout the course to refer each time to daily, weekly, monthly, and annual numbers. I shall therefore simplify the exposition by referring to one kind of number at a time leaving it to you to realize that what is said of monthly numbers by way of illustration would apply equally to numbers representing seconds, hours, days, or weeks, or numbers representing years, decades, or centuries.

3. The Forces Creating Time Series

In connection with your study it will be helpful if you realize that any time series may be influenced by (a) growth forces, and/or (b) periodic forces, and/or (c) random forces, in any combination. In addition, any time series expressed in dollars (for example sales in dollars) is influenced by changes in price. Price, in turn, is influenced from time to time by inflation. The combination of all these forces creates the events which are recorded by the time series.

(a) Growth Forces.

The term growth forces refers to the forces, whatever they are, which make time series increase or decrease gradually over a period of time. Growth may be positive or negative. Positive growth refers to gradual increase over a period of time. Negative growth refers to gradual decline over a period of time. Some time series are not influenced by growth forces at all--for example, barometric pressure.

That part of the behavior of a time series which results from the growth forces acting upon it is called, interchangeably, the growth element, the growth component, or the growth factor.

The growth factor is often called the growth trend. As you know, trend means the prevailing tendency or direction. In statistics it refers to a curve which changes its direction slowly, or to the series of numbers which the curve represents.. The underlying growth is called secular trend, from a Latin word meaning, among other things, an age or a century.

The growth element of a time series needs but little discussion. The population of our country increases gradually over the years. The population of your town may also be increasing--or may be gradually on the decline (negative growth). Your business is doubtless growing. The government is growing. Taxes are growing.

The figures with which you are working must usually be adjusted for growth before you can find and/or isolate the rhythmic tendencies which are present.

(b) Periodic Forces

The term periodic forces refers to forces, whatever they are, which influence a time series to fluctuate in regular waves or oscillations. The force that tends to make the winter cold and the summer hot is a periodic force.

A time series may be influenced by two or more periodic forces at one and the same time. Consider the tides. A time series representing water levels in New

York harbor is influenced by one periodic force which results from the rotation of the earth, another periodic force which results from the rotation of the moon around the earth, another periodic force which results from the eccentricity of the earth's orbit, and other periodic forces as well, all periodic forces acting concurrently.

Although, strictly speaking, a periodic force is perfectly regular, it will be convenient, later in the course, to extend the meaning of the term to include forces that change their periodic characteristics gradually. Such forces may become stronger or weaker or have a wave length which increases or decreases over a period of time. Such forces usually offer no insoluble problems to the cycle analyst, as long as the irregularities behave in a regular way.

At this point it may be well to review the definition of the words cycle, rhythm, periodicity.

The word cycle comes from the Greek word meaning circle. Hence, in its most general meaning, it refers to a complete sequence of events, returning to the original state. Insects, in their life cycle, go through the states of pupa--larva--adult and then to the next generation, pupa again. In the business cycle, we have a succession of events starting with depression, recovery, boom, recession, and depression again. In the cycle of the seasons, we have spring, summer, fall, and winter--and spring again.

The word cycle, by itself, does not imply any uniformity of time interval for the course of events. One business cycle may be three years from depression to depression. The next may take seven years to go through the same sequence of boom and bust.

When you want to imply uniformity of time interval you should use the word rhythm, from a Greek word meaning measured time. If you speak of the rhythm of business, for example, you would imply that depressions or recessions come at more or less equal time intervals. This course is primarily concerned with rhythm. It really should be called Rhythm Analysis.

The word cycle, preceded by a term to indicate a time interval, as a 6-year cycle, is often used to mean a rhythm of the indicated length.

The word periodicity refers to the state or quality of being regularly recurrent, i.e., periodic.

Thus you have three words to describe the regularity with which a succession of events repeats:

Perfect regularity--periodicity.

Tending toward regularity--rhythm.

Here coming around again to the place of beginning with no necessary regularity--cycle. But the word cycle is often used popularly in the other two senses, as well. When you hear or read the word cycle you have to judge by the context whether it is being used strictly or in the popular sense of rhythm or periodicity.

That part of the behavior of a time series which results from the periodic forces acting upon it is called, more or less interchangeably, the periodic element, the periodic component, or the periodic factor.

The word factor means any of the elements or quantities which, when multiplied together, form a product. The fact that the various elements of a time series are ordinarily multiplied together explains why they are often called factors.

Some time series are not influenced by periodic forces. But many times series which you would not expect to be influenced by periodic forces act as if they were influenced by such forces, as you will presently see.

(c) Random Forces

Random forces are haphazard forces.

There is no infinite variety of random forces, each with their own characteristics. If you put into a hat 100 cards numbered 1 to 100, draw at random, and plot on a chart the various numbers you draw, you will get one sort of curve. If ten of your 100 cards are numbered 1, ten are numbered 2, ten are numbered 3, and so on, up to ten cards numbered 10, your chart will be quite different. Within the framework of their characteristics, however, random forces are unpredictable, and so of course are the fluctuations which they cause in a time series.

Random fluctuations may happen to come at more or less regular time intervals, but if they do it is merely by chance. The regularity will not continue, except by chance.

That part of the behavior of a time series which results from the random forces acting upon it is called, more or less interchangeably, the random element, the random component, or the random factor. The term random factor being more strictly reserved for those situations where the elements are multiplied together.

4. Synthesis--The Combination of Growth,,Periodic, and Random Components

I think it will help you to know how to look for cycles, and how to break a curve down into its component parts, if we start with synthesis. Synthesis is the process of putting things together. When the characteristics of each part of the synthesis are known, such a synthesis consists of controlled data.

If your controlled data consist only of random numbers you know that the cycles which may be present are random. If the controlled data consist in part of periodic components you know that a significant cycle is present, even though it may be too faint to be detectable.

Once you know about putting the component parts together into a synthesis, the reverse process--analysis--is duck soup.

Growth, periodic, random price, and inflation components can be present in a time series singly or in any combination. In the following sections we will work out some examples to show the synthesis or combination of various components as follows: (a) growth and random components, (b) periodic and random components, (c) two or more periodic components, (d) growth and periodic components, and (e) growth, periodic, and random components. The matter of price and inflation components will be discussed later.

(a) The Combination of Growth and Random Components.

Let us now construct a series of numbers which combines growth and random components. This is done in Table 1. The components and their combinations are charted in Fig 1.

Growth takes place in a variety of ways, as I have already told you. For our example, however, let us use the simplest possible form of growth, a series of numbers increasing from month to month by a constant amount. The numerical values of such a growth element are listed in Column A of Table 1 and are plotted as Curve A in Fig. 1.

This particular set of random numbers was obtained by rearranging in random order the amount by which the monthly sales of a manufacturing company were above or below trend. The random numbers so obtained are listed in Column B of Table 1 and are plotted as Curve B in Fig. 1.

Random numbers and the numbers representing growth can be combined either by multiplication or by addition. For this example let us use the simplest possible way and combine them by addition, placing the result in Column C of Table 1, and plotting it as Curve C of Fig 1. (In real life, you should combine by multiplication, as we shall see in section (e) below.)

(b) The Combination of Periodic and Random Components

For our second example to illustrate the synthesis of the basic elements of which any time series may be composed, consider several combinations of periodic and random elements, as computed in Table 2 and plotted in Fig. 2.

Here again, for the sake of simplicity, and to enable you to follow the arithmetic as easily as possible, the two components have been added.

Note that the combination of a perfectly regular periodicity and a series of random numbers creates a rhythm--a succession of ups and downs which repeat with a beat.

Before starting to discuss periodicity we are going to need some explanation of terms as follows:

A wave is one complete undulation or cycle of a periodicity or a rhythm.

The top of a wave is the highest point of the wave and is called high, top, crest, peak, interchangeably.

The bottom of a wave is the lowest point of the wave and is called low, bottom, trough, valley, interchangeably.

Amplitude is the height of the top of a wave above the horizontal axis, or the depth of the bottom of a wave below the horizontal axis. When it measures the height of the top above the axis it is called positive amplitude. When it measures the depth of the bottom below the axis it is called negative amplitude. The vertical distance from top to bottom is called the overall amplitude. It is *the* sum of the positive and negative amplitudes, disregarding sign.

The axis is a straight line, real or imaginary, around which the parts of a wave are symmetrically arranged. Axes can be either (1) horizontal or (2) vertical but in cycle analysis the word axis refers to the horizontal axis unless specifically indicated otherwise.

The length of a regular cycle (periodicity) is measured (a) from the top of one wave to the top of the next, or from some other fixed point such as (b) the bottom of the wave or (c) the place where the curve crosses the axis in an upward direction in going from left to right (upward crossing), or (d) the place where the curve crosses the axis in a downward direction in going from left to right (downward crossing). Wherever you choose it, the fixed point for starting a wave is called the epoch. Astronomers and physicists generally take the upward crossing of a curve as the epoch. Business statisticians, biologists, and climatologists usually take either the top of the wave or its bottom as epoch.

The term period is usually used in scientific writing to designate the interval of time required for periodic motion or phenomena to create a cycle and begin to repeat itself. When you are dealing with a rhythm instead of a true periodicity, the interval of time from epoch to epoch, of course, differs from wave to wave. However, the various waves have an average length. This average length is called the period of the rhythm.

Having got these definitions out of the way, let us proceed. You will remember we were about to combine periodic and random components.

The random numbers are listed in Column A of Table 2. They are the same random numbers we used in Table 1.

To the series of random numbers let us now add, in sequence, the successive numerical values of a perfectly regular zigzag shaped cycle with a period or length of eight months and with a strength or amplitude of $\frac{1}{2}$. The successive values of this periodicity are recorded in Table 2 in Column B. The sum of the random numbers and the periodicity is recorded in Column C. Curves charting these various values are plotted in Fig. 2, Curves A, B, and C, respectively.

Let us now add to the random number with which we started (Column A) a perfectly regular periodicity with an amplitude of 20 as posted in Column D, putting the sum of the random numbers and the periodicity with an amplitude of 20 in Column E and plotting the values of both Column D and Column E in Fig. 2 as Curves D and E.

Similarly, to the random numbers we will add 8-month cycles with amplitudes of 40 and 80 respectively as recorded in Columns F and H, posting the results in Column G and I, and plotting each pair of curves in Fig. 2 as Curves F and G, H and I.

In each of the composite series just constructed (composites of random numbers and a true periodicity) and in the curves charting them, namely, C, E, G, and I, you know there is an 8-month cycle. You know this because it is there by construction. Each of these curves, therefore, looks the way a real series of events would look if it were influenced simultaneously month by month by random forces of the indicated magnitude and characteristics and by a perfectly regular 8-month cycle.

A perfectly regular cycle plus random numbers gives us a rhythm--a tendency toward regularity. In Curve C of Fig. 2 this tendency is so faint that you are better at seeing cycles than I am if you can see any 8-month cycle in the composite. On the other hand, if you cannot see that Curve I of Fig. 2 goes up and down with rhythm or a beat, you had best stop at this point until you can see it.

You should study the composite Curves C, E, G, and I in Fig. 2 to acquaint yourself with what a real cycle of various relative intensities actually looks like when obscured by random variation.

(c) The Combination of Two or More Periodic Components

Let us now construct some series of numbers which combine two or more periodic components.

To keep the arithmetic simple and easy to follow, let us make the combination by addition.

First let us combine 3-month, and 10-month periodicities, each of zigzag shape. The pattern will repeat in 30 months ($3 \times 10 = 30$).

Then let us combine 3-month, 10-month, and 14-month periodicities, all of zigzag shape. The pattern will repeat in 210 months ($3 \times 5 \times 2 \times 7 = 210$).

The computations are made in table 3 and are plotted in Fig. 3.

Additional Examples. As the subject of synthesis and its reverse, analysis, is so important, it seems desirable to include some additional examples. Consider the synthesis of a perfectly regular 4-month cycle and a perfectly regular 6-month cycle as illustrated, with the arithmetic, in Table 4 and in Fig. 4.

In this illustration the combination or synthesis is effected by means of addition. The 4-month cycle, illustrated by Curve A, starts out at $+4$. Two months later it reaches a trough at -4 , an over-all move of eight points in two months, or four points per month.

The 6-month cycle, illustrated by Curve B, starts with the value of $+3$ and goes down to a value of -3 , three months later. This constitutes an over-all move of six points in three months or two points per month.

The successive values of Curve A are $+4$, 0 , -4 , 0 , $+4$, etc. The successive values of Curve B are $+3$, $+1$, -1 , -3 , -1 , $+1$, $+3$, etc.

The combination is effected by simple addition as shown in Table 4.

Fig. 5 shows the combination of 3- and 4-year cycles.

In Fig. 5 the 3-year cycle starts out at $+12$, one and a half years later it is down to -12 , an overall move of 24 points in a year and a half, or 16 points in one year.

Sixteen points down from $+12$ is -4 and therefore the value for one year after base is -4 ; the value for 2 years after base is also -4 ; 3 years after base it is -12 again, and so on.

The 4-year cycle also happens to start out at $+12$. It goes down 24 points in 2 years, or 12 points in a year. This gives one year after base a value of 0 ; 2 years after base a value of -12 ; 3 years after base a value of 0 ; 4 years after base a value of $+12$, and so on.

To get a crude value for the combined effect of both waves, one merely adds the two cycles together as shown in Table 5.

Fig. 6 shows the combination of 6- and 8-year cycles.

The 6-year cycle starts out at -20 and 3 years later has reached $+20$, 3 years later or 6 years from the beginning, the 6-year cycle is down to -20 again. Thus, in each year the 6-year cycle goes up or down $13\frac{1}{3}$ points. Consequently, in the first year after base its value is $13\frac{1}{3}$ points above -20 or $-6\frac{2}{3}$. In the second year it has a value of $+6\frac{2}{3}$, in the third year it has a value of $+20$, in the fourth year a value of $+6\frac{2}{3}$, etc.

The 8-year cycle shown also goes from -20 to $+20$ or an overall move of 40 points in four years, or 10 points a year. Consequently, one year after base, this 8-year cycle is at -10 , and two years after base, it is at 0 , three years after base it is at $+10$, four years after base it is at $+20$, five years after base at $+10$, etc.

To combine these two waves crudely, you merely add these values algebraically, as shown in Table 6.

For accurate work and especially where the waves are large, the values should be expressed as percentages and should be multiplied together. That is, instead of adding $+20\%$ to $+20\%$ and calling it $+40\%$, you should get the value of the crest of two 20 waves by taking 120% of 120% which, of course, is 144% ($+44\%$). Similarly, the troughs of two 20 waves coming together would be 80% of 80% or 64% , (or -36%) of trend. You should make similar computations for the value of each wave at each point of your series.

(d) The Combination of Growth and Periodic Components.

Let us now construct a series of numbers which combines growth and periodic components.

For the growth element we could use a variety of curves, but, for simplicity, we will use a straight line trend as in the example combining growth and random numbers, posting the values in Column A of Table 7 and plotting them as Curve A of Fig. 7.

Our periodic component also could have any one of a variety of shapes, the only requirement at this point being that whatever the shape it repeat with perfect regularity, time after time. I have posted the values of the periodic components in Column B of Table 7, and plotted them as Curve B of Fig. 7.

The combination of growth and periodic elements in this instance, as in the combination of growth and random numbers, is made by simple addition so that you can follow the arithmetic more easily. The result is posted in Column C of Table 7 and plotted in Curve C of Fig. 7.

In numbers representing real situations, the various elements generally combine by multiplication. That is, usually the growth element should be multiplied by the random element and/or the periodic element expressed as percentages.

The example just given, computed in Column C of Table 7 would, therefore, have been more realistic if we had multiplied each value representing the growth element by the corresponding value of the periodic element, expressed as a percentage, as recorded in Table 7, Columns D and E and plotted in Fig. 7, Curves D and E.

Note that if you combine your elements this way (by multiplication) when the growth element is small the periodic element has a small absolute amplitude; when the growth element is large the absolute amplitude of the periodic element is large. However, plotted on ratio scale, the periodic element has a uniform amplitude throughout the series.

If you will stop a moment to think, you will realize that this conforms to your own experience. You start in the ice cream business. Your sales are \$500 a month in winter, in summer \$1,000 a month. That is, \$500 a month more, or double. The business grows. Ten years later your sales in winter are \$5,000 a month, in summer \$10,000 a month. The periodic seasonal pattern increases in proportion to the growth--double in summer whatever it is in winter. It does not increase by a constant amount. It is not \$500 more in summer for example, regardless of the size of the business.

(e) The Combination of Growth and Periodic and Random Components

To complete the illustration, let us now combine all three elements, doing it this time by multiplication as it should be done to get a result more nearly in conformity with usual actual life experience. The computations are given in Table 8, illustrated in Fig. 8. We divide the product of growth, the periodic factor and the random factor by 10,000 because both periodic and random factors

were expressed as percentages, which in each case introduce a distortion of a hundred fold in the result. We adjust for this distortion by dividing by 100 times 100 or by 10,000--that is, by pointing off the answer four more places to the left. If we had expressed the periodic factor and the random factors as ratios (by pointing each of them off two more places to the left) we would not have needed to make this adjustment in Column D.

5. Analysis--The Separation of Growth, Periodic, and Random Components

In the previous section you have been concerned with the synthesis or putting together of the three basis components of time series. You will remember that in any time series the three chief components are--or may be--the growth component, the periodic component, and the random component.

In this section we will consider analysis.

The process of analysis is merely the reverse of the process of synthesis. Where the combination of the components was made by adding you can take the combination apart by subtracting; where the combination was made by multiplying, you can take the combination apart by division.

In dealing with controlled data the process is very simple, because you know in advance the characteristics of each of the components. The analysis of controlled data is therefore merely, like synthesis, a matter of simple arithmetic.

In dealing with data from real life you have no such easy problem. You have to estimate what the trend factor probably was. You have to estimate the probable length, shape, amplitude, and timing of the periodicities, if any, which are elements in the data. Further, you have to figure out how these various elements have combined--for example whether by addition or multiplication, or in some other way. (However, in the absence of evidence to the contrary, you can assume combination by multiplication.)

Finally, you have to estimate the probability that the observed rhythms are the result of periodic forces which can be expected to continue, or are the result of random forces which can be expected not to continue.

The rest of the section will outline in a general way how to go about the problem. Lessons which follow will go into detail in regard to methods of finding and isolating the periodic components.

(a) The Separation of Growth and Random Components

If you can be sure no periodic elements are present in a time series, the separation of the growth and random components is quite easy.

With no periodicities present the growth factor can usually be determined with a good deal of accuracy by methods which will be discussed later. It is also relatively easy to estimate the way in which the random element and the growth element combined--whether by addition or by multiplication or in some other

way. To unscramble the two elements (growth and random) is then merely a matter of adjusting each item of the combined series for the corresponding item of the trend to get the random element as a residual.

Thus, if we take the combined series of controlled data in Table 1, and adjust by subtracting the known trend, we will get the random element as a remainder.

However, except in controlled data I know of no way to be sure that a periodic component is lacking. It now looks as if economic affairs, weather, population growth, the incidence of disease, crop yields and many other phenomena might be subject to periodic fluctuations of one kind or another. We will get into this matter more fully later.

(b) The Separation of Periodic and Random Components

Having determined the characteristics of the periodic elements it is very easy to remove the periodicity by subtraction or division. Such a calculation is merely the reverse of the process we used above (Table 2) to combine them. The random element is left.

(c) The Separation of Two or More Periodic Components

The process of separating two or more periodic components, like the two processes named above, is merely a matter of arithmetic once the characteristics of the periodic elements and the way they are combined has been discovered. In fact, if you have but two components, you need to know the characteristics of but one. When it is removed the other will be the remainder. Similarly, with three components you need to know but two, and so on.

(d) The Separation of Growth and Periodic Components

In separating growth and periodic components you can remove the growth component first, which will leave the periodic component as a remainder, or vice versa. The only trick is to find out the characteristics of the components and how they are combined. In the absence of evidence to the contrary you can assume they are combined by multiplication.

(e) Separation of Growth and Periodic and Random Components

This example is the one that is nearest to the behaviors of actual life.

In controlled data, knowing the growth and periodic factors, and how they are combined to form each value of the synthesis, you can subtract or divide each value of the synthesis by the corresponding values of the growth trend and the periodic elements to obtain the random component as a remainder (quotient). That is, the process is simply the reverse of the process of synthesis.

From the above comments it should be clear that the process of analysis is a very simple matter of elementary arithmetic, if you know the trend and/or the cycle involved.

The trouble is, in actual life, you do not know either the trend or the periodicities--you have to find them.

The rest of this course will be largely concerned in telling you how to find them, how to isolate them, and how to evaluate them.

6. The Concept of Rate of Change

When cycles are combined with other cycles and/or trend we often have the situation where the trend and/or the other cycles are more powerful than the cycle in which you are interested. In such a situation the cycle may express itself merely as alternation in the rate of change of the combined series. That is, the combined series will go up faster and less fast, or go down faster or less fast.

You will be given a problem to illustrate this fact.

If the trend and/or the other cycles are stronger than the cycle in which you are interested, your cycle simply cannot come through to a crest. It will express itself merely in rate of change.

Conversely, when you are looking for cycles in a curve (or a series of numbers) which increases or decreases rapidly, you do not look for crests and troughs or even areas of high and low, but for periods of more rapid rates of growth followed by periods of less rapid rates of growth.

* * * * *

After you have studied this lesson and worked the problems you will have a pretty good mastery of some of the basic concepts in regard to cycles. The next lesson will tell you how to get hints of cycles in series of figures which may or may not be influenced by periodic forces.

TABLE 1

A Series of Numbers Representing a
Growth Component (Trend), A Series
of Random Numbers, and their
Combination by Addition

Month num- ber	(A)	(B)	(C)	Month num- ber	(A)	(B)	(C)
	Trend	Random Numbers	Trend plus random (Col. A plus Col. B)		Trend	Random Numbers	Trend plus random (Col. A plus Col. B)
1	60	1	61	43	102	9	111
2	61	9	70	44	103	-11	92
3	62	15	77	45	104	33	137
4	63	-18	45	46	105	-10	95
5	64	-20	44	47	106	4	110
6	65	8	73	48	107	-25	82
7	66	-24	42	49	108	-2	106
8	67	-3	64	50	109	12	121
9	68	2	70	51	110	-5	105
10	69	-8	61	52	111	3	114
11	70	-58	12	53	112	5	117
12	71	25	96	54	113	4	117
13	72	66	138	55	114	14	128
14	73	8	81	56	115	-19	96
15	74	-17	57	57	116	13	129
16	75	6	81	58	117	-21	96
17	76	17	93	59	118	19	137
18	77	-19	58	60	119	-6	113
19	78	-1	77	61	120	-13	107
20	79	-5	74	62	121	12	133
21	80	-9	71	63	122	7	129
22	81	5	86	64	123	-27	96
23	82	-4	78	65	124	13	137
24	83	-9	74	66	125	57	182
25	84	-58	26	67	126	-12	114
26	85	12	97	68	127	10	137
27	86	-8	78	69	128	6	134
28	87	-42	45	70	129	-3	126
29	88	26	114	71	130	61	191
30	89	-12	77	72	131	-19	112
31	90	-52	38	73	132	68	200
32	91	14	105	74	133	16	149
33	92	-1	91	75	134	10	144
34	93	-8	85	76	135	-8	127
35	94	-21	73	77	136	3	139
36	95	11	106	78	137	19	156
37	96	-10	86	79	138	-4	134
38	97	15	112	80	139	-2	137
39	98	9	107	81	140	-9	131
40	99	30	129	82	141	14	155
41	100	12	112	83	142	-7	135
42	101	-6	95	84	143	-18	125

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TABLE 2

A Series of Random Numbers, Together with a Variety of 8-month Periodicities of Various Amplitudes, and Their Combination by Addition.

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)
Month number	Random numbers	An 8-month cycle with an ampli- tude of 4	Sum of random numbers and the 8-month cycle with ampli- tude of 4	An 8-month cycle with an ampli- tude of 20	Sum of random numbers and the 8-month cycle with an ampli- tude of 20 (Col. A plus Col. D)	An 8-month cycle with an ampli- tude of 40	Sum of random numbers and the 8-month cycle with ampli- tude of 40 (Col. A plus Col. F)	An 8-month cycle with an ampli- tude of 80	Sum of random numbers and the 8-month cycle with an ampli- tude of 80 (Col. A plus Col. B)
1	1	0	1	0	1	0	1	0	1
2	9	2	11	10	19	20	29	40	49
3	15	4	19	20	35	40	55	80	95
4	-18	2	-16	10	-8	20	2	40	22
5	-20	0	-20	0	-20	0	-20	0	-20
6	8	-2	6	-10	-2	-20	-12	-40	-32
7	-24	-4	-28	-20	-44	-40	-64	-80	-104
8	-3	-2	-5	-10	-13	-20	-23	-40	-43
9	2	0	2	0	2	0	2	0	2
10	-8	2	-6	10	2	20	12	40	32
11	-58	4	-54	20	-38	40	-18	80	22
12	25	2	27	10	35	20	45	40	65
13	66	0	66	0	66	0	66	0	66
14	8	-2	6	-10	-2	-20	-12	-40	-32
15	-17	-4	-21	-20	-37	-40	-57	-80	-97
16	6	-2	4	-10	-4	-20	-14	-40	-34
17	17	0	17	0	17	0	17	0	17
etc.									
82	14	2	16	10	24	20	34	40	54
83	-7	4	-3	20	13	40	33	80	73
84	-18	2	-16	10	-8	20	2	40	22

TABLE 3

The Combination of 3- and 10-month Periodicities and 3-,
10-, and 14-month Periodicities.

Month number	(A) 3-month cycle	(B) 10-month cycle	(C) Combination of 3-month plus 10-month cycles	(D) 14-month cycle	(E) Combination of 3-month, 10-month, and 14-month cycles
1	6	10	16	14	30
2	-3	6	3	10	13
3	-3	2	-1	6	5
4	<u>6</u>	-2	4	2	6
5	-3	-6	-9	-2	-11
6	<u>-3</u>	-10	-13	-6	-19
7	6	-6	0	-10	-10
8	-3	-2	-5	-14	-19
9	<u>-3</u>	2	-1	-10	-11
10	6	<u>6</u>	12	-6	6
11	-3	10	7	-2	5
12	<u>-3</u>	6	3	2	5
13	6	2	8	6	14
14	-3	-2	-5	<u>10</u>	5
15	<u>-3</u>	-6	-9	14	5
16	6	-10	-4	10	6
17	-3	-6	-9	6	-3
18	<u>-3</u>	-2	-5	2	-3
etc.					
82	6	6	12	2	14
83	-3	2	-1	6	5
84	-3	-2	-5	10	5

TABLE 4

A 4-month Cycle, a 6-month Cycle, and Their Combination by Addition

Base Month	(A) 4-mo. cycle	(B) 6-mo. cycle	(C) Combination by addition (Col. A plus Col. B)
Base Month	4	3	7
2nd "	0	1	1
3rd "	-4	-1	-5
4th "	<u>0</u>	-3	-3
5th "	4	-1	3
6th "	0	<u>1</u>	1
7th "	-4	3	-1
8th "	<u>0</u>	1	1

etc.

TABLE 5

A 3-Year Cycle, a 4-Year Cycle, and Their Combination by Addition

		(A) 3-year cycle	(B) 4-year cycle	(C) Combination by addition (Col. A plus Col. B)
Base Year		12	12	24
2nd	"	-4	0	-4
3rd	"	-4	-12	-16
4th	"	12	0	12
5th	"	-4	12	8
6th	"	-4	0	-4
7th	"	12	-12	0
8th	"	-4	0	-4
9th	"	-4	12	8
etc.				

TABLE 6

A 6-Year Cycle, an 8-Year Cycle, and Their Combination by Addition

		(A) 6-Year cycle	(B) 8-Year cycle	(C) Combination by addition (Col. A plus Col. B)
Base Year		-20	-20	-40
2nd	"	-6 $\frac{2}{3}$	-10	-16 $\frac{2}{3}$
3rd	"	6 $\frac{2}{3}$	0	6 $\frac{2}{3}$
4th	"	20	10	30
5th	"	6 $\frac{2}{3}$	20	26 $\frac{2}{3}$
6th	"	-6 $\frac{2}{3}$	10	3 $\frac{1}{3}$
7th	"	-20	0	-20
8th	"	-6 $\frac{2}{3}$	-10	-16 $\frac{2}{3}$
9th	"	6 $\frac{2}{3}$	-20	-13 $\frac{1}{3}$
etc.				

TABLE 7

The Combination of Trend and a Periodicity by Addition
and by Multiplication.

Month number	(A) Trend	(B) 8-month cycle expressed as plus and minus values	(C) Combina- tion of trend and cycle by addition (Col. A plus Col. B)	(D) 8-month cycle expressed as per- centages	(E) Combina- tion of trend and cycle by multipli- cation (Col. A times Col. D ÷ 100)
1	10	8	18	108.	10.80
2	11	4	15	104.	11.44
3	12	0	12	100.	12.00
4	13	-4	9	96.	12.48
5	14	-8	6	92.	12.88
6	15	-4	11	96.	14.40
7	16	0	16	100.	16.00
8	17	4	21	104.	17.68
9	18	8	26	108.	19.44
10	19	4	23	104.	19.76
11	20	0	20	100.	20.00
12	21	-4	17	96.	20.16
13	22	-8	14	92.	20.24
14	23	-4	19	96.	22.08
15	24	0	24	100.	24.00
16	25	4	29	104.	26.00
17	26	8	34	108.	28.08
etc.					
82	91	4	95	104.	94.64
83	92	0	92	100.	92.00
84	93	-4	88	96.	89.28

TABLE 8
Growth, Periodic, and Random Factors and Their
Combination

	(A)	(B)	(C)	(D)
Month number	Growth factor	Periodic factor (an 8-mo. cycle) expressed as per- centages	Random factor expressed as per- centages	Growth factor, times perio- dic factor, times random factor. (Col. A times Col. B times Col. C ÷ 10,000)
1	10	100.0	101.0	10.1
2	11	120.0	109.0	14.4
3	12	140.0	115.0	19.3
4	13	120.0	82.0	12.8
5	14	100.0	80.0	11.2
6	15	80.0	108.0	13.0
7	16	60.0	76.0	7.3
8	17	80.0	97.0	13.2
9	18	100.0	102.0	18.4
10	19	120.0	92.0	21.0
11	20	140.0	42.0	11.8
12	21	120.0	125.0	31.5
13	22	100.0	166.0	36.5
14	23	80.0	108.0	19.9
15	24	60.0	83.0	12.0
16	25	80.0	106.0	21.2
17	26	100.0	117.0	30.4
		etc.		
82	91	120.0	114.0	124.5
83	92	140.0	93.0	119.8
84	93	120.0	82.0	91.5

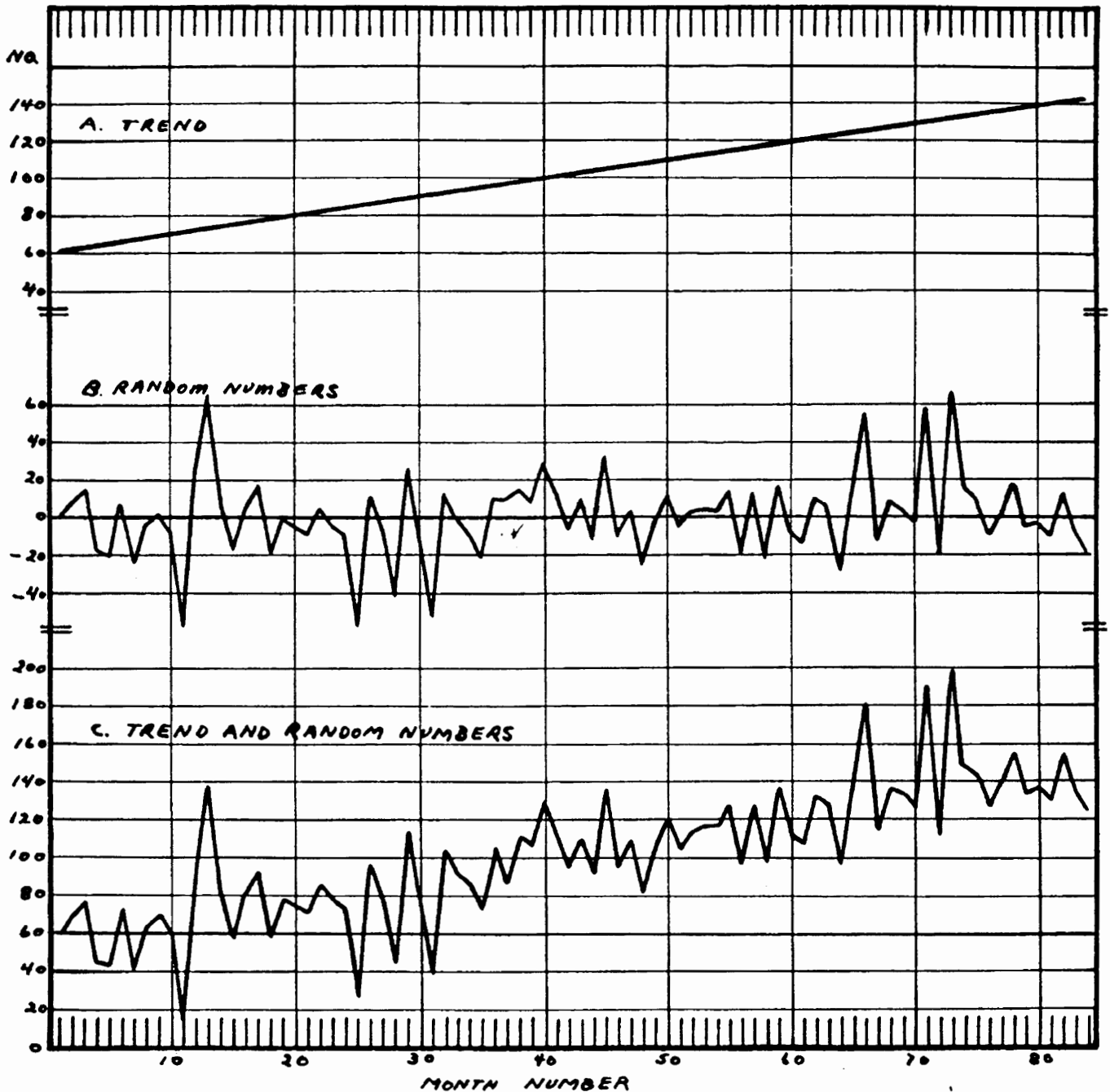


Fig. 1. Growth Component, Random Component, and Their Synthesis

Curve A Trend.
Curve B Random numbers.
Curve C Trend plus random (Col. A plus Col. B).

This figure also illustrates analysis or the breaking down of a curve into its parts. If we had started with Curve C and adjusted for the effect of trend (Curve A), we would have obtained the random elements (Curve B) as a residual.

Source: Table 1, Lesson V

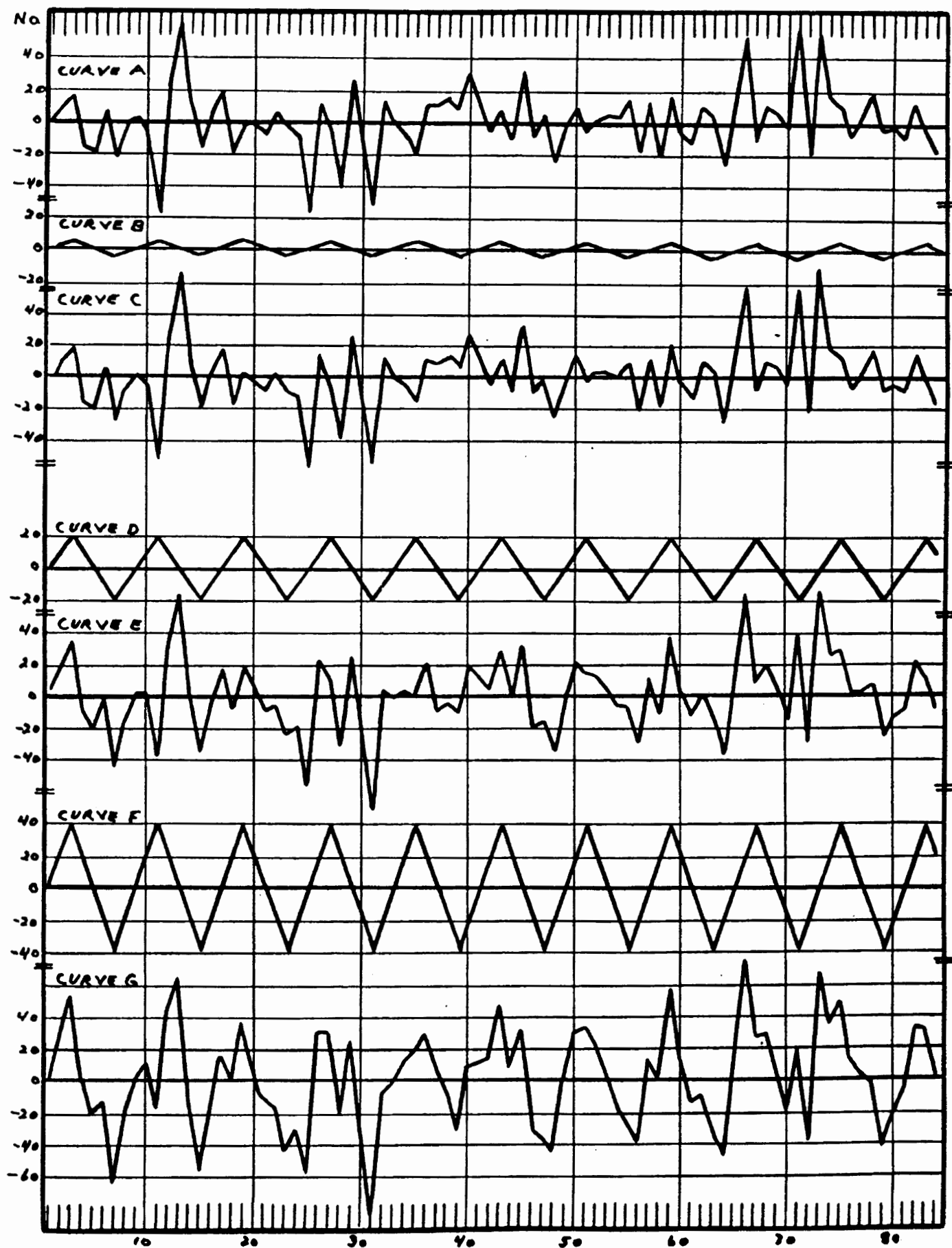


Fig. 2. Continued on Page 25.

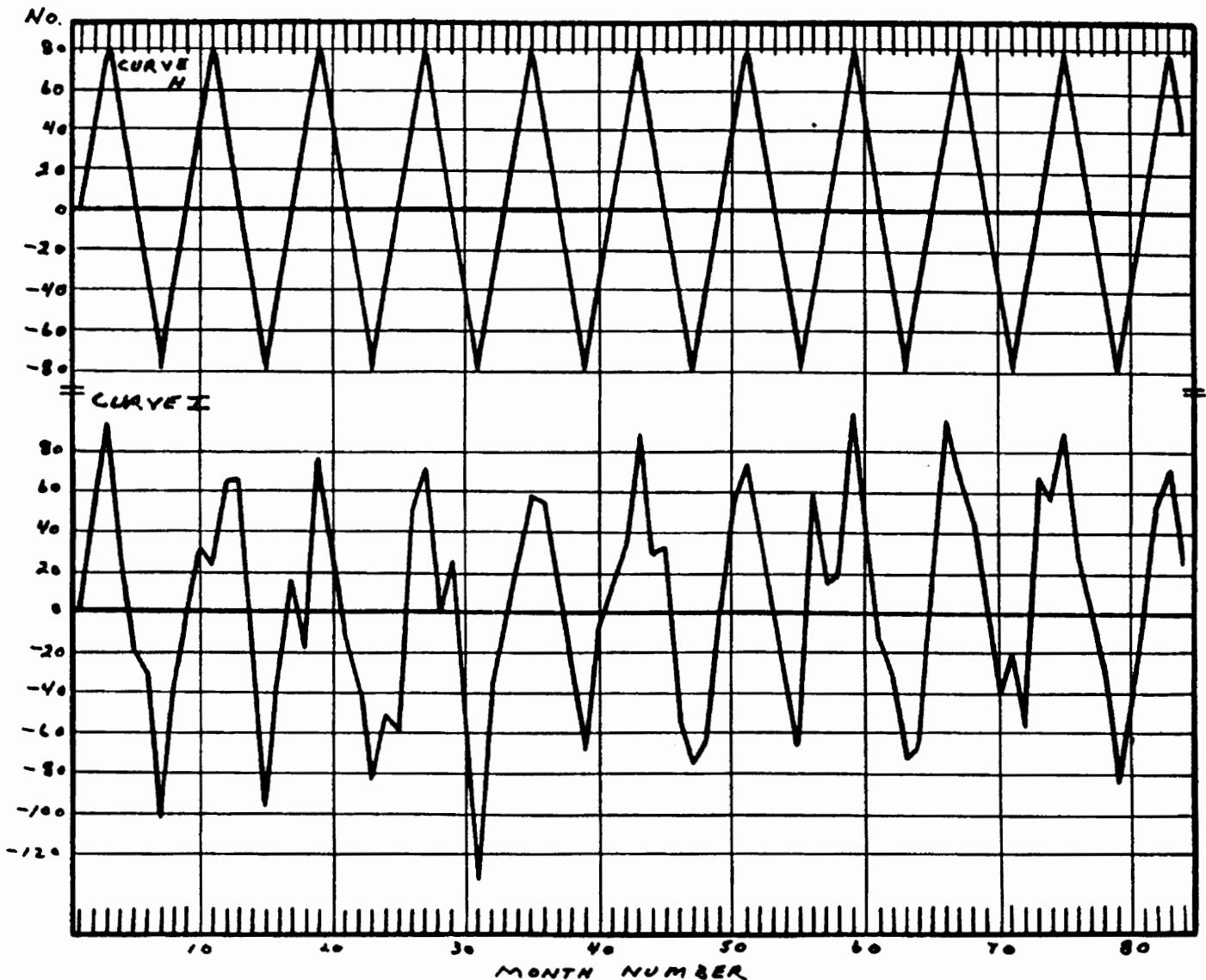


Fig. 2. A Series of Random Numbers, Together with a Variety of 8-Month Periodicities of Various Amplitudes, and Their Combination by Addition.

- Curve A** A Series of random numbers.
- Curve B** An 8-month cycle with an amplitude of 4.
- Curve C** The sum of the random numbers and the 8-month cycle with an amplitude of 4.
- Curve D** An 8-month cycle with an amplitude of 20.
- Curve E** The sum of the random numbers and the 8-month cycle with an amplitude of 20.
- Curve F** An 8-month cycle with an amplitude of 40.
- Curve G** The sum of the random numbers and the 8-month cycle with an amplitude of 40.
- Curve H** An 8-month cycle with an amplitude of 80.
- Curve I** The sum of the random numbers and the 8-month cycle with an amplitude of 80.

If we had started with any of the combined cycles and adjusted for the given cycle, we would obtain the random element, Curve A.



Fig. 3. The Combination of 3- and 10-month Periodicities and 3-, 10-, and 14-month Periodicities. The Combination Was Made by Addition.

- Curve A A 3-month cycle.
- Curve B A 10-month cycle.
- Curve C Combination of 3-month plus 10-month cycles.
- Curve D A 14-month cycle.
- Curve E Combination of 3-month, 10-month, and 14-month cycles.

This chart, like Figures 1 and 2, illustrates analysis as well as synthesis. If we had started with Curve E and subtracted Curve D we would have obtained Curve C. If, from Curve C, we had subtracted Curve B we would have obtained Curve A.

Source: Table 3, Lesson V

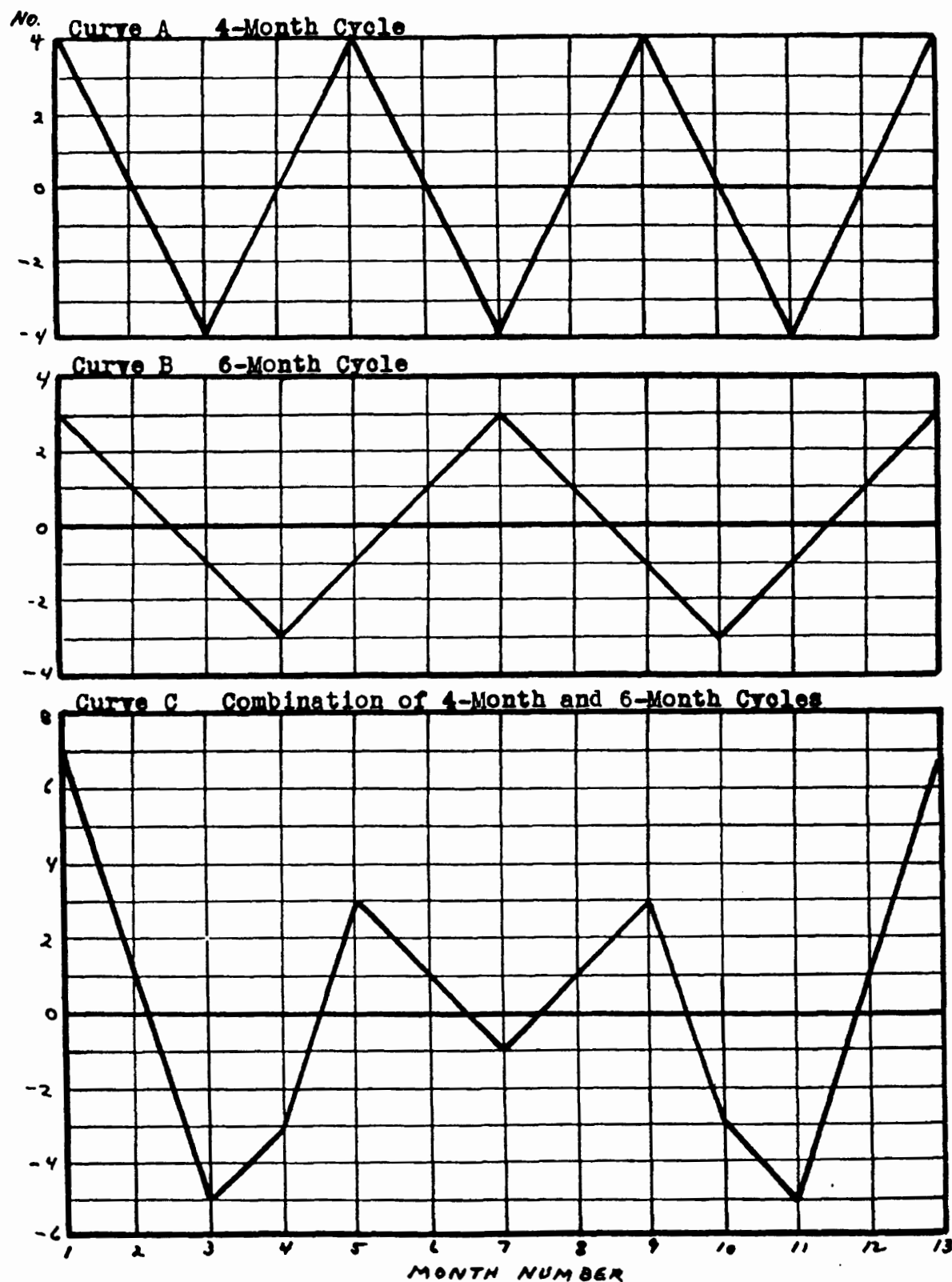


Fig. 4. A 4-Month Cycle, A 6-Month Cycle, and Their Synthesis by Addition.

The pattern will repeat at the end of 12 months.

Source: Table 4, Lesson V

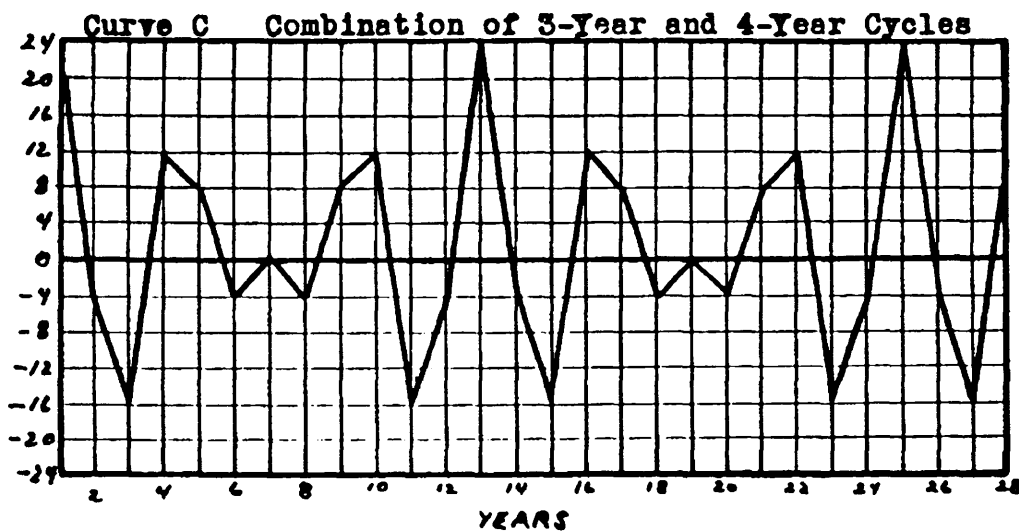
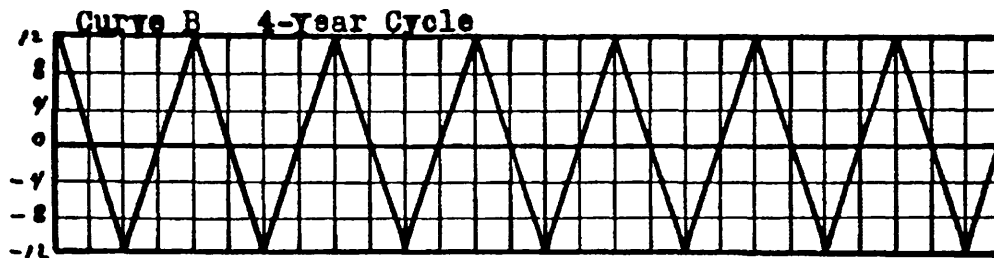
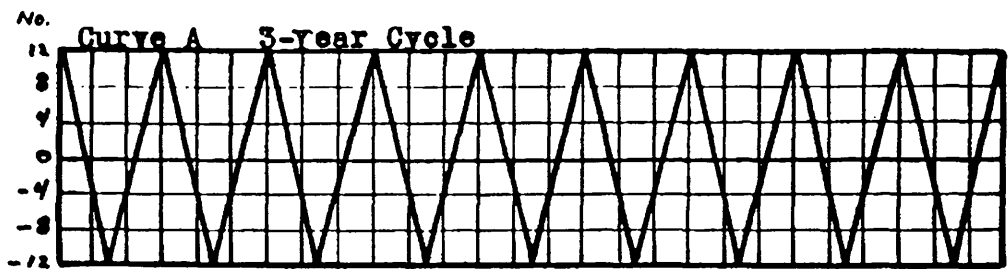


Fig. 5. Regular 3-year and 4-year Cycles and Their Combination.

The sum of the two repeats every 12 years. (After Warren and Pearson.)

Source: Table 5, Lesson V

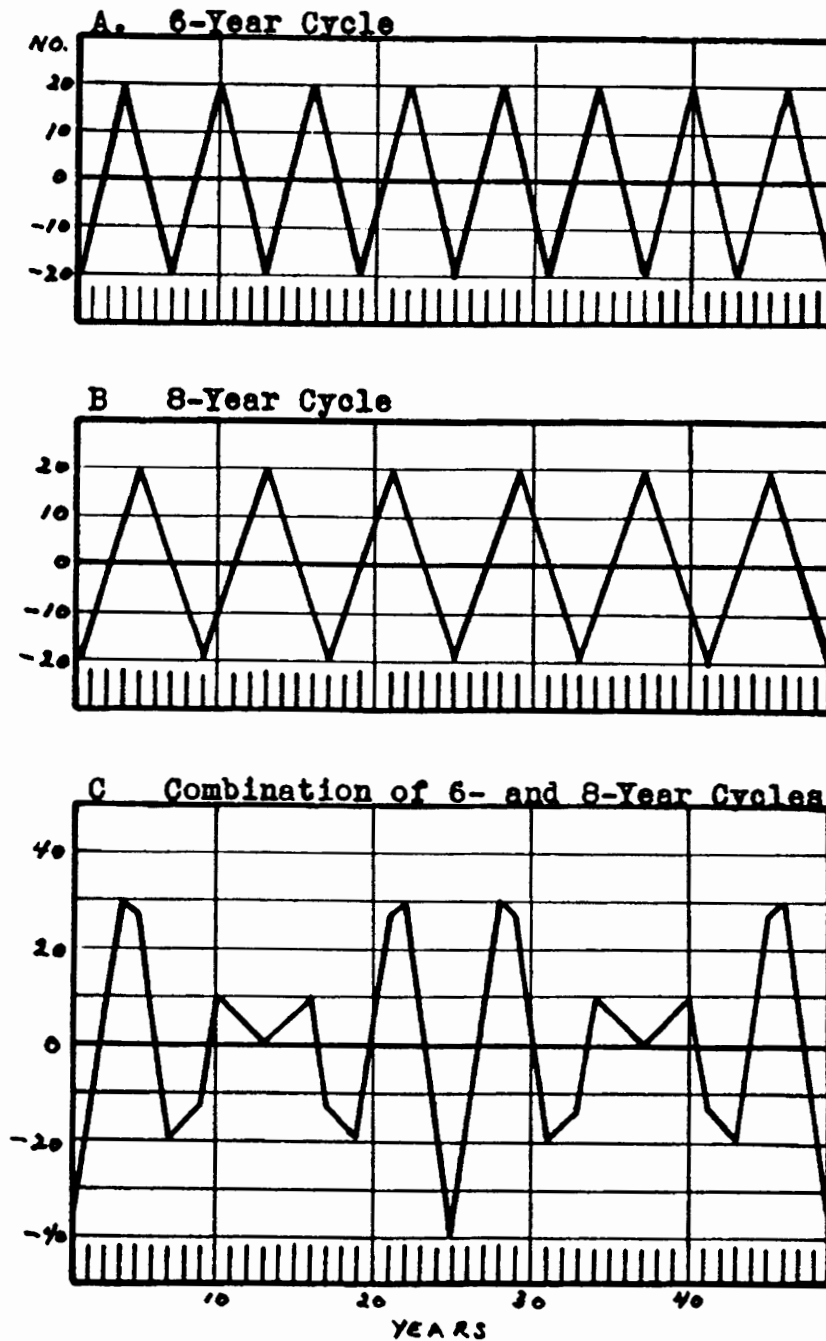


Fig. 6. Synthesis of Two Cycles

Regular 6-year and 8-year cycles and the two combined. The sum of the two repeats every 24 years. (After Warren and Pearson.)

Source: Table 6, Lesson V

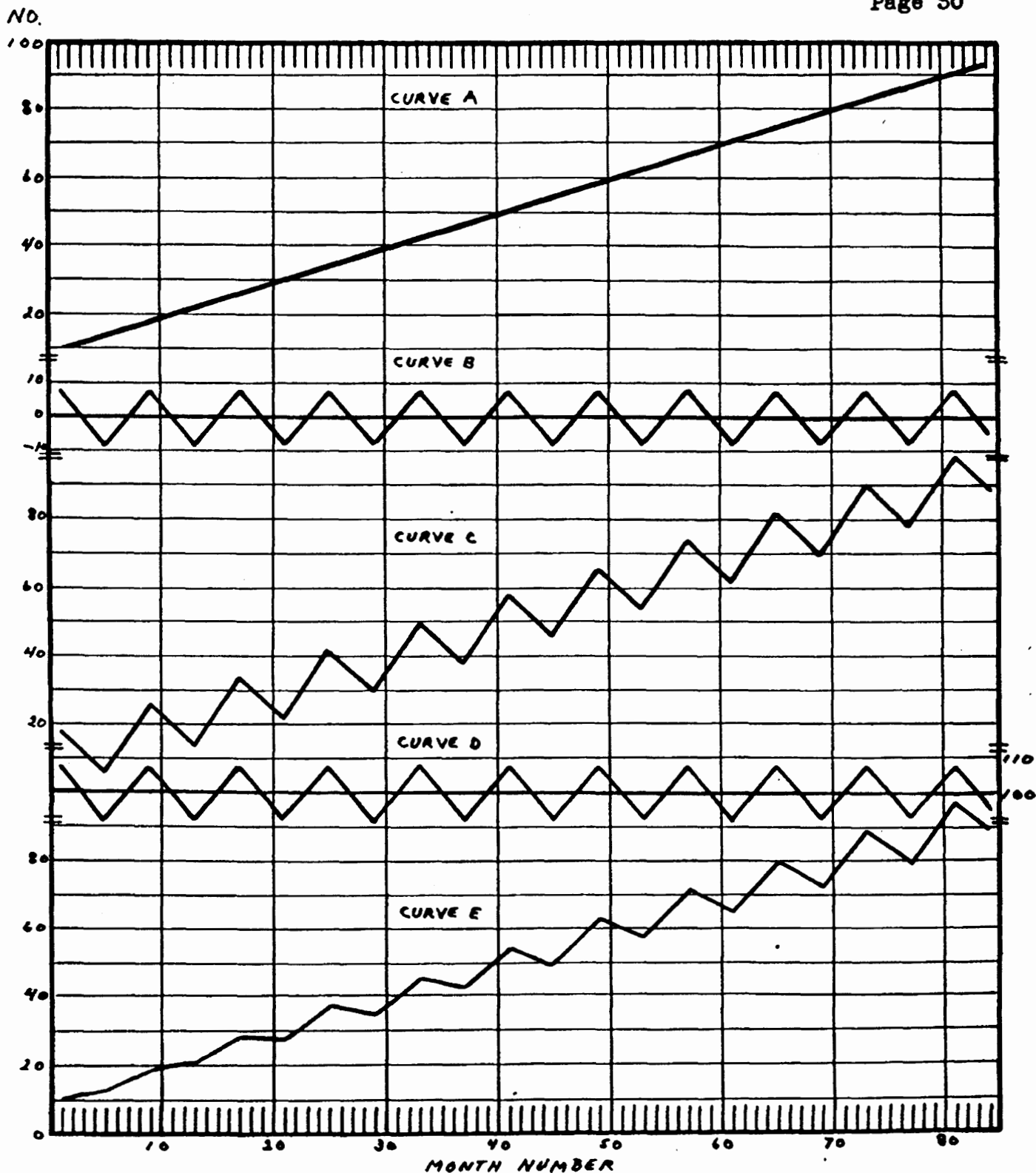


Fig. 7. The Combination of Trend and a Periodicity by Addition and by Multiplication.

- Curve A Trend.
- Curve B An 8-month cycle expressed as plus and minus values.
- Curve C The combination of trend and cycle by addition.
- Curve D An 8-month cycle expressed as percentages.
- Curve E The combination of trend and cycle by multiplication.

Source: Table 7, Lesson V

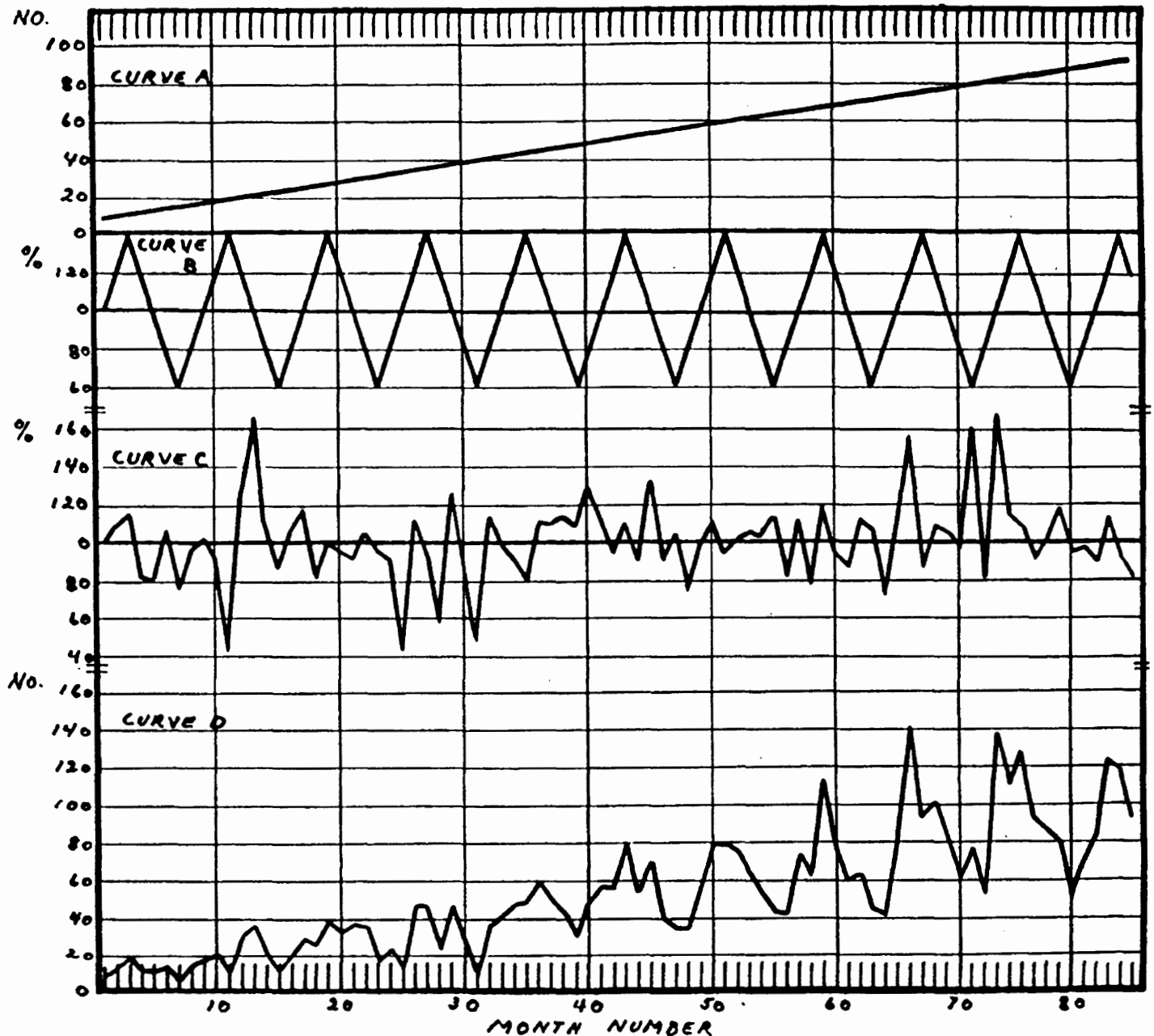


Fig. 8. Growth, Periodic, and Random Factors and Their Combination

- Curve A Growth Factor.
- Curve B Periodic factor (an 8-month cycle) expressed as percentages.
- Curve C Random factor expressed as percentages.
- Curve D Growth factor times periodic factor times random factor.

This chart, like the ones preceding, serves to illustrate the process of analysis as well as the process of synthesis. When you divide the values of Curve D by the corresponding values of Curve B and Curve A you obtain, as a residual, the random numbers in Curve C.

Source: Table B, Lesson V

Lesson V - Problems

Note: Please return these pages with the data filled in, along with the necessary charts.

Problem 1. In this problem logs of controlled data are used. Two cycles are given one four years long and one eight years long.

First: Combine the two cycles. Remember that cycles are combined by multiplication, which means that here you add the logs and subtract 2.000. Subtracting 2.000 is the same as dividing by 100.

Second: Assume that trend is 2.000 in the first year and increases by .0005 a year. Combine this trend with the two cycles. Space is provided below.

DATA					
Year Number	A 4-Year Cycle	B 8-Year Cycle	C Synthesis 4 and 8 Year Cycles	D Trend	E Synthesis Cycles and Trend
1	2.000	2.004	2.004		
2	2.004	2.008			
3	2.000	2.004			
4	1.996	2.000			
5	2.000	1.996			
6	2.004	1.992			
7	2.000	1.996			
8	1.996	2.000			1.9995
9	2.000	2.004			
10	2.004	2.008			
11	2.000	2.004			
12	1.996	2.000			
13	2.000	1.996			
14	2.004	1.992			
15	2.000	1.996			
16	1.996	2.000			
17	2.000	2.004			
18	2.004	2.008			
19	2.000	2.004			
20	1.996	2.000			
21	2.000	1.996			
22	2.004	1.992			
23	2.000	1.996			
24	1.996	2.000			
25	2.000	2.004			
26	2.004	2.008			
27	2.000	2.004			
28	1.996	2.000			
29	2.000	1.996			
30	2.004	1.992			
31	2.000	1.996			
32	1.996	2.000			

Problem 1 continued.

Third: Plot the data on an $8\frac{1}{2} \times 11$ sheet of arithmetic paper (enclosed). You will have Curves A, B, C, D, and E to match the columns of data. Be sure to use the same vertical scale for all the curves.

Question: In the 8-year cycle the top is _____% above the axis and the bottom is _____% below the axis.

If these data represent an index of production, the index in the highest year on record is _____.

Problem 2. These data are also controlled logs.

First: Complete the columns, performing any necessary computations.

DATA

Month Numbers	4-Month Cycle	6-Month Cycle	Synthesis 4- & 6- Month Cycles	5-Month Cycle	Synthesis 4- & 5- Month Cycles	Synthesis 4-, 5-, & 6- Month Cycles
1	2.000	1.995	_____	2.001	_____	1.996
2	2.004	1.998	_____	2.005	_____	_____
3	2.000	2.001	_____	2.001	_____	_____
4	1.996	2.004	_____	1.997	_____	_____
5	2.000	2.001	_____	1.997	_____	_____
6	2.004	1.998	2.002	_____	_____	_____
7	2.000	_____	_____	_____	_____	_____
8	1.996	_____	_____	_____	_____	_____
9	2.000	_____	_____	_____	_____	_____
10	2.004	_____	_____	_____	_____	_____
11	2.000	_____	_____	_____	_____	_____
12	1.996	_____	_____	_____	_____	_____
13	2.000	_____	_____	_____	_____	_____
14	2.004	_____	_____	_____	_____	_____
15	2.000	_____	_____	_____	_____	_____
16	1.996	_____	_____	_____	_____	_____
17	2.000	_____	_____	_____	_____	_____
18	2.004	_____	_____	_____	_____	_____
19	2.000	_____	_____	_____	_____	_____
20	1.996	_____	_____	_____	_____	_____
21	2.000	_____	_____	_____	_____	_____
22	2.004	_____	_____	_____	_____	_____
23	2.000	_____	_____	_____	_____	_____
24	1.996	_____	_____	_____	_____	_____

Second: Plot curves A, B, C, D, E, and F on an $8\frac{1}{2} \times 11$ sheet of arithmetic chart paper enclosed.

Question: The value of the bottom of the 5-month cycle is _____, and it occurs at the _____ month.

Problem 3. Following is a 4-week cycle and a series of random numbers. They are expressed as per cent of trend.

First: Combine the cycle and the random series.

Week Number	DATA (Per Cent of Trend)		C Synthesis: 4-Week Cycle & Random Numbers
	A 4-Week Cycle	B Random Numbers	
1	103%	110%	_____
2	100	105	_____
3	97	90	_____
4	100	103	_____
5	103	95	_____
6	100	110	_____
7	97	93	_____
8	100	93	_____
9	103	85	_____
10	100	93	_____
11	97	103	_____
12	100	103	_____
13	103	90	_____
14	100	93	_____
15	97	103	_____
16	100	110	_____
17	103	110	_____
18	100	110	_____
19	97	110	_____
20	100	105	_____
21	103	90	_____
22	100	110	_____
23	97	90	_____
24	100	110	_____

Second: Plot Curves A, B, and C on $8\frac{1}{2} \times 11$ paper (enclosed).

Question: Although the bottom of the cycle is expressed as 97% of trend, the actual bottom of cycles and randoms is _____ % of trend.

What happened to the cycle in Curve C?

Problem 4. There are two perfectly regular cycles to be combined. Data are in logs. The first cycle is 5 weeks long and has a value at top of 2.025 in the second week. The bottom has a value of 1.975, $2\frac{1}{2}$ weeks later.

Problem 4 continued.

It goes like this:

Week	Cycle Value
1	2.005
2	2.025
3	2.005
4	1.985
5	1.985

and repeat

The actual bottom of the cycle (value 1.975) falls half way between position 4 and 5.

The second cycle is 5.2 weeks long and has a value at top of 2.026 in the second week. The bottom of the second cycle has a value of 1.974. The bottom comes 2.6 weeks after the top.

You will observe that the 5.2-week cycle inches ahead by .2 week per cycle. In two cycles it will have inched ahead .4 week, in three cycles, .6 week, and so on. By the time it has gone 5 cycles it will have inched ahead a whole week. Five cycles are, therefore, 26 weeks long. The pattern then repeats.

It will go like this:

Week Cycle	Value	Week Cycle	Value	Week Cycle	Value	Week Cycle	Value	Week Cycle	Value
1	2.006	6	2.002	11	1.998	16	1.994	21	1.990
2	2.026	7	2.022	12	2.018	17	2.014	22	2.010
3	2.006	8	2.010	13	2.014	18	2.018	23	2.022
4	1.986	9	1.990	14	1.994	19	1.998	24	2.002
5	1.982	10	1.978	15	1.974	20	1.978	25	1.982
								26	1.986

First: Set the cycles up on tabulation paper (enclosed). The first column will be for week numbers. Column A is for the 5-week cycle, Column B the 5.2-week cycle and Column C the synthesis.

Continue until the combination has come around again to the position from which it started.

Second: Chart the two cycles and the combination on an 11x17 chart (paper enclosed). Allow one space per week on the chart, and continue the operation until the combination has come around again to the position from which it started.

Question: Does the above problem enable you to generalize regarding the length of time required for two (or more) cycles to come around again to the same phase relationship? If so, do so.

Problem 5. On Friday, September 10, 1954, Bethlehem Steel closed at $76 \frac{5}{8}$. On Monday, September 13th it closed at 79. Interpolate closing prices for September 11th and September 12th.

Problem 6. Combine the following:

A trend which starts at 2.000 and increases .003 per year, and

A 4 year cycle with an amplitude of $\frac{1}{1001}$ starting at crest. In other words cycle values will be 2.001, 2.000, 1.999, 2.000, and repeat.

Plot the result on arithmetic paper.

How long did it take you to study the lesson?

How long did it take you to work the problems?

Is the lesson material clear? If not, underline in red the phrases or sentences that were hard to understand.

Do you want more problems?

Do you want harder problems?

Please keep us posted in regard to all these matters with each lesson, so that we can regulate the course to suit your needs and desires.

LESSON V

Supplement 1

COMBINING CYCLES

Question:

I can't seem to figure out the length of time required for two or more cycles to come around to the same phase relationship. Can you help me?

Answer:

When cycles are combined and the cycles are even periods of time, the least common multiple of the lengths will be the time required for the combination to come around again to the place (position) of beginning.

Thus we have, on page 11 of Lesson V:

3 times 10 times 14, or
3 times (5 times 2) times (7 times 2), or
3 times 5 times 2 times 7 equals 210.

At the 211th month all the cycles will be at the point of beginning. This is the first time after month number one that this will happen.

210 divided by 3 equals 70)
" " " 10 equals 21) times each cycle will be used
" " " 14 equals 15)

Or, take another case - combine cycles 2, 6, and 18 months long. Obviously, it will take 18 months before the combination repeats.

Now, when you have combinations involving cycles of fractional length, the process is the same except that for the fractional cycle the factor is not the length of the cycle, but the time it takes for it to repeat itself. It takes 26 weeks for the 5.2-week cycle in problem 4 to get around again to the point of beginning. One of the factors is 26. The other factor is 5, the length of the other cycle. 26 times 5 equals 130, the number of weeks before the combination will repeat.

Take another case:

<u>Combine cycles</u>	<u>Cycle will repeat</u>	<u>Pattern will repeat</u>
5.2	in 26	
5.0	in 5	in 26 times 5 times 17.
3.4	in 17	or 2210

If this is not clear, please write again.

Supplementing this answer. one student writes:

Problem: Determine the secondary cycles that are generated when two regular cycles of different lengths are combined.

Rule When two signals of different frequency are combined two new frequencies
(from are generated:
elec- (a) a frequency equal to the sum of the two original signals
tronics): (b) a frequency equal to the difference of the two original signals.

Appli- (Note that a 4-month cycle has a frequency of $1/4$ cycle per month)
cation:

A 4-month cycle combined with a 6-month cycle will generate—

- (a) a cycle having the frequency of $1/4 + 1/6 = 5/12$
which means 5 cycles in 12 months or a 2.4-month cycle.
- (b) a cycle having the frequency of $1/4 - 1/6 = 1/12$
which means a 12-month cycle.

Similarly a combination of 4 and 5-month cycles will generate a 20-month cycle and a 2.2-month ($9/20$) cycle.

Combination of 5 and 6-month cycles will produce a 30-month cycle and a 2.7-month ($11/30$) cycle.

LESSON V

Supplement 2

REVERSING CYCLES

When two cycles of closely related wave lengths are present at one and the same time in the same series of figures they combine into what is known as a compound cycle or a reversing cycle. The reversing cycle has a false length midway between the lengths of the cycles which combined to create it.

As the two real cycles get closer and closer into phase with each other the amplitude of the combination increases until the cycles are exactly together, at which time it is at a maximum. The amplitude then decreases until, when the cycles are out of phase, if their amplitudes are equal, the cycles practically wash each other out. The cycles in combination then gradually reappear, but in reverse phase (upside down in relation to their previous phase (timing)).

Problem 4 of Lesson V asked you to combine 5.0- and 5.2-week cycles. As you may have noticed, the combination created a 5.1-week reversing cycle. That is, if you determine the cycle length of the combination over the first half of the series you created you will find it to be 5.1 weeks long. However, if you project a perfectly regular 5.1-week cycle onto the second half of your computation you will find that crests come where the projection calls for lows, and vice versa.

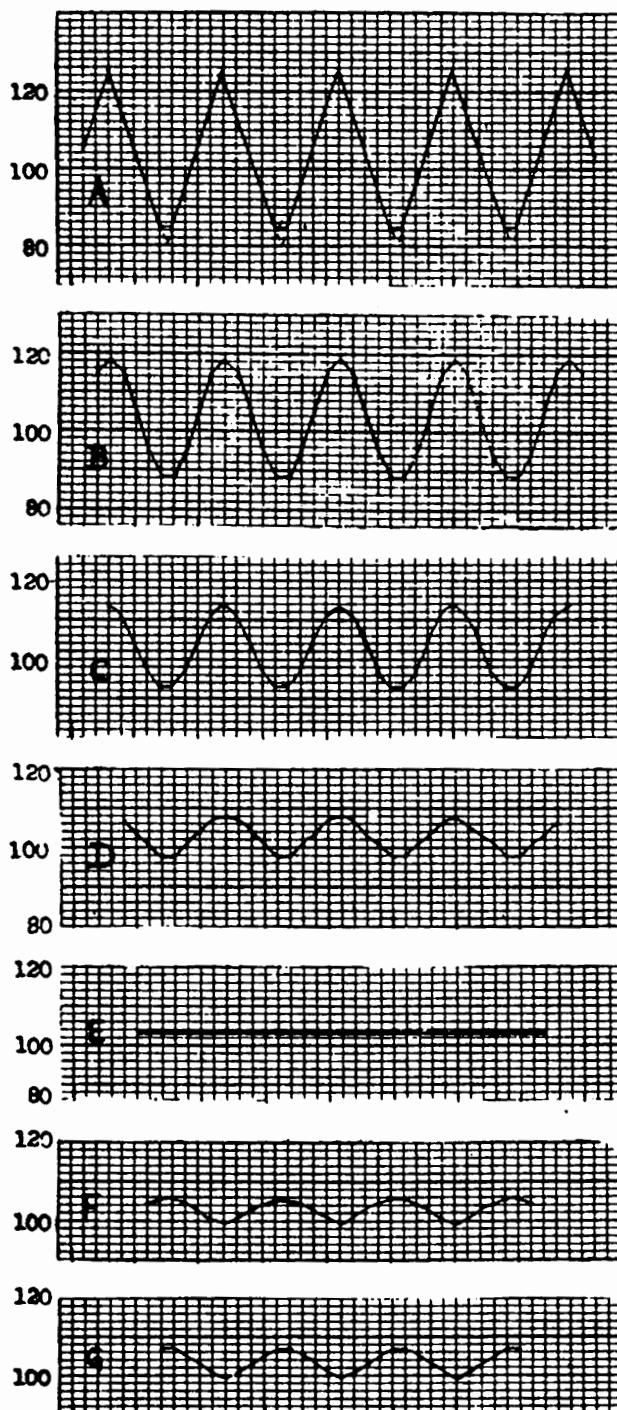
The time required for two cycles to come around to the same phase relationship is called the synodic period of the two cycles. Physicists call the intermittent strength resulting from the coming together of two cycles a beat.

When you find a reversing cycle you can determine the length of the cycles which may have created it as follows:

1. Count the waves from one place where they vanish (the node) to the next such place.
2. Multiply the number of waves obtained by step one by the false (apparent) length. This procedure will give you the time span from node to node, i. e. the synodic period.
3. Multiply the synodic period by two.
4. Divide twice the synodic period (as obtained by step 3) by twice the number of waves between nodes plus one. This computation will give you the length of one of the component cycles.
5. Divide twice the synodic period (as obtained by step 3) by twice the number of waves between nodes minus one. This computation will give you the length of the other component cycle.

Additional material in regard to reversing cycles, reprinted from Cycles of December 1950 is being sent you herewith.

Fig. 8



- A. A series of 9-year rectilinear waves
- B. Their 3-year moving average
- C. Their 5-year moving average
- D. Their 7-year moving average
- E. Their 9-year moving average
- F. Their 11-year moving average
- G. Their 13-year moving average

Note how, as the moving average gets longer the waves get flatter until, when the length of the moving average is the same as the length of the wave, they disappear. As the moving average gets longer still, the waves reappear in reverse phase (upside down).

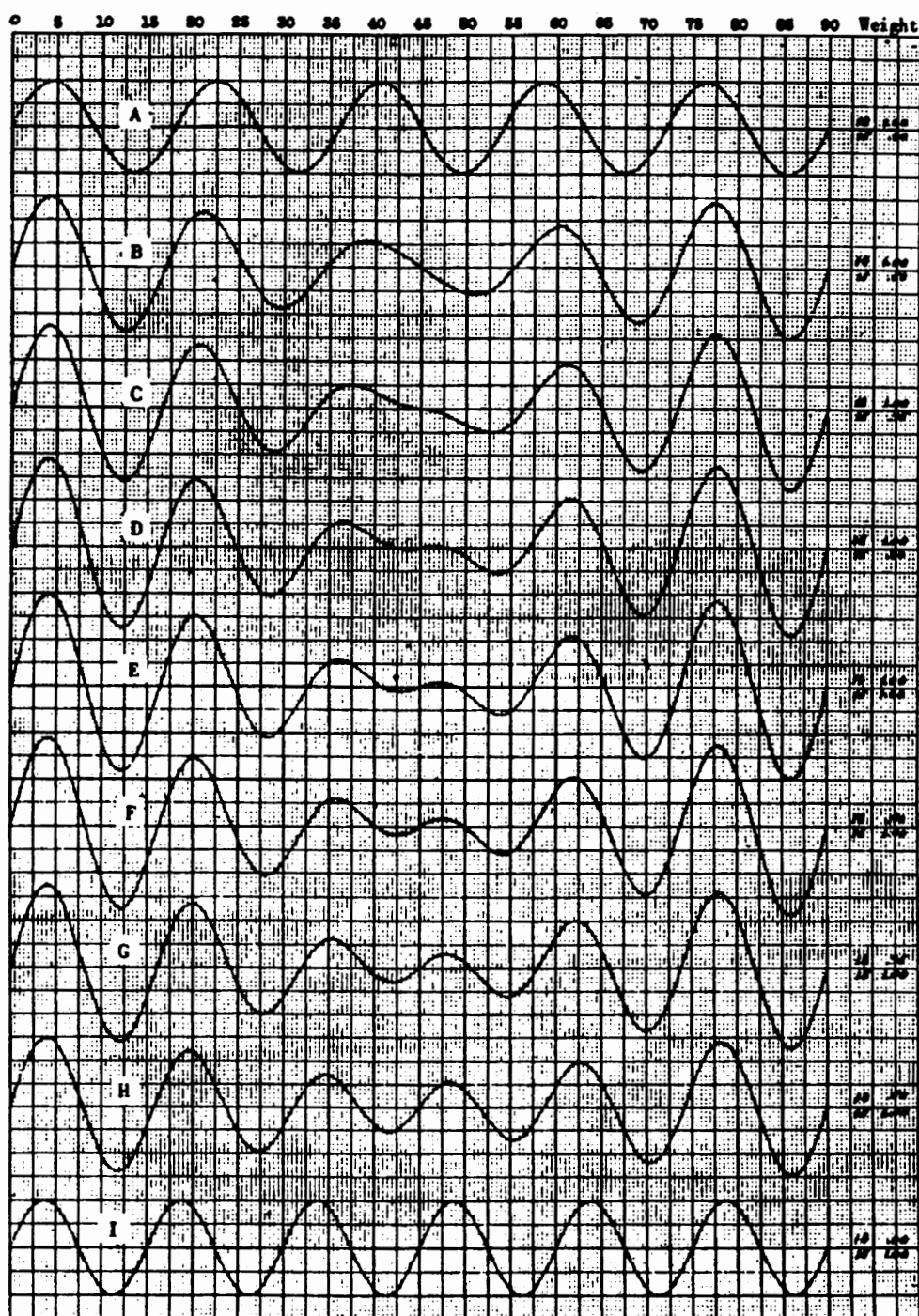


Fig. 1. Summation of 18-month and 15-month sine waves for their full synodic period of 90 months. Curve A shows the 18-month wave with an amplitude of 1.00 and the 15-month wave with an amplitude of .00—that is the 18-month wave all by itself. Curve I shows the 15-month wave all by itself. Curves B to H inclusive show various combinations of these two wave lengths with the respective amplitudes or weight as indicated in the right hand column.

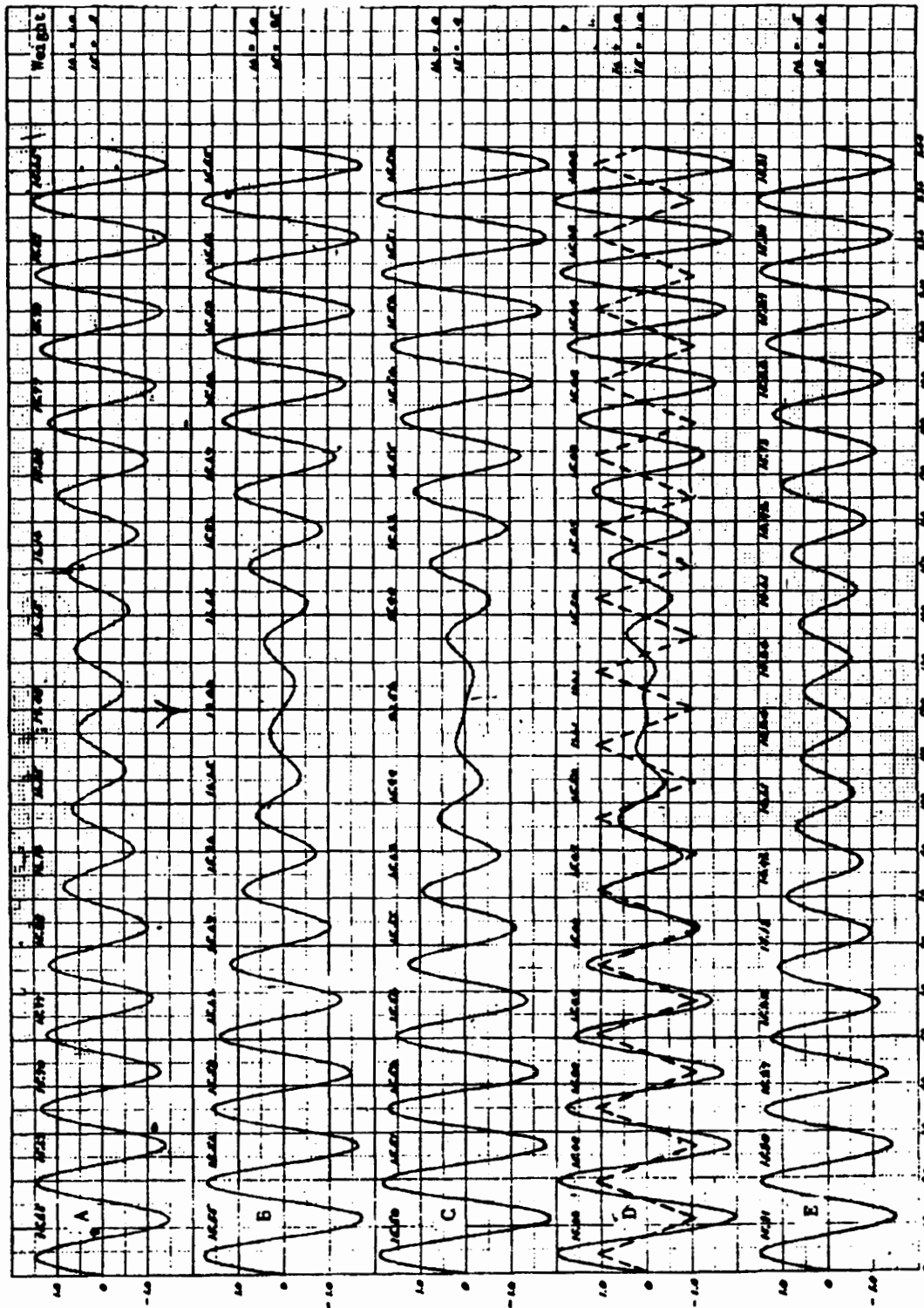


Fig. 2. Summation of 16-month and 15-month sine waves for their full synodic period of 240 months. The various curves show the results when the indicated weights and amplitudes are changed as indicated in the right hand column. The broken line in Curve D represents a perfectly regular 15.48-month cycle, as might have been erroneously assumed from the behavior of the first 110 months.

LESSON 7

Supplement 3

CYCLES IN COMMODITY FUTURES

Question:

How do I go about finding cycles in commodity futures?

Answer:

The simplest way to answer your question is to say at the beginning, "I do not know." I can, however, make some general observations:

Futures run for less than a year. If, in the early part of the year, you wish to have advance knowledge of the inherent cycles—assuming this to be possible—it is clear that you must base your knowledge on some other series of figures, for the current future provides no experience to go on.

Even for the latter part of the futures year one has had only 6 or 7 months of experience in that particular future. This length of time does not provide enough repetitions of any usable cycle to throw much, if any, light on the last few months of the futures behavior.

Of course, you could string a number of futures together into a chronological series (with gaps between the closing of one future and the opening of another). For example, you could plot May corn 1948, follow this with May corn 1949, follow this with May corn 1950 and so on, but such a series of figures would, it seems to me, have no meaning.

First of all the futures for each separate year would refer to a separate thing. The May 1948 futures consist of estimates made day by day from the middle of May 1947 to the middle of May 1948 of what corn will be selling at in May of 1948. The May 1949 futures consist of estimates, day by day, from May 1948 until May 1949 of what corn will be selling at in May of 1949. Hence, the series is not homogeneous.

Even if it were homogeneous the gaps would cause considerable trouble. For example, corn futures for May 1948 closed in May of '48 at \$2.35. May 1949 futures opened in May of 1948, a few days later, at \$1.65.

Another difficulty caused by the gaps is that the new future does not always open immediately after the closing of the old one. For example, corn for May 1954 closed (of course) in May 1954 but the corn future for May 1955 did not open until July 1954, a gap of two months. This might not make cycle analysis impossible but it would make it more difficult.

I think if I wanted to study futures I would plot (on log paper) all the different futures (May, June, July, etc.) for the commodity in which I was interested, one above the other, on a chart which also showed the daily spot prices. I would then study this chart to see if a future knowledge of spot prices would be any help to me in futures trading. If so, I would make my analysis of spot prices where homogeneous series of figures are available over a long period of time.

Incidentally, speaking of corn reminds me that there are cycles in corn prices. Samuel Benner discovered a $5\frac{1}{2}$ year cycle in these prices back in 1973 and it has kept on coming true ever since. As nearly as I can measure it, its length is 5.55 years. What other cycles there may be in corn prices I do not know.

Question:

Why are cycles not always equiamplitudinal?

Answers:

Because the axis isn't always in the middle! This isn't a smart Alec answer. Its straight goods.

It isn't the cycle that's off, it's the axis or trend. The question should really be, "Why doesn't the trend always bisect the cycle?"

There are many answers to that one, depending upon how you construct the trend. However, it is enough to say at this point, that, if you wish, you can always adjust the trend up or down so that the cycle will be equiamplitudinal.

LESSON V

Supplement 5

ABOUT PROBLEM 4

Comment: In actual practice there is only one reading per week for weekly data - this is the nature of the data.

In drawing a chart of a wave with tops and bottoms which come between the weekly readings, you can make a flat top, or you can draw in the assumed peak or trough - although you would never have a point to match in the data.

If you are starting with a curve which is a combination of two waves (the end of this problem is usually the beginning of an actual one) and are trying to break it down into its components, you assume that in some cases the peaks and troughs will come between the plots. You idealize the component cycles on this assumption.

Comment: When you are analyzing a time series you do not start with the ideal wave. You might start with a line similar to Curve C in problem 4.

By the nature of weekly data you would have only one value per week and that is all you could plot, and you would plot these only at weekly intervals.

But in breaking Curve C into its component cycles you could very well use a wave of fractional length which assumed tops and bottoms between the readings. In drawing a picture of such an ideal wave you would ordinarily show the top as a peak regardless of whether it fell at the precise weekly time or not.

LESSON VI

HOW TO GET HINTS OF CYCLES

HOW TO MAKE AND USE THE TIME CHART

A. HOW TO GET HINTS OF CYCLES

It is usually easy to find rhythm in a chart of a series of numbers, or even in the numbers themselves.

What to Look For

A study of the curves so far presented where we know a true periodicity is present--because we introduced it by construction--shows us what to look for when we examine a series of figures of actual events which may have been influenced by rhythmic forces--or may not.

Cycles ordinarily increase gradually over a period of time and then gradually decrease again. Ordinarily they are not sudden upsurges and downsurges.

Moreover, cycles are ordinarily pulled one way or the other and/or pointed up by random elements and by other cycles. Therefore, you cannot expect perfect regularity, or expect the cycle to prevail consistently. Turn back to Lesson V, Figs. 2, 3, 4, 5, 6, 8 for examples in controlled data of a true periodicity distorted by other periodicities and/or random numbers.

The thing to look for is the perfectly regular patterns (periodicities) to which the ups and downs of your time series most nearly conform. This is the concept for lack of which many cycle students have gone wrong.

To illustrate, suppose you have a time series with the following values for successive years: 0, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1, -1, -1, -1, -1, 0, 1, 0, -1, 0, 1, 0. This series is charted in Fig. 9. It has waves 4, 4, 8, and 4 years long, counting crest to crest, respectively. Counting crests we have five crests. This gives us four cycles, first crest to last crest. It is 20 years from the first crest to the last one ($4+4+8+4 = 20$). With four cycles this gives an average cycle length of five years. Fig. 10 shows the 5-year cycle in comparison with the original curve. Except at the two ends, a five-year cycle will fail to conform to anything. But a perfectly regular four-year cycle will conform exactly to three of the four cycles--with one complete miss. See Fig. 11. The given figures conform more nearly to a four-year pattern, in spite of the miss, than to a five year one.

Let me give you another example. Fig. 12 shows a street plan of the city of A----. Let us predict, on the basis of cycles, the probable spacing of the streets to the east.

It is perfectly obvious that the streets come, according to the scale, 200 feet (every half inch on the map) except at position 4 on the scale, where some big building on a public square interrupt the regular sequence. The typical

spacing is 200 feet, and the correct projection of probabilities is given by the broken lines in Fig. 13, which shows the streets to the east falling at 200 foot intervals. This is only rudimentary common sense.

However, I have seen cycle studies by noted scientists which take a series of events like those pictured in Fig. 12 and proclaim that the cycle length is the average length of the cycle. The average distance between streets in Fig. 12 would be six times 200 divided by 5, or 240. Establishing cycle length on such a basis is nonsense.

The thing you are looking for is the effect that would be created by a perfectly regular underlying periodic force, regularly increasing and regularly decreasing, obscured though it may be by random variation of one sort or another and by other cycles. As you can see by reference to the various Curves of Fig. 2 in Lesson V, such a force expresses itself in the form of areas of high and areas of low, more or less equally alternating. You do not look for high and low values as such, except as these give you hints of high and low areas, and the more or less adjacent turning points of the real underlying cycles which you are postulating.

In this connection, note particularly that when someone else has combined two numbers such as 2 and 6 to get 8, we have no way of knowing for sure just how the 8 was obtained. The 2 and the 6 have been welded so that we cannot see the joint. For all we know, seeing only the answer, the 8 might have been made up by adding 1 and 7, or $3\frac{1}{2}$ and $4\frac{1}{2}$, or $1\frac{1}{2}$ and -6, or any two numbers the sum of which equals 8.

When we come to analysis, therefore, and look at a curve like Curve I of Fig. 2 the most we can say is that it looks as if it had an 8-month component—it looks as if it were a series of random numbers, influenced by a regular 8-month periodic force, four months up, four months down. Curve I of Table 2 looks as a curve would look if that is the way it had been created. In this instance we know our supposition is correct, but in a series of unknown composition the observed regularity could have come about by accident. Not easily, perhaps, but it could have happened. The smaller degree of regularity found in Curve 6 could be found more easily by accident than the degree of regularity found in Curve I, and so on progressively (Curves E, and C) the smaller the amplitude of the periodic component, relative to the size of the random numbers (the length of the series, remaining the same), the more easily the resultant rhythm could have come about as a result of the random forces just happening to fall that way.

In all cases, therefore, except where we are dealing with controlled data, we have two problems: One, we must find the perfectly regular periodicity (or periodicities) which could most reasonably have caused the observed behavior, and second, we must come to some conclusion in regard to the probability that the observed regularity could have come about by chance.

In the first instance, the analyst is not concerned with whether the observed regularity is there by chance or not. He is merely trying to compare the observed behavior with perfectly regular periodic patterns to see the periodicity to which it most nearly conforms.

Here are some of the simplest of the methods that may be employed to do this:

1. Inspection
2. Counting intervals
3. Thumb and pencil
4. Graduated scale
5. The Time Chart

1. Inspection As a Means of Getting Hints of Cycle Length.

The easiest way to find rhythm in the ups and downs of a curve is by simple inspection. This method takes a little practice but in many instances, I find it adequate.

If you look at Curve I in Fig. 2 (Lesson V) it should be perfectly clear to you by inspection that the curve fluctuates in a rhythm about 8 months long.

The same cycle should be almost equally apparent in Curve G in the same figure.

The top Curve (A) of Fig. 14 of this lesson reproduced Curve G of Fig. 2 (Lesson V). Below it (B) is the same curve with a pencil line to show what the trained eye sees immediately.

It is sometimes easier to look for lows only, as in Fig. 14 Curve C, or for highs only as in Fig. 14 Curve D.

As this subject is so important let us look at more examples.

Fig. 15 shows Curve E of Fig. 2 (Lesson V) as it is and as the trained eye sees it, either from looking from low to low and from high to high. Hereafter, to save space, I shall show what the eye sees low to low or high to high. You should realize however that the cycle analyst will look at every curve from both points of view as in Fig. 14 and Fig. 15.

You can often see several rhythms in the same curve at the same time. Fig. 16 is a synthetic curve made up of a 9-year cycle and a 54-year cycle in combination. You can look at either cycle at your option, disregarding the other. The separation is very easy when one cycle is several times as long as the other.

When the component rhythms are of more nearly the same length, as in Curve A of Fig. 17, the separation is more difficult, but with a little practice it is not too hard, either. Curves B, C, and D in Fig. 17 show the three different rhythms which the trained eye sees easily in this curve.

Seeing different cycles in the same curve is something like focusing your eyes on different objects in the same view. Hold your hand in front of you and focus on it; the other side of the room is blurred. Focus on the window curtain; your hand becomes blurred. Focus on the view out the window; both the hand and curtain become blurred. It is the same when you look at one rhythm and put other rhythms out of your eyes, as it were.

Fig. 17 is interesting for another reason. It is made up purely of random numbers. The observed rhythms are perfectly real but they are not significant. They will not continue. They are however nearly as regular as the rhythm in Curve I in Fig. 2, and much more regular than the rhythm in Curve C in Fig. 2, where, in both instances, we know the rhythm is significant, because it is there by construction.

In Fig. 2 we know the rhythm is significant. In Fig. 17 we know it is not significant. How can you tell in the figures with which you are working, whether the cycle is significant or not? This subject will be discussed later. At the moment all we are trying to do is to learn how to get hints of cycles, significant or not. I introduced cycles in random numbers at this point to give you a sense of proportion in connection with this whole subject. When you get a hint of a cycle, you may have made an important scientific discovery, or you may merely be having fun with figures.

Some people find it helpful to supplement simple inspection by laying a number of pencils on the curve to accent the various areas of high (or low), and by moving the pencils back and forth until they accent areas of high (or low) that are reasonably equidistant from each other. The method has the advantage that the position of the pencil can be changed readily. I rarely use this method myself, but some cycle analysts find it helpful.

2. Counting Intervals Between Turning Points As a Means of Getting Hints of Cycle Lengths.

The simplest possible way in which to get a hint of the length of a rhythm that is present in a series of figures--if this length is not immediately obvious to you by inspection--is to count the intervals between turning points of a chart of the figures.

For preliminary purposes take every turning point. That is, in a series of monthly figures, if you are counting intervals between crests, count as a crest every month which is higher than the month before and the month after. If you are counting lows, count as a low every month which is lower than the month before or the month after. Record these values on the chart.

If you can discover any tendency toward regularity, such as 3, 3, 4, 3, 4, 3, 3, 3, 3, 4, 4, 3, 3, 3 months between turning points, you have a hint of a rhythm--in this case a little over 3 months long--which may be present in the numbers. Sometimes you can get a suggestion of a rhythm by combining intervals. To oversimplify what I mean, suppose you have the following series of intervals, 3, 4, 3, 4, 3, 2, 2, 3, 3, 4, 3, 4. It is obvious that by grouping the two 2's into one 4 we get a regular alternation of 3's and 4's, suggesting a $3\frac{1}{2}$ -month cycle.

Some people argue that we must take turning points as we find them. This is not so. Turning points of themselves have no significance. They are largely the result of random forces, or a shorter cycle, or longer cycles and/or trend which transforms cycles into variations in rate of change. Remember what we are trying to do. We are trying to find the perfectly regular repetitive pattern to which the various ups and downs of your curve--in the order of magnitude which you are investigating--most nearly conform.

The method of counting intervals from crest to crest (or trough to trough) cannot be used except to suggest short cycles, that is, cycles which are only a few units long. The method is of no use for finding 20- or 30- or 40-month cycles in monthly data. It is of use in helping to find 2-, 3-, 4-, 5-, or 6-month cycles in monthly data or 2-, 3-, 4-, 5-, or 6-year cycles in yearly data. I have rarely found it helpful in getting hints of cycles more than 12 units long.

Now, let us consider an actual example. Consider Fig. 18 Curve A. Counting crest to crest we get the values 3, 3, 4, 4, 2, 3, 4, 3, 3, 4, 2, 2, 3. These values have been posted to the chart and the terminal points of the various waves have been marked by a vertical line.

If you now group the two 2's into a 4 we get 3, 3, 4, 4, 2, 3, 4, 3, 3, 4, 4, 3, or very considerable regularity indeed. From the top of the first crest to the top of the last crest is 40 years. In those 40 years we have 12 rhythms. The average length of the cycle, from end to end, is $40/12 = 3\frac{1}{3}$ years. The figures will fit a perfectly regular $3\frac{1}{3}$ year periodicity with dramatic correspondence. (See Fig. 18 Curve B). But here again I have fed you a series of random numbers. Fig. 18 is identical with Curve A of Fig. 2, as you may have noticed.

As Fig. 18 is random the $3\frac{1}{3}$ "year" cycle, although real, is not significant and will not continue, as you can see for yourself by counting intervals between crests on the rest of the curve in Fig. 2.

3. "Thumbing" as a Means of Getting Hints of Cycle Length.

To help the eye to see whether or not successive crests (or tops) are reasonably equidistant, i.e., rhythmic, it is often convenient to measure off, by means of your thumbnail against a pencil, a fixed distance from the pencil point so as to get a first approximation of the length of the cycle. Fig. 19--shows Curve E of Fig. 2 repeated successively with the thumb on 9 years from the pencil point.

It is obvious that this length is too long. When the length is reduced to 8 years the successive positions of the cycle, as measured on the pencil, fit the curve much better.

Thumbing is my favorite method for preliminary reconnaissance.

4. The Graduated Scale as a Means of Getting Hints of Cycle Lengths.

Once simple inspection or other methods have indicated a cycle length the indicated length should be tested by means of a graduated scale.

A graduated scale is merely a strip of paper on the edge of which, with a pencil, you have made a series of marks at equal intervals. You make the marks as far apart as the wave you wish to test is long. Thus, to test for a $5\frac{1}{2}$ -month cycle you make your marks as far apart as $5\frac{1}{2}$ months would be on your chart paper. It is a good plan generally to make every 5th mark longer and to number the marks from 0 up.

The graduated scale should be as long as the chart to be tested, or a little longer.

The advantages of the graduated scale are (a) it provides a perfectly regular cycle with which to compare your actual behavior, (b) it can be slipped back and forth so as to compare turning points at every possible position, (c) it can be slid up and down so that you can compare either highs or lows, (d) it is easily and quickly made, (e) it is inexpensive.

Fig. 20 gives an example of a curve to which a graduated scale of the proper length has been applied to reveal first highs, and then lows.

5. The Time Chart as a Means of Getting Hints of Cycle Lengths.

The Time Chart is a diagram of highs and lows set up in such a way as to give you hints of rhythms that may be present in your time series. It is a most important adjunct for cycle analysis. It was devised by a business consultant of New York City and a pioneer in rhythm analysis.

The Time Chart depends for its usefulness upon an objective method for determining the "highs" and "lows" for any given wave length being sought. The Time Chart itself is then merely a method of diagramming these highs and lows in such a way as to throw light upon the length and amplitude of the rhythm or rhythms that may be present in the figures.

The Time Chart is important enough to rate a separate lesson if it were not such an integral part of the subject of how to get hints of cycles. As it is however, I shall handle it merely as a separate section of Lesson VI.

Summary: Remember that at this stage of things, you are merely trying to get hints of cycles. You start out by postulating the possibility of cycles of any and all lengths. You are trying to find the perfectly regular periodicity which fits the data best. At this point you do not need to care whether the best fit is good enough to mean anything or not.

For instance when I fitted 21, 32, and 58 item periodicities to Fig. 17 I was saying in effect, "Although you could fit periodicities of any length to this curve, you get a better fit for the lengths 21, 32, and 58 than for other lengths". This does not mean that cycles of other undiscovered lengths may not be present. This does not mean that cycles of the lengths you have found are significant. It merely means that of all possible cycle lengths, certain lengths fit the data best.

Let me restate: This is not the point in the analysis to judge whether you have discovered anything of value. You are merely seeking hints of cycles. You are merely trying to find the periodicities that fit the data best. Whether best is good enough is another story.

B. HOW TO HAVE AND USE THE TIME CHART

As I have said above, the time chart involves two processes, namely (1) an objective method of determining highs and lows, and (2) a technique, better than any other I know about, to reveal rhythms.

1. How to Determine Highs and Lows

Consider the Curve plotted in Fig. 21. Four 18-month waves are clearly evident. They have been diagrammed for you by means of a broken line.

You readily accept my statement that there are four waves even though there

are six crests, lettered A, B, C, D, E, and F. (A crest, for our present purposes, is defined as a value higher than the value immediately preceding or the value immediately following.) You say, and rightly, that C and D are so close to each other in time that they should count as one.

If two of the crests were as close to each other as the crests D and E, it would be utterly silly to say we had five waves of 14.4 ($72 \div 5$) months each just because we had six crests or turning points in the 72-month interval between A and F. There are crests and crests.

How can you tell which crests you should count in marking off waves, and which you should ignore? In a case as clear as that illustrated in Fig. 21 your common sense will tell you the answer, but not all cases are as clear. In difficult cases you might say one thing, I another. An objective method is needed.

An objective method is one which is impersonal. That is, it is a method by which you and I, working separately, can get the same results.

The objective method of determining the "highs" and "lows" can best be explained by an illustration. But first we must introduce a concept that will be new to many. Consider the Curve in Fig. 22. Each point on the curve has a value, depending upon its height above the base line. Thus the point for the year 1920 has a value of 850. But the point for the year 1920 has another characteristic. It is higher than any of the points of the three preceding years. That is, a horizontal line projected backward from the point at 1920 until it intersects the curve between years 1916 and 1917, as indicated by the fine dotted line, would skim over three points. Similarly, a horizontal line projected backward from the point at year 1922 would be lower than the ten preceding points--but not lower than the eleventh. Those numbers--the whole years or time units backward on a horizontal line that are in the clear before the curve is intersected, or the end of the series reached--three and ten in the examples given, I call the clearspan number of the point.

If the given point has a value greater than that of the point next preceding, the horizontal line skims above preceding points and the clearspan is indicated by a black number. If the given point is below the point next preceding, the horizontal line skims below the preceding points and the clearspan is indicated by a red number. When, as with the years 1902 and 1903, the values are higher than by any previous time, this fact is indicated by a \dagger sign after the clearspan, the plus to be read "or more". Thus the clearspan for 1902 would be $1\dagger$ (one or more). When the value of a point is exactly equal to the value of the point next preceding, the clearspan is 0, usually written in the opposite color from the clearspan number next preceding. Note that you count only the number of points that your imaginary horizontal line skims over or under. It cannot skim over or under a point of equal value. For example the clearspan number for 1925 is 8 because there are 8 years between it and 1916, a year of equal value.

The value and clearspan number of each point on the curve in Fig. 22 are given in the following table.

TABLE 9

Values and Clearspan Numbers of
Imaginary Data, Charted in Fig. 22

Year	Value	Clearspan Numbers	Year	Value	Clearspan Numbers
1900	--	--	1917	750	1
1901	100	--	1918	650	6
1902	400	1 1	1919	700	1
1903	800	2 1	1920	850	3
1904	700	1	1921	850	0
1905	600	2	1922	500	10
1906	500	3	1923	550	1
1907	600	1	1924	600	2
1908	400	5	1925	900	8
1909	300	7	1926	800	1
1910	200	8	1927	700	2
1911	400	2	1928	50	27+
1912	900	11 1	1929	200	1
1913	1,000	12 1	1930	250	2
1914	750	2	1931	200	1
1915	700	3	1932	100	3
1916	900	2			

The clearspan number can be obtained by counting the points that an imaginary backward horizontal projection would skim over or under until it intersects the curve, or passes beyond the first point of the curve.

The clearspan can be obtained more easily from the numerical data. This is done as follows: First notice if the value of the point for which the clearspan is being computed is over or under the value of the next preceding point. If it is above, the clearspan will be recorded in black. If below, in red. The numerical value of the clearspan will be the number of points back to a value as high or higher, less one, or as low or lower, less one, as the case may be. Thus the clearspan of the point at year 1920 is found by counting the years backward until one finds the first number with a value as great as or greater than 850, and subtracting 1. In this case, you must go back four years to 1916 to get a value as high or higher than 850. The number of the clearspan is therefore 4 less 1, or three, recorded in black. Again, the clearspan of the point at year 1922 is 11 less one, or 10, recorded in red.

If the points are numbered consecutively, as they are when the data are annual, you do not need to count--you can subtract. Thus, to get the clearspan of 1922 (value 500), you scan the value column (without counting) until you reach a value as low or lower than 500. You look for values as low or lower because the value for 1922 is below the value for 1921. Having found the value sought, you notice that it is at year 1911. Subtract 1911 from 1922; the difference is 11. Subtract 1 to get the clearspan of 10; record it in red. In practice, of course, you subtract the year after 1911, namely 1912, from 1922, and obtains 10 directly.)

In computing and recording clearspan numbers it is convenient to use a pencil with black lead in one end, red lead in the other. I use an Autopoint. It sells for 89¢.

If the data used are monthly, in order to make these computations practicable you must give consecutive numbers to all the months. Any consecutive numbers will do. One method uses December 1900 as month zero. This scheme makes December of 1901 month number 12, December of 1902 month number 24, etc. Thus December of any year is 12 times the number of years after 1900. For longer series, December 1900 can be taken as month number 1,000 or 3,000 and all other numbers raised accordingly. Standard month numbers from January 1901 to December 1957 are given on a separate sheet.

For the analysis of short term rhythms where weekly data are used, one system arbitrarily sets the week ending July 2, 1932, as week number zero. With this numbering plan, for example, the week ending December 31, 1949 would be week number 913, December 25, 1954, 1173. For series which go back prior to July 2, 1932 the week ending July 2, 1932 is called 4000.

For work with daily figures a daily number in which Saturday is seven times the week number could be used. Or you could use the Julian day number of the astronomers. In this system noon of August 15, 1954 is day number 243 5000.5.

We are now ready to proceed to a description of the objective method of determining highs and lows. "Highs" and "lows" can be defined as terminal points of cyclical rise and cyclical decline.

To meet "full standard" for a high or a low, a point must have a clearspan equal to at least one half of the length of the wave being sought, or the next smaller even number. Thus, for either 6- or 7-year waves, the clearspan of full standard highs or lows must be at least 3, black for the highs, red for the lows. The high is the highest point with the required clearspan between two lows of the required clearspan. Thus a high for 6- or 7-year waves is the highest point with a clearspan of black 3 or more between two lows with clearspan of red 3 or more; a low, the lowest point, with a clearspan of red 3 or more, between two highs with clearspan of black 3 or more.

For example, in the curve given in Fig. 22, if we are seeking 6- or 7-year waves, we could start at the beginning and go forward until we get a clearspan of 3 or more, either red or black. We find such a point at 1906. This year, with a clearspan of red 3, could mark (as far as clearspan is concerned) a full standard low for a 6-year wave, and is such a low, unless a lower value is present before the next clearspan of black 3. We then proceed to look for the next point with a clearspan of black 3 or more. We find such a point at 1912 with a clearspan of black 11½. The year with the lowest value between 1906 and 1912 is 1910 with a value of 200 and a clearspan of red 8. It is the low of the move.

We then go on from year 1912 until we find a point with a red clearspan of 3 or more, as at the year 1915. The highest value between 1910 and 1915 is at 1913 with a value of 1,000 and a clearspan of black 12½. This point is the high of the move. Similarly, year 1918 is a low, year 1920 is a high, year 1922 a low, year 1925 is a high, and year 1928 is a low.

Year 1903 is probably a high and year 1901 is possibly a low, and these facts may be indicated, but neither point has a known clearspan great enough fully to meet standards.

Suppose we were seeking 2- or 3-year waves. We would take as our full standard highs and the highest points with a clearspan of black 1 or more between points with clearspan of red 1 or more. By this rule, highs would be present at years '03, '07, '13, '16, '20, '25, and '30; lows at years '06, '10, '15, '18, '22, and '28. Lows are also perhaps at 1901 and 1932, but this cannot be known. (In practice one would hardly expect to find a 2-year wave through the use of annual figures, but we are concerned here with principles only.)

Thus, by this method, for any given length of rhythm, is it possible to make an objective determination of highs and lows. It is the only method I know of whereby this can be done.

2. The Technique for Revealing Rhythms.

Suppose we wish to use the time chart to scan the curve in Fig. 22 (the values of which are recorded in Table 9) for all possible wave lengths that would be revealed by this method. The longest wave that could repeat itself completely within the length of this series of figures would be half of 32 or 16 years in length. Full standard highs and lows for such a wave would have clearspans of black eight or more and red eight or more, and would be determined by the methods already described. From Fig. 22 or from Table 9 we see that highs of these standards occurred at 1913, and 1925, and that full standard lows come in 1910, 1922, and 1928. There is a possibility that 1903, exceeded as it is by only six values in the entire 32 years, is also a high, but you cannot know this without earlier figures.

These facts could be diagrammed on the grid shown in Fig. 23 using a black line for highs, a red line for lows. You can think of lows as being "in the red".

If high or low values come so near the beginning of the series that we do not know if they are full standard turning points, this fact is indicated by a parenthesis. In Fig. 22 the base year, 1900, is written in at the left and the lines are numbered to represent years after base.

Fig. 24 is the grid of Fig. 23 turned from sideways to up and down.

Another way to represent the time position of these highs and lows would be to cut the grid in two, as in Fig. 25 and place the second section beside the first. The numbers at the top of the column represent the base years, in this case 1900 and 1916; the numbers down the side represent the number of years after base. The lines in the first column represent the first sixteen years of the series, the lines in the second column represent years 16 to 32 inclusive. The last year represented in the first column is the base year of the second column.

The first red line comes at year 1910; the first black line at year 1913; the second red line comes at year 1922, or position 6 after base year of 1916. The next black line comes at year 1925, or 1916 plus 9; the last red line at year 1928, or 1916 plus 12. The crest at 1903 could be indicated by a black line with-in parentheses, (-----) to show that you do not know whether or not it is a full standard high.

Now, if there were a 16-year rhythm in this series of figures there would tend to be one high and one low in each column of 16 years; the highs would be about 16 years from each other and would therefore fall in about the same section of the grid. In other words, the black lines would tend to make a more or less horizontal line. The red lows would tend to do likewise. In Fig. 25 they do not do this. It is therefore obvious that in this series we do not have a 16-year rhythm.

But your time chart can be made to tell you more. As well as telling you of lengths that are absent it will tell you the approximate length of the rhythms that are present (if there are any).

Consider Fig. 26. Here I have extended the diagram of Fig. 25 by adding two additional 16-year sections based on 1884 and 1932 respectively. Let us now repeat in the 1884 section, three years after its end, the first black line "A" of 1903. Similarly, let us repeat the red line "B" of 1928 in the 1932 section, five years before its beginning. We can do this because any position in time can be plotted in any column if we extend the columns far enough one way or the other. We now have no more than one pair of highs and lows in each column, with all the lows on the same side (in this case above) all the highs next following. Let us connect highs and lows with vertical lines of their own color--black or red.

You see that the highs and the lows thus represented do line up after a sort, but not along a horizontal line as they would if the rhythms were 16 years long. To represent how they really fall, let us connect the midpoint of the first and last of the highs and the first and last of the lows by dotted lines. We note that for the red lines or lows, the dotted line slopes upward by 14 years in two cycles or 7 years ($14/2$) per cycle. This suggests a length of 16 years less 7 years or 9 years. The dotted line connecting the highs slopes upward 10 years in two cycles or five years ($10/2$) per cycle. This suggests a length of 11 years (16 years less 5 years). Averaging the two indicated lengths (9 and 11 years) we get a length of 10 years as an approximation.

Let us now make a 10-year time chart of the values (using a clearspan of five (V_5)). This is done in Fig. 27. As before, red lines "A" and "A" both represent an identical low point, in this case the low at 1922 (position 1910 plus 12 or position 1920 plus 2). This gives us the closest length that we can get by the time chart method for so few repetitions (if the series is long enough the time chart will indicate the length within 1% or less). Also it happens that a clearspan of 5 (V_5) instead of a clearspan of 8 (V_8) gives us no additional highs and lows in this instance.

Note that on the time chart there is a number above each of the lines and a number below each of the lines. The numbers above the lines give the value of the curve at the time indicated--i.e. the height above the base line; the num-

bers below the lines give the clearspan for the particular turning point at this position--written black or red, as the case may be.

This is how one would interpret such a chart:

There has been a tendency in the data plotted in Fig. 22 and recorded in Table 9 for highs to come at about ten year intervals and lows likewise. It is true that there have been but three repetitions of this wave, but there could not have been more repetitions of a wave of this length in a series as short as this. For the significance of such behavior we must resort to additional tests, but for what it may be worth we may observe directly from the time chart that no high of the 10-year cycle has ever had a value of less than 800. The most delayed high came only two years after median position. Also, insofar as we have the figures, no high has failed to be higher than all values for at least the preceding eight years. (All highs have a clearspan of at least 8.) The median high position is 3 years after base, and if this behavior continues, we may expect highs in the general neighborhood (perhaps two years one way or the other) of 1933, 1943, and 1953, etc.

As for the lows, no low has failed to have a value lower than any value in at least the eight preceding years. No low has ever had a value in excess of 500. The median behavior indicates a value of 200 as being closer to probabilities. No low has come more than two years before median timing, nor more than two years after such timing. (In longer series it is helpful to speak of the range within which the majority of the highs or lows have fallen.) If this behavior continues, lows may be expected two years one way or the other from 1940, 1950, and 1960.

Other methods can be used to refine these observations, to evaluate them, and to determine their significance.

Originally we started to look for whatever rhythm might be present at about half of our overall length of 32 years. We came up with a rhythm of about a third of our overall length. You might next look for a rhythm of a quarter of 32 years, or eight years, to see what you would find, then to a fifth of 32 years, a sixth of 32 years, and so on for as far as you wished. On the other hand, you might start by simple observation and use a time chart to see the regularity of some particular rhythm you thought might be present. Or you could start from a pre-conceived idea and explore the extent to which a nine-year or a $3\frac{1}{2}$ year or a six year wave might or might not be present. In any event, the time chart will be useful and will throw light on your problem.

Occasionally you find an extra pair of highs or lows within a cycle, as in Fig. 29. (Fig. 29 is a 4-year time chart of the curve plotted in Fig. 28.)

Often you fail to find highs and lows of full standard clearspan. In such cases you are compelled to fill in with sub-standard highs or lows. When this is done, this fact should be indicated by broken lines. If the clearspan of the point used for a high or low is less than full standard but over one-third of the proper clearspan the point is called a secondary high or low; it is indicated by a dashed line. If the clearspan is less than one third of the proper clearspan, but greater than one ninth, the high or low is called a tertiary high or low and it is indicated by a dotted line. The vertical connecting lines preceding secondary and tertiary turning points are also dashed or dotted respectively.

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chart. You can make a complete time chart analysis. You can start anywhere as a stab in the dark, and let the time chart itself lead you where it will. Or you can use inspection, thumbing, and the graduated scale for leads and use the time chart to help evaluate these leads. However, you do have to decide, one way or another, before you start, how long to make your time chart.

Having decided the length you must make your grid and most into it your highs and lows. These two steps are purely mechanical, but rate supplemental description.

Finally comes the matter of lining up the highs and lows. This step is subjective and is something of an art. It also rates a separate section.

How to Make the Grid of a Time Chart--Detailed Instructions.

- (a) Use unaccented quadrille paper, 5 lines to the inch each way.
 - (b) In setting up the grid, observe Standard Practice Instructions No. 7.
 - (c) Start your grid by putting a series of dots four spaces apart horizontally across the page, and two or three lines below the title. These dots will create a series of columns, and will serve to separate them.
- When a series of numbers is so long, relative to the length of the cycle that there are a great many repetitions of the cycle, you will sometimes want to put your dots 3 vertical lines apart, or 2, or even 1. However, best keep to the standard four lines in most instances.
- (d) In the intervals between the dots enter the base numbers of your cycles.

Use the third space from the left (i.e. between the 2nd & 3rd dots) for your first base number. The first base number is usually the number of the year or month before the beginning of your data. For example, if the curve of which you are making a time chart starts in 1858 your first base number would normally be 1857. However, you could use any base number you wanted, prior to the beginning of your series.

Between the next two dots to the right put your second base number. This number is one cycle length bigger than your first base number. If, for example you are making an 11 year time chart, and your first base number is 1857, your second base number would be 1868 ($1857 + 11$), your third 1879 ($1857 + (2 \times 11)$), and so on.

If the paper is not wide enough for all the cycles, paste on another piece.

Check the value of your last base number. Thus, if your first base number is 1857 and you wish to set up eleven 11-year cycles (10 cycles in addition to the first) your last base number would be $1857 + (10 \times 11)$ or 1967. Show by using a check mark that the value of your last base number has been checked.

If you are making a time chart of a fractional length, such as, 11.4-year, round your base numbers to the nearest whole number thus:

			<u>Round</u> <u>to</u>	<u>Interval</u>
1st base number	1857		1857	
2nd "	"	$1857 + 11.4 = 1868.4$	1868	11
3rd "	"	$1857 + 22.8 = 1879.8$	1880	12
4th "	"	$1857 + 34.2 = 1891.2$	1891	11

etc.

If your base number is a fractional length ending in .5, in time chart work always raise it when rounding (your text book to the contrary notwithstanding).

Each cycle of a time chart is normally an integral number of years (months, weeks, or whatever) long.

(e) Draw a horizontal line across the page about three inches from the top of a page. Draw it on one of the blue lines. Draw it in blue or green (or any color other than red or black). This line is your base line for all cycles.

(f) Enter your scale at the left, vertically, using the second column. (The first column is for finding.) The scale represents time units after base.

The scale will be as long as the cycle for which you are testing, i.e. it will equal the length or period of the time chart. Use one line for a year (month, week, or whatever units your data are in) if such a scale will cover no more than about a third of a page (perhaps 16 or 18 lines). If the number of terms in your cycle is greater than this, enter your scale by 2's or 5's, or 10's or whatever is necessary so that the entire cycle will occupy no more than about 1/3 of the page.

The scale will start with zero at the base line and run downward, for the length of the cycle.

If your time chart is of a fractional length, indicate this fact on the last entry of your scale. For example if you are making an 11.4-year time chart number the lines 0, 1, 2, 3, - - - -10, 11, 11.4 (for the 12th line). Do this as a reminder of the length of the cycle. However, the line so marked still represents 12 years after base.

(g) Now draw in the bottom of the grid. Use blue or green as for the top.

If your time chart is an integral number of years (months, weeks) long, the bottom of the grid is a straight horizontal line from the point on your left hand scale equal to the length of the time chart. That is, for an 11-year time chart your bottom line will be at 11 (the top line at 0).

If your time chart is of a fractional length the bottom line will be irregular. Thus, if your time chart is 11.5 years long, one cycle will be 12 years long, the next 11, the next 12, and so on. The bottom line of your grid will show this fact.

The bottom line of the grid always represents the same position in time as the base number of the next cycle. The area between top and bottom lines consti-

tutes your grid.

Your grid is now prepared. It has a position to represent each interval of time from the beginning of your data to the end, and a cycle or two into the future. The position five down on the scale, for example, represents five time units (years, or whatever) after each base year in turn.

Note that each column within the grid represents a particular section of time.

Note in addition that by going outside of the grid each column could also represent any other position in time; previous time above, subsequent time below. Two lines above the 0 of a certain column would represent 2 years before the base year of that column. Three lines after the bottom of the grid would represent three years after the end of the cycle--it would represent the same year as three lines (years) down in the next cycle--the next column.

And note finally that in time charts of fractional length the columns are of unequal length. The length of any column is always the same as the interval between the base number of that column and the base number of the column next to the right.

2. How to Enter Highs and Lows into the Time Chart

This process, like the making of the grid, is purely mechanical.

Indicate all full standard highs and lows on this grid by red or black horizontal lines, hereafter called bars, (each bar 4 blue blocks long) in the proper column and at the proper position.

That is, if you have a high at year 1911, you find the column into which 1911 belongs--i.e. between what base numbers it belongs. Then you find the number of years which 1911 is after the base year, and draw your bar on the blue line representing that year.

Thus if, in a 13-year time chart one base number is 1905 and another is 1918, you would draw your black bar representing 1911 at blue line 6, in column headed 1905, because 1911 is 6 years after 1905.

If the column were headed 1911 (had a base number of 1911) you would put your bar on the line called 0.

Now enter the value of the curve (in black) above the left hand part of the bar. Enter clearspan number (in proper color) below the right hand part of the bar.

Presumptive full standard highs & lows at either end of the series are also entered, but within parentheses, to show that you do not know whether they are full standard or not.

At this point in the process of making a time chart there are no bars outside of the grid, no broken or dotted bars within the grid, and no vertical red or black lines connecting the bars.

3. How to Connect Highs & Lows

Before discussing how to connect highs & lows, let's remind ourselves of a few fundamentals. A truly periodic curve of the same length as the time chart would ideally express itself as two horizontal lines across the chart, one red, one black. They would be connected because the bars would abut.

A truly periodic curve shorter than the length of the time chart would show series of bars which would rise toward the right. If you connected successive blacks & successive reds you would get a flight of steps rising from left to right.

A perfectly regular cycle longer than the length of the periodic table, would fall and connected, would show, left to right, a descending series of steps.

In any of the three instances given, the addition of random numbers will serve to distort the smoothness of the straight line or the flights of steps. If the distortion is bad enough it might even change full standard highs and/or lows into secondary or tertiary turning points.

Conversely, if, in an actual series, where the presence or absence of a cycle is not known, the highs and lows do tend to oscillate around horizontal lines, we have the suggestion that a cycle the length of the time chart may exist. If the highs and lows each tend to oscillate around straight lines which slope upward, the suggestion is for a shorter cycle. If the highs and lows each tend to oscillate around straight lines which slope downward, the suggestion is for a longer cycle.

Therefore, connecting the consecutive highs to each other (and connecting lows similarly) is merely an attempt to help the eye to see whether or not the turning points do or do not tend to oscillate around straight lines and, if so, whether these lines are horizontal, or slope up, or slope down, and if so, by how much.

Here is the procedure: (See Fig. 30)

(a) Connect to each other by solid black vertical lines all consecutive black bars which are not more than one vertical space apart. Do the same with solid red vertical lines for all consecutive red bars not more than one vertical space apart.

Never let lines of red cross lines of black.

This step may help you to see if the bars oscillate around a horizontal or sloping axis, and, if the axis slope, whether they slope upward or downward. It may also help you to see whether reds tend to be above blacks or vice versa.

(b) If Step (a) does not do either of these things, connect consecutive bars of the same color which are two vertical spaces apart.

(c) If Step (b) shows nothing, connect bars three spaces apart, etc.

(d) When you have an idea as to whether reds or blacks are predominantly on top, the bars which are out of place must be "moved." For example, suppose you decide that it will be easier to keep blacks on top. Suppose your first cycle has

black on top and red below, your second the same, your third has only a black, and your fourth has three bars, red, black, red. You would move the top red bar of your fourth cycle back into its proper place in the third cycle, (if, for example, it had been two spaces into the fourth cycle it would be put two spaces after the bottom of the third cycle, etc.). Having made this move you would have a black and a red in each cycle.

"Moving" a bar really means repeating it. You do not erase the bar from its original position. Thus after "moving" a bar you have two bars, each representing the same point in time.

(e) You then connect the bars you have moved to the adjacent bars of the same color.

When the duplicate bar outside the grid has been connected, the original bar inside the grid should have corresponding vertical lines run to the edge of the grid

(f) Any blanks which exist must be filled either by rearrangement of bars, or by adding a pair of highs and lows, one or both of which are secondary or tertiary. Do whichever tends to make the bars oscillate around the straighter line.

If connecting secondary bars, use a broken line preceding the bar. In connecting tertiary bars use a dotted line preceding the bar.

When you have to use secondary or tertiary bars you will often find the other bar of the pair to have full standards. If so it, of course, should be connected and inserted by means of a solid bar.

(g) If you have extra pairs of bars they must be discarded. That is, they are not connected.

(h) As a result of the foregoing steps you now have a red line and a black line going continuously across the page. Place a ruler, edge down, over each and note the degree to which it conforms to the straight line of the ruler. See if any other arrangement can be made which would conform better (straighter and/or with less variation) to some other straight line.

(i) Now fit straight lines to the red bars and to the black bars. These lines can be fitted (i) by eye, (ii) by means of a transparent ruler laid flat on the curve, (iii) by the method of semi-averages (described in Spurr, pages 324-25), and (iiii) by the method of least squares (Spurr, pages 334-40).

The method of least squares provides ⁵spacious accuracy and is not recommended except in exceptional cases. I generally use the method of semi-averages checked for common sense by means of a transparent ruler.

(j) Having found the straight lines to which the bars most closely conform, these lines should be drawn in with a sharp pencil.

(k) For now compute for each of your lines the length which it indicates.

To do this you find how far your line moves upward or downward per ~~cycle~~ and
column

subtract or add this number to the length of the time chart. Thus if you are working with an 11-year time chart and if measured from the middle of one cycle to the middle of 10 cycles later your line goes upward by 3 years, the length of the cycle is $11 - 3/10$.

In making this computation with time charts of fractional length compare cycles of the same length. For example, if you are working with an $11\frac{1}{2}$ year time chart choose cycles for measurement which are 11 years long or which are 12 years long. Do not have one of them 11 years and another 12 years.

(l) When the cycle lengths represented by the red lines and the black lines have been computed, average them to get the best approximation of the length of the cycle obtainable at this stage of the analysis.

(m) If the length indicated by the sloping line is importantly different from the length of the time chart, make a new time chart of the length indicated by the sloping lines, and proceed to re-determine the length.

(n) Search the time chart for indications of other lengths. For example, on 11 year time chart in addition to showing an $11\frac{1}{2}$ year cycle might also show an 8-year cycle and a 15-year cycle simultaneously. Draw in lines to represent these other cycles and note the results of your observations on the time chart itself.

#

Finally, a note of warning: Simple as its elements are, the time chart must be used with care, or unjustified conclusions will be drawn. It must be remembered that after all, the time chart is merely a tool for obtaining hints of rhythmic behavior. It does not reveal whether the indicated rhythms, if any, are significant or are merely the result of random forces. This determination must be estimated by other means.

The time chart and its adjunct, the objective method of determining highs and lows, are very valuable tools. I would like to give credit to their inventor, but I have reason to believe he would rather not be mentioned by name. All credit to him, anyway!

In the next lesson we will get into the matter of periodic tables.

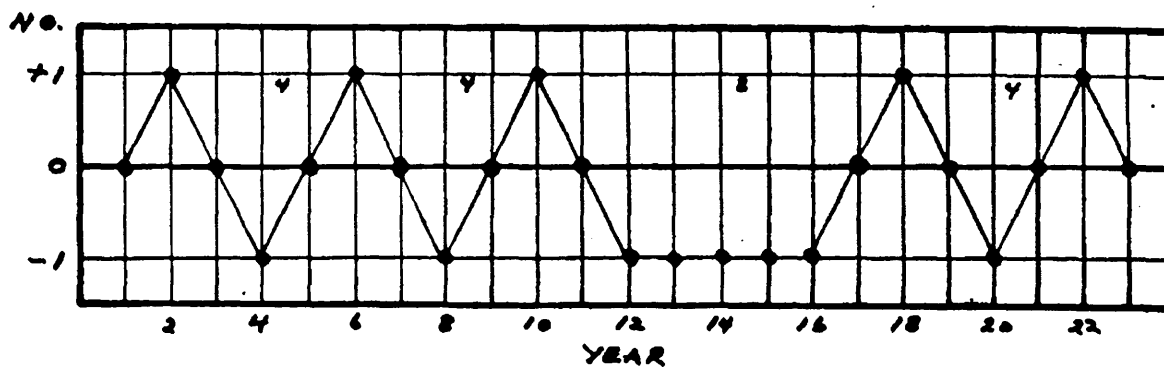


Fig. 9. Cycle of Unknown Length - Showing the Years Counted from Crest to Crest

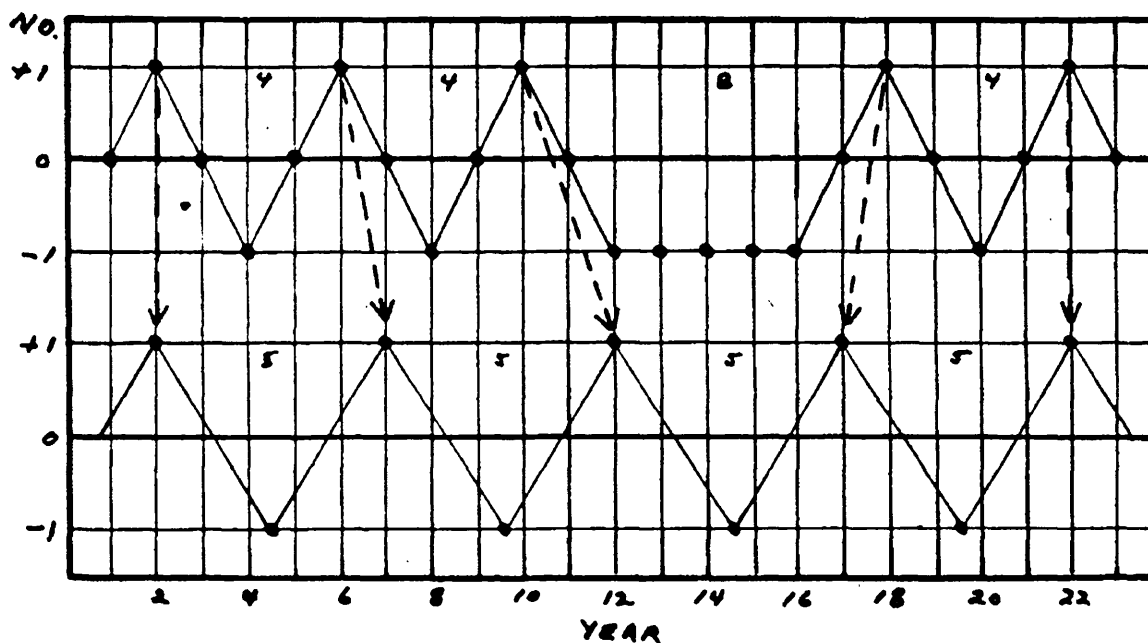


Fig. 10 - Cycle of Unknown Length (from Fig. 9) Compared to Regular 5-Year Pattern

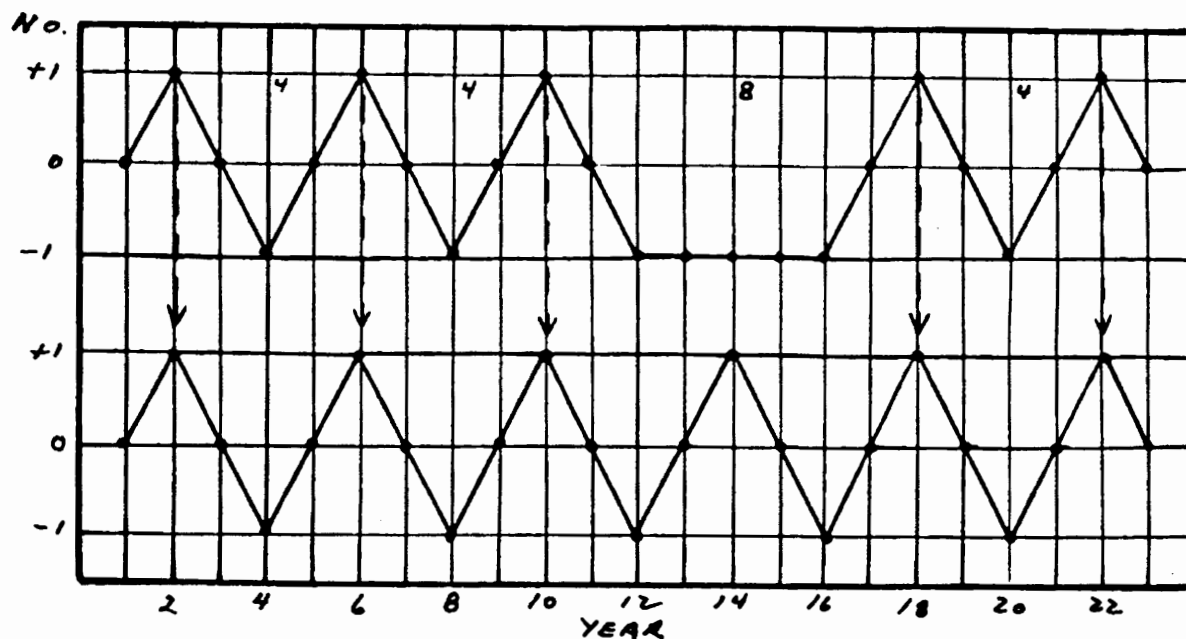


Fig. 11. Cycle of Unknown Length (from Fig. 9) Compared to Regular 4-Year Pattern

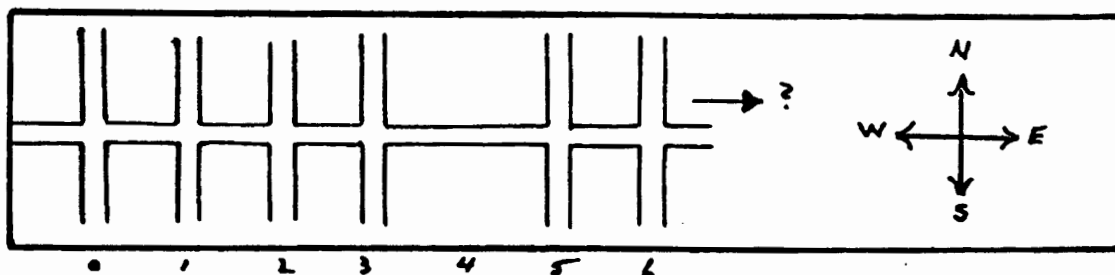


Fig. 12. Diagram of Uncompleted Street Plan

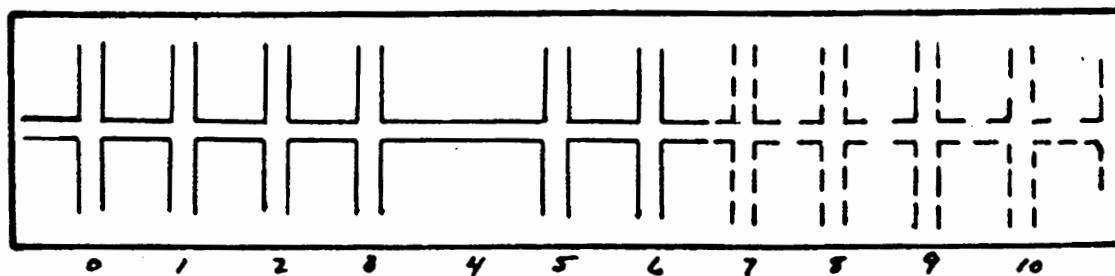


Fig. 13. Street Plan (from Fig. 12) With Probable Projection

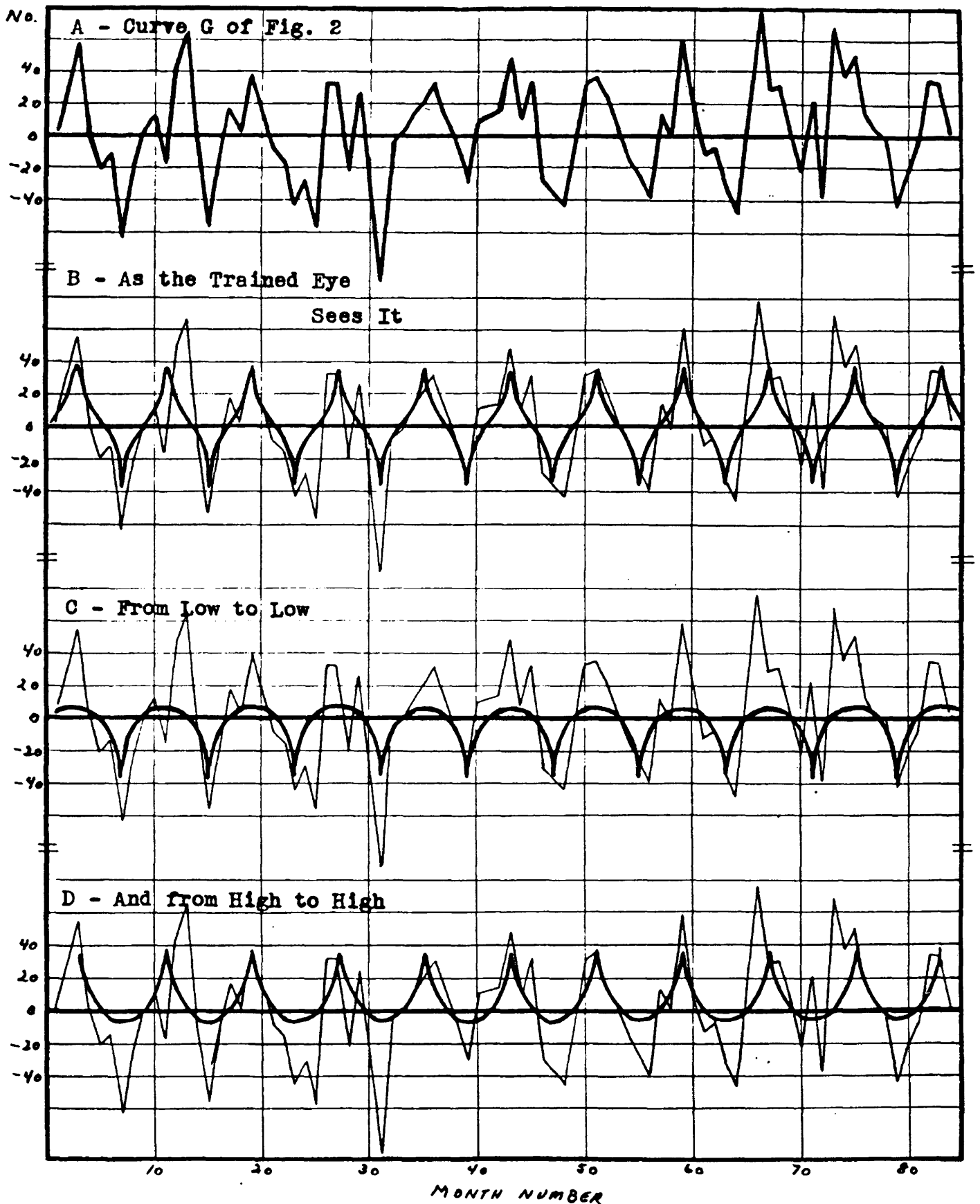


Fig. 14. Visual Analysis of Curve G of Fig. 2

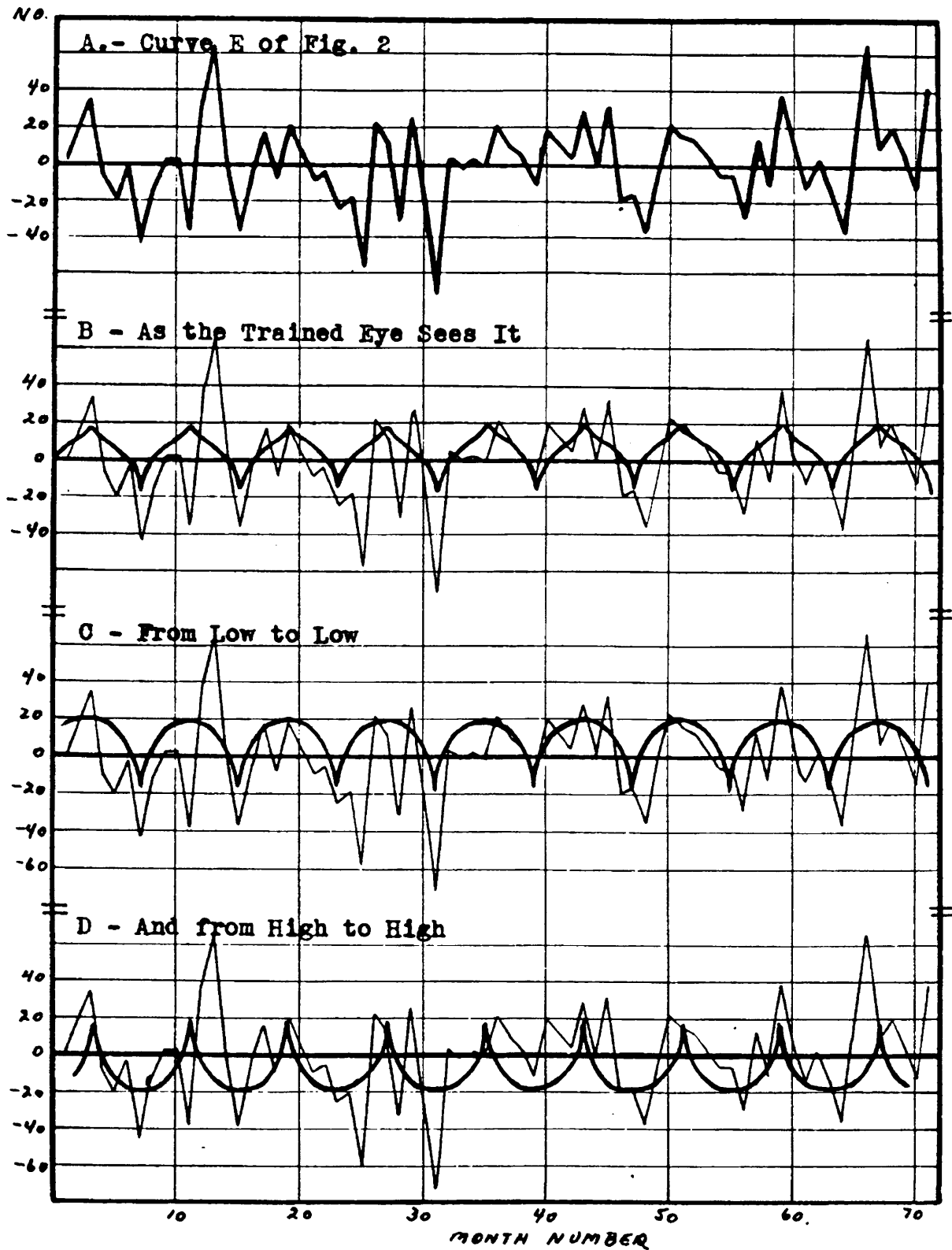


Fig. 15. Visual Analysis of Curve E of Fig. 2

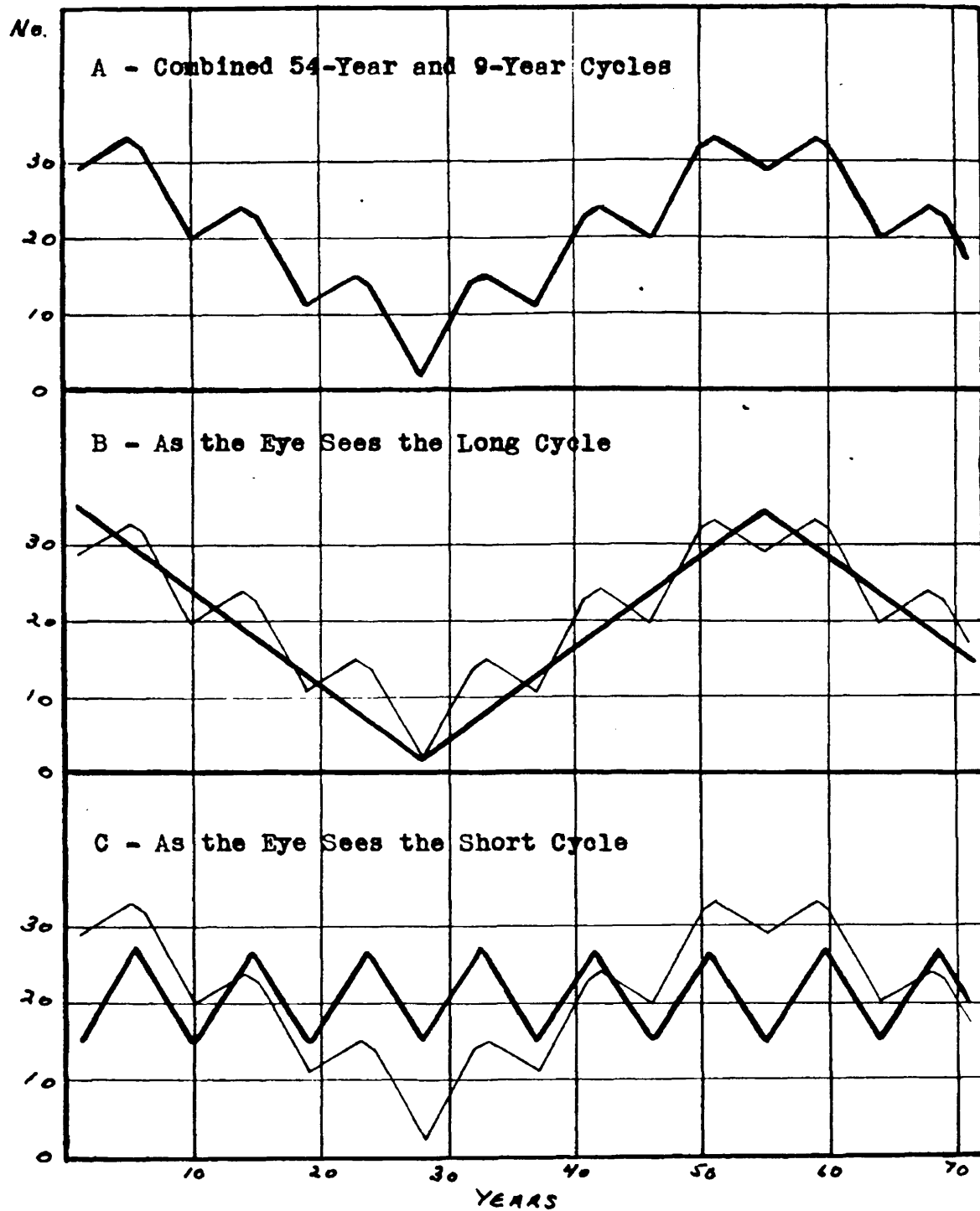


Fig. 16. Visual Analysis of Combined 54- and 9-Year Cycles

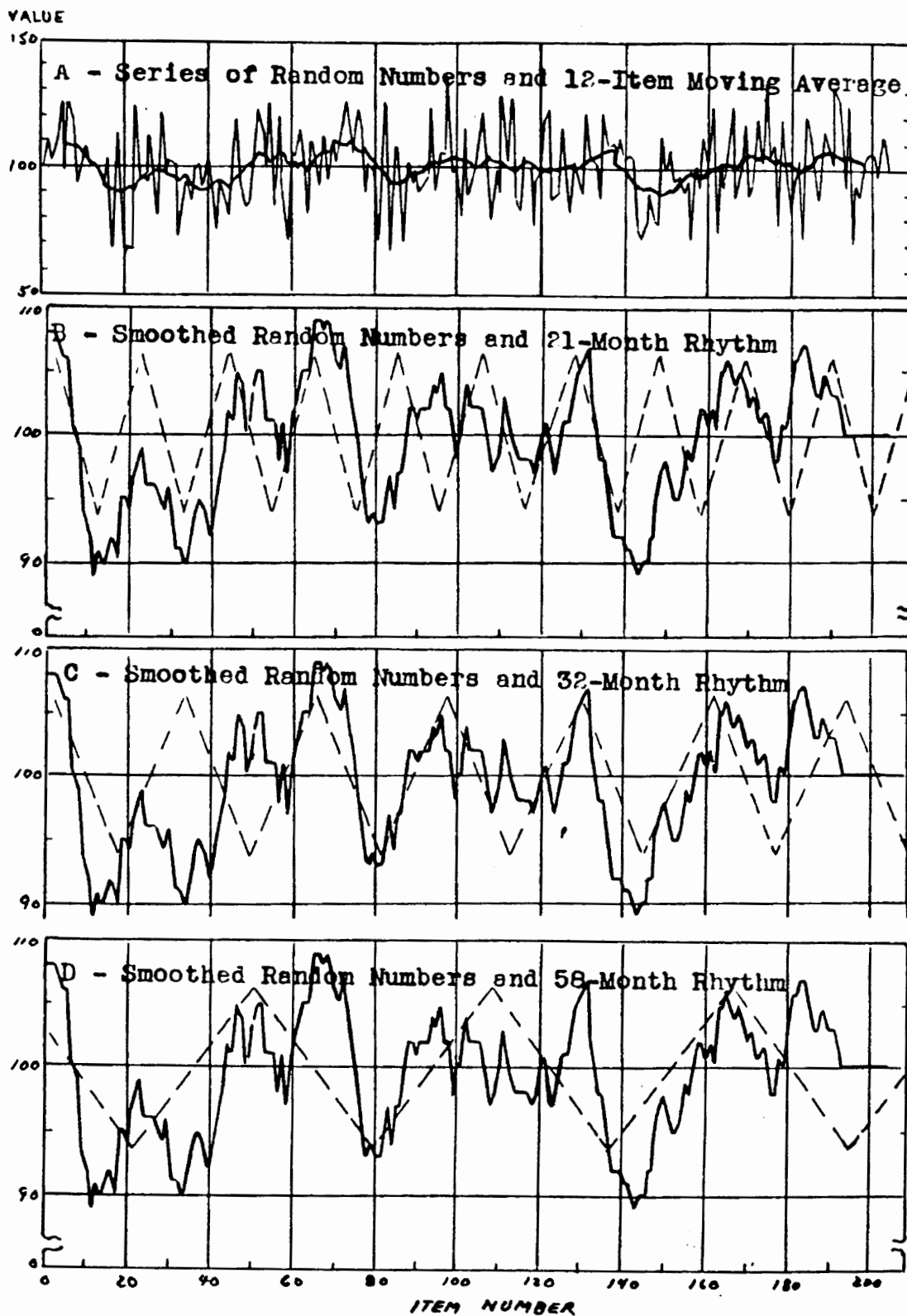


Fig. 17. Rhythms Found in Random Numbers

Source: Cycles, January, 1952

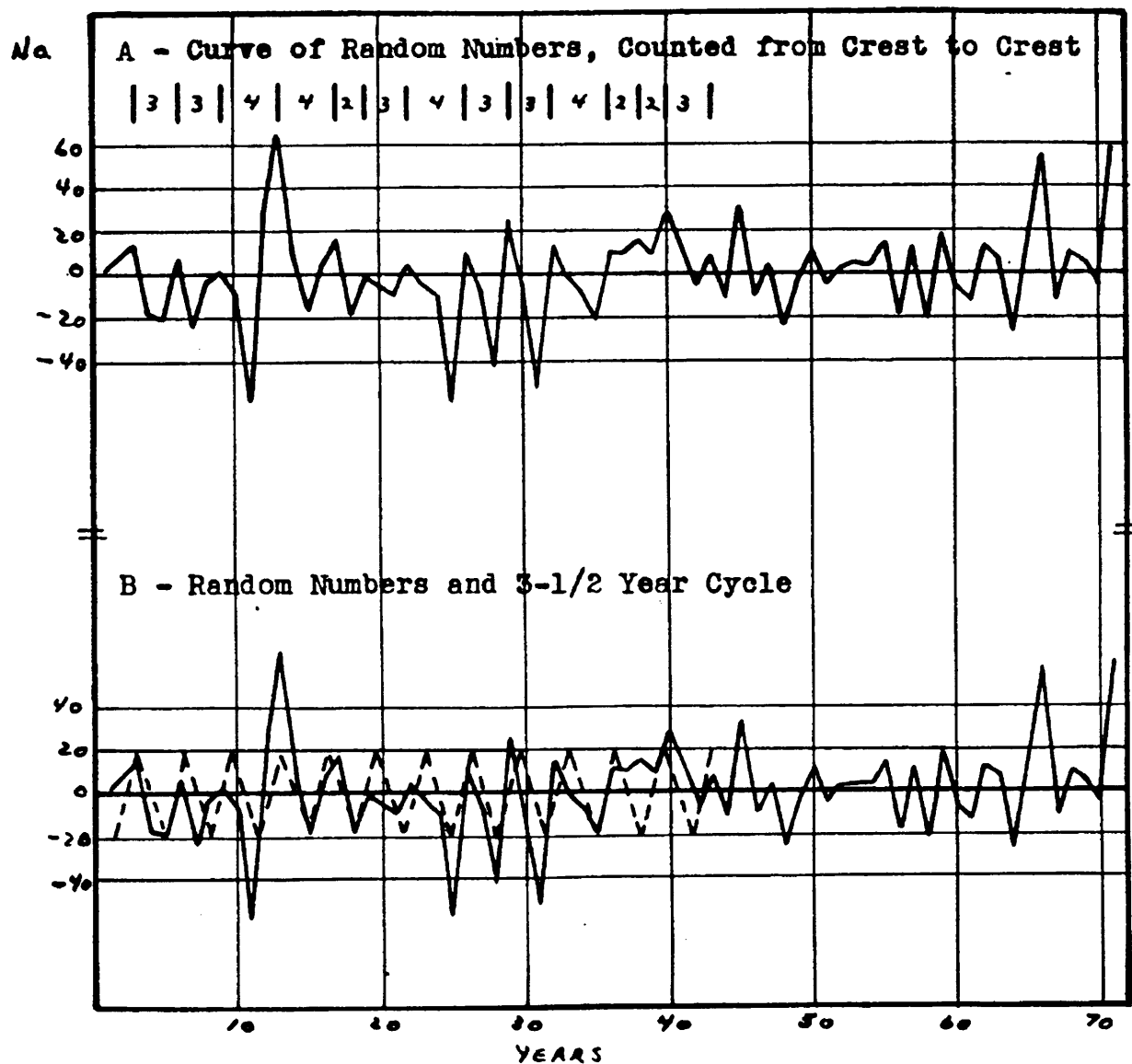


Fig. 18. Counting To Get Hint of Cycle Length

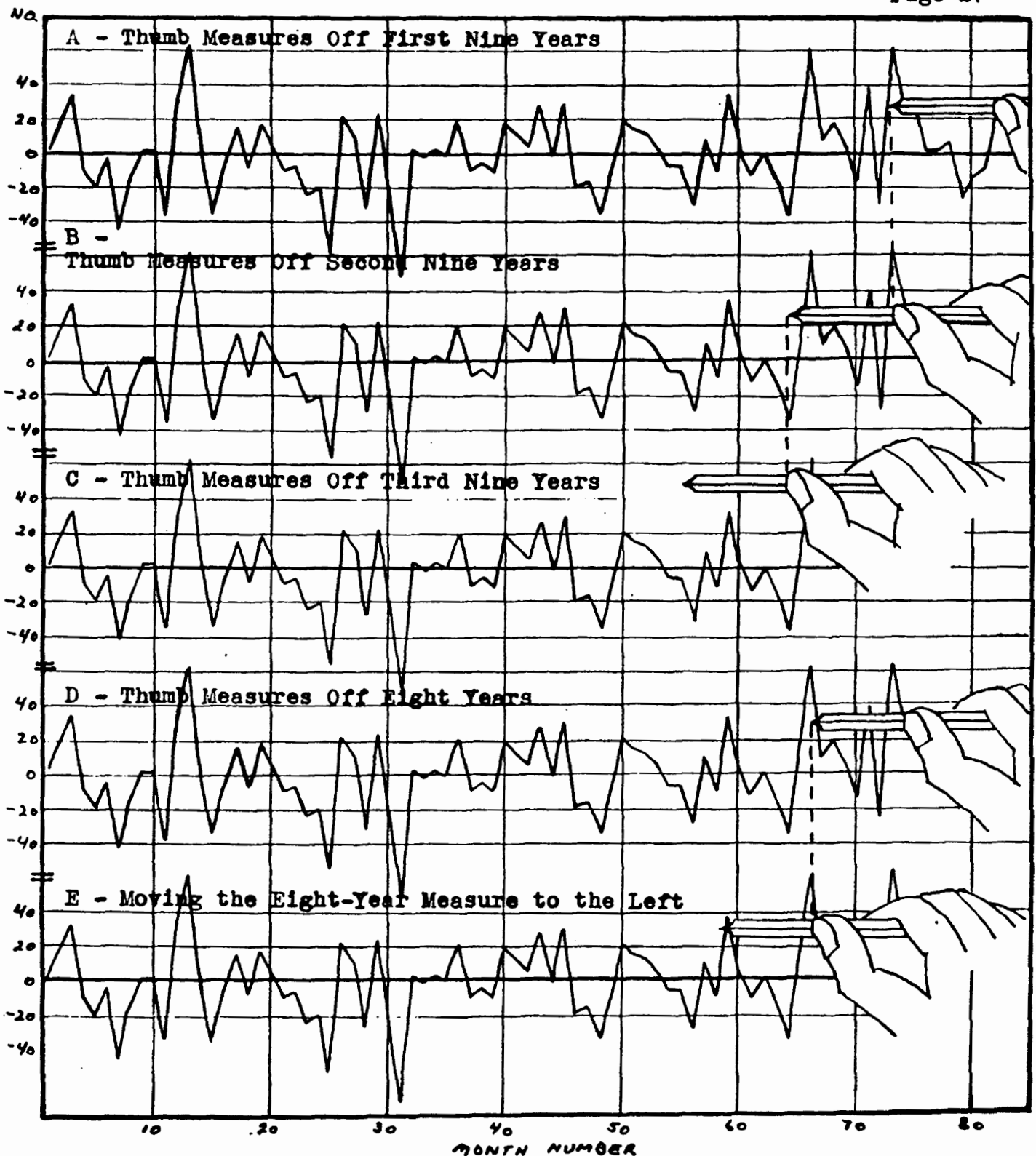


Fig. 19. Illustration of "Thumbing" as a Means of Getting Hints of Cycle Length

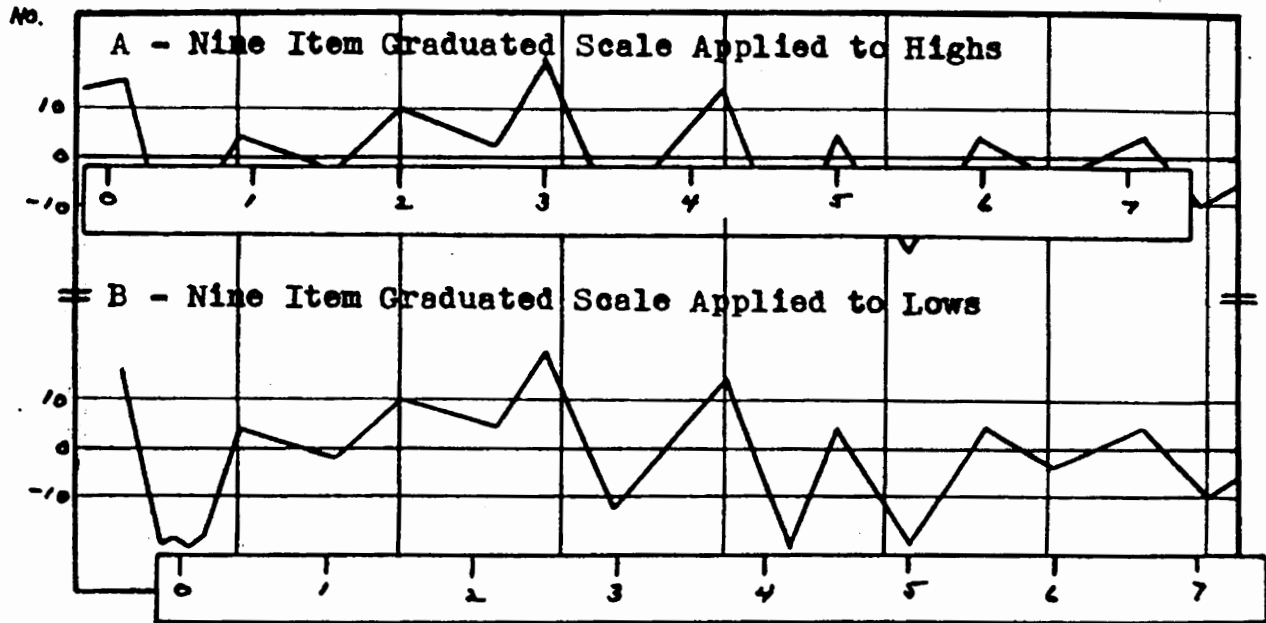


Fig. 20. Use of A Graduated Scale to Find Hints of
Cycle Length

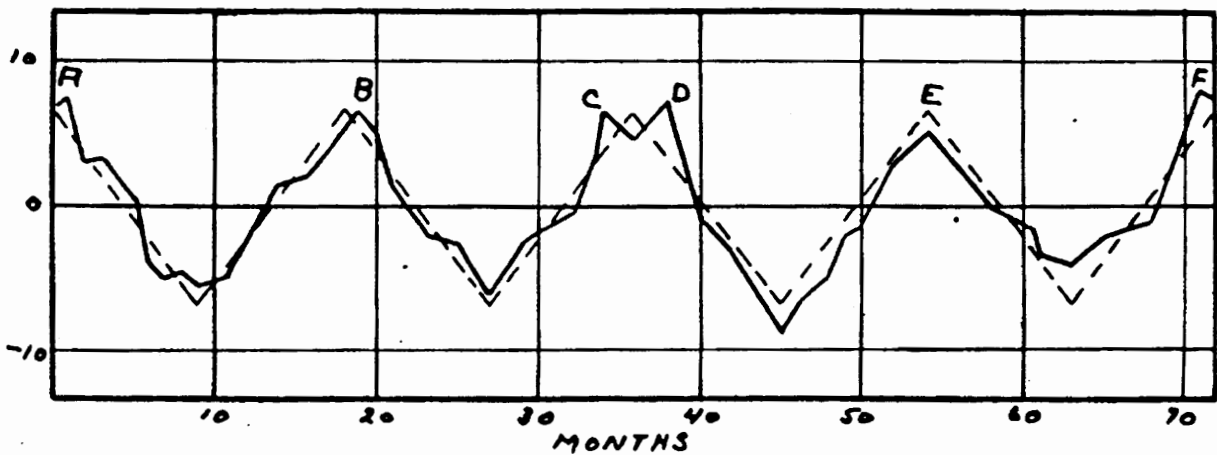


Fig. 21. Diagram of Four 18-Month Waves

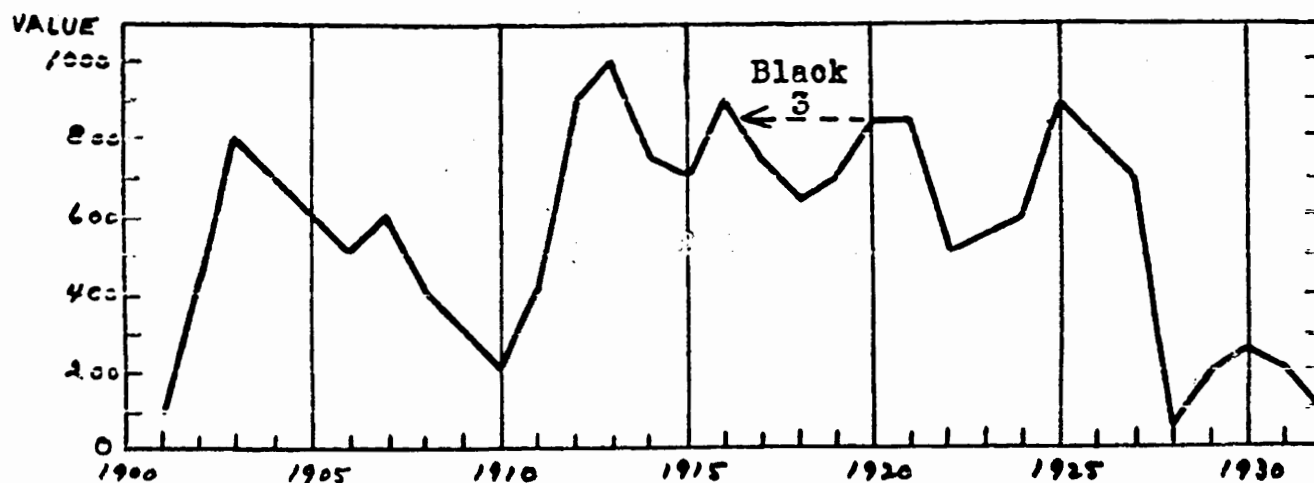


Fig. 22. This chart shows a hypothetical curve from 1901 to 1932 inclusive. The broken lines illustrate the concept of clearspan.

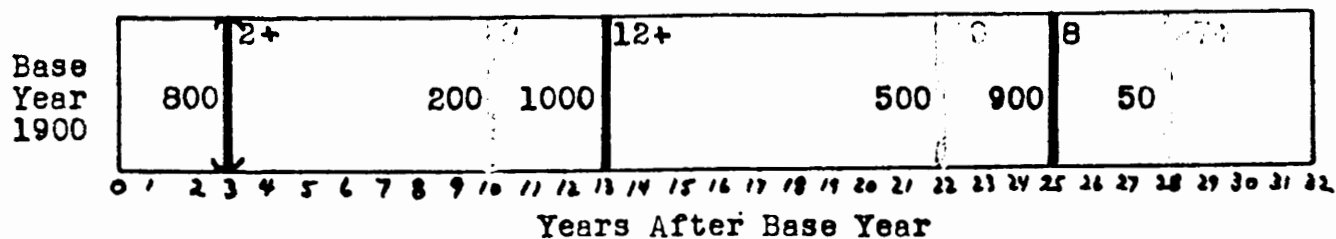


Fig. 23. This chart represents time from zero year (1900) to 32 years after 1900; in other words, 1900 to 1932. Heavy solid vertical lines indicate highs, using clearspan of 8; red vertical lines indicate lows similarly. The parentheses that surround the first high indicate that we do not know for certain that it is a standard high. The numbers to the left of the line refer to the value of the curve at the time indicated. The numbers to the right of the line indicate clearspan. This chart is merely a shorthand method of indicating the highs and lows with a clearspan of 8 or more, as found in Fig. 22, above.

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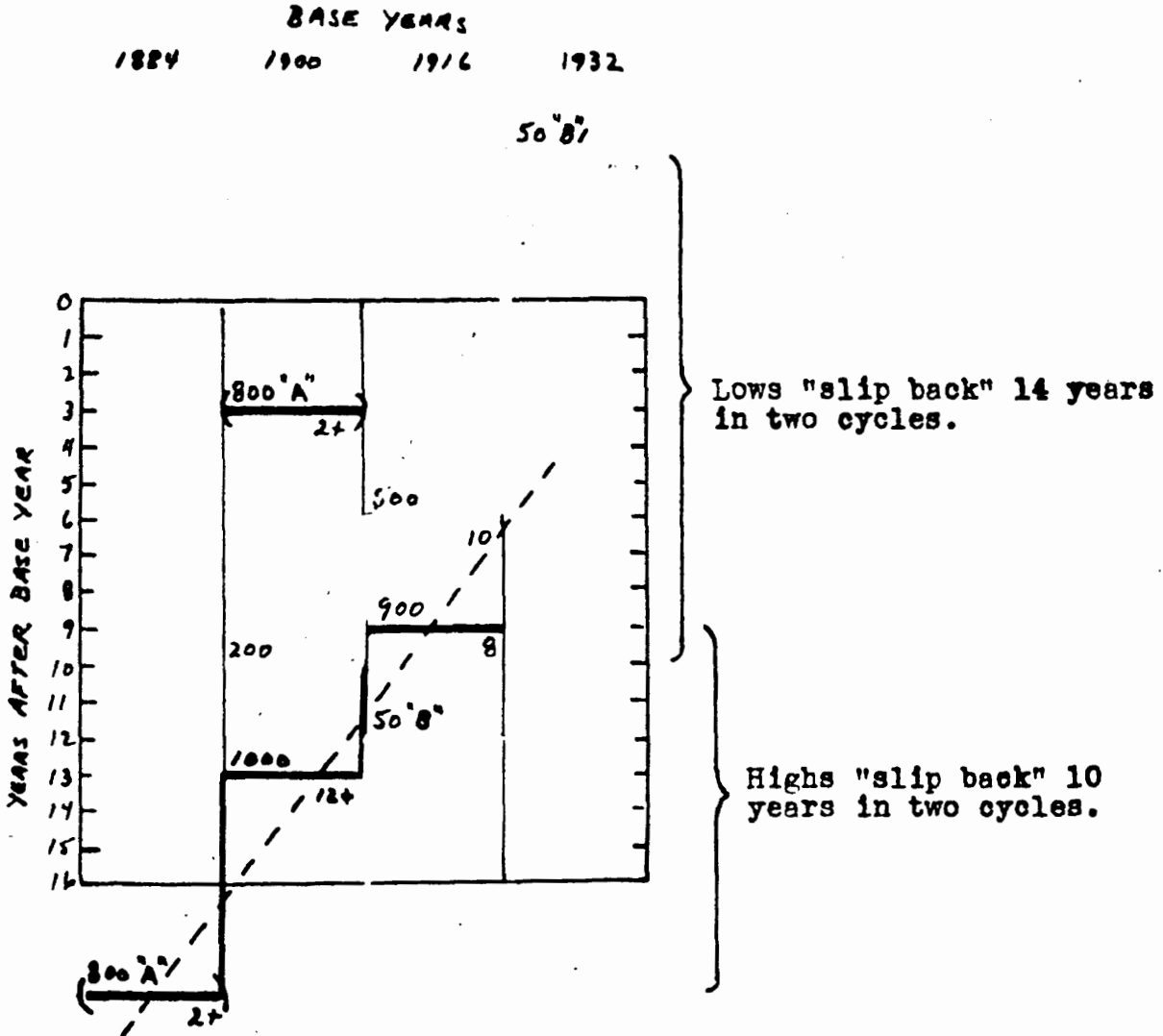


Fig. 26. This is a completed 16-year Time Chart of the data plotted in Fig. 22. Clearspan standard V_8 . Two more cycles have been added to the grid and the first high, "A", is then repeated in the new section based on 1884. The last low, "B", is then repeated in the new section based on 1932. You will note that both the first high and the last low fall outside of the grid. The highs and lows are then connected. Because the highs and lows do not fall opposite each other in a horizontal line, it is obvious that there is no 16-year rhythm in the given curve.

From the upward slope of the line one sees that there is a rhythm of something less than 16 years. As the lows drop back 14 years in two cycles, or 7 years a cycle, we see the indicated length based on lows is 16 minus 7, or 9 years long. As the highs slip back 10 years in two cycles, or 5 years a cycle, we see the indicated length based on highs is 16 minus 5, or 11 years. It is obvious that the length of the rhythm is approximately 10 years long (9 plus 11, and divided by 2).

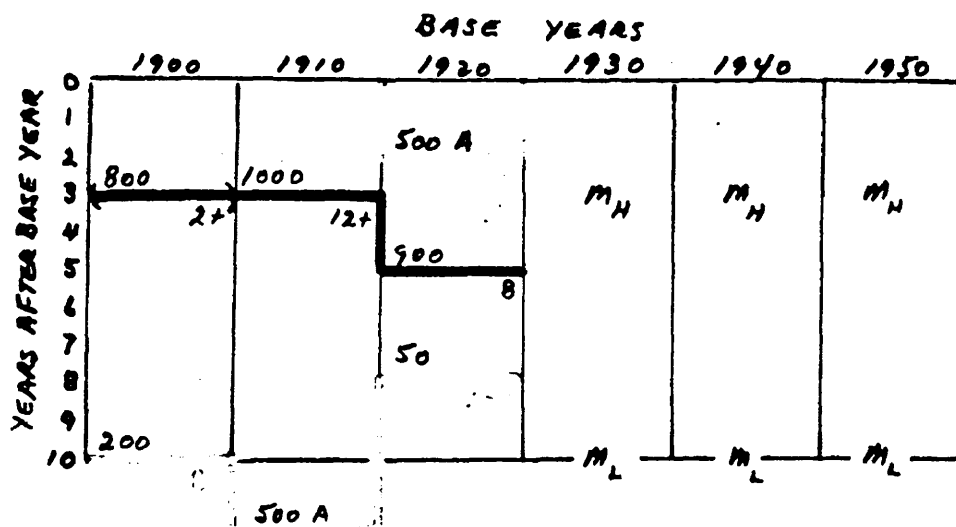


Fig. 27. A 10-year Time Chart of the data plotted in Fig. 22. Clearspan standard V_5 . Highs and lows connected. M_H signifies predicted median position of highs. M_L signifies predicted median position of lows.

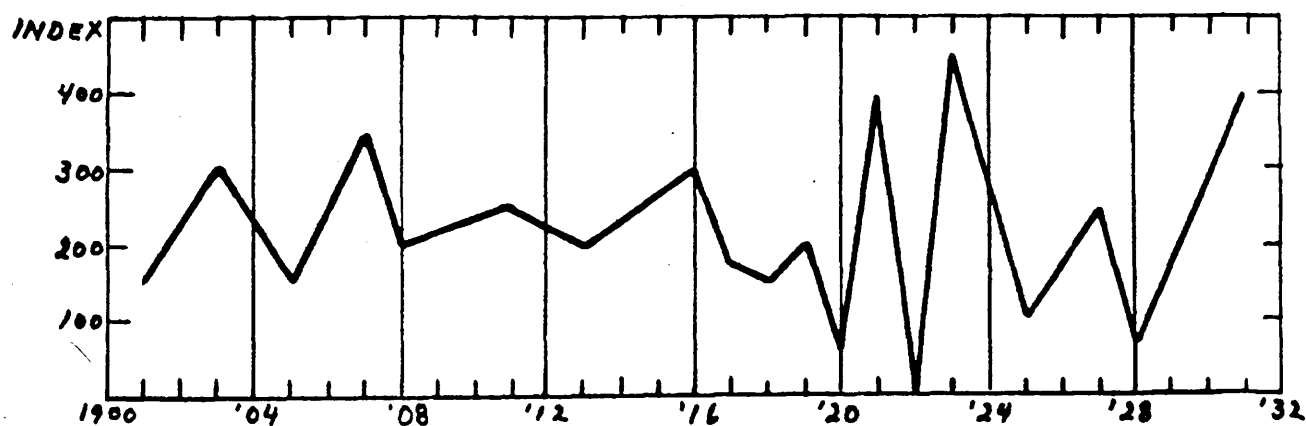


Fig. 28. This chart shows another hypothetical series of data, 1901-1931.

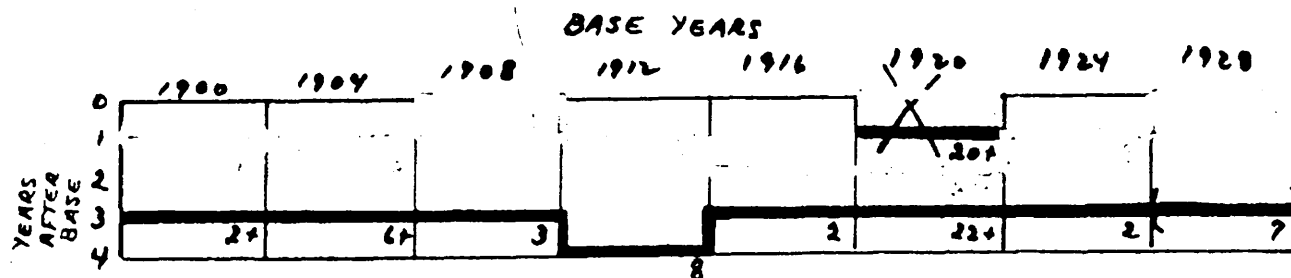


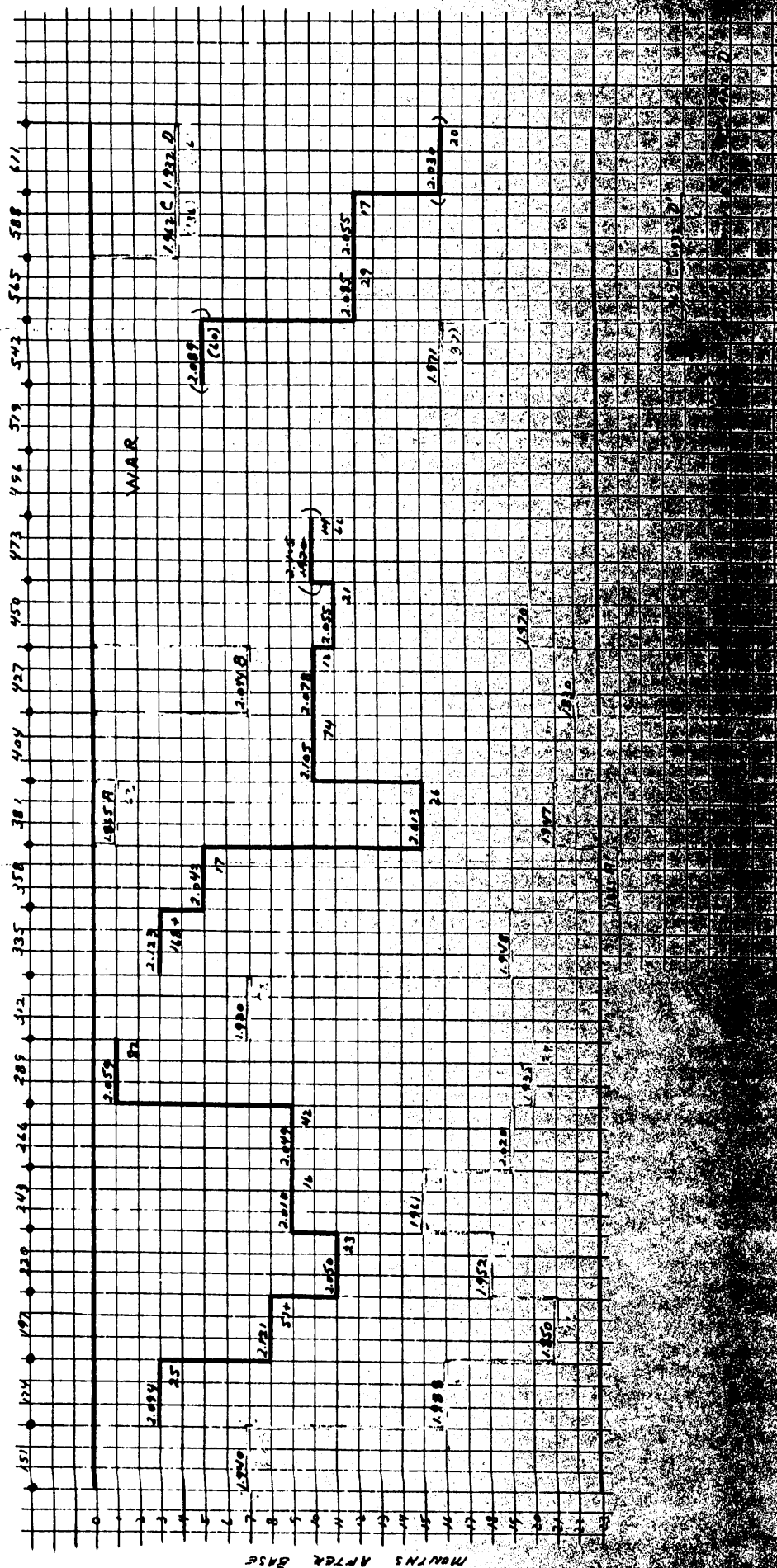
Fig. 29. A 4-year Time Chart of the data shown in Fig. 28.

Step III:

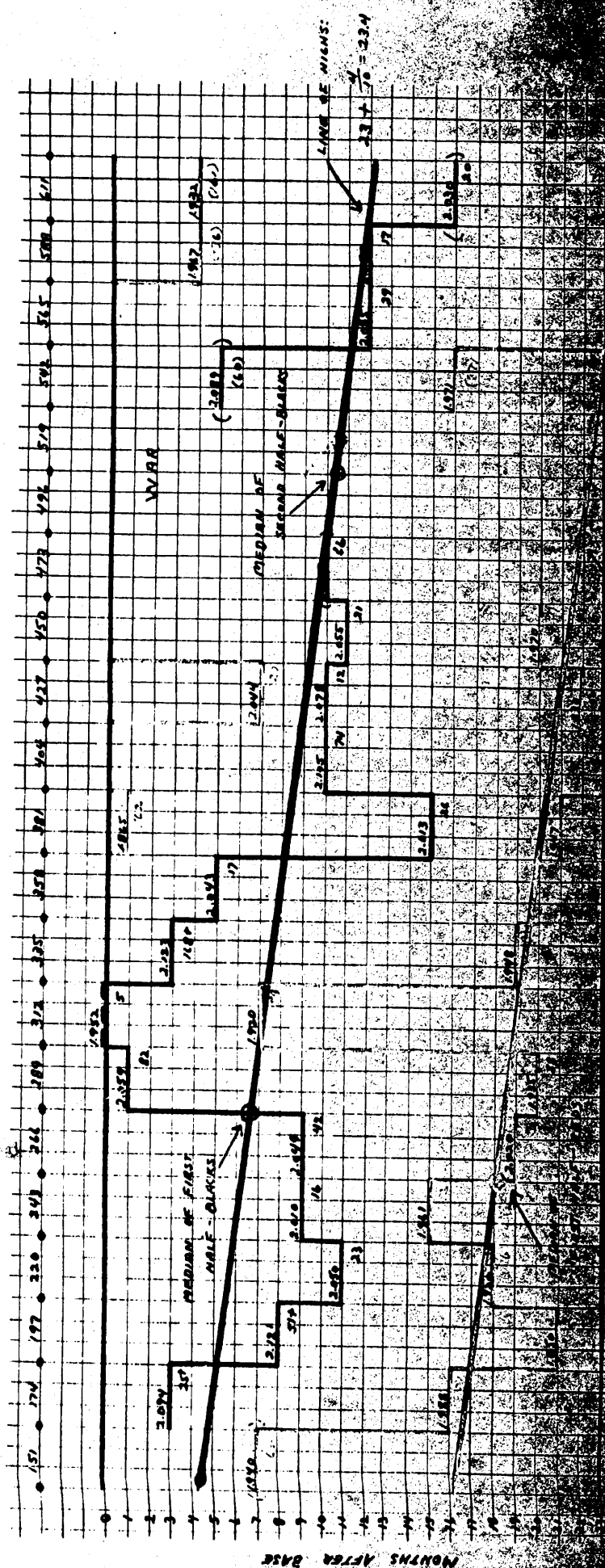
CONNECT HIGHS THAT ARE CLOSE. CONNECT LOWS THAT ARE CLOSE. START WITH LINES ONE SPACE APART, THEN CONNECT LINES TWO SPACES APART, ETC., UNTIL LINES MAKE A CIRCLE BETWEEN CONNECTED

STEP IV:

LINES MOVED AS NEEDED.



STEP VI: LINE UP HIGHS AND LOWS



Lesson VI - Problems

Problem 1. Compute clearspan numbers of logs of your stock market data (Col. B of S. & P.C.A., T.S.-1). Record in Column E of S. & P.C.A., T.S.-1.

Problem 2. Compute clearspan numbers of the 9-year moving average of logs of your stock market data (Col. C' of S. & P.C.A., T.S.-1). Record in Column F of S. & P.C.A., T.S.-1.

Problem 3. Compute clearspan numbers of the deviations of logs of your stock market data from their 9-year m.a. (Col. D of S. & P.C.A., T.S.-1). Record in Column G of S. & P.C.A., T.S.-1.

Problem 4. Using all methods given—inspection, counting, thumbing, graduated scales and time charts, make a thorough study of cycles from 7 to 13 years in length in the stock market data. Depend chiefly upon deviations of logs of data from this 9-year moving average.

In working this problem use not only Chart S. & P.C.A.-4, but use also Chart S. & P.C.A.-3.

In working this problem make at least six time charts of the deviations.

Be sure to record copious notes in your log.

Problem 5. Make a complete time chart analysis of the 9-year moving average of the stock market figures (S. & P.C.A., T.S.-1, Col. C, Clearspan numbers Col. E), lengths 15 years and up. In this connection study S. & P.C.A. Chart 3 where the 9-year moving average values are plotted.

Be sure to explain in your log just what you do and why you did it.

Problem 6. Make one time chart of logs of stock market data (S. & P.C.A., T.S.-1, Col. B, clearspan numbers Col. E) using for length the length which in Problem 4 seems best.

Make your notes full and complete.

* * * *

Send in S.P.C.A., T.S.-1, your log and all your time charts.

* * * *

P.S. This isn't make-believe any more. This is the real thing.

STANDARD PRACTICE INSTRUCTIONS

5. Indexes

You will need at least two indexes (really lists) for every time series to which you give serious study: (1) A list or index of line charts, (2) a list or index of time charts and periodic tables. Each index is merely a list in chronological order of the material concerned.

In each case the purpose of the index is primarily to tell you the number of (a) the last line chart or (b) the last time chart or periodic table you have made so that you will know the number to give to the next one.

However, the index of line charts has a secondary value. It is often easier to scan than the line charts themselves if you wish to know is there a chart of such and such a sort.

If line charts get too numerous, a cross index on 3x5 cards can be established and filed by content.

In the index of time charts and periodic tables these two sorts of material are entered interchangeably.

As time charts and periodic tables are filed by wave length it is easier to find them directly than by reference to the index.

Tabulation Sheets do not need to be indexed, but for big jobs it is sometimes necessary to set up a cross index for the columns of tabulation sheets, perhaps on 3x5 index cards.

STANDARD PRACTICE INSTRUCTIONS

6. The Graduated Scale

1. It is desirable to make graduated scales on heavy plain white paper or light cardboard.

2. Graduated scales should be about as long as your chart paper.

3. Graduated scales should be of some standard width. An inch is about right.

4. Do not use the end of the scale for the 0. Rather, put the line representing 0 in about half an inch from the left hand margin.

5. Be neat.

6. Make the lines representing each five cycles a little longer than the others.

7. Number the cycles from left to right, starting with 0.

8. Always put the length of the cycle and the name or number of paper to which the scale applies in the lower left hand corner. Example: "5 $\frac{1}{2}$ -yr. for Polypurpose".

9. Always--and this is important--always check the overall length of the graduated scale to make sure there are no cumulative errors. For example, if your scale shows twenty 5.4 year cycles its overall length (0 to cycle 20) should total 108 years. See that it does. Then mark "ckd" with your initial and date in the lower right hand corner.

10. File graduated scales (1) by kind of paper and (2) by length of cycle.

STANDARD PRACTICE INSTRUCTIONS

7. Time Charts

1. To make a time chart, use cross section paper, unaccented, 5 lines to the inch each way. If necessary, paste two or more sheets together, side by side, or use paper 11 x 17, or 11 by some longer length.

2. Place your initial and the date in the upper left hand corner.

The date certifies the time of your discovery of the cycle. However the chief use of the date here and elsewhere is for your own information later. A year from now, let us say, in reviewing your work, if your notes are poor, mislaid, or confused, you can often tell from the date, which of two time charts (periodic tables, or whatever) was made first. For example, if your 8.5-year time chart is dated 8-10-53, and your 8.4-year time chart is dated 8-11-53, it is obvious that you first thought the length was 8.5 and that, from this time chart you deduced that the length was more probably 8.4 and that you then made another time chart of the shorter length--instead of the other way around.

3. Place the title of the chart at the top of the sheet (i.e. top as it would be when filed in a loose leaf note book).

Always start your/^{sub}title with the length of the time chart. Make/^{sub}title full and complete. For example: "11-year T.C. of Dev. of logs of data from their 9-yr m.a. V_{5v_2} ".

4. Enter name of Time Chart in Index of Time Charts and Periodic Tables in consecutive numerical (and hence in chronological) order.

5. Place number of Time Chart in upper right hand corner of Time Chart. Get number from Index of Time Charts and Periodic Tables.

6. Next make your grid.

7. Now make your time chart proper.

8. Record on the time chart itself any computation regarding length of indicated rhythms, or any observations that may occur to you, for example "6- & $3\frac{1}{2}$ -yr. also?" or "Let's now try $13\frac{3}{4}$," or whatever.

9. Record on the time chart in the lower right hand corner the source of the data used in making it.

10. Do not neglect to record in the log whatever you are able to learn from the time chart.

11. File time charts intermingled with periodic tables in order of length, shortest on top. File by length of time chart and not by length of the indicated rhythm, but cross index by length of rhythm or rhythms if you wish. If you have a time chart and a periodic table of identical length, file time chart on top. Double and triple time charts are filed according to the length of the wave being doubled or tripled. Thus a $12\frac{1}{2}$ time chart would be filed under 6. So would an $18\frac{1}{3}$ time chart.

LESSON VI

Supplement 1

A Short-Cut or Bastard Method of Making a Time Chart

Although it is desirable to use the methods of constructing a time chart set out in Lesson VI, sometimes, when I plan to supplement the time chart with other methods, I use the short cut or bastard method described below:

1. Make a chart of the curve you wish to analyze, preferably on ratio scale unless you are charting the logs of the data.
2. Make your time chart grid.
3. Mark off on a piece of paper a length equal to your clearspan standard ($\frac{1}{2}$ of the length of the time chart, or the next smaller even number).
4. Using this paper to measure backward from successive possible crests and troughs, pick out the full standard highs and lows and post corresponding bars into the time chart grid.
5. Pick out pairs of secondary or tertiary turning points, as may be necessary to complete the bars of the time chart.
6. Connect, rearrange, and line up bars.

* * * *

Note that this method saves computation of clearspan numbers. On the other hand, it costs you the benefits that come from being able to read values and clearspan numbers direct from the time chart.

This method is not adequate if you are going to lean heavily on the time chart, but it will sometimes serve if the time chart is to be supplemented by other methods.

LESSON VI

Supplement 2

TIME CHARTS

In general, students have had less trouble with time charts than I expected. However, some points have come up which I would like to clarify.

Fitting Straight Slope-Lines to the Connected Bars.

Lines to show the slope of the bars should be fitted by eye. I often place a ruler edgewise over the bars and determine their slope by looking straight down on the upper edge of the ruler. More often I place a transparent ruler flat on the bars to get an idea of the sloped line which most nearly fits all of them.

I always compute the semi-averages and spot them on the time chart, but I never let myself be bound by them if, in spite of them, the eye says that a line of some other slope makes a better fit.

In figuring the semi-averages I always use the median bar, not the mean of the positions of the bars. If I have an odd number of bars I sometimes use the middle bar twice; sometimes drop it; sometimes make one "half" longer than the other.

Remember that if your series were long enough the red and black lines would always be parallel. Any divergence is therefore an error on the part of one line or the other, or both. You get rid of this error (in part) by averaging the slopes of the two lines, but you can also minimize it by keeping the lines as nearly parallel to start with as the situation permits.

Don't try to split eyelashes. Remember the time chart gives only an approximation of the length of the cycles. You make it to get a hint.

Connecting the Bars

One student wrote to say that you could connect the bars of a time chart to make them show any length you wished. This statement is not entirely so, but there is a good deal of truth in it notwithstanding. There are often several ways in which you can connect the bars of a time chart.

In fact one and the same time chart will often serve to indicate several different cycles, all present concurrently in the same series of figures. That is one of its advantages.

Keep in mind the three main uses of the time chart: One, to show the most promising cycle of the series, two, to help evaluate some particular cycle in which you are interested, and three, to give you hints of other cycles also present in the series.

If your object is the first one, you connect the bars to go where they will--as long as they oscillate around a straight line. If you have the second objective you try to hold the time chart to the length desired. (In any event you also look for other patterns into which the bars can be fitted.)

In any case, however, you must play the game and use the highs and lows of greatest clearspan. You can't throw out a bar with a big clearspan and use a bar with a little clearspan instead, just because the bar with the little clearspan falls where you want it to and the bar with the big clearspan doesn't.

Posting Clearspan Numbers and Values

One student wrote to say he could not see the use of posting values and clearspan numbers above and below the bars of a time chart.

The importance of posting such clearspan numbers is fourfold:

One, as both the clearspan and the value are important characteristics of your highs or lows, it aids you in the proper connecting of your time chart bars to have these values in front of you.

Two, the values and clearspan numbers help you to understand the characteristics of the curve. This is especially true when you have a time chart of the original data (or their lows) or have deviations from a straight line or other mathematical trend.

When, under these circumstances, you have values and clearspan numbers on the bars you can read at a glance, "The 9-year cycle has never had a high with a clearspan of less than so and so; never a high with a value of less than so and so. As the present values does not meet these standards it cannot be the high of the move unless it breaks all precedent."

Or, instead of "never" you can say "only once in five times," or whatever the facts are.

Three, the values and the clearspan numbers help you to squeeze additional information from your time chart. For example, every other low may tend to be lower, or have bigger clearspans, or whatever.

Four, it enables me to see with relative ease if you have made your time chart correctly. I cannot undertake to correct time charts without such postings, especially time charts of data of your own choosing.

LESSON VI

Supplement 3

TIME CHARTS--Continued

Question:

Item (k), page 18-19 in reference to measurements of cycle length as indicated by time-chart analysis reads in part: "Thus if you are working with an 11-year time chart and if, measured from the middle of one cycle to the middle of 10 cycles later," Where is the "middle" of a cycle on a time chart?

Answer:

The column of a time chart from one base number to another is itself a cycle. It would have been clearer, however, if I had said "Measured from the middle of one column to the middle of the column which comes 10 columns later." See diagram enclosed.

Question:

The last paragraph of item (k), page 19 in reference to measurement of cycle length as indicated by straight lines fitted to the bars of a time chart, reads as follows: "In making this computation with time charts of fractional length compare cycles of the same length. For example, if you are working with an $11\frac{1}{2}$ -year time chart choose cycles for measurement which are 11 years long or which are 12 years long. Do not have one of them 11 years and another 12 years." Since the measurement is along the straight lines which have been fitted to the bars, and since a straight line rises or falls a constant amount per cycle, I do not see how the length of the individual cycles could affect the results of the computation.

Answer:

My use of the word cycle to represent the successive sections of a time chart is again the cause of your confusion. If you substitute the words columns, I think the sentence will become clearer. Thus, "In making this computation with time charts of fractional lengths compare columns of the same length. For example, if you are working with an $11\frac{1}{2}$ -year time chart compare columns for measurements which are 11 years long or which are 12 years long."

In this connection refer to diagram 2.

Question:

In fitting a straight line to the bars of a time chart by the method of least squares, I assume that the base years (assuming annual data) are taken as the X values and the years after base as the Y values. Is that correct?

Answer:

As a rule it is undesirable to fit straight lines to the bars by means of least squares.

The only exception might be where you had a great many repetitions of the cycle. But even here the specious accuracy of a straight line fit is likely to be misleading.

However, if sometime you do want to fit a straight line by least squares, a quick easy way would be: For the Y values assign values to the bars based on their vertical position. For the X values assign values to the bars based on their horizontal position. See sketch 3.

Question:

In fitting a straight line to the bars of a time chart by the method of semi-averages, I assume you average the years after base (assuming annual data), not the year numbers. Correct?

Answer:

Yes, you average the years after base. Thus if you have three entries, one one year after base, one two years after base, and one three years after base, the middle one would be two years after base. Actually this can be done quickly by counting as in the illustration for the question immediately above.

Question:

In fitting a straight line by the method of semi-averages do you use the median or the mean?

Answer:

Use the median.

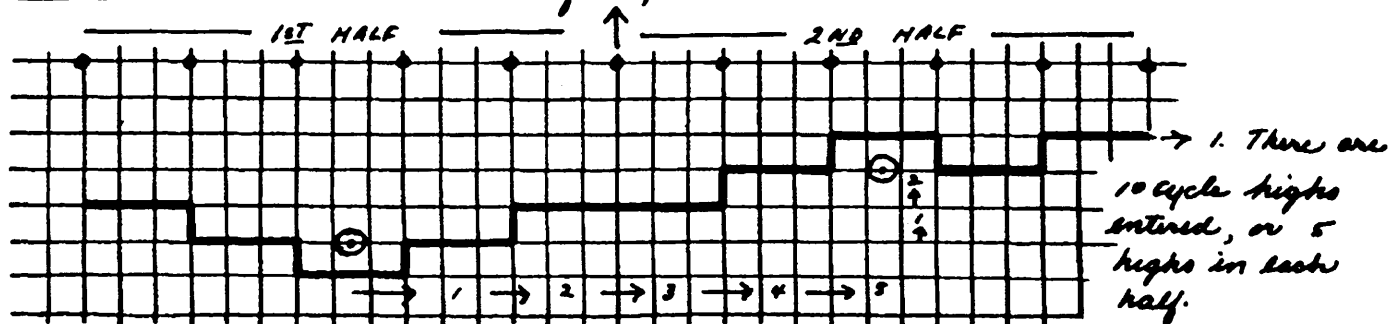
Question:

The last paragraph of item (f), page 15 reads in part: "For example if you are making an 11.4-year time chart number the lines 0, 1, 2, 3, —10, 11, 11.4 (for the 12th line). Do this as a reminder of the length of the cycle. However, the line so marked still represents 12 years after base." I should think this practice would be confusing, especially when fitting straight lines, measuring cycle length etc.

Answer:

I find it helpful to be reminded in this way of the length of the time chart. However, if you find it confusing, don't do it.

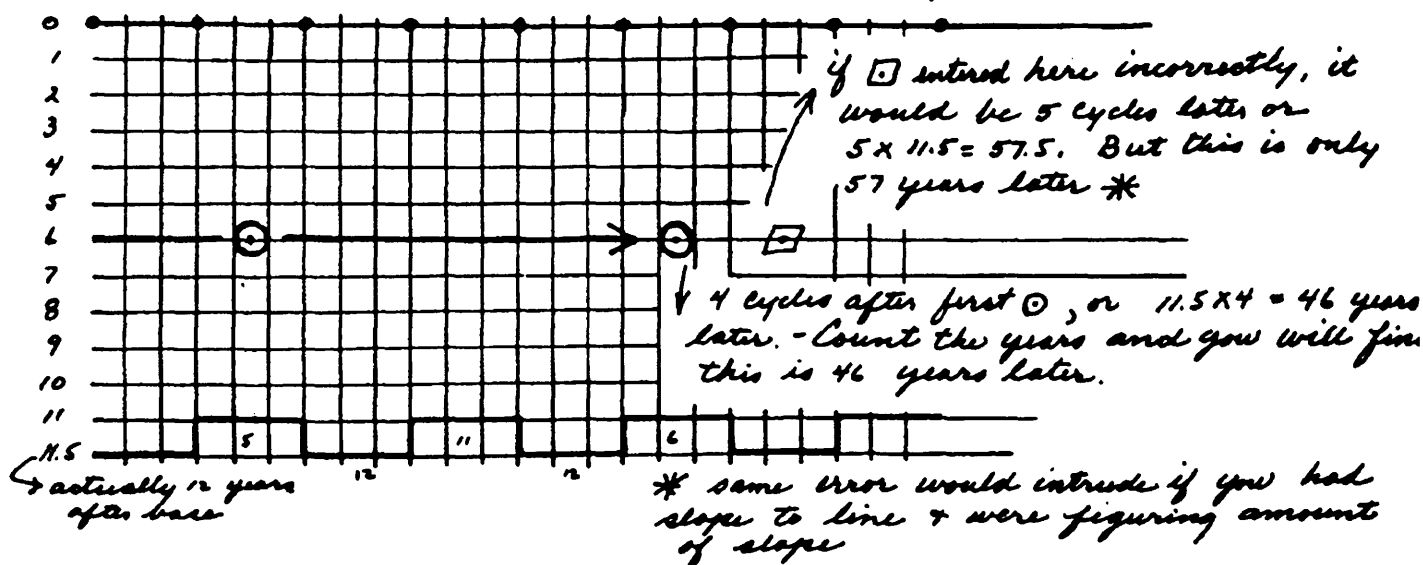
① Question: What is the "middle" of a cycle on a time chart?



2. Ⓢ Median of each half entered at the middle of the half. Median moves, as counted, up 2 in 5 cycles.
3. The point is to count correctly from the same relative positions.

② Question: How could length of individual columns affect result of computations?

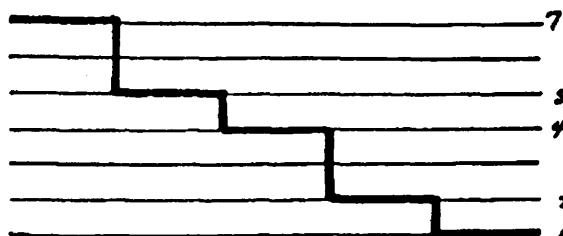
11 1/2 Year Time Chart - Assume perfect 11 1/2 year cycle



∴ Always figure slope by using relative positions.

③ Question: When you figure straight line by least squares, which are x and which are y values?

Assign values to the bars as entered for the y values, & have the origin of the x values equal 0. Thus:



x	y
2	7
1	5
0	4
-1	2
-2	1

$$a = \frac{\sum y}{N}$$

$$b = \frac{\sum (xy)}{\sum x^2}$$

$$y_c = a + bx$$

LESSON VI

Supplement 4

TIME CHARTS—Continued

There is only one way to determine the pairs of full standard highs and lows for a time chart of any given length. However, when it comes to deciding whether or not to drop out a pair of bars (leave a pair of bars unconnected) or whether or not to add a pair of bars, one or both of which are sub-standard, there are different ways in which this choice can be made. There is no single absolutely right way.

In part, what pair of bars you use depends upon what you wish to learn from your time chart. If you want to evaluate some particular cycle length you will use pairs of bars to reveal the extent to which highs and lows oscillate around an axis at or near that particular length. On the other hand if you are merely exploring, you will want to let the time chart tell its own story, and go up or down at will.

For example, on the stock market deviations with which you are working, the 7 year time chart can be allowed to take its own course, in which event it will reveal a cycle about six years long. However, if you already know of this cycle or if, for some other reason, you wish to investigate the possibilities of a 7-year cycle, you can use pairs of bars in such a way that they will throw light on the probability and length of any cycle in the general neighborhood of 7 years long.

Most of the time charts can be connected in at least two different ways. This fact is illustrated in the samples enclosed.

I find the chief errors made by students in connection with time charts to be as follows:

1. When students have to make use of a sub-standard bar, I find that some of them not only dash the sub-standard bar but the full standard bar that accompanies it. They should not do this. Use broken lines only for sub-standard bars and the vertical connecting line preceding them.

2. Some students seem to think that a bar of full standard clearspan which happens to come where you want it can be used in preference to the objectively determined high or low. This is wrong. You must, it is true, occasionally throw out pairs of full standard highs and lows, or add pairs, one or both of which are sub-standard, but you can never substitute a bar of lower clearspan for one of higher clearspan, even though there is a clearspan of different color between them.

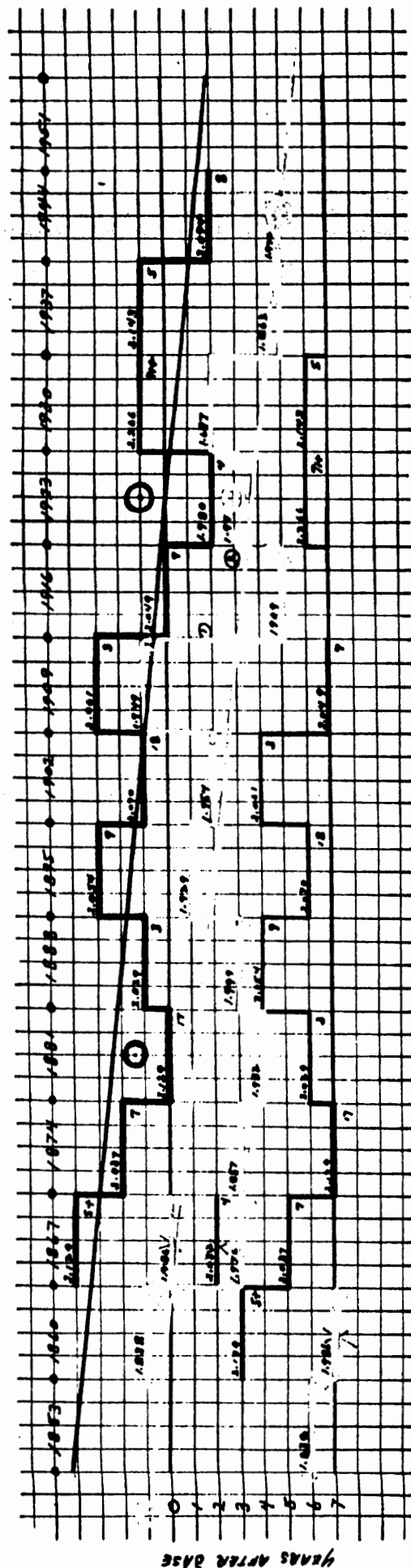
3. Another mistake frequently made by students in connection with time charts is in determining the slope of the line of highs and the line of lows.

I sometimes wish I had not mentioned the use of semi-averages. Fit your lines of high and the lines of low by EYE. For this purpose I find it best to use a transparent ruler laid flat on the time chart.

After you fitted the line by eye, find the median bar of each color in each half. Post a dot with a circle around it to mark this median position. Note whether, in the light of this additional information, you wish to revise your line of slope.

7-YEAR TIME CHART $\frac{1}{3}$ V

DEVIATIONS OF LOSSES OF DATA FROM 9-YEAR MOVING AVERAGES



① - MEDIAN FOR FIRST & SECOND HALF - DO NOT AGREE WITH LINE DRAWN BY EYE

② - LINE DRAWN BY EYE : $7 + \frac{1}{4}N = 7.43$ YEARS, POSSIBLE CYCLE LENGTH

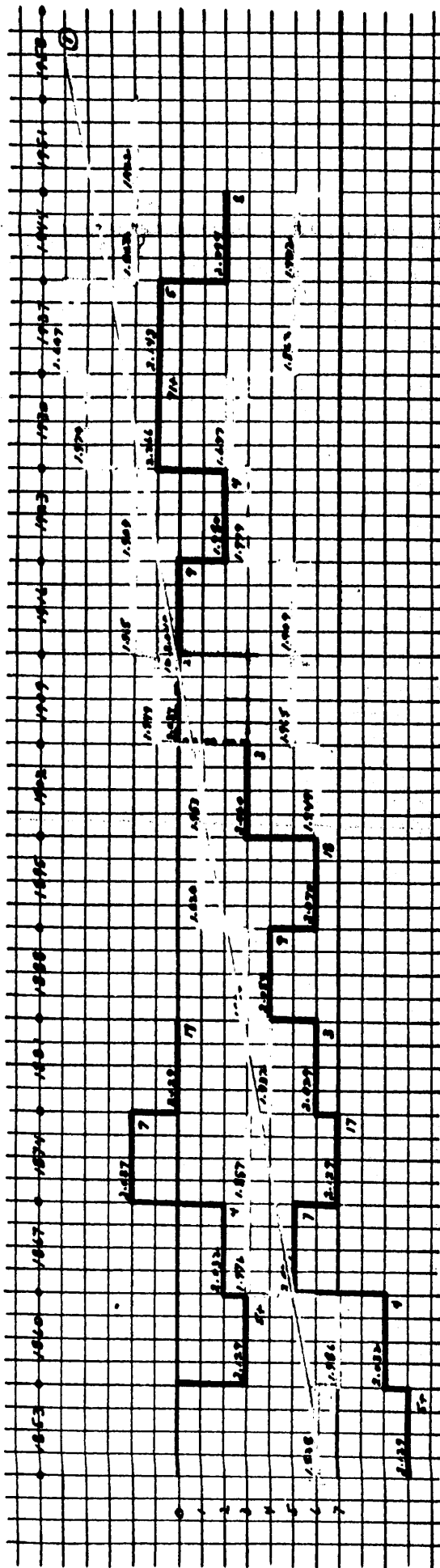
③ - DISTANCE BETWEEN TWO LOWS (OR HIGHS) GREATER THAN $\frac{1}{2}$ THE LENGTH OF THE COLUMN PUTS YOU ON NOTICE THAT AN ADDITIONAL ANGLE (HIGH & LOW) MIGHT CLARIFY THE SITUATION

④ - USE OF SUBSTANDARD BAR WEAKENS VALIDITY OF CONCLUSION

J & PCA

7-YEAR TIME CHART 1/3 1/

DEFINITIONS OF LOS OF DATA FROM 9-YEAR MOVING AVERAGE



THIS 7-YEAR TIME CHART SHOWS THE DATA MOVED TO BRING OUT 6-YEAR PATTERN

① RED LINE DRAWN BY EYE: 7- 1/5- 6.23 YEARS, POSSIBLE CYCLE LENGTH