# HOW TO MAKE A CYCLE ANALYSIS

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by
Edward R. Dewey

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# HOW TO MAKE A CYCLE ANALYSIS

## PREFACE

The subject of cycle analysis is a specialized branch of statistics. The book, "How to Make a Cycle Analysis" is a text at college level. I see no reason why any intelligent high school graduate with a good knowledge of arithmetic and some competence in the use of figures cannot use it successfully.

No mathematics except arithmetic are required as a prerequisite.

The text is a practical one designed to teach the elements of (a) how to find cycles, (b) how to determine their lengths, strengths, shapes, and timings, (c) how to isolate them, (d) how to evaluate them, (e) how to combine them, and (f) how to project them into the future.

Problems will be provided to be worked by the student. Students will make an actual analysis of the same figures I used for my stock market forecast of 1944. They will go through all the steps I went through to arrive at the results achieved.

This is the text used in the correspondence course given last year by the Foundation. There are twenty-three chapters which are followed by supplements which were added to illustrate or clarify portions of the lessons. The supplements also include questions asked by students and the answers given together with comments to students who made errors.

The text is designed to pass on to you as much as possible the knowhow that I have acquired in 20 years work in this field.

Edward R. Dewey, Director,
Foundation for the Study of Cycles,
East Brady, Pennsylvania.

# HOW TO MAKE A CYCLE ANALYSIS

by Edward R. Dewey

## OUTLINE

## PART I--ELEMENTARY STATISTICS

### Introduction and Lessons I--IV

This section is a review for those who have had a course in statistics; an introduction to statistics for those who have not had such a course.

Part I covers the following subjects:

- Definition of terms.
- Review of arithmetic. Machines. Short cuts. Some checking tricks.
- Preparation of the data for cycle analysis.
- Charting for cycle analysis - arithmetic charts, ratio charts. Interpolation.
- Averages - arithmetic means, geometric means, medians, modes. Which average to use.
- Index numbers - how to make them and how to use them in cycle analysis.
- Tabulation.
- Logarithms and how to use them for cycle analysis. Graphic logarithms.
- How to make and use moving average trends for cycle analysis. Geometric moving averages. Moving medians.
- How to remove trend for cycle analysis.

Part I has 100 pages written by me and is supplemented by 200 pages from Business and Economic Statistics by Spurr, Kellogg, and Smith, included by reference.

## PART II--CYCLE ANALYSIS

### Lessons V--XXIII

| | | |
|---|---|---|
| Lesson V | - | Cycle Analysis. How to make a cycle analysis of a series of numbers--a detailed outline. The forces creating time series. Synthesis. The proper way to combine cycles. How to combine growth, periodic and randon components. Analysis. How to separate growth, periodic, and random components. Reversing cycles. |
| Lesson VI | - | How to get hints of cycles. Inspection. Counting intervals. Thumbing. The graduated scale. The time chart. |

| | | |
|---|---|---|
| Lesson VII | - | How to make and use the periodic table to reveal the typical or average shape, strength, and timing of the cycle. How to rotate the periodic table to compensate for trend. Use of color in the periodic table. |
| Lesson VIII | - | How to use a periodic table to determine the length of the cycle. |
| Lesson IX | - | How to position the cycle. |
| Lesson X | - | Randoms: Three ways to minimize them. |
| Lesson XI | - | The effects of moving averages upon trend, upon cycles, and upon random numbers. |
| Lesson XII | - | Deviations of numbers from moving averages of various lengths. How to correct for distortion. |
| Lesson XIII | - | Three ways to definitize the cycle. Definitizing cycles of integral length and of fractional length. Determining the calendar timing of a cycle. |
| Lesson XIV | - | How to make a periodogram. |
| Lesson XV | - | Sine and cosine curves and how to fit them. |
| Lesson XVI | - | How to make a simple harmonic analysis. How to make a multiple harmonic analysis. Use and limitations of harmonic analysis. |
| Lesson XVII | - | How to compute moving percentages, moving ratios, and moving differences. Their effect upon cycles. How to use them to detect hidden cycles. |
| Lesson XVIII | - | The straight line trend. When to use it for cycle analysis. A short cut method for computing it. |
| Lesson XIX | - | How to use periodic tables to separate one cycle from another. |
| Lesson XX | - | Weighted moving averages. How to use them to reveal hidden cycles. |
| Lesson XXI | - | How to make a Streiff Analysis. |
| Lesson XXII | - | How to determine trend. How to project trend and cycles into the future. |
| Lesson XXIII | - | Tests for significance of cycles. |

Part II consists of about 500 pages.

In addition the lessons include, at appropriate places, standard practice instructions and supplemental material used by the author in the conduct of his own work.

# HOW TO MAKE A CYCLE ANALYSIS

## TABLE OF CONTENTS

# PART I - ELEMENTARY STATISTICS

## INTRODUCTION AND LESSONS I - IV

# INTRODUCTION

Cycle analysis is that branch of statistics which deals with the detection, isolation, and evaluation of rhythmic fluctuation in a succession of numbers.

Statistics is the science and art of dealing with masses of numerical facts. For the most part statistics is merely arithmetic and common sense. For this course you will not need to know any mathematics beyond eighth-grade arithmetic.

There are many good textbooks of statistics. However, none of them deals adequately with cycle analysis. There is no book you can buy, no collection of books even, which will show you adequately how to detect, isolate, and evaluate cycles.

On the other hand, many of the things you must know in connection with cycle analysis are adequately covered in any good statistical textbook. It seems foolish to duplicate anything good which has already been written. Therefore, for this course, I have adopted a textbook. It is Business and Economic Statistics by Spurr, Kellogg and Smith. I am sending you a copy under separate cover. I shall use this text as far as it goes. Where necessary I shall supplement it with my own material. Offhand, I would say that about a fifth of what you need to know about cycle analysis is to be found in the text; the other four-fifths you will get from the course material.

The course material is prepared and the main problem of the course is designed to help you make a complete cycle analysis of one series of figures. This series is Standard & Poor's Corporation Combined Index of Common Stock Prices. It is necessary sometimes to degress, but the occasional routine practice work is important to an understanding of the process followed in the main problem.

I will supply you with the following:

1. Textbook (the textbook includes a table of logarithms).
2. Lessons, including problems.
3. Standard Practice Instructions.
4. Reference materials.
5. Chart paper and tabulation sheets.

## Computation

Cycle analysis involves a lot of arithmetic. It is not feasible to do all of it in your head.

There are four aids to computation:

(1) Computing machines (which add, substract, multiply, and divide).
(2) Slide rules (which multiply and divide).
(3) Adding machines (which add and subtract).
(4) Logarithms.

Logarithms help because they enable you to substitute addition and subtraction for multiplication and division, and to substitute division for the extraction of roots.

( Do not let the word logarithm scare you. Logarithms are as easy as pie. You can learn all you need to know about them in fifteen minutes. Operating a slide rule is easy, too.)

If you do not have access to a computing machine you can get along satisfactorily with a slide rule. You will need one or the other. You will not need both.

If you do not have access to a calculating machine you may wish to rent or borrow one. Comptometers are not suitable for cycle analysis. The Monroe , Friden, and Marchant companies all make good machines, but you will not find all models suitable for your work. My preference of make is the Friden.

Rather than spend money on machine rental, I myself would buy one of the little so-called executive desk model Monroes which are operated by hand. They can often be bought second hand for as little as $60.

But, as said above, you do not need a calculating machine. A slide rule will do your multiplying and dividing well enough. A slide rule and logarithms will do your roots, and an adding machine (or your head) will do your addition and subtraction.

If you do not have access to a computing machine you <u>must</u> have a slide rule. If you do not have one already, you can buy a pretty good one from any Sears Roebuck & Company mail order house for $3.75, plus postage. Instructions for operating come with it. Or, if you wish, I will lend you a student slide rule. Send $2.00 deposit, returnable at any time.

If you are pretty good at adding and subtracting you <u>can</u> get along without an adding machine. However, a machine will save you so much time that you may want to rent or borrow one. You can decide this later when you see how much work is involved and how much time it takes you.

If you do rent an adding machine, be sure to get one with a subtraction key. And make a deal so that the rental can be applied to the purchase price, if, later, you decide to buy.

## Charting

I will supply you with whatever chart paper you will need for the course. If you want more paper for your own purposes you can buy it direct from the Codex Book Company, Norwood, Massachusetts. I am asking this company to send you a catalogue. You will be surprised to see how many kinds of paper are available.

## Standard Practice

Experience has shown that certain standard practices pay in the end. Standard practice instructions will be given to you as occasion requires. Please follow them.

## Questions

Our mutual objective--yours and mine--is to teach you to make cycle analysis. However, you must take the responsibility for achieving this result. Like climbing a mountain, you must do the work. I will function as guide. I will assist you. But I am not a ski tow to carry you to the summit without work on your part. And remember that I cannot see you. If you do not call out when you need help, I may go on and leave you behind. So ask questions if there is anything you do not understand.

Questions will help both of us. Questions will help me to make the course better when I give it again. Questions will help you to get more out of the course than you otherwise would. Questions will help to keep you from being left behind. Ask all you want. The more the better.

Of course, I cannot undertake to answer questions outside the area covered by the course. And some of your questions may pertain to things that are going to be covered later in the course. If so, I will tell you.

## Plan of the Course

There are two ways to write a story--or a text book. The old fashioned way is to put things in orderly fashion. "Helen Robinson's grandfather was born at Hemstedshine, England, on July 18, 1865. . ." This way is extremely dull.

The modern way for stories--and it should be for a correspondence course, too, --is to start in the middle of things. Then to backtrack as necessary. " 'Let go of me, you beast,' shrieked Helen, but the burly man held her all the tighter." Because starting in the middle of things is more interesting, I have adopted the second method, as far as possible.

We will therefore start the course by making a cycle analysis of some stock market figures.

On a sheet enclosed you will find a series of stock market averages, 1854--1953.

(The words series means arrangement. A series of numbers is an arrangement of numbers. Numbers can be arranged in order of size, in order of time, and in many other ways. When the numbers are arranged in succession, one after the other, in order of time, the resultant series is called a time series. Successive hourly temperatures, daily stock prices, monthly production figures, yearly crop yields, etc., are all time series. Practically all cycle analyses are concerned with time series. In fact, cycle analysis is sometimes called time series analysis.)

You will proceed as follows:

The first thing you will do will be to chart the data.

(Data are numerical facts. The word is plural. (The singular, rarely used, is datum.) You never say "The data is.. . ." You always say "the data are . . ." The word data is pronounced day'-tah; not dah'-tah, or dat'-ah.)

You will chart the data for two reasons:

(1) You want to see if the series shows a trend.

(Trend is the tendency of a series of figures to increase or decrease gradually over a period of time.)

(2 ) You want to discover the approximate length of the cycles you want to study first.

(Cycles are oscillations--ups and downs. The word comes from a Greek word meaning circle-- a coming around again to the place of beginning, e.g., a low. You will want to study these up and down motions in the hope that they will be rhythmic.)

(Rhythm means to repeat with a beat. It comes from a Greek word meaning measured time.)

(Of course, you are hopeful that the rhythm will be so regular and will have repeated so many times that the regularity cannot reasonably be the result of chance. In this event you can expect it to continue. But this gets into the matter of evaluation, which will be considered later.)

If, in any instance, the data you are studying have no trend, you can proceed at once to hunt for cycles.

On the other hand if, as with the stock market figures, the data do show a trend, you have to remove the trend before you can proceed with the analysis.

You usually remove it as follows:

First, you find what the trend values are for each year ( a matter of simple arithmetic).

Second, you find for each year the percent that the actual value for that year is of the trend value for that year (this is merely the matter of computing a few percentages).

You now search these values adjusted for trend--these percentage of trend--for cycles.

If I had decided to give you, as the first problem, a series of barometric pressures, where there is no trend, the steps would have been two, as follows:

1. Chart the values.
2. Search for cycles.

Perhaps that would have been the best way to start. But I thought you would be more interested in the stock market so I started with a stock market problem. For this problem there are three steps, as follows:

1. Chart the data.
2. Remove the trend.
3. Search for cycles.

The problem of removing trend on its part breaks down into the three subdivisions as follows:

(a) Find trend.
(b) Express data as percentages of trend.
(c) Chart these percentages.

You will remember that I defined trend as "the tendency of a series of numbers to increase or decrease <u>gradually</u> over a period of time." Now that word <u>gradually</u> is pretty loose, so it should not surprise you to learn that there are a great variety of trends that can be fitted to any series of figures.

Perhaps the simplest trend, and one of the easiest to compute, is the moving average trend. It also happens, usually, to be the best trend for cycle analysis.

From the above preview of the analysis it should be fairly obvious that the first lesson will necessarily cover charting; the second will be a brief review of arithmetic; the third will tell you how to compute moving average trends; the fourth will cover adjusting for trend; and the fifth will get into the matter of hunting for cycles.

If you have had a college course in statistics you are likely to say, "I know all that stuff, up to Lesson V. I wish he would come to the point."

I'll grant you that, if you had a good course in statistics in college, you will know everything I am going to cover in Lesson I and II--or at least you did once! But you must remember that to those who did not have such a course, some of this material may be new. As for Lesson III and IV, I do not know of any course in statistics anywhere that covers moving average trends adequately from the standpoint of cycle analysis. No matter how many courses in statistics you have taken--even if you are a college professor of statistics--I'll venture you can learn something from Lessons III and IV !

Now, suppose we get to work.

STANDARD & POOR'S CORPORATION COMBINED INDEX, 1871-1953

Together with Clement Burgess Index (Adjusted), 1854-1870

(Annual Averages, 1935-39 = 100)

| Year | Index | Year | Index | Year | Index |
|---|---|---|---|---|---|
| 1854 | 22.1 | 1888 | 43.4 | 1922 | 71.5 |
| 1855 | 19.8 | 1889 | 44.4 | 1923 | 72.9 |
| 1856 | 20.3 | 1890 | 44.0 | 1924 | 76.9 |
| 1857 | 17.7 | 1891 | 42.0 | 1925 | 94.8 |
| 1858 | 14.5 | 1892 | 46.3 | 1926 | 105.6 |
| 1859 | 12.6 | 1893 | 39.8 | 1927 | 124.9 |
| 1860 | 15.4 | 1894 | 36.6 | 1928 | 158.3 |
| 1861 | 16.2 | 1895 | 37.8 | 1929 | 200.9 |
| 1862 | 21.3 | 1896 | 35.4 | 1930 | 158.2 |
| 1863 | 33.0 | 1897 | 37.2 | 1931 | 99.5 |
| 1864 | 35.2 | 1898 | 42.2 | 1932 | 51.2 |
| 1865 | 33.2 | 1899 | 52.6 | 1933 | 67.0 |
| 1866 | 35.2 | 1900 | 51.3 | 1934 | 76.6 |
| 1867 | 35.2 | 1901 | 65.5 | 1935 | 82.9 |
| 1868 | 39.9 | 1902 | 69.9 | 1936 | 117.5 |
| 1869 | 40.8 | 1903 | 60.2 | 1937 | 117.5 |
| 1870 | 36.4 | 1904 | 58.8 | 1938 | 88.2 |
| 1871 | 39.2 | 1905 | 75.1 | 1939 | 94.2 |
| 1872 | 42.0 | 1906 | 80.6 | 1940 | 88.1 |
| 1873 | 40.1 | 1907 | 65.6 | 1941 | 80.0 |
| 1874 | 38.2 | 1908 | 65.0 | 1942 | 69.4 |
| 1875 | 37.1 | 1909 | 81.2 | 1943 | 91.9 |
| 1876 | 33.9 | 1910 | 78.2 | 1944 | 99.8 |
| 1877 | 26.2 | 1911 | 77.3 | 1945 | 121.5 |
| 1878 | 28.2 | 1912 | 79.7 | 1946 | 139.9 |
| 1879 | 34.4 | 1913 | 71.1 | 1947 | 123.0 |
| 1880 | 43.5 | 1914 | 67.4 | 1948 | 124.4 |
| 1881 | 52.3 | 1915 | 69.9 | 1949 | 121.4 |
| 1882 | 49.3 | 1916 | 80.4 | 1950 | 146.4 |
| 1883 | 47.0 | 1917 | 72.1 | 1951 | 176.5 |
| 1884 | 39.5 | 1918 | 64.1 | 1952 | 187.7 |
| 1885 | 38.3 | 1919 | 74.6 | 1953 | 189.0 |
| 1886 | 44.8 | 1920 | 67.8 | 1954 | 226.7 |
| 1887 | 46.1 | 1921 | 58.3 | 1955 | 300.0 |
| | | | | 1956 | 341.5 |

Note: These data are derived from three series which have been spliced and put on a 1935-39 = 100 base to match currently published data.

Sources:
1854-1870: Clement Burgess Index.
1871-1917: Standard & Poor's Combined Index of Common Stock Prices, 1926 = 100.
1918-1953: Also Standard & Poor's Combined Index, but 1935-39 = 100.
Current monthly data are published in the Survey of Current Business.

STANDARD PRACTICE INSTRUCTIONS

1. Filing

You may wish to file your work papers in a three-ring binder, in ordinary file folders, or in some other way. I recommend three-ring binders. I shall send all material to you punched so that it will be ready to file this way, if you wish.

To start with, subdivide your binder--or set up your folders--as follows:

1. S. & P. C. A. (Standard & Poor's Combined Annual)
   a. Indexes
   b. Notes
   c. Charts (line charts)
   d. Tabulation Sheets
   e. Periodic Tables, Charts of Periodic Tables, Time Charts

2. Lesson Material and Assignments

3. Problems

4. Standard Practice Instructions

5. Reference Material

6. Supplies

If you use a binder, you can use thin cardboard or heavy paper to separate the sections and hold index tabs.

Later, as the binder fills up, you may wish to transfer the lessons and the other material to other binders or to correspondence folders of the sort used in vertical files. Or, you may wish to use such folders from the beginning. It is up to you.

# INTRODUCTION

## Supplement 1

## CALCULATING MACHINES

The following question, and my answer, may be of general interest.

QUESTION:

I am wondering why you prefer the Friden, what you think of the Marchant, whether you are familiar with the new models of the Marchant, and whether a 10 key or 8 key machine is needed?

ANSWER:

I think the Marchant is a good machine.

I am not familiar with the newest models of any of the machines.

As far as I know an 8 key machine would be adequate for any work in cycle analysis. We have 8 and 10 key Monroes and 10 Key Fridens.

My prejudice in favor of the Friden is frankly a prejudice. It is based on the following:

1. A repairman engaged exclusively in repairing Monroes told me the Friden was the best machine on the market.

2. The head of an accounting department of a large company which used Fridens, Monroes, and Marchants told me the Friden was the best of the 3 machines.

3. Dr. Leonard Wing, of the Foundation Staff, investigated Monroes, Fridens, and Marchants a year ago and decided in favor of the Friden. He said it was faster than the Marchant.

4. Comparing Friden and Monroe, I find the Friden gets out of order less easily.

5. I have been fairly lucky in price on Fridens. For example, the machine we bought for Dr. Wing (second hand, of course) cost only $325 for a 10 bank machine with fully automatic division, multiplication, negative multiplication, etc. etc.

I admit that the above is not enough to go on. I pass it on merely for what it is worth.

# INTRODUCTION

## Supplement 2

## HOMOGENEITY OF DATA

It's important that your data be as homogeneous as possible.

It is not always possible to have homogeneous data. For example, an index of general business, extending over any considerable period of time, will, at the present time, contain figures of industries such as television and aviation, not even in existence for the early years of the index. Similarly an index of production of an industry will, for the early years, necessarily omit sales of companies not yet created. Even for individual companies the figures for later years may not be homogeneous with the figures for earlier ones.

Homogeneity cannot always be secured, but at least you can be aware of the complexities introduced by its lack. To this end I am reproducing for you two articles which appeared in Cycles. The first, called "Cycles in the Stock Market," shows you that different sorts of stock have different cycles and that the indexes of stock prices we deal with are made up of very different industry components at different times. It was taken from Cycles for March 1951, pages 83-88.

The second article, a single page from Cycles for May 1951, page 178, illustrates a further difficulty which may arise even when the components of an index are characterized by cycles of the same length.

References to periodograms anticipate Lesson XIV, where periodograms are treated at considerable length.

# INTRODUCTION

## Supplement 2

## CONDUCT OF THE COURSE

In answer to the question, "Do you want the problems harder? Easier? About the same?" one of our more thoughtful students writes, "It is hard for even an active man to scale an eight foot wall. Yet by taking one easy step after another, each within the limits of his ability, even an ordinary person can climb a mountain. It is the job of the instructor to explain every step so that the student can cover the ground smoothly and easily."

I believe this man speaks for most of you and I shall be governed accordingly. I shall try to take you by the hand and lead you smoothly and carefully to the top of the mountain.

However, I would much rather do it the other way. I would rather make you skin your shins getting over 8-foot walls. I would rather you ended the course, not tourists bur commandos. I'd like to make you tough! I'd like to give you problems that would require you to think--problems that you might not be able to solve. H like to make you learn the hard way. But I don't think you want this and I shall bow to what I think are your wishes.

However, if you do come up against some problem that goes a little beyond the text, remember that it may have been introduced deliberately. See if you can't figure it out for yourself. (But if you can't, be sure to call for help!)

Also, keep me posted as to your reaction to what I am giving you. I can easily modify the material and the problems to meet the needs and desires of the majority, so please be vocal. And ask questions !

# Cycles in the Stock Market

BACK in 1943 and 1944 I had the time to do a good deal of work of a preliminary nature in connection with the cycles in the stock market.

I think it will interest you to learn about this work.

I have always liked working with stock market figures because some of them extend over a fairly long period of time, and because they seem to be relatively undistorted by war.

Stock market figures also have a great advantage over many figures in that they are available monthly, weekly, and daily as well as annually. Therefore, if one has the time and the patience, it is possible to determine cycle lengths with great accuracy especially if one is careful to choose homogeneous series of stock price figures.

It seems to me quite reasonable to accept stock market figures as a reliable measure of mass psychology. However, like so many ideas that are reasonable, this idea may not stand close scrutiny.

### Different Stocks Have Different Cycles

A powerful argument against the point of view that the stock market measures mass psychology is the fact that different stocks or groups of stocks evidence different cycles, or the same cycles with somewhat different weights, and at somewhat different timing.

As evidence of this fact I reproduce herewith as Fig. 1 six periodograms of various group averages prepared by E. S. C. Coppock of San Antonio, Texas, one of our members.

The periodograms represent the results of analyses of six different groups of common stocks as follows: Building Material Stocks, Canadian Gold Mining Stocks, Leveraged Investment Company Stocks, Motion Picture Stocks, Miscellaneous Low Priced Stocks, and Farm Equipment Stocks. All from January 1932 to May of 1950.

A periodogram is merely a chart showing the relative importance of various average cycles present in a series of figures. The different cycle lengths are shown on the scale at the bottom of the periodogram. The relative importance of the cycle is shown by the scale at the left. Speaking roughly, at or near every cycle length where the curve of a periodogram is high, there is in the figures that have been analyzed an **average** cycle of importance. And of course where there is an important **average** cycle there may be a **rhythmic** or repetitive cycle also.

For example, in the periodogram of Leveraged Investment Company stocks the peak at 30 months indicates an **average** wave at or very close to this length. In the same series another peak at 36 months indicates that, on the average, there is an even stronger wave of 36 months in length.

If you want more help in interpreting Mr. Coppock's periodograms, refer to the Technical Section of this report. But you do not really need to understand them fully in order to get their significance for our present purposes. This significance is that different stocks have different cycles.

For example, in Farm Equipment Stocks if there is a 30-month cycle it is obviously much less important than an average cycle of 29 months. And in Gold Mining Stocks an average cycle of 27 months is more important.

You can discover many other examples of difference if you study the periodograms on the two following pages.

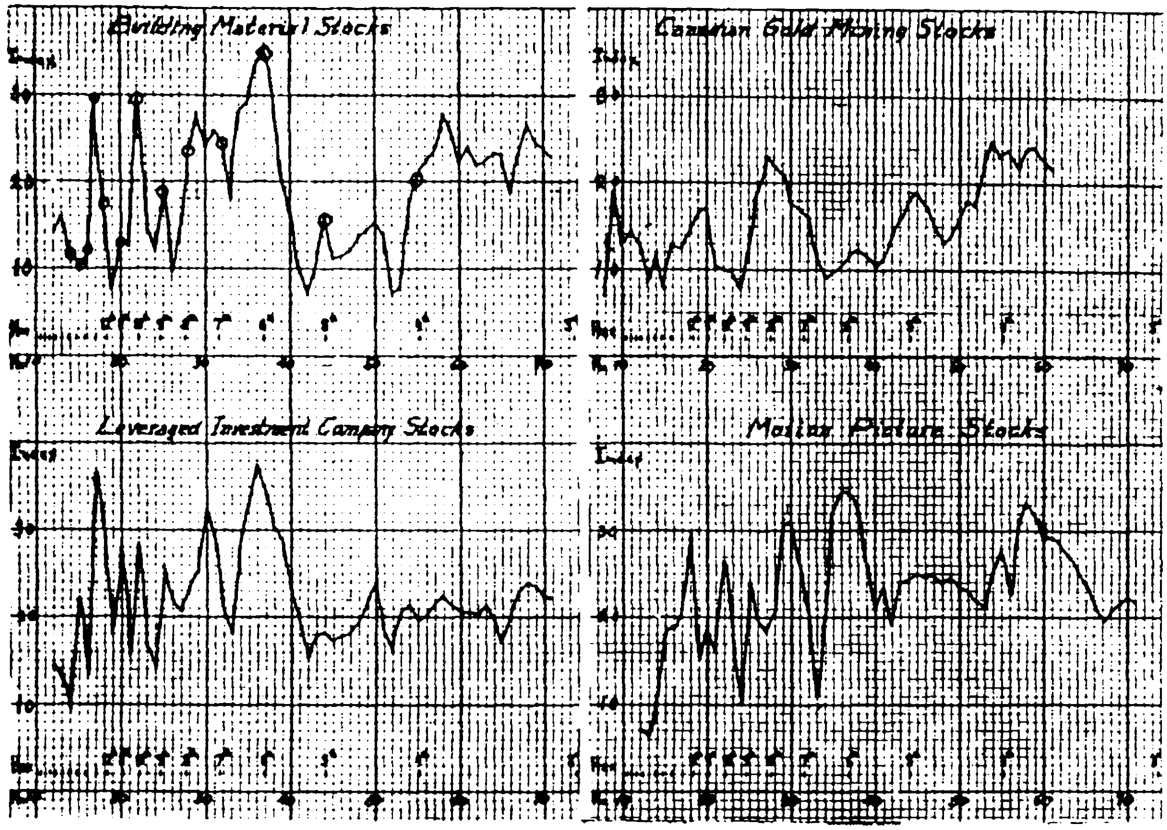


FIG. 1. PRELIMINARY PERIODOGRAM (DAVIS' DELTA (Δ) VALUES) FOR GROUPS OF STOCKS AS INDICATED. JAN. 1932 - MAY 1950. PEAKS INDICATE POSSIBLE CYCLES AT OR NEAR THE LENGTH IN MONTHS AS INDICATED. NOTE THAT THE CYCLES SUGGESTED DIFFER IMPORTANTLY FROM GROUP TO GROUP. CHARTS REPRESENT WORK OF E. S. C. COPPOCK OF SAN ANTONIO, TEXAS, AND ARE REPRODUCED THROUGH HIS COURTESY.

What all this adds up to is that the traders in different kinds of stock do not all behave the same way at the same time.

In other words, stock prices would seem not to be an index of mass psychology. The most that one can say is that the prices of groups of stocks may represent the intellectual and emotional attitudes of the traders in those particular groups.

Again, it might reasonably be supposed that cycles in the prices of stocks of companies or groups of companies corresponded to the cycles in the profits of those companies. But people who have studied the subject tell me that stock prices vary much more directly with the sales than with the earnings of the companies involved. I therefore imagine that we have here another example of something that "ought" to be so, but is not.

From the above observations I assume that the cycles we find in the prices of stocks in different companies and different industries probably corresponds to the cycles of the sales of those companies or industries, but I do not know this.

I make these comments chiefly to provide suggestions for future lines of research that you may find fruitful.

For present purposes what is important is the fact that different kinds of stocks show different cycles, or the same cycles with different amplitudes, or with different timing. Because of this fact any

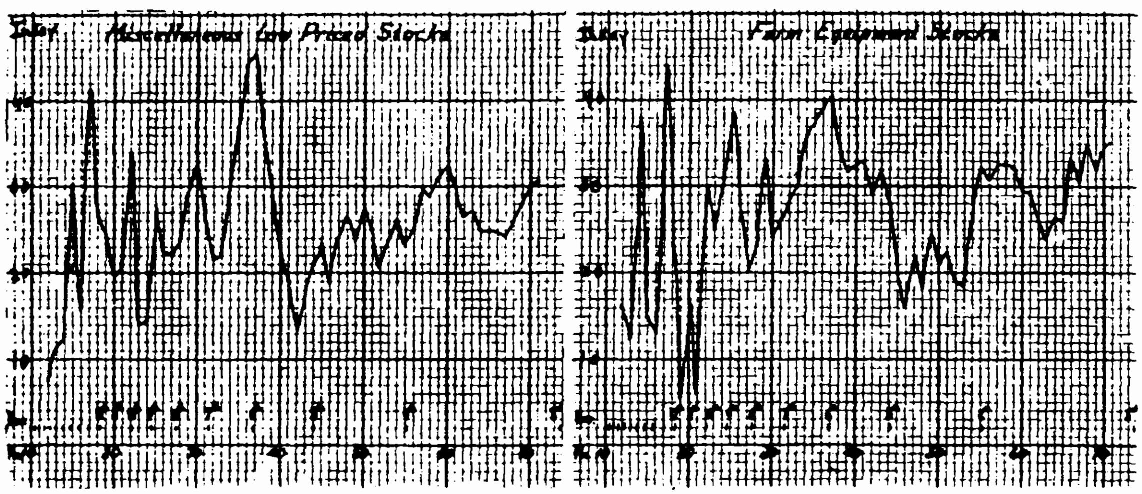


FIG. 1. CONTINUED

long series of figures representing the behavior of the stock market as a whole cannot be regarded as homogeneous.

**Stock Market Averages not Homogeneous**

This lack of homogeneity is due to the fact that stocks of new industries have been added from time to time--stocks which may be characterized by different cycles, or the same cycles coming a little earlier or a little later.

To show you this lack of homogeneity, I reproduce below as Fig. 2 a chart from **Common-Stock Indexes** by Alfred Cowles 3rd and Associates which shows the dates for which prices of the various kinds of stocks are introduced into the Cowles Commission—Standard Statistics Index. Roughly half of the industries go back no further than 1910.

You can easily see how confused the cycles in the overall index could be with new stocks, some of which may have different cycles, coming into the picture all the time. To analyze such a series is analagous to trying to make a chemical analysis of a vat of liquid into which new substances are constantly being introduced.

On the other hand there is a partially compensating advantage in dealing with a general index of all stocks. This advantage is the fact that in such an index the accidental variations of individual companies and individual industries are partly offset and thus minimized. However this advantage does not nearly offset the disadvantage of lack of homogeneity referred to above. **Any work on a combined index which extends over many years should be thought of as merely exploratory and preliminary.**

In my studies of the cycles in the stock market I did some work on the Dow Jones Industrials, but the greater part of the work was on the Cowles Commission Index P-2, Industrials, available by months from January 1871 forward.

**The Cowles Commission Index**

I chose the Cowles Commission Index for a number of reasons, some of which are contained in the book **Common-Stock Indexes** referred to above, in a section entitled "Criticism of Well Known Indexes." In addition, at least some of the cycles appear more clearly and more regularly in the Cowles Index. It was this latter fact that really caused the decision to use the Cowles Index.

Fig. 3 which, like Fig. 2, is reproduced with permission of the Cowles Commission from **Common-Stock Indexes**, shows you the differences between three of the best known indexes. The greatest difference is between the Cowles Index and the Dow-Jones Index, but this difference would be greatly reduced if the Dow-Jones figures were adjusted for the drastic change in the index made during 1914. This adjustment is of course always made by careful

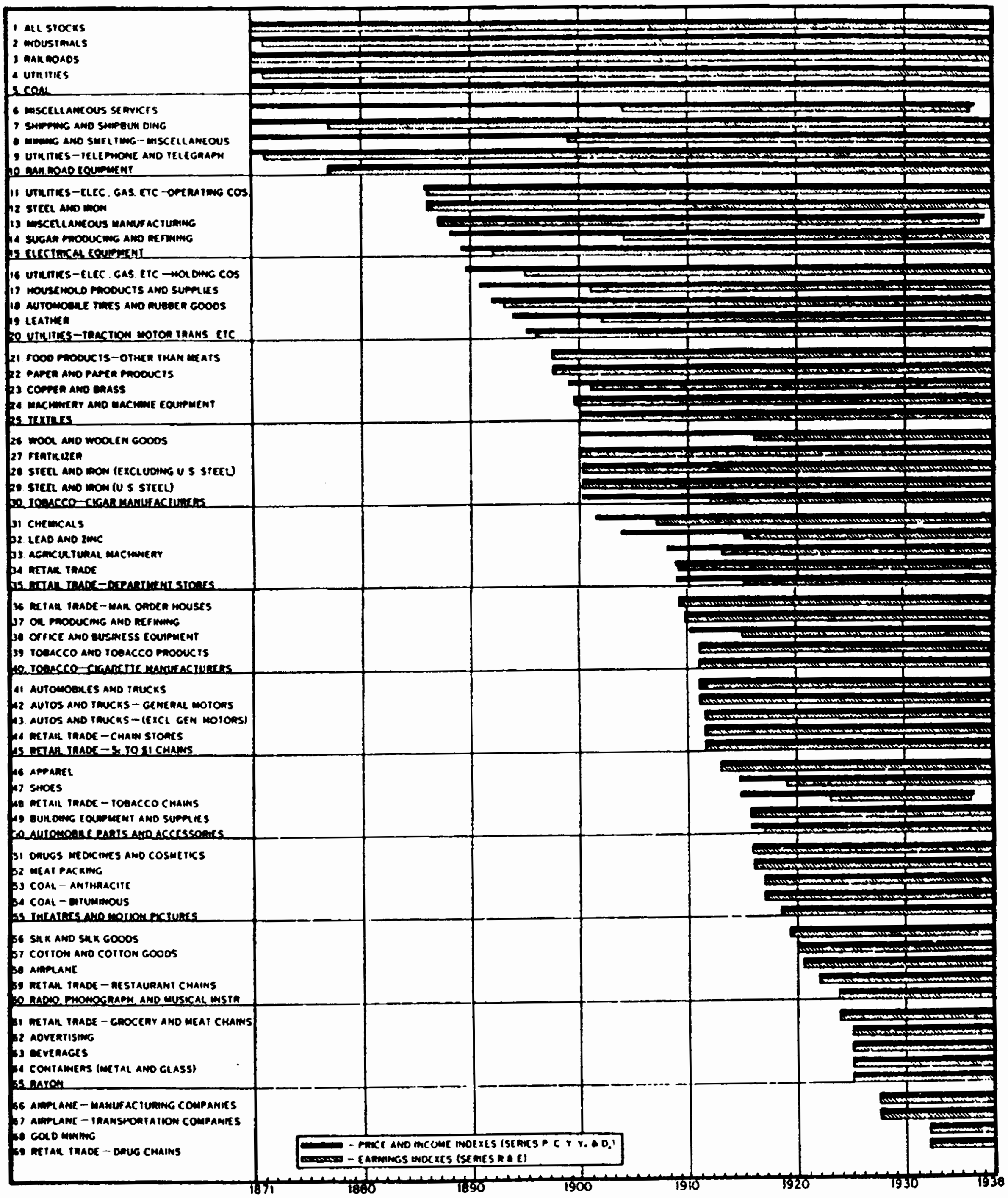



Fig.2. Periods covered by various Cowles Commission Indexes, 1871-1938. From **Common-Stock Indexes** by Alfred Cowles 3rd and Associates, Second Edition, 1939, reproduced with permission.
This chart gives dramatic illustration of the fact that no representative index of common stock prices, extending over a long period of time, can possibly be homogeneous.

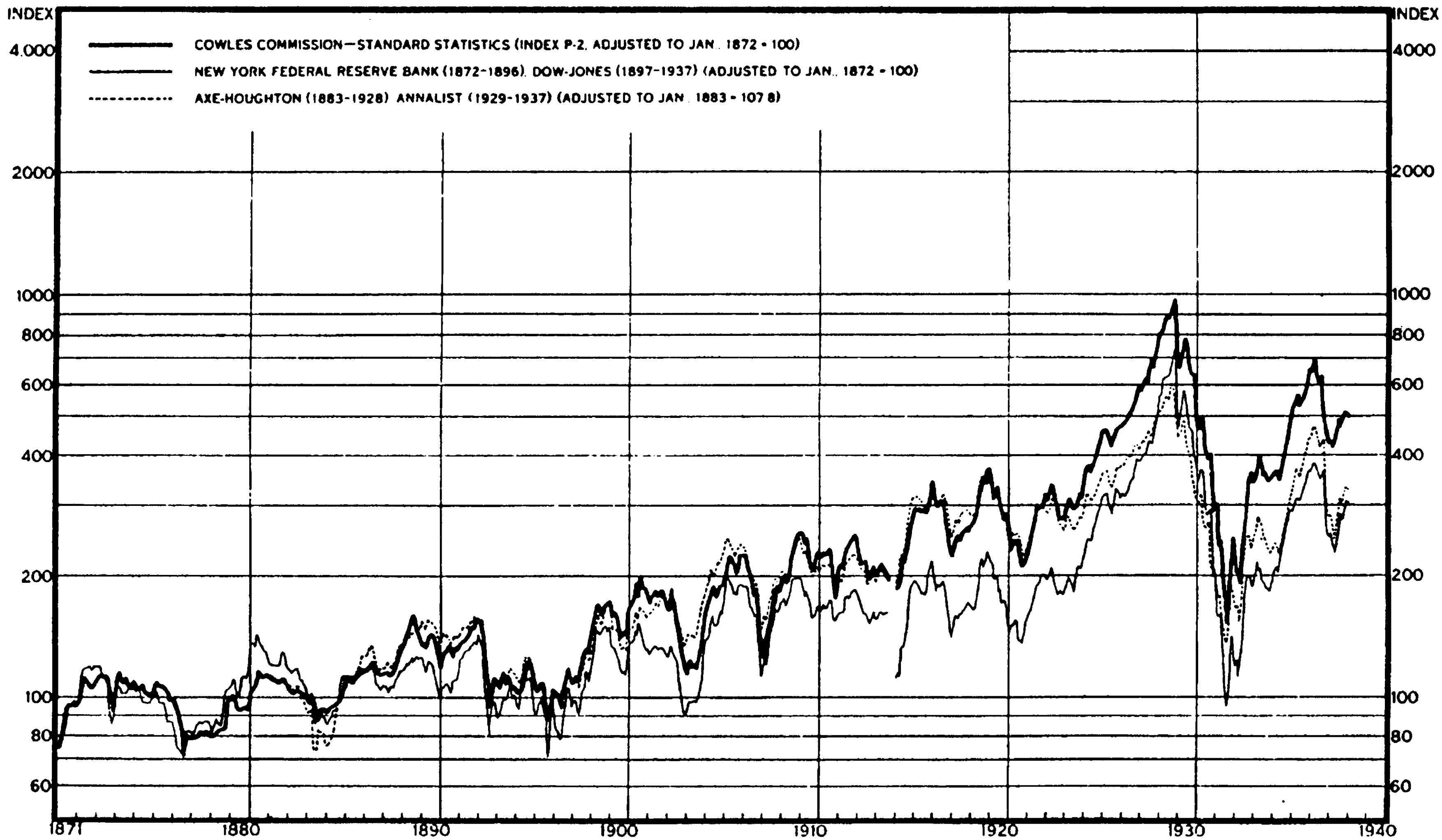



Fig.3. Comparison of Various Industrial Stock Price Indexes, 1871, 1938. From **Common-Stock Indexes**, by Alfred Cowles 3rd and Associates, Second Edition, 1939, reproduced with permission.
If adjustment is made for the drastic 1914 revision of the Dow-Jones index, as is always done by careful students, the Dow-Jones index and the Cowles index become much closer.

students. It eliminates the discontinuity of the curve between July 1914 when the exchange closed, and December 1914 when the exchange reopened.

As you doubtless know, the Cowles Commission Index ties in with the Standard Statistics weekly stock price indexes available from 1918 forward. (This index is now known as the Standard & Poor's Corporation index.)

In this connection, a word of warning is in order, because several years ago the Standard weekly indexes and their monthly averages were changed from a base of 1926=100 to a base of 1935-39=100. This means that the current values as you read them in the newspaper and in the **Survey of Current Business** are not directly comparable with the indexes as published previously, in **Common-Stock Indexes**, for example. It is a small matter to make the necessary adjustments, but it is important to realize that they need to be made.

I said at the beginning that my work on the stock market was of a preliminary nature. This is so not only because I worked with a combined index, but because I did not have time to do the work to make sure that the indicated cycles are **rhythmic** or regularly recurring. I found that a certain cycle was present **on the average** in the series as a whole and in each of its two halves or even in each third or each quarter. But this is very different from making sure that the waves repeat one by one time after time.

For example, if, for many years, I have put $300 in my savings account once every three years, this is very definitely an average of $100 a year, but no matter how well established the custom—and even if the behavior continues,—it does not indicate that I will deposit $100 next year and $100 the year after. Beware of mathematical averages unless you know what goes into them. Only when we have **rhythms** do we have any assurance of continuation.

When a cycle is present **on the average**, and especially when it is present on the average in each half or each third or each fourth of the series of figures, it provides you with a good **lead** or **clue** which you can then verify in other ways. But you should not use the information until you have verified the repetitive or rhythmic nature of the waves.

Perhaps this is enough in the way of preliminary explanation.

**Preliminary results of analysis**

What did I find as a result of my stock market work?

I found very definite hints of waves of many different lengths.

Some of the more important and more probable cycles have lengths as follows:

| | | | | |
|---|---|---|---|---|
| 2.975 | months | | | |
| 8.00 | " | | | |
| 8.80 | " | | | |
| 10.95 | " | | | |
| 14.40 | " | | | |
| 17.17 | " | | | |
| 18.17 | " | | | |
| 24.25 | " | | | |
| 29.28 | " | | | |
| 31.0 | " | | | |
| 35.5 | " | | | |
| 40.8 | " | or | 3.40 | years |
| 45.5 | " | " | 3.79 | " |
| 58.7 | " | " | 4.89 | " |
| 66.0 | " | " | 5.50 | " |
| 72.8 | " | " | 6.07 | " |
| 82.3 | " | " | 6.86 | " |
| 98.0 | " | " | 8.17 | " |
| 110.4 | " | " | 9.2 | " |
| 132.0 | " | " | 11.0 | " |
| 144.0 | " | " | 12.0 | " |
| 174.0 | " | " | 14.5 | " |
| 252.0 | " | " | 21.0 | " |

I hope I have impressed upon you the preliminary nature of this work. These are not exact lengths of discovered and confirmed cycles. They are merely areas where further work might be fruitful.

Each of these indicated lengths should be thoroughly investigated and the cycles either confirmed and accepted as real, modified, or discarded.

I shall tell you more about each of these cycles in future reports. I shall tell you the dates of crest and trough, the shape of the waves, the amplitude of the waves, and the techniques involved in the isolation.

I shall also tell you about many other cycles which seemed to be present, but about which I had greater question.

# Cycles in the Stock Market

THROUGH the great kindness of one of our members who has supplied the necessary money, and of the Mico Instrument Company of Cambridge, Massachusetts who have permitted us to use one of their machines, we are going to make for you a complete multiple harmonic analysis of the monthly stock market figures from January 1871 to December 1950.

The technique of multiple harmonic analysis was described in our reports for October and November 1950.

The figures we are going to use are the Standard & Poor's Corporation Monthly Industrial Stock Price Index (365 stocks), carried back to January 1871 by the Cowles Commission for Research in Economics, and adjusted for trend. These figures and charts of them were shown in the Data Section of the April report on pages 146-149.

As I explained in the March report, all long series of composite industrial stock price figures are unsuitable for cycle analysis because different industries are included at different times and different industries behave in different ways.

For example, airplane manufacturing company stocks are not included in the index prior to June 1928. If airplane manufacturing companies have some cycles that are unique to them, these cycles will be in the figures from June 1928 but not before. This situation will make the analysis very difficult.

Even worse, if airplane manufacturing companies have standard cycles which come ahead of or after standard timing, this fact will throw off your calculations in regard to length. To make it simple, look at Fig. 1:

Curve A shows a regular 4-year cycle in which five cycles add up to 20 years.

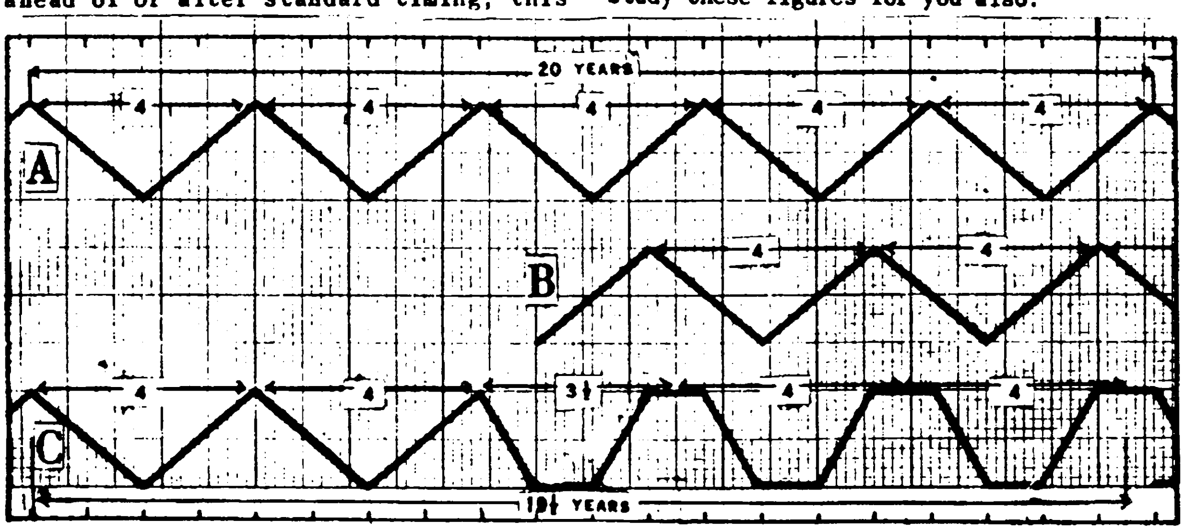
Curve B shows another regular 4-year cycle with crests and troughs which come a year ahead of the corresponding crests and troughs of Curve A.

In Curve C these two cycles have been combined. As B comes a year ahead of A, the crests and troughs in C will be halfway between, or one-half year ahead of A.

That is, whereas in Curve A five waves add up to 20 years, or an average of 4 years each, in Curve C five waves add up to 19½ years or 3.9 years each. Thus, what is really a 4-year wave in each series is made to look like a 3.9-year wave in the composite.

It is easy to see that a series of figures representing some one industry would be much better for cycle analysis than a series representing many industries combined. The only trouble is that if we limit ourselves to some one industry such as coal, the stock price figures for which are also available back to January 1871, we have something which is of only limited interest.

A compromise might be found in railroad stock prices. It would be these figures, in preference to industrials, which I would like to study if I could be governed exclusively by scientific considerations. If time and money permit I shall therefore study these figures for you also.



# INTRODUCTION

## Supplement 3

## ADJUSTING FOR INFLATION

As you know, the Double Deal (the New Deal plus the "Fair" Deal) diluted our currency so that it now takes two dollars to buy what used to be purchasable for one.

This fact may introduce lack of homogeneity in time series measured in dollars.

The techniques which will be discussed in this course pretty well overcome this difficulty for short cycles, but for long cycles it is often necessary to adjust price figures since 1932 to make them homogeneous with pre-1932 figures. This adjustment can be made in various ways. One way is to convert currency figures since 1932 to a gold basis.

With the thought in mind that you will occasionally want to make such adjustments, I am reprinting for you an article called "Prices 1933-1950 in Terms of Gold," which appeared in Cycles for June of 1951, pages 235-237, and two pages of correspondence in regard to this article which were printed in Cycles for September 1951, pages 267-268.

The values necessary to bring the article up to date are given below:

| Year | Black Market Price of Double Eagles in U. S. Paper Dollars, New York | Price of Double Eagles in Paper Dollars Divided by Twenty | Index of Average Wholesale Prices, all Commodities, in Paper 1926 = 100 | Index of Average Wholesale Prices, all Commodities, in Black Market Gold 1926 = 100 |
|---|---|---|---|---|
| 1951 | $49.35 | $2.47 | 180.4 | 73.0 |
| 1952 | 44.90 | 2.25 | *175.4 | 78.0 |
| 1953 | 42.50 | 2.13 | *173.0 | 81.2 |
| 1954 | 39.15 | 1.96 | *173.6 | 88.6 |
| 1955 | 39.00 | 1.95 | *174.2 | 89.3 |
| 1956 | 40.00 | 2.00 | *179.8 | 89.9 |

*Estimated prorated from the present index (1947 - 49 = 100)

Two points need to be made here:

(1) Stock market averages have, in my opinion, been affected by inflation, but I do not feel the above method is suitable for adjusting them. I have not yet worked out one that is.

(2) Although the adjustment can be applied rigidly to such things as tin prices, which are determined by world markets, it needs to be applied with judgement to things like pig iron prices, which, at least for the short run, are determined by purely national factors.

# Data

## PRICES 1933-1950 IN TERMS OF GOLD

STUDENTS of price cycles are confronted with a perplexing complication because our government in 1933 abolished free and unrestricted convertibility of currency into gold and later, in 1934, devalued our currency in terms of gold.

Of course we had been on a greenback basis before, from February 25, 1862 to January 1, 1879, but most long-term price series with which you come in contact have probably been adjusted to a gold basis for this relatively brief period of time.

I think it is fair to assume that any cycles discovered in figures from 1795 to 1932 (adjusted to gold when needed) are cycles of prices in terms of gold, and that any predictions based upon a knowledge of such cycles are predictions in terms of gold prices.

But the prices we read in the newspapers are not gold prices--they are prices in terms of " funny money" --prices in terms of inconvertible greenbacks.

How can we get on a " gold basis " for purposes of statistical homogeneity? That is, how can we convert our funny-money prices into prices as they would have been if we were still on the 1932 gold standard?

Probably there is no way in which this conversion can be made accurately, but one way of approaching the problem would be to divide the annual series you wish to convert by 1/20th of the corresponding New York City black market price of United

TABLE 2

| Year | A<br>Index of Average Wholesale Prices, All Commodities, in Paper Dollars | B<br>Open Market or Black Market Prices of 1/20 Double Eagles in U.S. Paper Dollars | C<br>Index of Average Wholesale Prices, All Commodities, in Gold (1926-32), or in Black Market Gold (1933-1950) (Col. A ÷ Col. B) |
|---|---|---|---|
| 1926 | 100.0 | $1.00 | 100.0 |
| 1927 | 95.4 | 1.00 | 95.4 |
| 1928 | 96.7 | 1.00 | 96.7 |
| 1929 | 95.3 | 1.00 | 95.3 |
| 1930 | 86.4 | 1.00 | 86.4 |
| 1931 | 73.0 | 1.00 | 73.0 |
| 1932 | 64.8 | 1.00 | 64.8 |
| 1933 | 65.9 | 1.31 | 50.5 |
| 1934 | 74.9 | 1.80 | 41.6 |
| 1935 | 80.0 | 1.74 | 46.0 |
| 1936 | 80.8 | 1.72 | 41.9 |
| 1937 | 86.3 | 1.73 | 50.0 |
| 1938 | 78.6 | 1.79 | 43.5 |
| 1939 | 77.1 | 1.81 | 42.7 |
| 1940 | 78.6 | 1.86 | 42.4 |
| 1941 | 87.3 | 1.73 | 50.4 |
| 1942 | 98.8 | 1.81 | 54.5 |
| 1943 | 103.1 | 1.81 | 57.1 |
| 1944 | 104.0 | 1.91 | 54.4 |
| 1945 | 105.8 | 2.03 | 52.1 |
| 1946 | 121.1 | 2.08 | 58.1 |
| 1947 | 152.1 | 2.16 | 70.3 |
| 1948 | 165.0 | 2.28 | 72.4 |
| 1949 | 155.0 | 2.24 | 69.3 |
| 1950 | 161.7 | 2.06 | 78.6 |

States double eagles ($20.00 gold pieces).

I am fortunate in being able to give you these black market prices through the very great courtesy of PICK'S WORLD CURRENCY REPORT, 75 West Street, New York City, a research organization specializing in statistical information on black, gray, and white market prices of currency and precious metals.

The prices are shown in Table 1.

Suppose, for an example, you apply this procedure to average wholesale prices for all commodities, as compiled by the Bureau of Labor Statistics in Washington. If you do this you will get the results shown in Table 2.

In Fig. 1 I have plotted for you the index in terms of black market gold as well as the index expressed in the usual way in funny-money inconvertible paper dollars.

Do not be misled into thinking that

TABLE 1.

| YEAR | BLACK MARKET PRICE OF DOUBLE EAGLES IN U.S. PAPER DOLLARS, NEW YORK | PRICE OF DOUBLE EAGLES IN PAPER DOLLARS, DIVIDED BY TWENTY |
|---|---|---|
| 1932 | $20.00 | $1.00 |
| 1933 | $26.10 | 1.31 |
| 1934 | 35.95 | 1.80 |
| 1935 | 34.73 | 1.74 |
| 1936 | 34.44 | 1.72 |
| 1937 | 34.54 | 1.73 |
| 1938 | 35.85 | 1.79 |
| 1939 | 36.13 | 1.81 |
| 1940 | 37.10 | 1.86 |
| 1941 | 34.69 | 1.73 |
| 1942 | 36.23 | 1.81 |
| 1943 | 36.10 | 1.81 |
| 1944 | 38.27 | 1.91 |
| 1945 | 40.65 | 2.03 |
| 1946 | 41.69 | 2.08 |
| 1947 | 43.27 | 2.16 |
| 1948 | 45.58 | 2.28 |
| 1949 | 44.75 | 2.24 |
| 1950 | 41.17 | 2.06 |

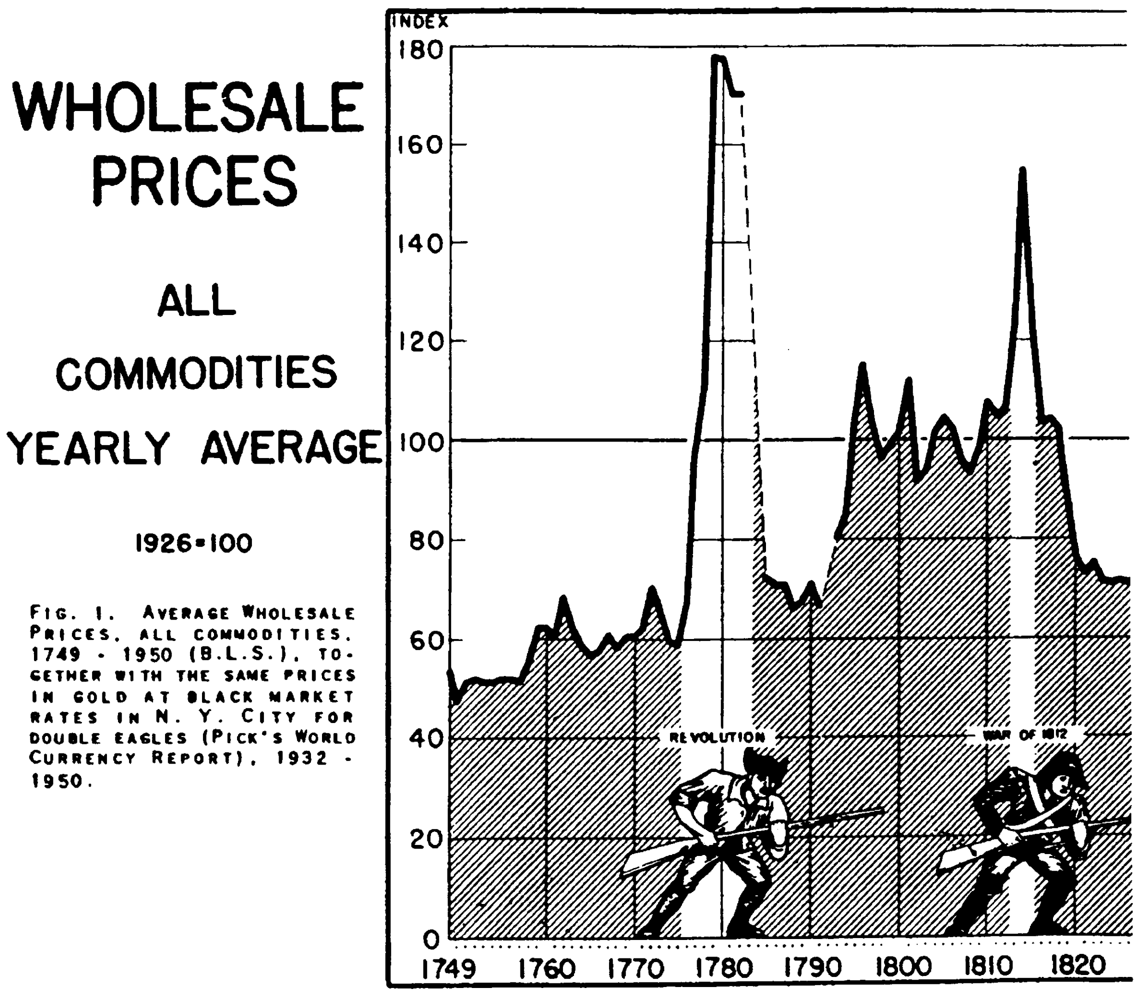



FIG. 1. AVERAGE WHOLESALE PRICES, ALL COMMODITIES, 1749 - 1950 (B.L.S.), TOGETHER WITH THE SAME PRICES IN GOLD AT BLACK MARKET RATES IN N. Y. CITY FOR DOUBLE EAGLES (PICK'S WORLD CURRENCY REPORT), 1932 - 1950.

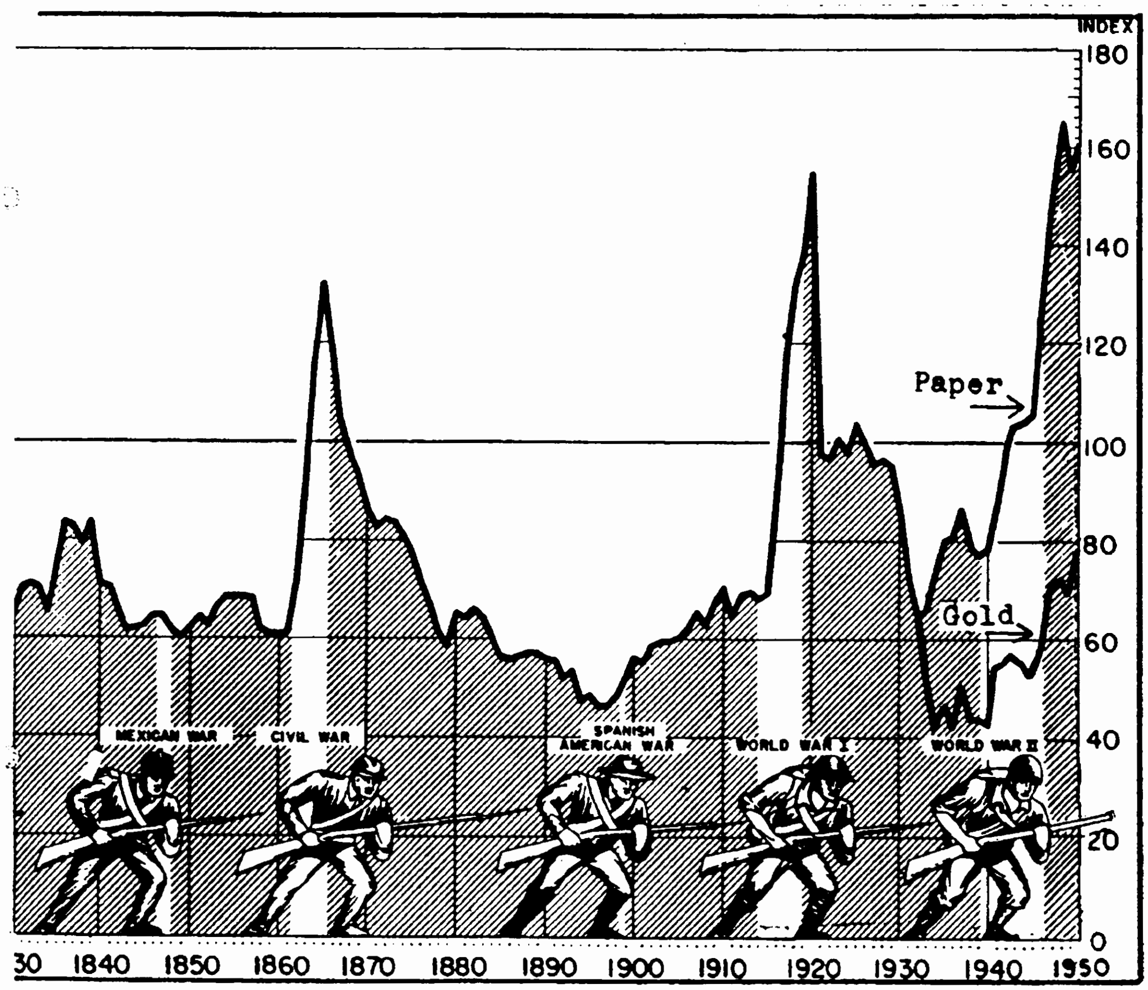
these average wholesale prices expressed in terms of black market gold are identical with the average wholesale prices in dollars as they would have been if we had never left the gold standard. As I said at the beginning, there is no way that I know of of making an exact determination of what these latter prices would have been. But I trust that as cycle students you will be helped by the expedient I have used to gain a truer knowledge of how the cycles are unfolding, and that these figures will also help you to make more exact determinations of the various cycles present in the commodity prices in which you are interested.

For example, when one looks at the average wholesale price curve converted to a gold basis one sees at once some preliminary evidence of a cycle of about 22 years in length with a low about 1940. Such a cycle would probably not appear from casual observation of the curve with prices from 1932 expressed in paper money, nor, in my opinion, could the exact characteristics of this cycle be as well determined from the paper money curve.

Similarly the price distortion of World War II and Post World War II appears in much more realistic proportions in the curve adjusted to a gold basis.

Every seller should, in my opinion, convert his present prices to a gold basis to see if, in terms of gold, his prices are above or below what they used to be. If this were done there would be some surprising revelations.

Table 1 provides a basis by which this conversion can be made. The next step is up to you! Many a company has cut its prices in terms of gold and is suffering from the pangs of financial malnutrition as a result. See to it that your company is not in this category.



You will be interested, I think, in a letter just received from Dr. Donald Thayer Bliss of the Naval Radiological Defense laboratory in San Francisco. Dr. Bliss' letter was written in response to an article in the June issue in which I attempted to chart average wholesale prices of all commodities from 1932 forward in terms of gold instead of in terms of paper dollars.

You will remember Dr. Bliss as the writer of a letter which appeared in the report for May 1951.

#### The Letter From Dr. Bliss

Dr. Bliss' letter follows:

Dear Mr. Dewey,

The June Report was most stimulating and intriguing; and that shall provide my excuse for intruding upon what must be, after all, a very busy Director.

A question lingers in my mind concerning the procedure adopted for converting 1933-1950 prices from paper to gold. It is my understanding that the possession of U. S. gold pieces is both illegal and a numismatist's ambition; hence, two diverse factors operate to elevate the price of "double eagles" above the "normal" level. The importance of the "double eagles'" numismatic interest may perhaps be gauged by the fact that raw gold was, if I remember correctly, being sold legally (because of certain loop-holes in the law) in this country at about $54 an ounce, only moderately above the established price. It seems to be possible, therefore, that a correction factor should be applied to the "double eagle" price index, a factor which would have the effect of reducing "double eagle" prices and of increasing "real" dollar prices. But, if a correction factor is necessary, what should be its magnitude?

I hope to be enlightened on this point, for if you can assure me that the numismatic correction factor is not large, I can hereafter with a clear conscience praise an Administration which displays so unique a concern for the monetary welfare of its citizens that it has graciously enabled them to possess more dollars by the simple and genial expedient of making dollars cost less.

Respectfully yours,
Donald Thayer Bliss

Berkeley, California

#### My Reply

In response I wrote as follows:

Dear Dr. Bliss:

You are absolutely right that a correction factor should be applied to the Double Eagle Price Index published in our June report, but I have reason to believe that this correction factor would not be large and that it is not enough to invalidate, for the cycle student, the figures I gave.

Before publishing my article I made extensive efforts to obtain the best possible figures. Among other things I attempted to obtain the price of gold in various foreign countries where free markets might be supposed to exist. Some of the difficulties in connection with free market prices are explained in a letter from Dr. O. Ernest Moore, Manager of the Research Department of the Federal Reserve Bank of New York.

#### Difficulties in Finding Free Markets

The gist of Mr. Moore's letter is as follows:

Prices for gold on the so-called free markets show widely divergent trends as between countries. The premia on gold, where they exist, reflect the monetary habits of the population, the efficiency and stability of the monetary system, and the degree of confidence in the national government, as well as the varying expectations as to social and international peace. The actual prices embodying these premia are quoted in local, inconvertible currencies. In the press and other published material, reference is often made to dollar quotations for gold on foreign markets, but these are merely computed figures arrived at by converting the local-currency price into dollars at such rates of exchange as may be available. For most local-currency quotations, different dollar equivalents can be obtained by converting the local currency price at different exchange rates (black market, official market, free market, etc.) and it is often difficult or impossible to know which rate should be used in calculating a realistic dollar equivalent. In those instances where the seller of gold wishes to make an actual conversion of his local currency proceeds into dollars, and is able to do so (such conversions being usually prohibited by law), the conversion is ordinarilly made through free or black markets at rates which are usually at a substantial discount from the official rates of exchange.

In continuing my investigations I received a number of letters on the subject from the various friends and members of the Foundation. Two of these letters follow:

#### Hong Kong Prices

"I regret the long delay in getting you the range of gold quotations you wanted. From Hong Kong all I have so far is the 1950 range, $44 3/4 high (December 29th), $36 7/8 low (June 9th) per Troy ounce of gold in Macao. I have written to London and hope to get more data...... .

"Expressed in Hong Kong dollars gold made a low there of 249 1/4 per tael (.945 fine) on June

9th and 329 1/2 on August 2nd and 3rd. The exchange rate for the Hong Kong dollar with ours made a high of 16.78 cents on December 29th (it went as high as 16.77 on June 6th) and a low of 15.60 on January 31st. The latest I have from Hong Kong is for April 28th when gold (Macao) was $45 (U.S.) per ounce, and $321 (H.K.) per tael locally with the Hong Kong dollar at 16.22 cents. One tael is 1.203 Troy ounces."

**European Prices**

" From a large firm of bullion brokers in London, I have obtained the following estimated range of gold prices since World War II.

| | 1946 | 1947 | 1948 | 1949 | 1950 |
|---|---|---|---|---|---|
| High | 38 | 45 | 47 | 52 | 40 1/2 |
| Low | 35 | 35 3/8 | 43 | 40 | 36 1/2 |

" These are prices in dollars per Troy ounce. no one in London, it appears, has kept a daily record of gold prices, and they have no record before 1946. These prices are for the gold in the main European centers, which probably show narrower fluctuations than in Hong Kong and other Eastern cities. I believe they are reliable.

" Before 1946 most of the present gold trading centers were in territory of belligerents and I do not think that quotations having any meaning could be obtained for the war years. From 1939 back to 1932 the United States Treasury's price can fairly be used.

" The South Africans recently announced that they were not selling more than 40% of their production in free markets for fear of breaking the price."

As you know, before devaluation (January 31, 1934) the value of gold was $20.67 per fine troy ounce. The present legal value is $35.00 per fine troy ounce.

**Black Market Prices**
**Probably Adequate for Cycle Students**

Taking all of these factors into account I think that for purposes of cycle study we can accept as adequate the black market prices of double eagles in New York as furnished by Pick's World Currency Report, 75 West Street, New York City, which were published in the June report. In other words, I believe the enhanced value of double eagles due to the illegality of their possession and due to their interest for coin collectors to be of moderate proportions.

At all events the prices given are vastly closer to the " true " prices than the prices in greenbacks which we are accustomed to read in the newspapers, and they are the best I am able to give you. If you ever come across better figures I would appreciate it if you would let me know.

**Legalized Robbery**

I enjoyed the last paragraph of your letter. Some people express the same idea more directly. For example, Dr. Walter F. Spahr, writing in *Monetary Notes* for June 1, 1951, wrote as follows:

" Apropos the looting of the savers in this country through depreciation of our currency, this author pointed out the following in an address in Washington, D. D., May 10, as an example of the state of affairs that prevails in this country:

" 'Perhaps one illustration will suffice to show the subtle aspect of the weakening process inherent in an irredeemable currency, and also how the public fails to react strongly because of lack of understanding of this process:

" 'The total loss, because of a depreciated dollar, on the average value of life insurance policies, time deposits in banks, and E, F, and G savings bonds for the years 1941-1950, in 1950 dollars as compared with 1941 dollars, amounted to $116,565,524,000. This huge loss, lightly regarded because so poorly understood, stands in sharp contrast to the officially estimated total loss of $1,901,000,000 by depositors in suspended banks during the years 1921-1933. Regarding the latter loss, extending over thirteen years, and which is only 1/61 of that over ten years on the three items mentioned, we still write and speak with emotion for the reason, apparently, that the meaning of that loss was brought home to us in a manner we could understand. But regarding a loss more than 61 times greater, on only the three items specified, we offer in general little more than platitudinous observations that reveal our small understanding of the devastating effects of a depreciating currency.' "

Very cordially yours,
Edward R. Dewey
Director

# INTRODUCTION

## Supplement 4

## HANDLING REVISED DATA

Question: . . . quite a number of the stocks listed on the New York Stock Exchange have been split. . . when this happens it seems to be the practice to go back and remake the chart taking into consideration the new ratio. For instance, when a stock is split 2 for 1 they go back and halve the data and make a new chart. What does this do to our cycle (analysis). Do we have to go back and re-calculate all the work, . . . or can we continue with the same data as previously used but make an appropriate adjustment to the current prices?"

Answer: You are right that it is customary to revise the previous figures, charts, etc., so that they will be directly comparable with current data.

It is a scurvy chore, but it is often more efficient (over the long haul) to have your work in terms of current data.

Of course you use your own good sense about how much of the figure work needs to be done over (if any).

For instance, if you have an analysis that you consider to be relatively complete, all you would bother to change would be the very final column - the antilogs of the synthesis.

On the other hand you might be part way through an analysis, and decide to handle it this way - translate the previous results to the current basis, and continue the work on the current basis.

That is, if you have (for example) four waves of which you are confident, you can apply the per cent amplitude of these waves to the new figures.

And, finally, you could make no change at all to any of the work, but put the current figures onto the old basis.

As for the actual adjustment you would make - if a stock selling for 52 split 2 for 1 and opened at 37 the new price on the old basis would be (37 times 2) or 74. Thus the data would jump 52 to 74. On the current basis the data would jump 26 to 37.

# INTRODUCTION

## Supplement 5

## OBTAINING DATA

Question: "I would like to secure daily stock prices for the last five years of a few specific stocks listed on the New York Stock Exchange. Can you tell me whether these are available, and if so, from what source?"

Answer: I do not know of any regularly published service which lists records of daily prices for specific stocks.

There may be services which would tabulate this information for you - at a price.

I called a couple of local (Pittsburgh) brokerage houses to find out if they kept such records. They keep records back about six months. If they want a price prior to their record they look it up.

The local Standard and Poor's office had no records but suggested writing the New York office of the Wall Street Journal.

The next step would be to inquire at the nearest library with a daily Wall Street Journal or New York Times file. They may have someone on the staff who would do the research for you.

One point - if you use daily figures for five years you will have 1300 figures for each series. This is a terrific volume of figures to handle. Why not use weekly figures, cover a longer period of time, and have fewer figures to handle? Weekly figures are available in Barron's.

Another point - when you are developing records to use in an analysis it is usually wisest to start with the latest figure and work backwards.

# LESSON I

## CHARTS
## AND HOW TO USE THEM IN CYCLE ANALYSIS

You almost always start a cycle analysis by charting the data.

Therefore, your first step in analyzing the stock market figures sent you with the Introduction is to plot them. The necessary paper is enclosed.

There are, however, some preliminaries. Before plotting the stock market figures I want you to know everything you need to know about charts, charting, and chart paper. More particularly I want you to--

1. Learn how to plot on ratio ruling.
2. Learn how to change scale on ratio ruling.
3. Learn the effect of plotting on ratio ruling vs. plotting on arithmetic ruling.
4. Learn when to use ratio ruling in cycle analysis charting and when not to use it.
5. Learn standard practice in regard to charting.

When you are master of the five points named above you are ready to plot the stock market figures and go on to Lesson II.

To teach you this, or make sure there are no gaps in your knowledge in case you already know it, I want you to study (a) this lesson, (b) an assignment in the text (this book is being sent you under separate cover) and (c) the standard practice instructions.

### Charts

You probably learned about charts and charting in grammar school.

There are many sorts of charts, but this discussion will be limited to charts used by the cycle analyst. These are chiefly line charts of time series.

In such charts <u>time</u> is indicated on the horizontal scale left to right. Values corresponding to the time are indicated on the vertical scale, bottom to top.

Time is <u>always</u> indicated on a uniform scale--for example, twice as much time is always represented by twice as much horizontal distance. For cycle analysis, using daily data, a week is always represented as seven times as long as a day.

Value (in points, numbers, tons, dollars, percents, or whatever) is indicated by the vertical scale. This vertical scale <u>may</u> be a scale with uniform intervals.

When it is, the scale is called an arithmetic scale. On an arithmetic scale, equal vertical intervals are used to represent equal absolute values. For a sample of paper with arithmetic scale see No. 3111 or No. 4111 enclosed.

For an example of a line chart on an arithmetic grid, see Chart 7-7 on page 139 of the text.

When plotting on an arithmetic scale the zero should almost always be shown.

Sometimes the fluctuations are so small that you could not see them adequately if you plotted on a chart running uniformly up from zero. Even so, you must show the zero, but the chart may be divided by a broken line. To see how this is done see Chart 13-2 on page 274 of the text.

The only time you do not need to show the zero when you use arithmetic scale is when you are showing percentage variation from trend. For an example of this sort see Chart 14-4 on page 301 of the text.

Ratio Charts

Much more useful for you as a cycle student--nay, almost indispensable (unless you use logs)--are charts with the vertical scale on ratio ruling. When you use such a scale equal vertical distances represent equal relative amounts (not equal absolute amounts). On such a scale the distance between 3 and 9 for example is the same as the distance between 1 and 3. This is so because the ratio in each case is the same. Nine is three times three; three is three times one.

A chart with vertical ruling of this sort is called a ratio chart, or a semi-logarithmic chart. It is often called a logarithmic or log chart although this is not quite correct.

(Paper with ratio ruling both ways is called log-log paper, but such paper is not used in cycle analysis.)

For a simple ratio ruling see paper No. 3134 or No. 4135 enclosed. A line chart using such a scale is illustrated in Chart 8-18 on page 169 of the text.

Now pick up a sheet of No. 3134 paper. It is one cycle ratio paper.

The horizontal scale is divided into equal arithmetic intervals, 12 per inch.

The vertical scale has no zero. It does, however, have heavy accented lines numbered, from bottom to top, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1. The top line really should be called 10. These heavy lines get closer and closer together as you go from bottom to top. There are not-so-heavy lines half way between the whole numbers.

This whole interval from 1 to 1 (or 10) is called a cycle.

Note that there are twenty subdivisions between 1 and 2; 1.1 would therefore be plotted two lines up from the bottom.

Note that there are also 20 subdivisions between 2 and 3, 3 and 4, and 4 and 5. However, by now the lines have got so close together that there is no longer room for 20 subdivisions.

So the makers of the paper put in only 10 subdivisions from here on up. Therefore 5.1 would be plotted 1 line above 5.

## Changing Scale on Ratio Charts

If you want to change the scale on ratio paper you do it by multiplication.

As set up, the scale runs from 1 to 10. You can multiply (or divide) each of these 10 numbers by any positive number. That is, you can multiply every value on the scale by 2 so that the scale will read 2, 4, 6, 8, 10, etc. to 20. You multiply by 30 so that it will read 30, 60, 90, 120, etc. to 300. You can divide by 10 so that the scale will run from 1/10 up to 10/10 or 1.

But you must never, never, never change a ratio scale by addition, adding 1 to 1 to make 2; 1 to 2 to make 3; 1 to 3 to make 4, etc.

## Ratio Charts of More Than One Cycle

If you were to paste two sheets of No. 3134 paper together, one above the other, so that the bottom 1 line of the one coincided with the top 1 line of the other, you would have some 2 cycle paper.

In this paper every position in the upper cycle would have a value just ten times the value of the corresponding position in the lower cycle. That is, 3 in the upper cycle would be 30, etc. The lower cycle would run from 1 to 10, the upper cycle would run from 10 to 100. If you pasted on another piece you would have three cycle paper. The third cycle values would run from 100 to 1,000, etc.

A sample of two cycle paper is enclosed. It is No. 3135. No. 3174 is a sample of four cycle paper.

You can of course change the scale on two cycle or three cycle or four cycle paper by multiplication also. Only whatever changes you make must be made for the entire scale. The values of each cycle are always ten times the values of the cycle below it.

In connection with the two cycle paper enclosed, note that the twenty subdivisions extend up only to 3. From 3 to 6 there are 10 subdivisions between the numbers from 6 to 1 (10) there are only 5. You always have to watch with log (ratio) paper to take account of these changing subdivisions.

The only other place where, in using log paper, you are likely to go wrong is this. Refer again to paper 3135. Suppose you have multiplied the scale by 10 so that the lower cycle represents 10 to 100, the upper cycle 100 to 1,000. You have plotted 98, one line below the 100 line in the middle of the page. Your next value is 102. It is not the next line up. This is why: The line next above called 2 represents 200--a value a whole 1 or higher. The twenty subdivisions between lines 1 (100) and 2 (200) thus each represent 5. The value 102 would thus be plotted 2/5 of a space above the center 1 (100) line.

Two Things to Keep in Mind

The two things you have to keep in mind in using log paper are therefore: (1) change scale by multiplication only and (2) watch out for the changing numbers of subdivision lines.

The Effect of Plotting on Ratio Ruling

You will get a pretty good idea of the effect of plotting on ratio vs. arithmetic ruling by working out the problems which go with this lesson.

The chief reason why, for cycle analysis, you have to use ratio ruling (or logarithms of the data on arithmetic paper, which is the same thing) is that, for the most part, cycles express themselves percentage-wise. For example, a certain business has a summer (July) peak of about 15% of the yearly average. This was true when the business was small; it is still true now that it is big. If you plot the monthly sales on arithmetic ruling when the business was small the summer peak will be too small to see. On ratio ruling all summer peaks will be the same size. What is true for the 12-month or seasonal cycle holds true for cycles of other lengths also.

A second reason for using ratio ruling (or logs of the data on arithmetic ruling) is this: Cycles are almost always isolated as percentages. But when we come to project percentages the questions arise, "What is the projection in dollars (or tons, points, index numbers, or whatever)?" To get dollars (or whatever) you are going to have to project the cycle in percent and the trend in dollars and multiply the trend by the values of the cycle.

In the example given above, to get dollar sales next summer you are going to have to say, "The cycle calls for July 15% above the yearly average. I estimate next year's yearly average at $100,000 a month. The cycle and trend together will therefore give us $115,000 for next July, if my guess of the yearly average is right and if the cycle continues."

Now this is not the place to get into trend projection. This is the place, however, to tell you that trend (smoothly changing values from year to year) can be estimated much better from values plotted on log charts (ratio ruling) than from values plotted on arithmetic charts. Thus, you have your second reason for using ratio ruling in cycle analysis.

When to Use Ratio Charts in Cycle Analysis

You always use ratio ruling for preliminary reconnaissance of (a) data containing trend or (b) data characterized by large fluctuations.

You can, of course, tell from the figures themselves the amount of trend or the amount of fluctuation involved.

If there is no trend and if the fluctuations are not large you would use arithmetic paper.

How big is large? Offhand I would say fluctuations as big as 20% up and down from the average of the group requires log paper. Fluctuations less than this can be plotted on arithmetic paper if there is no trend.

For serious work in cycle analysis, however, you will omit the use of log paper altogether. You will convert your data at once to logarithms and plot the logarithms on arithmetic ruling. You do this for two reasons: One, for ease of computation, and two, for flexibility in scale. (When you plot on log paper you have to use the scale provided by the manufacturers of the paper. When you plot the logs of the data on arithmetic paper you can choose any scale you like.)

* * *

All of this will become more clear after you study the assignment in the text and work the problems.

<u>Assignment</u>

1. Read Chapters 7 and 8 in the text <u>Business and Economic Statistics</u> by Spurr, Kellogg and Smith. This book is being sent to you under separate cover.

2. Study the section on "Ratio Charts" which begins at the bottom of page 158.

3. Solve the problems for Lesson I and send your answers to Miss Shirk. In doing this, please follow these instructions:

    a. Be sure to put your name in the upper left hand corner of every piece of paper you send.

    b. Title all papers and charts. Every piece of paper should have a name, telling what is on the page, and/or what work is done on that page. This should be at the top, centered, and separated from the body of the work on the page.

    c. Keep the work legible.

    d. Always use the problem number in connection with your answer.

    e. <u>Write as soon as you meet ANY</u> work which is not clear to you. Ask questions as often as necessary.

4. For each problem tell me how long it takes you. Also please tell me how long it takes you to cover (a) the lesson, and (b) the assignment.

5. Do you want assignments to be (a) longer (b) shorter?

6. Do you want problems to be (a) harder (b) easier?

## Lesson I - Problems

**Problem 1.** Compute the successive differences and the successive percentage differences of the following numbers and plot the numbers on both arithmetic and one cycle log paper.

a. Numbers Which Increase by a Constant Amount

| A<br>Number | B<br>One Item Moving Difference | C<br>One Item Moving Percent |
|---|---|---|
| 1.0 | | |
| 1.5 | .5 | 50 |
| 2.0 | .5 | 33 |
| 2.5 | etc. | etc. |
| 3.0 | | |
| 3.5 | | |
| 4.0 | | |
| 4.5 | | |

b. Numbers Which Increase at a Constant Rate

| A<br>Numbers | B<br>One Item Moving Difference | C<br>One Item Moving Percent |
|---|---|---|
| 1.00 | . | |
| 1.20 | .20 | 20 |
| 1.44 | .24 | 20 |
| 1.73 | etc. | etc. |
| 2.07 | | |
| 2.49 | | |
| 2.99 | | |
| 3.59 | | |

**Problem 2,** Plot the following data on arithmetic paper and on two cycle log paper.

Number of Pairs of Shoes Sold by General Store

| Month | Numbers of Pairs Sold 1952 | 1953 |
|---|---|---|
| January | 91 | 67 |
| February | 83 | 55 |
| March | 216 | 101 |
| April | 532 | 312 |
| May | 324 | 156 |
| June | 105 | 55 |
| July | 54 | 27 |
| August | 103 | 51 |
| September | 714 | 253 |
| October | 638 | 209 |
| November | 229 | 99 |
| December | 113 | 84 |

Problem 3. Make up a series of about ten figures and include the values 2.5 and 8,000 in your series. Plot the series on both arithmetic and log paper with a suitable number of cycles.

Problem 4. Plot this series of percentages on both arithmetic and one cycle log paper.

| | |
|---|---|
| 103.3 | 98.3 |
| 102.1 | 99.9 |
| 100.6 | 100.2 |
| 99.5 | 101.7 |
| 97.4 | 102.7 |

Problem 5. Plot the stock price figures given in Data, Page 1 on a sheet of 11" by 17" two cycle log paper and on an 11" by 17" sheet of a arithmetic paper.

On both charts for the time scale across the bottom give each space the value of one year. For convenience, in this instance, plot the data on the vertical lines rather than in the space between. Have the first vertical line represent 1850. This will place the 1854 value on the fifth vertical line.

For the arithmetic chart, choose your own vertical scale.

For the log chart use the first cycle 10 to 100, the second cycle 100 to 1,000.

## STANDARD PRACTICE INSTRUCTIONS

### 2. CHARTING

1. Always write on each chart the name of the time series involved. For example, Standard and Poor's Corporation Combined Index, Annual, 1935-39 = 100.

If a chart is more than one page long, put the name of the time series on each page.

2. Under the name of the time series put an exact description of what is being charted. For example, "Deviations of the data from their 9-year moving average trend."

3. Always code and number each chart for ready reference. If the chart extends over more than one page, use the same chart number for all pages, but add page numbers. Put code and chart number (and page number, if any) in upper right hand corner. For example, S. & P. C. A.--1, etc.

4. Always initial and date the chart using the upper left hand corner for the purpose.

5. Always put the source of your data on the chart. For example, Tabulation Sheet 1, Column G.

6. Before plotting figures enter number and name of chart onto index sheet.

7. Always check plotting and mark it "Ck'd" with an initial and date.

8. Plot points in the center of the time period involved if the data cover a period of time. Any departure from this practice should be noted on the chart.

9. Mark all scales clearly. Be sure to note what the scale is, (such as dollars and tons) as well as the numerical values of the scale.

10. If there are more than one line on a chart, be sure to label them.

# LESSON I

## Supplement 1

## INTERPOLATION OF DATA

Question from one of the students:

"Lesson I states, 'Time is always indicated on a uniform scale. Values for Saturdays and Sundays must be estimated and interpolated.' I understand from the statement that this is so. But I wonder why. Why would it not be better, say, in the case of the stock market figures to leave Saturdays and Sundays (and possibly all night-time hours as well) out of the time scale. It would seem to me that since the data are based only on the working hours of the Exchange, it would be much more accurate (and possibly even quite revealing to a cycle reader) to plot them on a time scale which accurately portrays the time factors involved. For, after all, time is only a relative term."

My answer:

The crux of the matter is this: Are the forces which produce regular significant cycles generated merely by the trading or during trading time, or are they produced irrespective of trading--that is, are they external to the trading? If the former, you should ignore time when the exchange is closed. If the latter you should interpolate to account for it:

To illustrate better what I mean, let me tell you of a little game my children used to play when they were young. One would say "Freeze!" whereupon everyone would stay stock still in whatever position he might be--fork halfway to mouth, for example, or one foot upraised--until the child who had said "Freeze!" said "Unfreeze!"

Now imagine a four day cycle in the market with a crest on a particular Tuesday. If it is symmetrical the low would be on Thursday. Friday it would be on the way up. If it is a four calendar day cycle it will reach its crest on Saturday, when the market is closed. If, however, it is a four market day cycle, it will be as if someone said "Freeze!" as the market closed on Friday and said "Unfreeze!" when the market opened on Monday or Tuesday or whenever. In this event the four day cycle would not crest on Saturday. It would be "frozen" and would "unfreeze" on the day the market opened--say Monday.

Now daily cycles in the market could behave this latter way. In fact, they may behave this way. I do not know. I have not done enough work in daily market cycles to find out. However, I assume that they do not.

I assume it because the longer cycles do not seem to immobolize when they suffer a gap. For example, during the war the production of passenger cars was completely suspended. "Freeze!" After the war production was resumed. "Unfreeze!" But the 41-month, 36-month, and 23 1/3-month cycles in these figures did not freeze. They came back into operation after the war just as if they had been operating in a vacuum all the time the war had been going on. They did not pick up where they left off. They picked up where they would have been if they hadn't left off.

Because the longer cycles behave this way I assume they are the result of ex-ternal forces which play upon the human beings involved. By analogy I assume that the shorter cycles, about which I know very little, also behave this way. But I may be wrong. (Every man is entitled to his first mistake!)

Shall we put it this way: In the absence of evidence to the contrary, I consider it more fruitful, in the first instance, to look for short cycles in the market on a calendar day basis than on a market day basis. However, in final analysis it isn't what I think. Nor is it what you think. It's what the facts are! The only way to settle the matter would be to study the market for cycles on both bases and find out which way turns out to be more productive.

I assume it is self evident that if short daily cycles in the market operate on a calendar day basis you won't be able to find them if you fail to interpolate values for the days the market is closed; and of course vice versa.

Finally, if I am right in thinking that the cyclic forces, like Cascarets operate "while you sleep," it follows that it would be almost useless to try to find hourly cycles in the market. With the market closed for three quarters of each day as well as on week ends, the job would be well nigh hopeless.

# LESSON I

## Supplement 2

## MORE ABOUT RATIO SCALE

It may help you to understand the difference between ratio scale and arithmetic scale if I reprint for you two articles which have appeared in Cycles.

The first article, called "The Ratio or Semi-Logarithmic Scale," is reprinted from Cycles for October, 1952, pages 278-281.

The second, called "Logarithms Confusing," was reprinted from Cycles for September 1952, page 245. It anticipates a little the subject of logarithms, to be treated in full in Lesson II-A.

## Technical

# THE RATIO OR SEMI-LOGARITHMIC SCALE

In a letter printed in the **Letters** section of our September 1952 report I promised a discussion of the ratio or logarithmic scale. In accordance with that promise I am printing herewith the following comments, taken in part from **Cycles—The Science of Prediction**, a book written a number of years ago by Mr. E. F. Dakin and myself.

The solid line in Fig. 3, printed below, shows the sales of a hypothetical business that grew 90 per cent in the first five year period, from 1905 to 1910, 80% in the next five-year period, 70 per cent in the next, and so on. The rate of growth, that is, declines 10 per cent in each five-year period, and falls to a level of 20 per cent in the five-year period from 1940 to 1945.

The curve in Fig. 3 gives little or no **visual** warning of the drastic fall-off in sales that lies immediately ahead (as shown by the broken line in this chart) if the growth pattern continues in the future as in the past. If this pattern does continue, the sales in 1950 will be only 10 per cent more than the sales in 1945, and the sales in 1955 will show no growth whatever when compared to 1950.

In order to make this situation visually clear, many businessmen and students of trends prefer to plot all data pertaining to growth on what is known as a **ratio (or semi-logarithmic) scale**. On such a scale equal vertical distances represent equal **percentage** change.

Let us illustrate by charting the growth of another business using both the arithmetic and ratio scales.

The business starts at a sales volume of $2,000 a year; the second year it doubles its sales to $4,000; the next year it doubles again to $8,000.

On the arithmetic scale these facts would be represented in a chart laid out as in Fig. 1.

On a ratio scale, the same facts would be represented on a chart laid out as in Fig.1a.

Notice that on a ratio scale the numbers on the vertical scale represent equidistant percentage values, and come closer and closer together, in a way that interposes the same vertical distance between 4 and 8 as between 2 and 4 (because 8 bears the same relation to 4 that 4 bears to 2).

A straight line plotted on a ratio scale means that the **rate** of growth is constant. (Such a state of affairs, of course, never exists for long in any real situation.)

When the **rate** of growth declines as the business or the organism gets older, this decline is represented on a ratio scale by a bending-over in the line.

Thus, if the business cited here grew from $2,000 in its first year to $4,000 in its second year, and then grew an equal **amount**—from $4,000 to $6,000—from its second to third year, these facts would be shown on **ratio** scale as in Fig. 2a.

On the other hand, using the arithmetic scale, the line would continue from the second to the third year at the same slope as from the first year to the second, giving no visual suggestion that the rate of growth was declining. The same figures plotted on the arithmetic scale would look as shown in Fig. 2.

Further to compare the difference between arithmetic and ratio scales, the sales of the hypothetical business, discussed first above, are charted in Fig. 3a.

One advantage of such a plotting is that it shows directly the falling off in the rate of growth, and permits making a visual or freehand projection into the future that is much more likely to be fulfilled than any similar projecting of a chart on the arithmetic scale.

Ratio scales are usually labeled as such. If not, they can be recognized from the fact that the numbers on the scale come closer and closer together as they grow larger, as in Fig. 1a & Fig. 2a above; or else, for equal intervals, the numbers get proportionately larger and larger, as in Fig. 3.

The interval from one number—let us say 1—on the vertical scale of a ratio chart to the number ten times higher, in this instance 10, is called a "cycle." The same distance is used to represent the next tenfold growth, from 10 to 100, and so on. In reading a curve plotted on ratio scale, one should always look at the vertical scale to see how many cycles are involved.

So that you may make additional comparisons of data charted on ratio and on arithmetic scales, Three other series of figures have been plotted on both ratio and arithmetic rulings, namely Figs. 4 and 4a, 5 and 5a, and 6 and 6a.

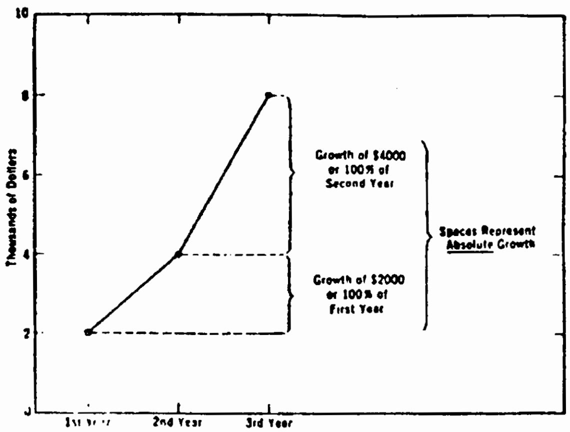



Fig. 1. Equal Rate of Growth—Arithmetic Scale.

Sales of a hypothetical business organization showing constant rate of growth.

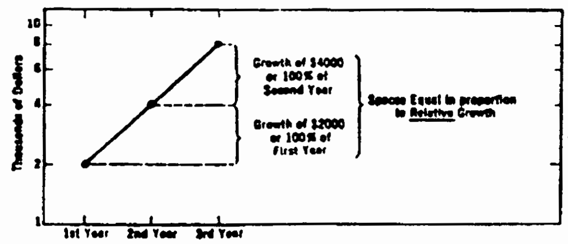



Fig. 1a. Equal Rate of Growth—Ratio Scale

Sales of a hypothetical business organization showing constant rate of growth.

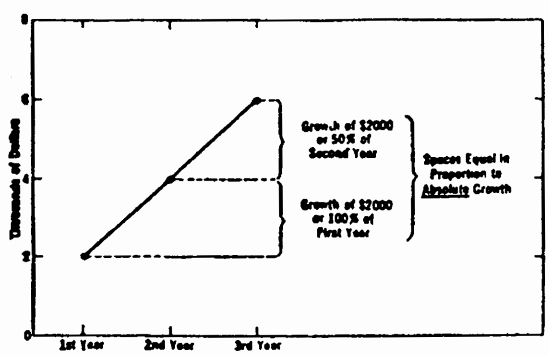



Fig. 2. Equal Amounts of Growth—Arithmetic Scale.

Sales of a hypothetical business organization showing constant amount of growth.

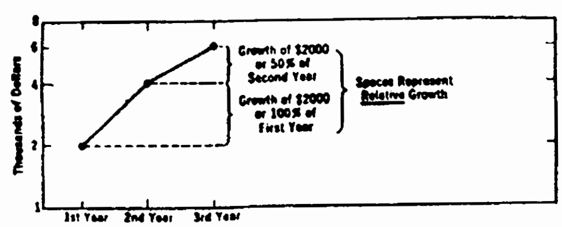



Fig. 2a. Equal Amount of Growth—Ratio Scale.

Sales of a hypothetical business organization showing constant amount of growth. Ratio scale.

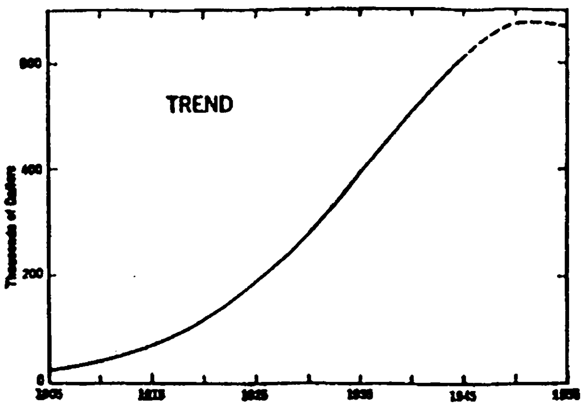



Fig. 3. Trend of a Hypothetical Business Organization

Data—1905-1945, with a projection to 1955. The projection is based on the assumption of a continuation of the constant decline of the rate of growth.

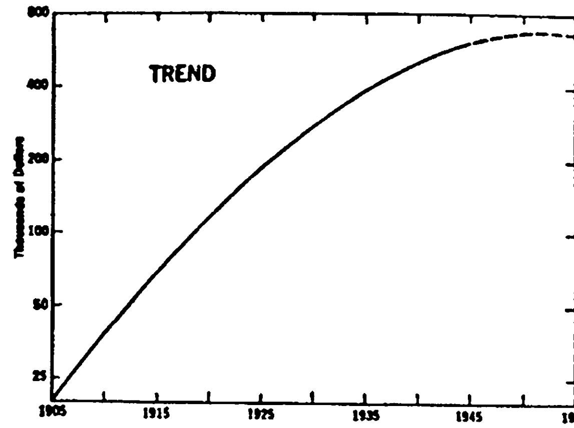



Fig. 3a. Trend of a Hypothetical Business Organization. **Ratio scale.**

Data 1905-1945, with a projection to 1955. The projection is based on the assumption of a continuation in the constant decline of the rate of growth.

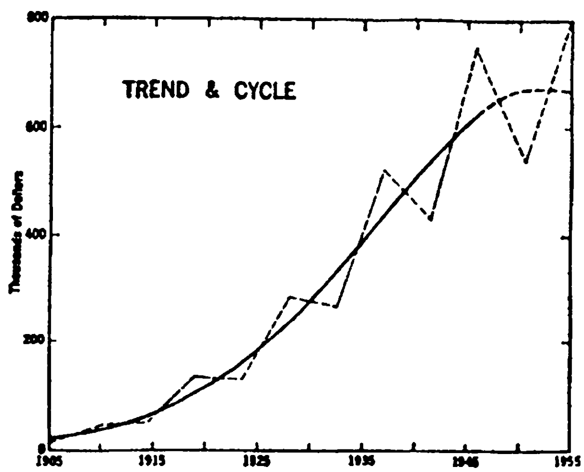



Fig. 4. The Trend and the Cycle

Trend of a hypothetical business organization as shown in Fig. 3 with a regular 9-Year Cycle of 20 per cent amplitude superimposed.

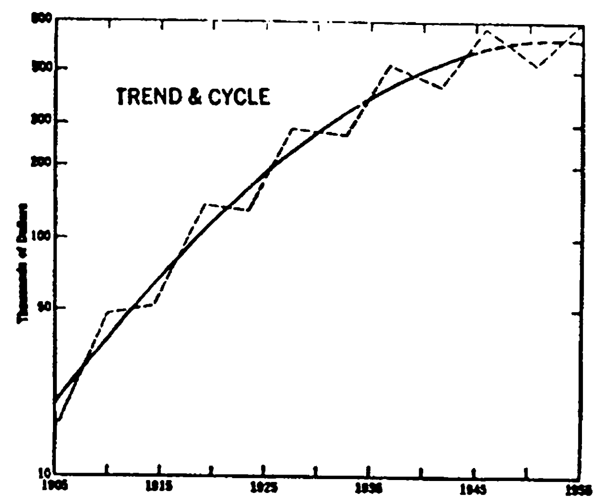



Fig. 4a. The Trend and the Cycle.

Trend of a hypothetical business organization, as shown in Fig. 3, with a regular cycle of 20 per cent amplitude superimposed. **Ratio scale.** Note that when one uses ratio scale, the wave has equal absolute magnitude throughout.

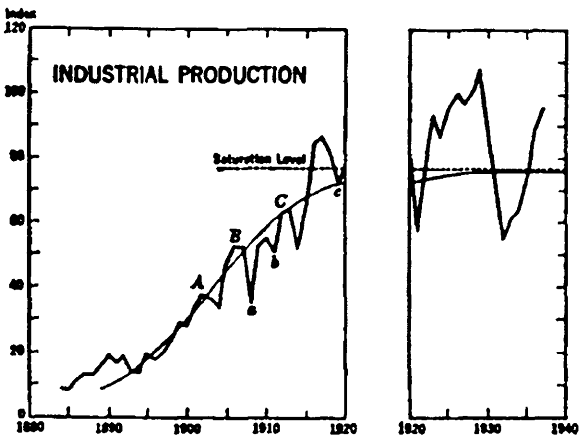



Fig. 5. Growth of Industrial Production in the United States.

Data—1854-1937, together with Trend (after Davis). The chart has been split into two parts to emphasize the changing character of the trend.

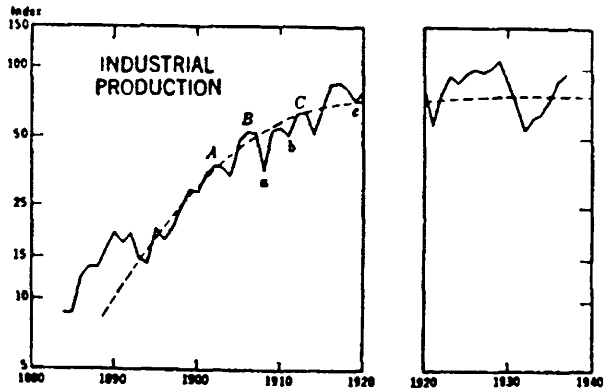



Fig. 5a. Growth of Industrial Production in the United States. **Ratio scale.**

Data 1884-1937, together with trend. (After Davis.) The chart has been split into two parts to emphasize the changing character of the trend.

On ratio scale all booms and depressions are shown in their true relative proportions.


Fig. 6. Index of Spanish Trade.

Data—1530-1650 (after Davis and Hamilton) together with Trend.

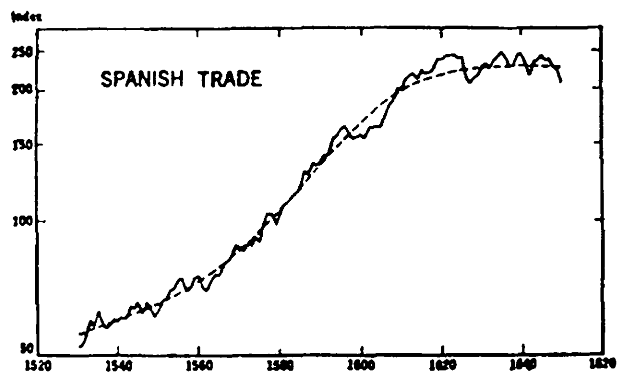



Fig. 6a. Index of Spanish Trade. **Ratio scale.**

Data 1530-1650, (After Davis and Hamilton), together with trend.

# Letters

## LOGARITHMS CONFUSING

Dear Mr. Dewey:

Although I have been subscribing for a number of years, the mechanics of the preparation of charts, particularly logarithm, etc., is very much over my head. Someday I hope to struggle out of the absolute novice class and be able to render a little genuine appreciation of the fine work I think you are doing.

Meanwhile, I do get some ideas out of some of the charts that are presented.

Very truly yours,

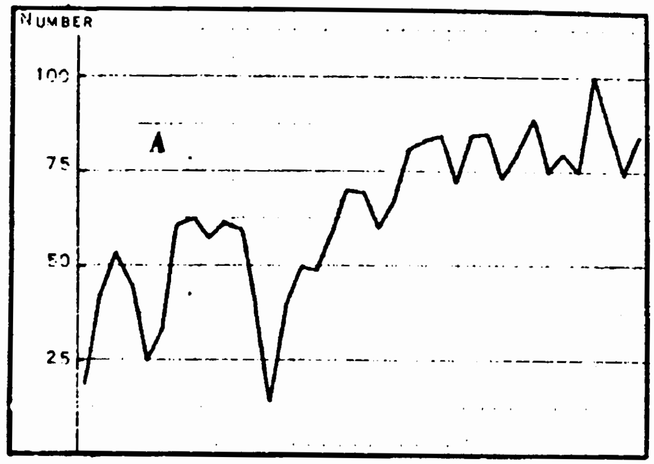
Dear Mr. -

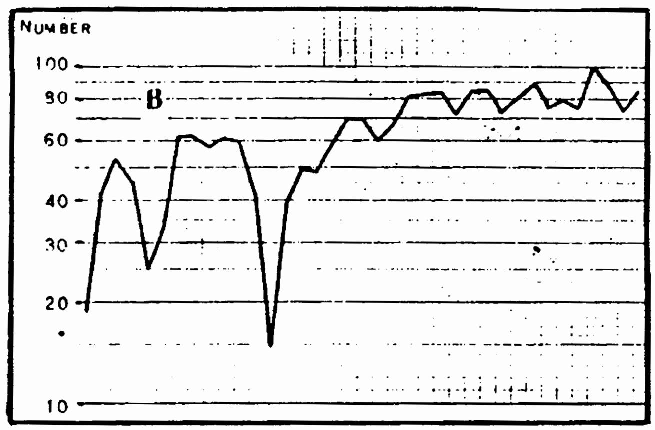
I will try to give the subject of logarithmic charts a little space in a future report. In the meantime, it may help you to remember that a chart of the logarithms of a series of numbers looks just the same as a series of numbers plotted on log paper. In either event, as far as cycles are concerned the effect is merely to pull down the peaks and deepen the troughs. For example, if you had a series of waves that ran from 75 to 133 (that is, from 25 below the axis to 33 above) and plotted this curve on log paper, the crest at 133 would show up the same distance from the axis or 100% line as the 75% troughs would be below it.

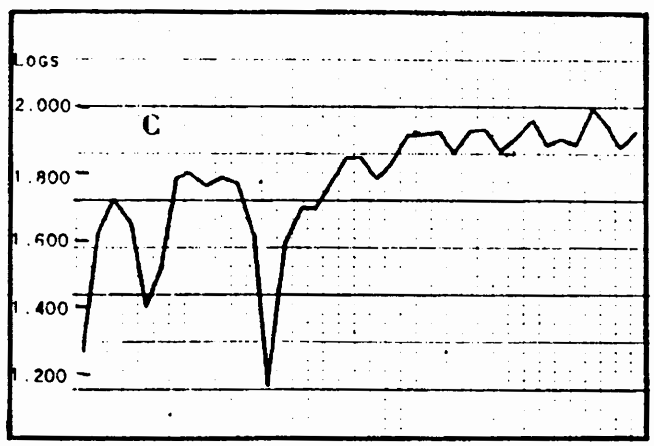
For most purposes you can forget that my charts are plotted on log paper or are logarithms plotted on arithmetic paper and read them in much the same way as you would read charts plotted on arithmetic paper.

Cordially yours,

In order to illustrate the above letter the three charts on the right are reproduced for your convenience. Curve A represents a series of figures plotted on arithmetic scale. Curve B shows the same figures plotted on ratio or semi-log scale. And in Curve C the logarithms of the same series are plotted on arithmetic scale. Note that Curves B and C are identical.



LESSON I

Supplement 3

ABOUT PROBLEM 1

The idea here was to illustrate how a series of numbers increasing by constant amounts compares to a series increasing at a constant rate (per cent) when plotted on arithmetic paper and when plotted on log paper.

The numbers increasing by a constant amount make a straight line on arithmetic paper, but the line bends over on log paper as the percentages become smaller.

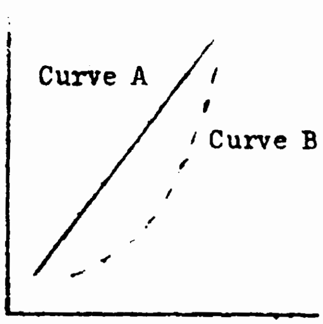
The constant per cent line on the arithmetic paper will bend upward, but will be a straight line on log paper.

Thus, by using logs on arithmetic paper, or numbers on log paper you get the same result - which is to have equal percentage changes cover equal areas on the chart.

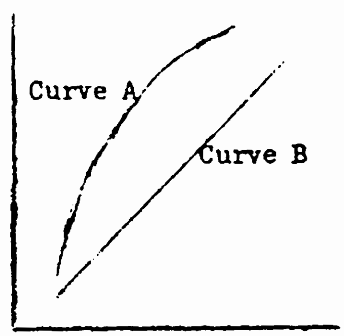
The application in the course is that we use logarithms - plotted on arithmetic paper - and we want to be sure you know what it's all about.

Here are two scratch charts which illustrate the point.

On Arithmetic Scale:




On Log Scale:




Curve A increases by a constant amount.
Curve B increases by a constant per cent.

# LESSON II

## USE OF NUMBERS
## AVERAGES
## INDEX NUMBERS
## TABULATION

We are making a cycle analysis of certain stock market averages.

The first step in making a cycle analysis is, almost always, to make a chart of the data on log paper.

You have done this for the stock market figures for part of your work in connection with Lesson I.

From this chart you were able to see by inspection that the figures include a trend factor. They do not oscillate up and down around a horizontal axis like a curve of barometric pressure, for example. They tend to increase gradually over a period of time. This gradual increase is called the trend. It must be removed.

As I stated in the introduction, there are many possible trends that we could use. For the stock market figures, however, I want to use a moving average trend.

The computation of moving average trends involves averages. Averages involves arithmetic. Many of the time series you will deal with in cycle analysis are in the form of index numbers. Time series of this sort are usually best made by posting the figures into a tabulation.

There is nothing much in averages, in the use of numbers, or in index numbers that you did not learn by the time you reached the eighth grade, or that is not a simple corollary of what you had learned by that time.

However, I want to make sure that you have not forgotten anything. I want you to realize the pertinent corollaries of what you have learned. I want to be sure you know some short cuts. Therefore, I am using Lesson II for a hurried review.

Because Messrs. Spurr, Kellogg and Smith have covered the subject as well as I could (and probably better), I am going to save myself work by referring you to them for the text for this lesson.

Bon voyage!

## Assignment

Study the following parts in Business and Economic Statistics by Spurr, Kellogg, and Smith

1. Chapter 2: "How to Handle Numbers." This begins on page 17. However, omit the section on square roots.

2. Chapter 10: "Averages." Because we are working with time series rather than Static data, omit the sections about grouped data, and study the following:

   a. Page 197 through to, but not including, the subdivision "Grouped Data" on page 199.

   b. Page 203, the section on "The Median" through to the subdivision "Grouped Data" on page 204.

   c. Page 207, the section on "The Mode" through to the subdivision "Grouped Data" on page 208.

   d. Page 210, "The Geometric Mean" to "Grouped Data" on page 212.

   e. Page 212, "Which Average To Use?" to the bottom of page 215.

3. Chapter 12: "Index Numbers." Study pages 242-247. Read as much of the rest of this chapter and the next one (Chapter 13: "Some Important Indexes") as your interest dictates. The more you know about the construction of index numbers the better.

Copy the Standard and Poor's Corporation stock market averages, 1854-1953, sent you with the "Introduction," onto 11" by 17" tabulation sheets in accordance with Standard Practice Instructions No. 3, enclosed. Tabulation sheets are also enclosed.

Keep this tabulation. Do not send it in yet.

How long did it take you to study each of the three assignments listed above? How long did it take you to copy and compare the stock market averages?

STANDARD PRACTICE INSTRUCTIONS

3. TABULATION SHEETS

(A) Annual Data

1. Always write on every page of each tabulation sheet the name of the time series involved. For example, Standard and Poor's Corporation Combined Index, Annual, 1935-39 = 100. Best use No. 3 pencil or ink.

2. Always code and number each page of each tabulation sheet for ready reference. Put code and tabulation sheet number and page number in upper right hand corner. For example, S. & P. C. A.—T. S. 1, Page 1, etc. No matter how many pages are involved the whole set of pages is called Tabulation Sheet 1, or whatever.

3. Always initial and date the tabulation sheet, using the upper left hand corner for the purpose.

4. Leave the first column of your tabulation sheet blank for binding holes.

5. Into the second column of your tabulation sheet enter the years in succession. Enter every year from end to end even if you have no data for that particular year. This column is called the stub. Head it with the word Year. Fill the page solid. Do not leave any empty lines. Do not leave the last line on the page blank.

6. Into the third column of your tabulation sheet enter the data. This column is called the first column of your table (the binding column and the stub are not counted). Head it with the letter A and the word Data. The remaining columns will be lettered B, C, D, etc. and will contain various manipulations of Column A, or other numbers.

7. Always head every column clearly to show exactly what the numbers are which it contains.

8. Always put source of data on tabulation sheet. When the numbers in any other column of a tabulation sheet come from some other source, enter the source at the end of the column, at the bottom of the last page of the tabulation sheet or on a separate page.

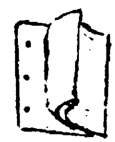
9. Initial and date each column as the work is completed. Place these initials and dates at the bottom of the last page of a tabulation (or on a separate page).

(The initial is not important unless two or more people are at work on the job, but it is good practice anyway.)

10. All copying should be compared. All manipulation should be checked. After the comparing and checking of each column, note "compd" or "ckd" and the date and your initial.

11. When figures in any column of a tabulation sheet are charted always put chart number at the head of the column for cross reference.

12. In filing your tabulation sheets, fold them as follows: so that the folded sheet will be $8\frac{1}{2}$" by 11" and the number of the tabulation sheet can be read.



13. Tabulation sheets should be filed in numbrical order in the folder or section of the three-ring binder devoted to this particular time series and labeled "Tabulation Sheets."

(B) Monthly, Weekly, and Daily Data

Instructions for making tabulation sheets for monthly, weekly, and daily data are the same as for annual data except that two columns must be left for the stub. Use the first column for the name of the month and year or date. Use the second for the month, week number, or day number.

I will furnish you with standard month numbers, week numbers, and day numbers as you need them.

Always leave a line for every month, week, or day even if you have no data. For example, in listing daily stock market data leave blank lines for Saturdays, Sundays, and holidays.

Do not leave any blank lines in your stub. Fill every line of the stub solid. For example, do not leave a blank line at the end of a year. January 1947 must follow on the next line after December, 1946.

Do not bother to put the year after any month except January. Thus:

N
D
J 1947

etc.

# LESSON II-A

## LOGARITHMS AND HOW TO USE THEM IN CYCLE ANALYSIS

The use of logarithms in cycle analysis makes the work a lot easier, and for some aspects of the work is essential.

As you can learn all you need to know about logarithms in a few minutes, you might as well use them in spite of the formidable name.

### Logarithm

Every number has a corresponding logarithm, or log. Conversely, if the log of a number is given, the number itself (antilog) can be determined. A table of logs is given in the text on pages 546, 547.

1. If you add the log of one number to the log of another number, you will get a log of a number which is the product of the tow numbers with which you started.

For example, the log of 100 happens to be 2. The log of 1,000 happens to be 3. If you add 2 to 3 you get 5. Now 5 is the log of 100,000, the product of 100 and 1,000. Thus, by using logs you can substitute addition ofr multiplication.

Another example: Multiplu 28,184 by 33, 113.

| | |
|---|---|
| The log of 28,184 is | 4.450 |
| The log of 33,113 is | 4.520 |
| Add: | 8.970 |

The number of which 8.970 is the log is 933,260,000. This number is very close to the product of 28,184 and 33,113 as you can easily confirm by actually multiplying the numbers. (The actual product is 933,256,792, which we could have obtained by using more decimals for our logs.)

2. the second characteristic of logs follows from the first. It is this: If you subtract the log of one number from the log of another number you will get the log of a number which is the quotient of the first number divided by the second number. That is, by using logs you can substitute subtraction for division.

Using our same examples:

| | |
|---|---|
| The log of 100,000 is | 5 |
| The log of 1,000 is | 3 |
| Subtract: | 2 |

The number of which 2 is the log is 100. This is the same answer that you would get if you divided 100,000 by 1,000.

Similarly.

Divide 933,260,000 by 28,184.

| | |
|---|---|
| The log of 933,260,000 is | 8.970 |
| The log of 28,184 is | <u>4.450</u> |
| Subtract: | 4.520 |

The number of which 4.520 is the log is 33,113.

If you will actually divide 933,260,000 by 28,184, you will quickly see the advantage of logs.

3. The third characteristic of logs is this: If you multiply the log of a number by any number you get the log of a number which is the corresponding <u>power</u> of the first number.

Thus if you multiply the log of 12, which is 1.0792, by 2, you will get 2,1584, which is the log of 144. As you know, the 2nd power or square of 12 (12 times 12) is 144.

If you multiply the log of 12, which is 1.0792, by 3, you get 3.2376, which is the log of 1,728, the 3rd power or cube of 12 (12 times 12 times 12).

4. Conversely, if you <u>divide</u> the log of a number by a number, the result will be the log of the corresponding <u>root</u> of the number.

For example, of you divide 2.1584, the log of 144, by 2, you will get 1.0792, the log of 12, which is the square root of 144.

Thus, the use of logs enables you, at small effort, to substitute (a) addition for multiplication, (b) subtraction for division, (c) multiplication for the raising to a power, and (d) division for the extraction of a root.

<u>Advantages of Logarithms</u>

From the standpoint of cycle analysis, if you have access to modern calculating machines, with automatic multiplying and automatic dividing attachments, the use of logs for multiplication and division is not as great as it is if you do not have access to such machines. In cycle analysis the raising of numbers to powers (multiplying them by themselves) is not often needed. but for computing geometric moving averages, a frequent operation in cycle analysis, logs are essential. Even modern machines cannot give you the 9th root or the 17th root of a number without the aid of logs.

Logs have another advantage from the standpoint of cycle analysis: They avoid the use of log paper, which is otherwise indispensable. This comes about by reason of the fact that a chart of the logs of a series of number, when plotted on arithmetic grid, gives you the same curves you would get by plotting the actual numbers on log grid. Not only is the plotting on arithmetic grid easier, but it is more flexible, for you can choose any scale you wish instead of being limited to the scales provided by log paper manufacturers.

To summarize: Except for preliminary reconnaissance, the use of logs for cycle analysis is always desirable and in any instances essential. As all you have to learn, in addition to what I have told you above, is how to use a log table--and this is very simple--the effort is easily worth while.

## HOW TO USE A LOG TABLE

Common logs are divided into two parts, (1) the part to the left of the decimal point, called the characteristic, and (2) the part to the right of the decimal point called the mantissa (so named by John Wallis in 1693 from a Latin word meaning an addition or make weight).

The characteristic you have to compute: the mantissa you look up in the table or read from a chart. You proceed thus:

First, to compute the characteristic:

For numbers, 1 or more, the characteristic is positive and is one less than the number of digits in the number. For numbers less than 1, the characteristic is negative and one more than the number of zeros to the right of the decimal point. Thus--

| Number | Characteristic |
|---|---|
| 45610. | 4 |
| 4561. | 3 |
| 456.1 | 2 |
| 45.61 | 1 |
| 4.561 | 0 |
| .4561 | -1 |
| .04561 | -2 |
| .004561 | -3 |
| .0004561 | -4 |

Second. To find the mantissa:

Find your number, disregarding the decimal point, in a table of mantissas (commonly called a table of logarithms). Find and record the corresponding mantissa.

You already have a reference to your table of logs (mantissas). All the mantissas given are understood to be preceded by a decimal point.

The table on pages 546 and 547 is a four-place table. In it you locate the number you are looking up in the stub at the left and in the column headings across the top. The corresponding mantissas are in the body of the table.

Starting at the upper left-hand corner of page 546 you will see that:

| | | | |
|---|---|---|---|
| If the number is | 100, | the mantissa is | .0000 |
| | 101, | | .0043 |
| | 102, | | .0086 |
| | 110, | | .0414 |
| | 199, | | .2989 |

When a number falls between the ones shown on the table, (that is, if the number has four significant figures), you must interpolate to get a more exact mantissa. If the number you have is 101,200, the mantissa would be two-tenths of the way between .0043 and .0086. OR, .2(.0086 - .0043) .0043 = .0052.

A five place log table would enable you to read .0052 directly from the table, and would probably spell out the proportionate parts of the mantissas for numbers having five significant figures.

Third, combine the characteristic and the mantissa.

In making the combination, a positive characteristic precedes the mantissa. A negative characteristic either (a) follows the mantissa, or (b) has its negative sign printed above it to emphasize that it is only the characteristic which is negative, not the mantissa, or (c) it is written in a combined positive and negative form as shown below.

For example:

| Number | Characteristic | Mantissa | Log |
|---|---|---|---|
| 45610. | 4 | .65906 | |
| 4561. | 3 | .65906 | 4.65906 |
| 456.1 | 2 | .65906 | 3.65906 |
| 45.61 | 1 | .65906 | 2.65906 |
| 4.561 | 0 | .65906 | 1.65906 |
| .4561 | -1 | .65906 | 1.65906 or 0.65906-1 or 9.65906-10 |
| .04561 | -2 | .65906 | 2.65906 or 0.65906-2 or 8.65906-10 |
| .004561 | -3 | .65906 | 3.65906 or 0.65906-3 or 7.65906-10 |
| .0004561 | -4 | .65906 | 4.65906 or 0.65906-4 or 6.65906-10 |

In cycle analysis it is usually possible to avoid negative logs completely, but you should know that if you want to divide a negative log by a figure, you should make the negative characteristic this number or a multiple of it by suitable adjustment, as follows:

| Log | Division | Log Rewritten | Log Divided |
|---|---|---|---|
| 0.65906-1 | 2 | 1.65906-2 | 0.82956-1 |
| 0.65906-4 | 3 | 2.65906-6 | 0.88656-2 |

Of course mantissas do not have to be positive. For example:

$$.0650 - 1 = -.9350$$

but, if you use negative mantissas you will have to reconvert to positive mantissas before looking up values in the tables, as all mantissas in tables are positive.

If one number is bigger than another, the logarithm of the bigger number is always bigger than the logarithm of the smaller number, but not quite as much bigger.

Logarithms cover all values in the range from zero to infinity. A negative number cannot have logarithm.

## Graphic Logarithms

Tables of mantissas are also obtainable in graphic form in a book called A Graphic Table Combining Logarithms and Anti-Logarithms by Lacroix and Ragot, which you can buy through any bookstore.' For my part, I prefer the graphic logs. The book contains a four-place graphic log table and a five-place graphic log table. with care you can read the four-place table in five places and the five-place table to six places. If you are doing a good deal of work with logarithms, I suggest that you add this book of graphic log tables to your library.

In using graphic log tables, you find the number in the left-hand part of the stub in the column marked N, and on the scale above the line. You find the mantissa on the right-hand part of the stub in the column marked L, and on the scale below the line.

## Anti-logarithms

Anti logarithms are merely the numbers corresponding to logarithms.

You look up the mantissa of the log in a logarithmic table and see the number to which it corresponds. You then place the decimal point as indicated by the characteristic. For example, if your logarithm is 2.4639, you find .4639 in the log table. It is on page 546 on line 29 and in column headed 1. The digits of your number are 291 and the characteristic, 2, tells you that the decimal place is after the third digit; thus, 291.

If the mantissa of your logarithm had been .4644, you would have had to interpolate from the table of proportional parts to obtain the number 291.3.

## Rounding Logarithms

The four-place mantissas, given in the text, are more accurate than you will usually need for cycle analysis. Where the cycles are of important amplitude, percentage-wise, you can round the logarithms as given in the table to three places and usually have sufficient accuracy. Never indulge in unnecessary accuracy. It is specious.

For cycles of small amplitude, four-place logarithms are required.

## Charting Logarithms

You chart logarithms on ordinary arithmetic paper. The curve you obtain is the same as the curve you would get by plotting the data on logarithmic paper with a corresponding scale. See Lesson I, covering the subject of charts and charting.

In general, the curves of logarithms look much the same as curves of the numbers except that (a) curves concave upward often become convex upward and that (b) cycles in the lower part of the curve are accentuated; cycles in the higher part of the curve are flattened. You will have demonstrated this to you own satisfaction by your work in Lesson I.

## NOTE

Spurr, Kellogg, and Smith logarithm table is set up for only three-place numbers. (it gives four-place logarithms but only three-place numbers.) Thus, if you want to know the log of 123, for example, you can read it directly from the table, but if you want to know the log of 1237 you have to interpolate.

It is a lot easier to use log tables set up for four-place numbers but naturally they are ten times as long (20 pages instead of 2 pages).

If you plan to do a good deal of work with logs you might want to pick up at a second-hand book store somewhere a statistical textbook containing one of the longer tables.

Incidentally, my clerks all prefer my book of graphic log tables. From such tables you read the number on one scale and read the log on the other one. A book called A Graphic Table Combining Logarithms and Anti-Logarithms by Lacroix and Ragot (MacMillan) gives four- and five-place tables for both numbers and logs. With care the five-place tables can be read to six places. I think the book costs $3.50. Any bookstore can get it for you if you find yourself interested.

## Assignment

Study the section in the text on logs. This is on pages 543-545.

Work the following problems:

Problem 1. Find the logs of each of the following numbers:

89,400
.00643
4.41

Find the numbers of which the following are logs:

1.9518
8.8425-10
3.3936

**Problem 1:** (Continued)

Solve the following, using logs:

$$(10)^2 \times .001 \times \sqrt{10,000}$$

(Prove by arithmetic)

**Problem 2:** Construct on a piece of blank paper the vertical grid for a one-cycle log chart, allowing 6 inches from the top to the bottom. Show only the major divisions 1, 2, 3, 4, etc.

**Problem 3:** The trend of a series increases annually at the rate of 3% a year, compounded. The trend value in 1900 was 55. What is the trend value in 1950? In other words, find Trend, where Trend $= 55(1.03)^{50}$. (That is, multiply 55 by 1.03 multiplied by itself 50 times.) Solve by logs.

Look up the logs for the stock market series and enter in Column B of the tabulation sheets. Use 3-place mantissas.

# LESSON II-A-1

## THE USE OF LOGARITHMS

**Question:**

Is it all right to use negative logs?

**Answer:**

Certainly, if you wish. I sometimes do, but mostly I find it better not to.

Here are (A) a series of figures, (B) their logs, and (C) the three month moving average of their logs. Now, if you want deviations from the moving average, you can express these deviations either as shown in Col. D or as shown in Col. E or in Col. F.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Year | Data | Log of Data | 3-Year M. A. of Log | Deviations (+ 2.000 form) | Deviations (+ form) | Deviations (Short form) |
| 1901 | 25 | 1.398 | | | | |
| 1902 | 30 | 1.477 | 1.424 | 2.053 | +.053 | +53 |
| 1903 | 25 | 1.398 | 1.392 | 2.006 | +.006 | + 6 |
| 1904 | 20 | 1.301 | 1.310 | 1.991 | -.009 | - 9 |
| 1905 | 17 | 1.230 | 1.277 | 1.953 | -.047 | -47 |
| 1906 | 20 | 1.301 | 1.310 | 1.991 | -.009 | - 9 |
| 1907 | 25 | 1.398 | 1.392 | 2.006 | +.006 | + 6 |
| 1908 | 30 | 1.477 | | | | |

When I use the + form or the short form I always put negative numbers in red.

As stated in the text, you can never look the negative log up in a log table until you make the mantissa positive.

**Question:**

How many places of logs should I use?

**Answer:**

No more than you have to!

You'll have to figure it out for each case, until you get used to using logs. Even then, when something new comes up, you may have to figure it out.

Suppose your data are 11, 12, 13, 95, 96 and 97. Let's convert to logs and then let's convert the logs back to antilogs.

| Number | 2-place Log | 3-place Log | 4-place Log |
|---|---|---|---|
| 11 | 1.04 | 1.041 | 1.0414 |
| 12 | 1.08 | 1.079 | 1.0792 |
| 13 | 1.11 | 1.114 | 1.1139 |
| 95 | 1.98 | 1.978 | 1.9777 |
| 96 | 1.98 | 1.982 | 1.9823 |
| 97 | 1.99 | 1.987 | 1.9868 |

| Original values | Antilogs of 2-place logs | Antilogs of 3-place logs | Antilogs of 4-place logs |
|---|---|---|---|
| 11 | 10.96 | 10.99 | 11.00 |
| 12 | 12.02 | 11.99 | 12.00 |
| 13 | 12.88 | 13.00 | 13.00 |
| 95 | 95.50 | 95.06 | 94.99 |
| 96 | 95.50 | 95.94 | 96.01 |
| 97 | 97.72 | 97.05 | 97.01 |

Several things should be clear from this table:

1. Antilogs of big logs like .98 do not correspond to the original numbers as well as antilogs of little logs like .04 or .08. Hence, in testing for the consequences of using logs of various sizes always test with the big logs. That is, if a certain number of places are good enough for the big logs, they will surely be good enough for the little ones.

2. In the example given, two place logs will not always round back to the original number. Therefore, they cannot be used. Three place logs will always give antilogs that will round back to the original figures. Hence they are plenty good enough. In this example the use of 4 place logs is silly.

# LESSON II-B

## USE OF NUMBERS

### A. Check by Common Sense

Always check all copying, all computations, and all charts from the standpoint of common sense. Is the answer reasonable? Have the values jumped too much according to what is usual? Is the result exceptionally large or exceptionally small?

Here are some examples of the sort of thing you should always do:

| Year | Data | Mental Comments |
|---|---|---|
| 1854 | 22.1 | |
| 1855 | 19.8 | about 3 down |
| 1856 | 20.3 | " 1 up - Reasonable |
| 1857 | 17.7 | " 3 down - " |
| 1858 | 14.5 | " 3 " - " |
| 1859 | 21.6 | " 7 up Wow! Number immediately after is down to 15 too so (this is quite a peak, 50% up in one year. Things don't (behave this way. Let's investigate--Right number is |
| 1860 | 15.4 | O.K. (12.6. The clerk who did this is a reverser--posted (21 for 12. Who did it? (Look at end of column), M.F. (Watch M.F. very carefully hereafter and replace her if |
| 1861 | 16.2 | O.K. (opportunity occurs. Reversers are made that way and (are hopeless! |
| 1862 | 21.3 | up 5 - O.K. but check just to make sure. Yes, o.k. |
| 1863 | 33.0 | up 11 Wow! But the numbers that follow are up there too, (and it's Civil War time, so I guess it's all right, (but I'll check to make sure (never take anything for |
| 1864 | 35.2 | O.K. (granted)--Yes, o.k. at 33.0. |
| 1865 | 33.2 | O.K. |

Another example:

| Year | Data | Log | Mental Comment |
|---|---|---|---|
| 1854 | 22.1 | 1.344 | Log of 2 is .301 so this seems reasonable. |
| 1855 | 19.8 | 1.297 | Both go down together. |
| 1856 | 20.3 | 1.308 | Both go up a little. |
| 1857 | 17.7 | 1.248 | Both go down a little. |
| 1858 | 14.5 | 1.611 | Number goes down, log goes up. Error!! Look up log of 14.5. It is 1.161. Same damn clerk again, I'll bet. Yes (looking at initials at end of column) H.F. We'll have to get rid of her as soon as we can. |

Another example:

| Year | A<br>Data | B<br>Col.A times 2 | Mental Comment |
|---|---|---|---|
| 1854 | 221 | 442 | twice two hundred and a quarter is about four hundred and a half, o.k. |
| 1855 | 198 | 386 | twice nearly two hundred is nearly four hundred, o.k. |
| 1856 | 203 | 406 | twice two hundred is four hundred, o.k. |
| 1857 | 177 | 354 | twice 1 3/4 is 3 1/2, o.k. |
| 1858 | 145 | 190 | twice 1 1/2 is -- what's this? It ought to be about 300, not 190. Oh yes, should have been 290. Who did it. J.C. Teach her to be more careful and to check by common sense, if she has any.<br><br>I also note that, as the multiplier is 2, all values in Col. B should be even. They are. |

Observe now that the value for Col. B for year 1855 is wrong, it should have been 396. Checking by common sense won't catch <u>all</u> the errors, but it should catch all the <u>big</u> ones, and they are the ones that do the most damage. Also they are the easiest to miss when checking in the ordinary way.

Another example:
Multiply 146.78324 x 46.40768
The answer obtained is: 68,118.696312832

It is obviously wrong, for 146 by a little less than half a 100 is, by common sense, a little less than half 14,600.000 etc. or about 7,000.000 etc.

The decimal is in the wrong place. The answer is 6,811.869 etc.

<u>Always</u> check <u>everything</u> from the standpoint of common sense.

## B. Some Checking Tricks

Reversed numbers. The difference between two numbers, two digits of which have been reversed, is always divisible by 9. Thus, if two columns of figures which should total the same, differ by an amount, say 63, which is exactly divisible by 9, look for a reversed number in one of the columns. (For example 92 copied as 29.) If the difference is a number not divisible by nine, say 62, there is an error which is not a reversed number.

Casting out 9's. This method is easy and fairly reliable. It has been known for 1000 years (but not by me! I learned it only recently from the section on Arithmetic in the Encyclopedia Brittannica).

Here is how it works. Consider the problem 57 plus 2 = 59.
To check, divide all terms by 9 to get remainders as follows: $57 \div 9 = 6$, and 3 remainder; $2 \div 9 =$ zero, and 2 remainder; $59 \div 9 = 6$ and 5 remainder.
The check: 3 plus 2 = 5

57 times 2 = 114
To check divide by 9 to get remainders as follows: $57 \div 9 = 6$, and 3 remainder; $2 \div 9 =$ zero and 2 remainder; $114 \div 9 = 12$ and 6 remainder.
The check: 3 times 2 = 6

Short cut:

| | |
|---|---|
| Add digits of 1st number (57) | $5 + 7 = 12$ |
| add digits of sum | $1 + 2 = 3$ |
| This is the remainder (as above) | |
| Add digits of 2nd number (2) | 2 |
| This is the remainder | |
| Add digits of 3rd number (59) | $5 + 9 = 14$ |
| add digits of sum | $1 + 4 = 5$ |
| This is the remainder | |

3, and 2 and 5 are the remainders
The check: 3 plus 2 = 5
or, in the problem on multiplication get remainder of first two terms above.
add digits of product (114) $1 + 1 + 4 = 6$
3 and 2 and 6 are the remainders
$3 \times 2 = 6$

In this short cut method note that if you get 9 as a "remainder," you must subtract 9 to get true remainder of 0. Thus, if you find square root of 1578.42 is 39.72 with remainder of 7.416, to check let a = 157,842, b = 3972, r = 7416.

The first addition of digits gives you
a = 27, b = 21, and r = 18.

The next addition of digits gives you
a = 9, b = 3, and r = 9

Your check is $a = b^2 + r$
This is not $9 = 3^2 + 9$
but rather $0 = 0 + 0$

Here is another example:

Suppose you multiply 25.78 x 45.7 and get for your answer 1,178.146. Is the answer right?

To find out:

First add the digits of the multiplicand: 2 + 5 + 7 + 8 to get 22
add the digits of the sum: 2 + 2 to get 4 remainder
Then add the digits of the multiplier: 4 + 5 + 7 to get 16
add the digits of the sum: 1 + 6 to get 7 remainder
Then add the digits of the product: 1+1+7+ 8 + 1 + 4 + 6 to get 28
add the digits of the sum: 2 + 8 to get 10
add the digits of the sum: 1 + 0 to get 1 remainder

Now multiply the remainder 4 x 7 to get 28
add the digits of the product: 2 + 8 to get 10
add the digits of the sum: 1 + 0 to get 1
1 = 1

Hence the multiplication is probably numerically correct. (The manipulation is correct unless the error in your multiplication (or in your check) is divisible by 9.) This method does not check the decimal point. If the problem does not check, either the problem or the check contains a numerical error.

# cycles

Cycles are the simplest thing in the world, at least in principle. Let me give you an example.

Suppose you are visiting my house and, looking out of the window, notice a bus pass by at 10:00 a.m. Half an hour later at 10:30 you notice another bus pass. At 11:00 you see another one. "Ah ha!" you say. "Buses here run every thirty minutes." You have discovered a cycle all by your little lonesome, without benefit of slide rule or gobbledegook. Cycles are just as simple as that.

Now what?

Well, first of all, you have a basis for predicting the probabilities of the future. If we go in to lunch and come out of the dining room at 1:05 you will know that you probably missed the 1:00 o'clock bus and that, *if the cycle is continuing*, your next bus will pass in 25 minutes. So you chat for about 20 minutes and leave at 1:25 so as to have to stand in the wind and the rain the least amount of time.

Of course the schedule may have changed, or the bus have been delayed by an accident. You can't count for sure on a bus at 1:30. You are merely playing *probabilities*.

### Regularity Gives Predictability

Where you have regularity you have predictability—at least to the extent that the regularity governs, and is not present by chance.

Now let's do some more supposing.

You are overlooking a street near the center of a small town. Every ten minutes or so 10 or 15 people come along, more or less in a bunch. Another cycle! Again you can predict (with qualifications). More than this, with this regular result appearing before you, you have a right to assume a cause—if the time intervals have been regular enough and have repeated enough times so that the behavior cannot reasonably be the result of chance.

You don't know the cause but there is no law against guessing. So you guess that there is a bus station around the corner and that every ten minutes a bus comes in and discharges its passengers. If you then find out that there *is* a bus station, and that a bus does come in every ten minutes, your guess is bolstered. It is bolstered still more if you find that the buses arrive just about the time your bunches of people have been coming. But you still don't *know* that your people are coming from the buses. You would have to go out on the street and go around the corner to find out for sure.

### Other Cycles

You continue to watch and count the people. You see other patterns. Every other ten-minute group of people is bigger. Perhaps there is a second bus with a twenty-minute schedule that reinforces the crowd from the first bus.

This is really the whole story about cycles.

You can predict that part of the traffic that comes in regularly recurring bunches. You can *not* predict by your 10- or 20-minute cycles the crowd of people that will come along when the feature of the local movie house is over. Your predictions are only *partial*. But they are good as far as they go *if* the cycles keep on coming true, and *if* you have timed them right. Moreover, by finding something else (in the example given, a bus schedule) with regularly recurring cycles of the same length you get a clue as to possible cause and effect and relationship. But to make sure, you need to run your clue down.

Where you have partial regularity you have partial predictability. For example, suppose you have a chart recording your heart beats over the period of a minute. Now look at the first half of it only. If you count 30 beats in the 30 seconds you have under examination you will not be far wrong if you forecast another beat at 31 seconds, another at 32 seconds, etc.

Perhaps, if your heart speeded up a little, you would be quite wrong if you made your projection ten beats ahead, but by making your forecast only a beat at a time and revising your forecast each beat, you could probably make your prediction come out fairly well.

Another example of partial predictability: Suppose you knew that February 1st is, in New York City, on the average, the coldest day of the year. Suppose, on the basis of this information, you forecast February 1, 1952 as the coldest day of next winter. It is almost certainly true that February 1st will be colder than August 1st, but in any given year it may not be the very coldest day by as much as one or two months. The cycle of the year influences the weather but it does not completely dominate it. From a knowledge of the cycle of the year alone you could forecast the coldest day only with wide tolerences, yet the cycle of the year is very real.

Almost everything fluctuates with rhythm—that is, in more or less regular cycles. Putting it another way, almost everything acts as if it were influenced by regularly alternating up and down forces which first speed it up and then slow it down.

### Random Ups and Downs Too

Everybody would know this fact were it not for two things: First, in addition to the cycles there are accidental (random) fluctuations in things, too. These randoms hide the regularities so that at first glance you do not see them. Second, things act as if they were influenced simultaneously by several different rhythmic forces, the composite effect of which is not regular at all.

If we had several moons, the ups and downs of the tides would be very irregular. All the other moons would mix things up. It might have taken us much longer to find out about the tides.

### Separating Cycles Easy

If you have a long enough series of figures with which to work it is not too hard to separate the different regular cycles from each other. When this has been done, you can project each regular cycle into the future: Then you can easily find out the combined or composite future effect of all the various cycles. When you have done this you have a preview of what is going to happen (a) *if the cycles continue,* and (b) *except as the cycles may be upset or distorted by accidental random noncyclic events.*

"Why wouldn't the cycles continue?" you may ask.

I'll give you one reason: The cycles may have been present in the figures you have been studying merely by *chance.* The ups and downs you have noticed which come at more or less regular time intervals may have just happened to come that way. The regularity—the cycle—is there all right, but in such circumstances it has no significance. If you are zealous enough you can find regularity in almost anything, including random numbers where you know that the regularity has no significance and know it will not continue.

### Many Repetitions Needed

How can you tell in any given instance whether or not the regularity you see is the result of a real underlying cyclic force which will continue to fluctuate regularly in the future?

The answer is, if the cycle has repeated *enough* times with *enough* regularity and with *enough* dominance, the chances are that it is the result of real cyclic forces.

Let me give you an example: Pick up a pack of playing cards and start to deal. The first card is red, the second is black, the third is red, the fourth is black. You have two waves of a regular cycle, red, black; red, black. But this sequence could easily come about by chance. You continue to deal: red, black; red, black. Four times in a row now, this regular alternation. It could still be chance, but it couldn't be chance very often. Continue to deal. Red, black; red, black; red, black. Seven times now! It could still be chance, but it is less and less likely. It begins to look as if somebody had stacked the cards. You go through the entire deck. Twenty-six times! "Somebody certainly stacked the deck," you say. "It couldn't happen this way by chance once in a million times."

### Less and Less Likelihood of Chance

Exactly the same sort of reasoning applies to the cycles you see in the ups and downs of the stock market, or the sales of your own company, or the weather, or anything else in which you may be interested. The more the cycle has dominated, the more regular it is, *and the more times it has repeated,* the more likely it is to be the result of a real cyclic force that will continue. If it has not dominated enough, or has not been regular enough, you must have more repetitions to get equal assurance.

Well then, supposing that there are these rhythmic cycles in something. Suppose further that you have some knowledge of cycles. So what?

By using nothing more complicated than simple arithmetic you can find the cycles. By seeing how many times each cycle has repeated in the past you can have a pretty fair idea of its significance (i.e. whether or not it will continue). And by projecting all the significant cycles into the future you can get some light on what is ahead *insofar as the cycles govern.*

### Cycle Forecast Like a Weather Forecast

In weather we are used to forecasts in terms of probabilities. "The probability for the Pittsburgh area is for snow." When you hear such a forecast on the radio you don't rush out to put on your chains. The weather man may be wrong. You wait for the snow to fall. *Then* you put on your chains. But the forecast warns you that snow is *likely*, and on the strength of this fact you do make sure that you have your chains with you. If you are to make use of cycles in your business or your stock market forecasting you are going to have to use the same approach. Moreover, just as you refrain from shooting the weather man when he is wrong, I hope you will refrain from shooting the cycle analyst too.

### Cycles Indispensable

Cycles are not the whole answer, but on the other hand they are indispensable in attempting to arrive at the whole answer.

Cycles remind me of women. Women are not perfect (with individual exceptions, I hasten to add). But they are the best thing so far invented for the purpose. Until something better comes along we will have to make shift with them as they are—or else miss the values they have to offer. Let's find out all we can about them!

### Cycles Show Us Our Ignorance

The second reason that the study of cycles is important arises from the fact that wherever you have rhythmic variation—or for that matter pattern of any sort—you probably have a cause. For example, if you scatter iron filings on my desk you would expect them to be distributed at random over its surface. If, to the contrary, you find them arranging themselves into a pattern, you have a right to assume that some unknown force is present—perhaps a powerful magnet in my top drawer.

Similarly, if you find pattern, or more specifically rhythm (reasonably regular cycles), in the alternate thickness and thinness of tree rings or rock strata, or in the abundance of insects, or in the prices of common stocks, you may be sure that there is a cause for such behavior. Also you may be sure that if you do not know that cause you do not fully understand the behavior with which you have to deal. Thus the study of cycles reveals to us our ignorance, and is therefore very disturbing to people whose ideas are crystalized. "If there are regularly recurring ups and downs in business or in prices, all I have ever learned is wrong," an eminent economist once told me; and he added, in a moment of unusual candor, "I simply cannot afford to accept such an idea. All my life's work would be ruined." Many doctors in the days of Pasteur must have felt much the same way about germs.

### Identical Cycles Suggest Interrelationship

The study of cycles has a third value. Such study can indicate possible cause-and-effect relationships, thus helping to solve the very problems it reveals.

For example, when I was a boy I noticed that the moon came up about an hour later each night. When I visited the seashore I noticed also that the tide came in half an hour or so later each twelve hours. I did not have the wit to go on from there to note that two consecutive tides were delayed by exactly the same amount of time as was the moon, and therefore that there was perhaps some connection between the two. I did not have the wit to make this observation, I say, but one of the older astronomers did. In doing so he demonstrated the third value of a knowledge of cycles: Where the rhythms in two different phenomena have the same average time span, you are justified in suspecting a possible direct or indirect interrelationship between the phenomena.

### Scientists Interested

For these three reasons many scientists are interested in the study of cycles. First, it is the business of science to predict; second, it is the business of science to solve mysteries and to learn the "how" of things; and third, the true scientist welcomes any tool that gives him hints as to possible cause and effect relationships. Perhaps 3,000 scientists, the world over, have concerned themselves with the subject, and have written scientific papers embodying the results of their observations.

Scientists however, like all the rest of us, are pretty self centered. Thus the mammalogist interested in cycles in animal abundance is usually not much interested in geological cycles, or in astronomical cycles, or in business cycles. He is primarily, and usually exclusively, a mammalogist. Likewise the men in each of perhaps thirty different aspects of natural and social science are likewise specialists, each involved in his own little field.

However, the techniques of cycle analysis are the same whether one is studying biological data or financial data. And—much more important—it is only by studying cycles in all sorts of phenomena, and finding which phenomena have cycles of identical average time span, that we are likely to get hints of cause and effect relationships.

### A New Science

These two facts cry aloud for the creation of a new science—the science of cycles—which is concerned with rhythmic fluctuation *per se*, which will develop techniques of cycle analysis, which will isolate cycles in all the 30 or 40 different branches of science where cycles are important, and which, having assembled enough facts, will perhaps someday venture to advance some theories in regard to cause and effect.

The Foundation for the Study of Cycles was created, in 1940, to found such a new science, and to develop it to the point where it could serve mankind. In the eleven years which have elapsed since its creation, the Foundation has made slow but steady progress.

It should be clear from the above remarks that the Foundation is purely an educational and scientific body and in no sense a commercial organization. It exists, not to make money but to serve mankind.

An eminent group of scientists and administrators have lent their names to the Committee of the Foundation. Various scientific societies have appointed advisors to help with its awards, and many individual scientists have joined the Foundation as scientific members. On its part, the Foundation has tried to maintain the high scientific standards of the many universities and institutions here and abroad which are, through their professors, connected with it, and I think we have pretty well succeeded.

### Cycles Can Help You, Too

The research of the Foundation is of immediate practical value to the average citizen, too. By uncovering cycles in production and trade it throws light on the probabilities of booms and depressions. By uncovering cycles in international conflict it throws light on the probabilities of war. By uncovering cycles in the prices of commodities and of securities it throws light on the probabilities of panics and of other financial disturbances.

The results of the Foundation's research are made available to the general public by means of reports, bound together and issued ten times a year in the form of a magazine called *Cycles—A Monthly Report*.

For further information, address Foundation for the Study of Cycles, 680 West End Avenue, New York 25, New York.

# LESSON III

## HOW TO MAKE AND USE MOVING AVERAGE TRENDS FOR CYCLE ANALYSIS

### Trends

Trend, as I have said before, is the tendency of a series of figures to increase (and/or decrease) gradually over a period of time.

There are short term trends and long term trends. You can say, "The trend of the market has been downward for the last few days." In the next breath you can say, "But over the last century, the trend has been up."

Trend is often a blend of (a) physical factors, (b) price factors, and (c) inflation factors. As these elements change in different ways it is sometimes necessary to adjust values (for example, sales) to take account of price changes, or to adjust prices to take account of inflation. However, you do not need to bother about this complications at the present time. We shall come back to it later.

When you are talking about trend over a long period to time (several decades or more) you should use the term <u>secular trend</u> (<u>secular</u> means existing or continuing for ages or centuries).

If you will refer to your chart of stock market prices, 1854-1953, it is obvious that the values plotted evidence secular trend. They get bigger and bigger as the years roll on.

Now this concept of getting bigger and bigger is a bit fuzzy. In order to deal with it statistically you have to definitize it--you have to get numerical trend values for each year or other period of time for which you have data. This process is called <u>trend fitting</u>.

There are various ways to fit trends. Each way will give you different values. It should therefore be clear that the process of trend fitting is subjective--that is, it depends upon <u>you</u>.

Also, the process of trend fitting depends upon the purpose you have in mind.

Are you definitizing trend so as to remove it in order to study the cycles better? If so, you might use one trend line, Or are you definitizing trend so as to learn the law of growth of a particular time series in order to make a long term forecast? If so, you might use a different trend line.

If you want to get rid of secular trend so that you can study cycles better, usually the best way to definitize trend is to use a <u>moving average</u>.

A moving average is a series of overlapping averages. The number of terms averaged gives you the length of the moving average. (Each item of a time series is called a term.) Thus, if your annual data are 10, 12, 11, and 13, a three-year moving average of these numbers would be a succession of overlapping averages, grouped by threes. These averages would be 11 (the average of the first three values, $\frac{10+12+11}{3}$), and 12 (the average of the next three values $\frac{12+11+13}{3}$).

Each term of your moving average is plotted at the midpoint of the numbers being averaged. Thus your value of 11, above, is plotted at the second year; your value of 12 is plotted at the third year. There would be no 3-year moving average values for the first and last years.

In searching for cycles you generally compute a moving average trend with a length more or less equal to the length of the cycle in which you are interested. Thus, if you wish to study 8-,9-, or 10-year cycles you would compute a 9-year moving average trend; if you wish to study 18-, 19-, or 20-year cycles you would compute a 19-year moving average trend.

As, in the process of detecting and isolating cycles, you are generally going to use moving average trends, this lesson will concern itself largely with moving averages. However, as you do not always use moving average trends for cycle determination, you should at this point, have at least a general idea of the other sorts of trends that can be fitted. These are, among others,

(1) Graphic trends, i.e., trends fitted freehand to a chart of the data. The chart may be either arithmetic or ratio. The trend may be either a straight line or a curved line. In either case, the line is called a curve.

(2) Mathematical trends.
(a) Straight lines, fitted mathematically either to the data or to logs of the data, often by the method of least squares.
(b) Curved lines, fitted similarly.

Later, when you wish to project your cycles into the future, it will be necessary to know more about these other trends in order to determine and project secular trend. Therefore, when you get into the matter of cycle projection, it will be necessary to come back to this subject.

In the above exposition I have left two matters open ended. I said that you "generally" use moving average trends in cycle analysis work, I said you "generally" use moving averages of about the same length as the length of the cycle being studied. When don't you do these things? The answer to this question will be easier if I wait until you have made a thorough study of moving averages.

## Moving Averages

The use of moving averages in cycle analysis is covered in a paper I once wrote, a copy of which is enclosed herewith. Consider it as part of this lesson and study it down to the section called "C. The use of the Moving Average in Cycle Analysis."

## Assignment

In addition to studying "Cycle Analysis: The Moving Average" down to "C. The Use of the Moving Average in Cycle Analysis." read the following assignment in Spurr:

Chapter 14: Page 293 through the first paragraph on page 298.

Chapter 15: Page 320 through the first paragraph on page 325.

Page 327, "Graphic versus Mathematical Methods," through second line on page 334. Note that the advantages and disadvantages of the moving average refer to their use in determining secular trend, not to their use for determining cycles.

Page 351, "Summary," down to "Problems" on page 353.

## Problems

1. To further your stock market study please now compute a centered 9-year moving average of the logs of the index. If you have access to a computing machine use the short-cut method and post your answer in Column C. If you have only an adding machine and a slide rule post the moving totals into Column C., post 1/9 of each of these totals into the next column and call it Column C' (to be read C prime). Head Columns C and C' accurately.

Note that in computing a 9-year arithmetic moving average of the logs of the data you have actually computed a 9-year geometric moving average of the data themselves. Consider the first term of your moving average: When you added the nine logs together it was equivalent to multiplying all nine index values together. When you divided the sum of these logs it was the equivalent of taking the ninth root of the product of all these numbers.

2. Now plot on a sheet of 11" x 17" arithmetic polypurpose paper the logs of the stock market data (S. & P. C. A.,--Chart 3).

3. Now plot, on the same chart, using the same scales, the centered 9-year moving average of the logs of the data. Plot the moving average by means of a broken line.

Do not send this chart in yet. I will call for it after Lesson IV.

4. How long did it take you to study this lesson and the assignment?

5. How long did it take you to work Problem 1? Problems 2 and 3?

# JOURNAL OF CYCLE RESEARCH

Vol. 3 No. 2

SPRING 1954

# JOURNAL OF CYCLE RESEARCH

LEONAPD W. WING, EDITOR

CONTENTS, SPRING 1954



The Journal of Cycle Research is published quarterly at 680 WEST END AVENUE, NEW YORK 25, NEW YORK, by

THE FOUNDATION FOR THE STUDY OF CYCLES



Subscription to the Journal of Cycle Research costs $5.00 a year. Single copies are $1.25 each. Send your check to the publication office, Foundation for the Study of Cycles, 680 West End Av. New York 25, New York.

# Cycle Analysis: The Moving Average

By Edward R. Dewey
Director, Foundation for the Study of Cycles

The moving average is a mathematical tool of great use to students of cycles. As there is confusion in the minds of some people in regard to the use of this tool, it seems wise to issue a bulletin on the subject.

Some sections of this bulletin are merely a restatement of what you can find in any good text book of statistics. In other sections, however, you will find material, some of which is not, as far as I know, available readily, if at all.

## I. DEFINITIONS AND DESCRIPTION OF METHODS

### (The Simple Arithmetic Moving Average)

**Averages**

Everybody knows that an **average** is a typical value which tends to sum up or describe a number of figures. There are at least five different kinds of averages commonly used by statisticians; but the one which ordinary folk think about when they hear the word **average** is the one computed by adding all the items together and dividing the total by the number of items. Thus, if we have four items, 10, 12, 11, and 13, the average of these items is 10 + 12 + 11 + 13 (46) divided by 4, or 11½. (Statisticians call an average computed this way the **arithmetic mean**, but you do not need to remember this term, because I shall not use it again.)

**Time Series**

An arrangement of numbers is called a **series**. When the numbers with which we deal represent events which occur one after another in time, the arrangement is called a **time series**. Thus, in the example above, if 10, 12, 11, and 13 represent the price of cotton for each of four consecutive years, or represent the number of accidents on each of four consecutive days, you would call the numbers by this name—a time series.

You could still average the numbers and say, for example, that the average price for all four years was 11½ cents, or that during the period there was an average of 11½ accidents per day, as the case might be.

You could also say that the average price for the first three years was 11 cents, (10 + 12 + 11 (33) divided by 3) and that the average price for the last three years was 12 cents (12 + 11 + 13 (36) divided by 3).

**Moving Averages**

A moving average is merely a succession of averages secured from a series of numbers by dropping the first number (item) in each group averaged and including the next number in the series after the group, thus obtaining the next group to be averaged, and so on.

Thus, when you averaged the first three numbers of our time series (10, 12, and 11) and got 11, and then dropped the first number (10) and added the fourth number (13) and averaged again and got 12, you were constructing a moving average. Easy, wasn't it?

Because you were averaging three items at a time, you would call the result a 3-item or 3-term moving average. If the items represented yearly values you would call the result a 3-year moving average. If the items represented daily values, you would call the result a 3-day moving average.

**Moving Totals**

The moving total is the series of successive totals from which the moving average is computed.

For example: When, above, you added 10, 12, and 11 to get 33, and then added 12, 11, and 13 to get 36 (as a step in the task of getting 11 and 12, the two terms of the moving average), you were computing a moving total.

It was so easy that you did it without knowing it!

The moving total, like the moving average, should be posted in a table or plotted on a chart against the middle item of the group of items being totalled, as will be explained below.

**Plotting or Posting Moving Averages**

Each item or term of a moving average is always properly posted or plotted against the center of the group of items being averaged.

Many otherwise intelligent people fool themselves into thinking that if they plot or post an average against the last figure of the group of figures being averaged they somehow are getting later values. Of course this is nonsense.

If the values for 1933, 1934 and 1935 were 10, 12, and 11 respectively, the average for these three years is 11, whether we say 11 for the three years **beginning** in 1933, or 11 for the three years **centering** on 1934, or 11 for the three years **ending** in 1935. In talking about an average, we could choose any one of the three ways with equal propriety, as long as we made it clear which way we had chosen. But when averages are posted to a table, or plotted as a point on a chart, they **must** be posted or plotted against the middle of the group of figures being averaged, otherwise convention will be violated and, much more important, distortions are introduced into all further work. (The reasons for this will appear later.) Let me repeat, moving averages must **always** be posted or plotted against the central item of the items being averaged.

**Two Examples**

To make the process doubly clear, let us work out teo examples:

TABLE 1.
COMPUTATION OF 3-YEAR AND OF 7-YEAR MOVING AVERAGES

| | | Computation of a 3-Year Mov. Aver. | | Computation of a 7-Year Mov. Aver. | |
|---|---|---|---|---|---|
| | A | B | C | D | E |
| Year | Data | 3-Year Moving Tot. of Col. A | 3-Year Moving Av. of Col. A (Col. B÷3 or Col. B x 1/3) | 7-Year Moving Tot. of Col. A | 7-Year Moving Av. of Col. A (Col. D÷7 or Col. D x 1/7) |
| 1933 | 10 | . | . | . | . |
| 1934 | 12 | 33 | 11 | . | . |
| 1935 | 11 | 36 | 12 | . | . |
| 1936 | 13 | 40 | 13 1/3 | 91 | 13 |
| 1937 | 16 | 43 | 14 1/3 | 98 | 14 |
| 1938 | 14 | 45 | 15 | 109 | 15 4/7 |
| 1939 | 15 | 46 | 15 1/3 | 127 | 18 1/7 |
| 1940 | 17 | 55 | 18 1/3 | 149 | 21 2/7 |
| 1941 | 23 | 69 | 23 | 173 | 24 5/7 |
| 1942 | 29 | 87 | 29 | . | . |
| 1943 | 35 | 104 | 34 2/3 | . | . |
| 1944 | 40 | . | . | . | . |

It is obvious that with a 3-year moving average there are no values to place against the first and the last items of the series. With a 7-year moving average there are no values to place against the three first and the three last items of the series. In constructing a moving average one always loses one or more terms at each end.

**Formulas**

The formula for a 3-year moving average is

$$MA_b = \frac{a + b + c}{3}$$

where a to c represent successively each three consecutive terms of the data and $MA_b$ stands for the 3-year moving average to be posted against the central term, b.

The formula for a 7-year moving average would be

$$MA_d = \frac{a + b + c + d + e + f + g}{7}$$

where a to g represent successively each seven consecutive terms of the data and $MA_d$ stands for the 7-year moving average to be posted against the central term, d.

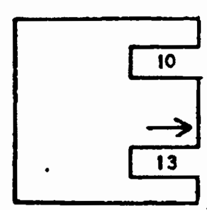
**Mechanical Details of Computation**

As has been explained, to get a 3-year moving average, one first computes a 3-year moving total, and divides each item of the moving total by 3.

To get the first figure of the moving total, add together the first three items of the data. To get the next figure of the moving total you subtract the first item of the data, and add the fourth. This process gives you the sum of items 2, 3, and 4. You proceed in this way successively.

In actual practice it is hard to pick out which items to add and which to subtract. You get mixed up.

To make the calculation foolproof, cut two slots out of a card or piece of paper so as to expose the first and fourth items in the series, but not the second and third. In our example in Table 1, it would look like this: (The lines **between** slots must always be one less than the number of terms in the moving average.)



Place this screen over the data so that the first item (in this instance 10) appears in the upper slot and the fourth item (in this instance 13) in the lower one.

Now, from 33, the sum of the first three figures, already posted in Col. B, subtract whatever appears in the upper slot (10) and add what you see in the lower slot. This gives you 36 which you enter in Col. B opposite the arrow. (The arrow is placed against the middle figure of the three items whose total is obtained by this method.)

```
  10 •
  12
  11
  33 S
  10 -
  13
  36 S
  12 -
  16
  40 S
  11 -
  14
  43 S
  13 -
  15
  45 S
  16 -
  17
  46 S
  14 -
  23
  55 S
  15 -
  29
  69 S
  17 -
  35
  87 S
  23 -
  40
 104 •
 ====
  29
  35
  40
 104 •
 ====
```

Now, slip your screen down a line so that 12 shows in the upper slot and 16 in the lower one.

From 36, subtract 12 and add 16 to get 40, the third item of your moving total.

Continue in this way until you have dropped 23 and added 40 to come up with the final item in the moving total, namely 104.

Now, add together the last three items of the data-- 29 + 35 + 40, to get 104 as a check on the accuracy of your work.

If you use an adding machine with direct subtraction, your tape will look like the figures shown to the left:

I find it better to run the entire tape before posting any values to Col. B. One reason is that it is quicker to do your posting all at once. Another reason is, if you should make an error you will not need to erase from Col. B all the figures from the error forward.

In doing long columns of figures, I also find it a good idea to check every 50 or 100 items by adding up the proper number of items of the original data to see if the total agrees with the subtotal on my tape.

Now that we have our moving totals (Col. B) we compute the moving average either by dividing each figure of the moving total by 3 or by multiplying it by the reciprocal of 3. This latter method is often easier. The reciprocal of a number is 1 divided by the number. In this case it is .333333.

Actually, .333333 x 33, the first figure in Col. B is 10.999989 which, of course, rounds to 11. To get the 11 in the machine directly, I always record the last digit of the reciprocal as one more than it really is. In this case we would therefore have .333334 x 33 or 11.000022. The error is twice as large, but the excess is dropped anyway and this method saves the need of rounding.

**Alternate (Short-Cut) Method**

You can compute a moving average directly without computing the moving total. When a calculating machine is available, this method is usually preferable. The method is a little hard to describe but very easy to compute. Proceed as follows:

Place the reciprocal of the number of items of the moving average in the machine. As we are computing a 3-item moving average, we put in the machine the reciprocal of 3 which is 1/3 or .333333 (only, as above, we call it .333334). We lock this figure into the machine for the whole operation.

We first multiply this reciprocal by the first item of our data, 10, and obtain 3.33334. **Without removing this product**, we then multiply the reciprocal by 12 (add it in 12 times) and obtain a total of 7.333348. **Without removing the product** we then multiply the reciprocal by 11 to obtain a grand total of 11.000022 or 11, which is the first figure of our moving average. (1/3 of the first item + 1/3 of the second item + 1/3 of the third item is the same as the sum of the first three items divided by 3.)

We then remove 1/3 of the first figure by subtracting the locked-in reciprocal 10 times to get 7.666682 and multiply (add the reciprocal in) 13 times to obtain 12.000024 or 12, the second figure of our moving average. This process is continued right down the column until 23 times the reciprocal has been removed and 40 times the reciprocal has been added in to obtain 34.666736 or 34 2/3 for the moving average value for 1943. This value is checked by adding together 29 times the reciprocal, 35 times the reciprocal, and 40 times the reciprocal, or adding 29, 35, and 40 and dividing by three.

If we were computing a 7-year moving average, we would, of course, use the reciprocal of 7, which is .142858 (the last figure has been raised by one). To get the first item (or term as it is more usually called) of our moving average, we add together the sum of this reciprocal times each of the first seven items of the data, and then add and subtract products of the reciprocal as above.

**Moving Averages With an Even Number of Items**

You may have noticed that so far we have talked exclusively about moving averages with an odd number of terms—3 or 7.

When we compute moving averages with an even number of terms such as 2 or 4, we run into a slight complication, due to the fact that the moving average must always be posted or plotted against the middle of the group of data being averaged, and the middle of an even number of items fall between two of the items.

We could post or plot a 4-year moving average **between** the years and this is sometimes done, as in the table on the following page:

TABLE 2.
COMPUTATION OF A 4-YEAR MOVING AVERAGE

| YEAR | A<br>DATA | B<br>4-YEAR MOV. TOT. OF COL. A | C<br>4-YEAR MOV. AVER. OF COL. A (COL. B÷4 OR COL. B x .25) |
|---|---|---|---|
| 1933 | 10 | | |
| 1934 | 12 | | |
| | | 46 | 11½ (AT POSITION 1934½) |
| 1935 | 11 | | |
| | | 52 | 13 (AT POSITION 1935½) |
| 1936 | 13 | | |
| 1937 | 16 | | |

However, the results of this method of posting are very awkward to describe in words and preclude any comparison between the moving average and the original data. Therefore, in practice it is almost **universal** to compute a 2-item moving average of the even-term moving average in order to center the moving average exactly, thus:

TABLE 3.
COMPUTATION OF A 2-YEAR MOVING AVERAGE OF A 4-YEAR MOVING AVERAGE

| YEAR | A<br>DATA | B<br>4-YEAR MOVING TOT. OF COL. A | C<br>4-YEAR MOVING AV. OF COL. A (COL. B÷4 OR COL. B x .25) | D<br>2-YEAR MOVING TOT. OF COL. C | E<br>2-YEAR MOVING AV. OF COL. D (COL. D÷2 OR COL. D x .5) |
|---|---|---|---|---|---|
| 1933 | 10 | | | | |
| 1934 | 12 | | | | |
| | | 46 | 11½ | | |
| 1935 | 11 | | | 24½ | 12¼ |
| | | 52 | 13 | | |
| 1936 | 13 | | | 26½ | 13¼ |
| | | 54 | 13½ | | |
| 1937 | 16 | | | | |
| 1938 | 14 | | | | |

You note that the first item of the 2-year moving average of the 4-year moving average is centered exactly against 1935; the second item is centered exactly against 1936.

In practice one would have computed a 4-year moving total, then a 2-year moving total of the 4-year moving total, and divided these values by 8, thus:

TABLE 4.
COMPUTATION OF 2-YEAR MOVING AVERAGE OF 4-YEAR MOVING AVERAGE—PREFERRED METHOD

| YEAR | A<br>DATA | B<br>4-YEAR MOVING TOT. OF COL. A | C<br>2-YEAR MOVING TOT. OF COL. B | D<br>2-YEAR MOV. AVER. OF 4-YEAR MOV. AV. OF COL. A (COL. C ÷ 8) |
|---|---|---|---|---|
| 1933 | 10 | | | |
| 1934 | 12 | | | |
| | | 46 | | |
| 1935 | 11 | | 98 | 12¼ |
| | | 52 | | |
| 1936 | 13 | | 106 | 13¼ |
| | | 54 | | |
| 1937 | 16 | | | |
| 1938 | 14 | | | |

Or one would have used the short-cut method and computed a 4-year moving average directly, as explained above, without bothering with the 4-year moving total. One could also have computed the 2-year moving average of the 4-year moving average directly by the same means:

TABLE 5.
COMPUTATION OF 2-YEAR MOVING AVERAGE OF 4-YEAR MOVING AVERAGE—SHORT-CUT METHOD

| YEAR | A<br>DATA | B<br>4-YEAR MOV. AVERAGE OF COL. A, COMPUTED DIRECTLY | C<br>2-YEAR MOV. AVER. OF COL. B, I.E. A 4-YEAR MOV. AVER. OF COL. A, CENTERED |
|---|---|---|---|
| 1933 | 10 | | |
| 1934 | 12 | | |
| | | 11½ | |
| 1935 | 11 | | 12¼ |
| | | 13 | |
| 1936 | 13 | | 13¼ |
| | | 13½ | |
| 1937 | 16 | | |
| 1938 | 14 | | |

Also, in actual practice, to save space, one posts the 4-year moving average in either the second or third position but marks it clearly to indicate that it is not truly centered as it should be, thus:

TABLE 6.
COMPUTATION OF THE 2-YEAR MOVING AVERAGE OF A 4-YEAR MOV. AVERAGE--POSTED AS IT IS DONE IN ACTUAL PRACTICE

| YEAR | A<br>DATA | B<br>4-YEAR MOV. AV. POSTED TO THE SECOND POSITION (CENTERED MINUS ½ YEAR) | C<br>2-YEAR MOV. AVER. OF 4-YEAR MO. AV. I.E. A CENTERED 4-YEAR MOV. AVER. OF COL. A |
|---|---|---|---|
| 1933 | 10 | | |
| 1934 | 12 | 46 | |
| 1935 | 11 | 52 | 12¼ |
| 1936 | 13 | 54 | 13¼ |
| 1937 | 16 | | |
| 1938 | 14 | | |

In any event, the final column is called a 2-year moving average of a 4-year moving average or, more usually, a centered 4-year moving average.

**Formulas**

For those who like to have relationships expressed in formula form, it may be stated that the formula for a centered 4-year moving average is:

$$MA_c = \frac{a + 2b + 2c + 2d + e}{8}$$

or more simply:

$$MA_c = \frac{\frac{1}{2}a + b + c + d + \frac{1}{2}e}{4}$$

where a to e represent successively each five consecutive items of the data and $MA_c$ stands for the 4-year moving average to be plotted against c, the center term or item.

## II. THE USE OF THE MOVING AVERAGE

This second section of the report will tell you how to use the moving average in statistical procedure, with particular emphasis upon its use in cycle analysis.

The moving average is used (a) to smooth time series, (b) to approximate the trend of time series, and (c), in cycle analysis, to help us (i) to separate cycles and (ii) to obtain a more exact estimate of the characteristics of each of the various cycles that may be present.

### A. The Use of the Moving Average to Smooth Time Series

The chief use of moving averages in ordinary statistical procedure is for the smoothing of time series. As this use of the moving average as such does not particularly concern the cycle analyst, it will be touched upon here only very briefly.

A smooth curve is one which does not change its slope in a sudden or erratic manner. The student interested in smoothing time series is referred to Frederick R. Macaulay's classic, **The Smoothing of Time Series**, published by the National Bureau of Economic Research (New York) in 1931. This book is now out of print but you can occasionally pick up a copy ($5 to $10) in second-hand book stores and of course you can always consult it at any good library.

**The Effect of Moving Averages Upon Random Fluctuations**

It should be obvious that the effect of moving averages upon random fluctuations is to average out the irregularities. It should be equally obvious that the more items that are combined into the moving average, the smoother will be your result and the closer it will approximate the average value of the successive numbers. One example should be enough to make this perfectly clear.

In Col. A of Table 7 are shown 30 digits taken at random from the New York City telephone book. For demonstration, these digits have been smoothed by a 3-item moving average, a 7-item moving average, and a centered 10-item moving average. See Fig. 1 on page 307.

It is obvious by inspection that as we increase the number of items of the moving average, the closer all terms of the moving average approach the average value of all the digits, which is 5.07.

TABLE 7.
VARIOUS MOV. AVERAGES OF A SERIES OF RANDOM NUMBERS

| | A | B | C | D |
|---|---|---|---|---|
| ITEM | RANDOM NUMBERS (TAKEN FROM THE TELEPHONE BOOK) | 3-ITEM MOV.AVER. OF COL.A | 7-ITEM MOV.AVER. OF COL.A | CENTERED 16-ITEM MOV.AVER. OF COL.A |
| 1 | 6 | • | • | • |
| 2 | 6 | 6.67 | • | • |
| 3 | 8 | 6.67 | • | • |
| 4 | 6 | 4.67 | 5.00 | • |
| 5 | 0 | 2.33 | 5.28 | • |
| 6 | 1 | 3.00 | 5.28 | • |
| 7 | 8 | 5.67 | 4.43 | : |
| 8 | 8 | 7.33 | 3.71 | • |
| 9 | 6 | 5.33 | 5.00 | 5.19 |
| 10 | 2 | 3.00 | 5.57 | 4.81 |
| 11 | 1 | 4.00 | 5.14 | 4.66 |
| 12 | 9 | 5.00 | 5.00 | 4.66 |
| 13 | 5 | 6.33 | 5.28 | 4.84 |
| 14 | 5 | 5.67 | 5.00 | 5.09 |
| 15 | 7 | 6.67 | 4.86 | 5.09 |
| 16 | 8 | 5.00 | 4.86 | 5.06 |
| 17 | 0 | 2.67 | 4.86 | 5.12 |
| 18 | 0 | 3.00 | 5.14 | 5.12 |
| 19 | 9 | 4.67 | 4.43 | 5.03 |
| 20 | 5 | 7.00 | 4.28 | 5.00 |
| 21 | 7 | 4.67 | 5.43 | 5.06 |
| 22 | 2 | 5.33 | 6.57 | 5.09 |
| 23 | 7 | 5.67 | 5.28 | • |
| 24 | 8 | 7.67 | 4.57 | • |
| 25 | 8 | 5.33 | 4.86 | • |
| 26 | 0 | 2.67 | 5.57 | • |
| 27 | 0 | 3.00 | 5.14 | • |
| 28 | 9 | 5.33 | • | • |
| 29 | 7 | 6.67 | • | • |
| 30 | 4 | • | • | • |

(SEE FIG. 1 ON PAGE 307)

Of course the process of smoothing also has the effect of minimizing cyclic fluctuations that may be present in the data as well as of smoothing out random fluctuations. It would not seem necessary to illustrate this fact at this point.

**Weighted Moving Averages**

In connection with smoothing a time series, one often gets better (i.e. smoother) results by the use of several successive smoothings. For example, if one took a 2-year moving average of a 6-year moving average of a 9-year moving average of a time series, one would obtain a much smoother curve than could be obtained by any of these moving averages taken separately.

The compound effect of such a series of consecutive moving averages could be expressed by the following formula:

$$MA_h = \frac{a+3b+5c+7d+9e+11f+12g+12h+12i+11j+9k+7l+5m+3n+o}{108}$$

Such a moving average is called a weighted moving average because for each item of the moving average each of the terms is used a different number of times and therefore with different weights.

In the formula described, for any one term of the moving average each of the items g, h, and i have 12 times the effect or weight in the composite as do items a or o, which are used but once.

Macaulay reports upon many formulae which have been developed by various investigators in order to achieve particular purposes. For example, "....take a 3-months moving total of a 5-months moving total of an 8-months moving total of a 12-months moving total of the data. To the results apply the following extremely simple set of weights: + 2, -3, 0, 0, 0, 0, 0, + 3, 0, 0, 0, 0, 0, -3, + 2. Divide the final results by 1440."

A weighted moving average of the sort described above, **with negative weights** near the ends, if properly designed, will overcome the tendency of the ordinary moving average to stay too low at cycle tops and too high at cycle bottoms.

Some of these formulae become rather complicated For example:

" Take successively a 3-months moving total of the data, a 5-months moving total, another 5, an 8, and a 12-months moving total. To the results apply the following set of weights: + 1,331,771, - 1,949,056, 0, 0, 0, 0, 0, + 2,175,370, 0, 0, 0, 0, 0, 0, - 1,949,056, + 1,331,771. Divide each of the final results by 6,773,760,000. "

In view of the fact that Macaulay has covered the subject so admirably and the further fact that we here are not interested in smoothing as such but only in the smoothing that may result as we use the moving average in the detection and isolation of cycles, it seems unnecessary to give further attention at this time to this aspect of the subject.

## B. The Use of Moving Averages in Trend Determination

The moving average is often used to give an approximate measure of the trend of a time series.

**Trend** may be defined as the tendency of data in a series to increase or decrease over a long period of time. How long is " long " depends upon circumstances.

The table below shows, by means of controlled data, three trend lines and, in connection with each, its 5-year moving average.

The trend shown in Col. A increases by a **constant amount**. The trend shown in Col. C increases by **amounts that get progressively greater** as time goes on. The trend shown in Col. E increases by **amounts that get progressively less** as time goes on. Cols. B, D, and F show a 5-year moving average for each of these three trend lines. The trends, with their moving averages superimposed by means of broken lines, are shown in Fig. 2 on page 307.

TABLE 8.
5-YEAR MOVING AVERAGES OF CONTROLLED DATA SHOWING EFFECT UPON THREE DIFFERENT TYPES OF TREND

| YEAR | A<br>A TREND WHICH INCREASES BY A CONSTANT AMOUNT | B<br>5-YEAR MOVING AVERAGE OF COL. A | C<br>A TREND THAT INCREASES BY AMOUNTS THAT GET GREATER | D<br>5-YEAR MOVING AVERAGE OF COL. C | E<br>A TREND THAT INCREASES BY AMOUNTS THAT GET SMALLER | F<br>5-YEAR MOVING AVERAGE OF COL. E |
|---|---|---|---|---|---|---|
| 1ST | 40 | . | 0 | . | 0 | . |
| 2ND | 80 | . | 5 | . | 80 | . |
| 3RD | 120 | 120 | 15 | 20 | 155 | 150 |
| 4TH | 150 | 160 | 30 | 35 | 225 | 220 |
| 5TH | 200 | 200 | 50 | 55 | 290 | 285 |
| 6TH | 240 | 240 | 75 | 80 | 350 | 345 |
| 7TH | 280 | 280 | 105 | 110 | 405 | 400 |
| 8TH | 320 | 320 | 140 | 145 | 455 | 450 |
| 9TH | 360 | 360 | 180 | 185 | 500 | 495 |
| 10TH | 400 | 400 | 225 | 230 | 540 | 535 |
| 11TH | 440 | 440 | 275 | 280 | 575 | 570 |
| 12TH | 480 | 480 | 330 | 335 | 605 | 600 |
| 13TH | 520 | 520 | 390 | 395 | 630 | 625 |
| 14TH | 560 | 560 | 455 | 460 | 650 | 645 |
| 15TH | 600 | 600 | 525 | 530 | 665 | 660 |
| 16TH | 640 | . | 600 | . | 675 | . |
| 17TH | 680 | . | 680 | . | 680 | . |

(SEE FIG. 2 ON PAGE 307)

It will be noted by comparing the moving averages with the trend that where, as in Col. A, the trend increases by a constant amount the moving average coincides with it. Where, as in Col. C, the trend increases by amounts that get greater as we go from year to year, the moving average lies above the trend. Where, as in Col. E, the trend increases by amounts that get less from year to year the moving average lies below the trend.

**The Geometric Moving Average and Its Use**

For growth curves that increase by an increasing amount, such as the curve set forth in Col. C above, we can usually get a better fit by computing the **geometric moving average.**

In fact when the growth increases by increasing amounts such that the **rate** of growth is constant, the geometric moving average will give a perfect fit.

The geometric moving average is merely the **nth root** of the terms **multiplied** together instead of the **nth** of the terms **added** together. For example, for a 5-year **geometric** moving average, instead of successively **adding** together each five consecutive terms and **dividing** by five, you successively **multiply** together each five consecutive terms and take the fifth **root** of the product. The formulae for a 5-year arithmetic moving average and a 5-year geometric moving average are as follows:

The arithmetic moving average:

$$MA_c = \frac{a + b + c + d + e}{5}$$

The geometric moving average:

$$GMA_c = \sqrt[5]{a \times b \times c \times d \times e}$$

Of course, in practice, to get a geometric moving average, one merely looks up the logarithms of the data in a table of logarithms, records them as in Col. B in Table 9, which follows below, computes the arithmetic moving average of the logs, and reconverts by looking up the antilogs, all as demonstrated in the table.

TABLE 9.
COMPUTATION OF A 5-YEAR GEOMETRIC MOVING AVERAGE
CONTROLLED DATA

| YEAR | A<br>DATA<br>(TREND, WITH CONSTANT 6% RATE OF GROWTH) | B<br>LOGS OF COL. A | C<br>5-YEAR ARITHMETIC MOV. AVER. OF THE LOGS | D<br>ANTILOGS OF COL. C I.E. 5-YEAR GEOMETRIC MOV. AVER. OF COL. A |
|---|---|---|---|---|
| 1ST | 100.00 | 2.0000 | • | • |
| 2ND | 106.00 | 2.0253 | • | • |
| 3RD | 112.36 | 2.0506 | 2.0506 | 112.36 |
| 4TH | 119.10 | 2.0759 | 2.0759 | 119.10 |
| 5TH | 126.25 | 2.1012 | 2.1012 | 126.25 |
| 6TH | 133.82 | 2.1265 | 2.1265 | 133.82 |
| 7TH | 141.85 | 2.1518 | 2.1518 | 141.85 |
| 8TH | 150.36 | 2.1771 | 2.1771 | 150.36 |
| 9TH | 159.38 | 2.2024 | 2.2024 | 159.38 |
| 10TH | 168.95 | 2.2227 | 2.2227 | 168.95 |
| 11TH | 179.08 | 2.2530 | 2.2530 | 179.08 |
| 12TH | 189.83 | 2.2784 | 2.2784 | 189.83 |
| 13TH | 201.22 | 2.3037 | 2.3037 | 201.22 |
| 14TH | 213.29 | 2.3290 | 2.3290 | 213.29 |
| 15TH | 226.01 | 2.3543 | • | • |
| 16TH | 239.66 | 2.3796 | • | • |

As the 5-year geometric moving average is seen by inspection to be the same as the data, there seems to be no need to chart the result.

When the **rate** at which the curve increases is **decreasing**, the geometric moving average will lie below the curve. When the **rate** at which the curve is **increasing**, the geometric moving average lies above the curve. When the **rate** of growth is constant, as in the example above, the geometric moving average lies on the curve. I find the geometric moving average very useful, and use it a great deal.

Even though very few curves grow at an absolutely constant rate of growth, it is true that many growth curves tend to increase this way and are concave upward when plotted on arithmetic paper; in other words, they grow from year to year in an absolute amount which increases with each successive term. This is one reason why, in most cycle analyses it is usually desirable to deal with the logarithms of the data instead of with the data themselves.

## C. The Use of the Moving Average in Cycle Analysis

How can a knowledge of moving averages be used to assist you in cycle analysis—that is, (i) to help you detect and separate cycles that may be present in the data you are studying, and (ii) to help you to obtain a more exact knowledge of their characteristics than would otherwise be possible?

Every time you compute a moving average of a time series you affect cycles of every length that may be present in that series. But, you influence cycles of different length in very different ways. And this fact in turn has an effect upon the comparison that you may make between two different moving averages or between the original data and the moving average.

Therefore, where there are several cycles present concurrently in a time series, by a suitable selection of moving averages, you can minimize or even eliminate some of these cycles and leave others virtually unchanged or, if you wish, magnified.

To see how to make these manipulations, you must first examine the effect of moving averages of different lengths upon a perfectly regular cycle that we can use for purposes of demonstration.

This brings up the question of the shape of the cycle that we should use. However, before we begin to talk about cycles and wave shapes, we will need to have in mind a few more definitions of terms. With these out of the way we can return to a discussion of the proper shape of wave to use for our demonstration, without the need of interrupting the discussion to define terms as we go along. From that point we can go on to a discussion of the effect of moving averages upon the wave shape we have chosen.

## Definitions of Certain Terms Used in Cycle Analysis

**Cycle**, coming from a Greek word meaning circle, implies coming around to the place of beginning. Strictly speaking, in the word itself there is no necessary implication of regularity, but the word is often used loosely to denote rhythm or periodicity.

**Rhythm**, coming from a Greek word meaning measured time, implies a beat, or a tendency toward perfect regularity or periodicity. It

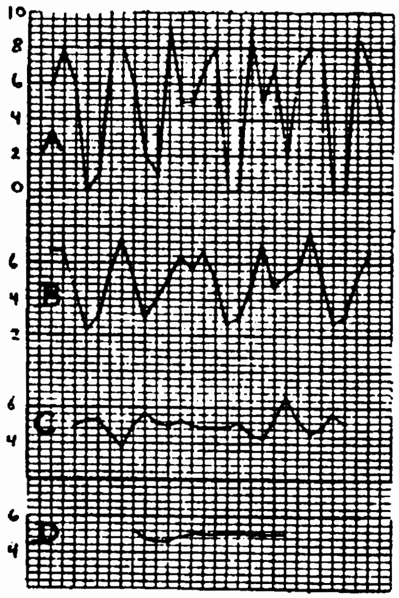



Fig. 1

A. Random Numbers
B. Their 3-Year Moving Average
C. Their 7-Year Moving Average
D. Their 16-Year Moving Average

Note that the longer the moving average, the smoother the curve.

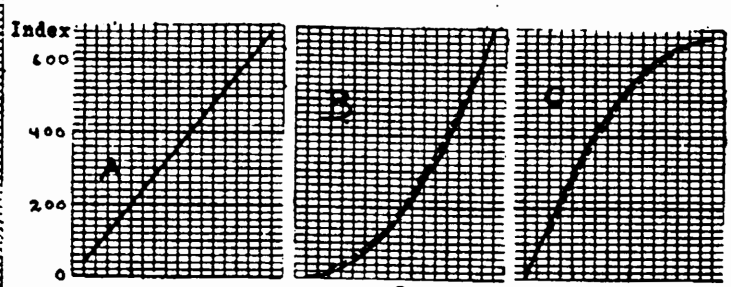



Fig. 2

A. Trend that increases by a constant amount together with its 5-year moving average. (The moving average does not show because it coincides with the trend.)
B. Trend that increases by increasing amounts and, broken line, its 5-year moving average. Note that the moving average lies above the trend.
C. Trend that increases by decreasing amounts and, broken line, its 5-year moving average. Note that the moving average lies below the trend.

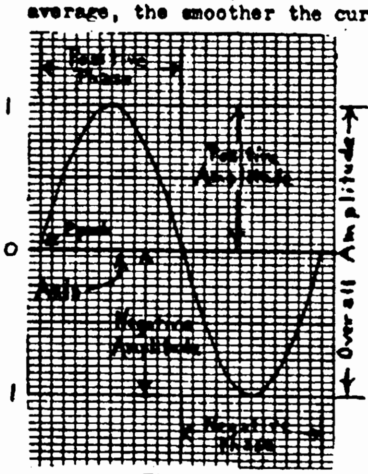



Fig. 3

Sine Wave

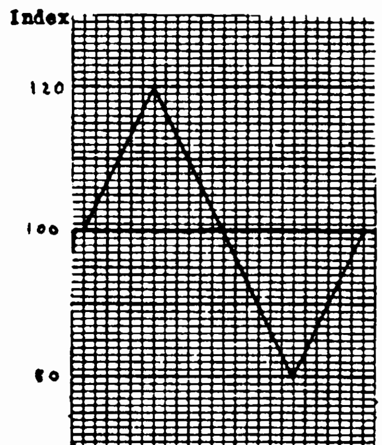



Fig. 4

Rectilinear Wave

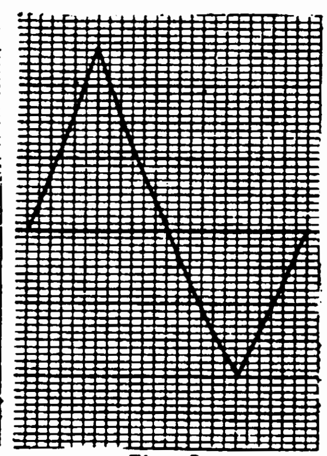


Fig. 5

"Compound Interest" Wave

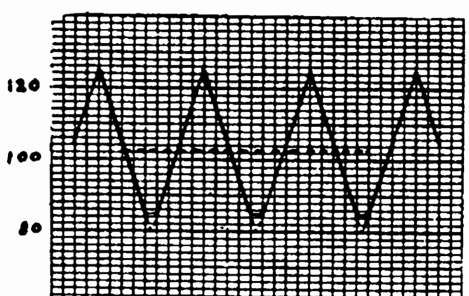



Fig. 6

9-Year Rectilinear Wave
and, broken line,
Its 9-Year Moving Average

Note that a 9-year wave is completely eliminated by a 9-year moving average.

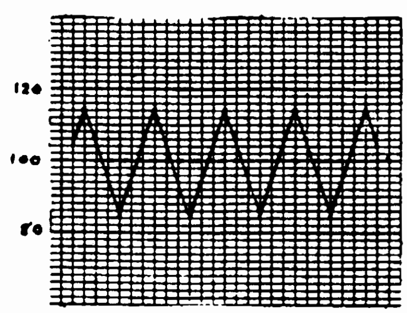



Fig. 7

6-Year Rectilinear Wave
and, broken line,
Its 6-Year Moving Average

Note that a 6-year wave is completely eliminated by a 6-year moving average.

is what we really mean on most of the occasions when we use the word cycle.

**Cycle analysis**, as we are using the term in this bulletin, should really be called rhythm analysis, as we are concerned with **rhythmic cycles**—cycles that recur with a beat.

**Periodicity**, in the strict sense, is the quality of being **regularly** recurrent. It is a quality not often found in nature. The ideal cycles that we shall presently construct for purposes of demonstration, however, are true periodicities.

A **wave** is one single cycle or undulation. Waves have **frequency**, **amplitude**, **period**, and, at least when they represent harmonic curves, **phase**.

**Frequency** is the number of complete vibrations to and fro—i.e. waves—per second. It is a term not used by cycle analysts when dealing with cycles that are over a second in length.

**Amplitude** is the range on one side or the other from the axis around which the wave oscillates. **Positive amplitude** is the distance above the axis, **negative amplitude** is the distance below the axis, **overall amplitude** is the sum of the positive and negative amplitudes. Amplitude may be expressed in absolute units or as a percentage of the axis or trend.

**Period** is the interval of time required for a periodic motion to complete a cycle and begin to repeat itself. It is the length of the wave from crest to crest or trough to trough or from some other point on the curve taken as the **epoch**. (The **epoch** is the point on the curve chosen as the beginning of the wave. In physics and astronomy it is usually taken as the point where the curve crosses the axis on its upward motion, but it may be any other point as well.)

**Phase**, in a simple harmonic curve, is the point or stage in the period to which the oscillation has advanced considered in relation to a standard position or assumed instant of starting. It is measured along the axis, usually in degrees. By extension of meaning, **positive phase** is therefore the part of the wave above the axis or trend, and **negative phase** is the part of the wave below the axis or trend. When the crests (or troughs) of two or more different series of waves come at the same time, the waves are said to be **in phase** with each other. When the crests of one series of waves coincides with the troughs of another series, the series are spoken of as **in reverse phase**.

A **simple harmonic curve** referred to once or twice above, is the curve you would get by tracing the motion of a pendulum upon a piece of smoked paper that was moving at uniform speed at right angles to the direction in which the pendulum was swaying back and forth. It is perfectly simple, regular, and symmetrical and in mathematical study, is usually referred to as a sine curve. A single oscillation is called a **sine wave**. This curve, and many of these definitions are illustrated in Fig. 3 on page 307.

A **rectilinear or saw-tooth wave**, on the other hand, is a wave the sides of which are straight lines; in other words, zigzag. See Fig. 4 on page 307. In electrical engineering a rectilinear wave usually refers to a square wave of this shape , but the term has a more general application also.

## Wave Shapes Usually Found

Because sine waves are so simple in shape, so easy to combine with each other and so satisfactory to handle mathematically, and because they are the shape taken by sound waves and many other kinds of waves with which the physicist deals, it is assumed by many students of cycles in climatology, biology, economics, and other fields that the waves with which they deal **ought** to be sine shape too.

Unfortunately things are not always what they "ought" to be. It has been my experience that waves in economic and biologic time series seem **never** to be sine shape (but this does not mean that the next wave I study might not be of this shape).

It is hard to be sure of the exact shape of a wave. There are almost always variations of length and of amplitude, as we go from one wave to the next. Also there are usually several rhythms present concurrently, and they mix each other up. Finally, there are random factors that enter into the picture which sometimes cannot be removed easily without distorting the wave shape. Therefore I cannot say I am sure of the **exact** mathematical average shape of the waves in any rhythm I have ever studied.

However, if I were forced to express my best guess, I would say that the waves we find in weather, biology, medicine, economics, hydrology, geology, etc., are likely, on the average, to be approximately rectilinear, that is saw-tooth or zigzag. It is not an accident that the " ideal " waves that I have diagrammed in many of the charts that have been published are of this shape.

More exactly, I would put it that the **logarithms** of the data seem, on the average, to conform to a zigzag shape. The result of this fact, of course, is that the average wave shape is saw-tooth when the raw data are plotted on semi-logarithmic paper. This is another way of saying that the sides of the average wave seem to follow the shape of the compound interest curve. That is, the **percentage** rise from the trough to the axis is the same as the **percentage** rise from the axis to the crest

For example, if the trough is at 50 and the axis is at 100, the crest would be at 200 (not 150); one hundred is twice 50, and two hundred is twice one hundred. A wave that follows this law is illustrated in Fig. 5 on page 307.

(The characteristics just described offers another reason why it is usually so highly desirable, in subjecting a series of figures to a rhythm analysis, to convert the raw figures into logarithms before starting work, and to work with them throughout the course of the analysis.)

May I hasten to say that these beliefs are entirely the result of observations as to how the waves in general actually do behave, and are in no sense the result of theories as to how the waves "ought" to behave. I do not yet know enough to talk "oughts."

A second characteristic of the average waves of the rhythms I have studied is that with most of them the upward movements and the downward movements seem to be symmetrical. That is, the lows tend to fall midway between the highs, and vice versa. This characteristic is so generally true that I have come to suspect as possibly spurious any average wave which, without a reason, fails to conform to this pattern.

On the other hand, I have come across undoubted rhythms where the average waves were very definitely neither symmetrical nor of simple zigzag or compound interest form. It is not safe to try to generalize too rigidly.

In discussing the effect of moving averages upon periodic waves, I have chosen for illustration a perfectly symmetrical rectilinear or zigzag wave, because for small amplitude waves this is a close approximation of the typical form and is in fact seemingly the exact form when the data are converted to logarithms.

## The Effect of Moving Averages upon Periodic Waves

### 1. Simple Waves

#### a. When the Length of the Moving Average Is the Same as the Length of the Wave

Suppose yuo have a time series that evidences a perfectly regular 9-year cycle that repeats itself time after time as in Fig. 6 on page 307. The figures for the annual value of such a time series are given below.

Let us compute the 9-year moving average of this series of figures as in Col. C of Table 10.

It is obvious from reference to Col. C that the wave has disappeared and that the moving average is merely a straight line. This straight line has been plotted as a broken line in Fig. 6.

A moment's reflection will explain the reason for this behavior. As the length of the moving average is the same as the length of the wave, the value of the item that is added is always the same as the value of the item that is dropped and, in consequence, the moving average remains unchanged.

It is possible to generalize the above observation and to say that when the moving average has the same length as any perfectly regular wave, its effect is to eliminate the wave completely.

TABLE 10.
A 9-YEAR MOVING AVERAGE OF A 9-YEAR WAVE IN CONTROLLED DATA

| YEAR | A<br>CONTROLLED DATA EVIDENCING A 9-YEAR WAVE | B<br>9-YEAR MOVING TOT. OF DATA | C<br>9-YEAR MO. AV. OF THE DATA (COL. B ÷ 9; OR TIMES 1/9; OR TIMES .111111. THE RECIPROCAL OF 9 |
|---|---|---|---|
| 1ST | 105 | • | • |
| 2ND | 115 | • | • |
| 3RD | 125 | • | • |
| 4TH | 115 | • | • |
| 5TH | 105 | 925 | 102.8 |
| 6TH | 95 | 925 | 102.8 |
| 7TH | 85 | 925 | 102.8 |
| 8TH | 85 | 925 | 102.8 |
| 9TH | 95 | 925 | 102.8 |
| 10TH | 105 | 925 | 102.8 |
| 11TH | 115 | 925 | 102.8 |
| 12TH | 125 | 925 | 102.8 |
| 13TH | 115 | 925 | 102.8 |
| 14TH | 105 | 925 | 102.8 |
| 15TH | 95 | • | • |
| 16TH | 85 | • | • |
| 17TH | 85 | • | • |
| 18TH | 95 | • | • |

To make the procedure doubly plain and to pave the way for a discussion of a method of separating compound cycles, let us work another example. Table 11, next following, gives a series of figures evidencing a perfectly regular 6-year wave. The data are given, together with their centered 6-year moving average.

Here again you get a complete elimination of the wave. The 6-year moving average of the 6-year wave is merely a straight line. It is charted in Fig. 7 by means of a broken line superimposed upon the 6-year wave with which we started. (See p. 307.)

It should also be obvious that if you had added the 6-year wave to a trend line that increased by constant amounts, the 6-year moving average of the combined wave and trend line would have reproduced the trend free and clear of the wave. (If the trend had increased by increasing amounts, the moving average would have lain above it; if by decreasing amounts the moving average would have lain below it; all as illustrated in an earlier section.)

TABLE 11.
A 6-YEAR MOVING AVERAGE OF A 6-YEAR WAVE

| YEAR | A<br>DATA EVIDENCING A 6-YEAR WAVE | B<br>6-YEAR MOVING TOT. OF THE DATA POSTED TO THE 3RD POSITION | C<br>2-YEAR MOVING TOT. OF COL. B POSTED TO THE 2ND POSITION | D<br>6-YEAR MOVING AV. OF THE DATA CENTERED (COL. C÷12) |
|---|---|---|---|---|
| 1ST | 105 | - | - | - |
| 2ND | 115 | - | - | - |
| 3RD | 105 | 600 | - | - |
| 4TH | 95 | 600 | 1200 | 100 |
| 5TH | 85 | 600 | 1200 | 100 |
| 6TH | 95 | 600 | 1200 | 100 |
| 7TH | 105 | 600 | 1200 | 100 |
| 8TH | 115 | 600 | 1200 | 100 |
| 9TH | 105 | 600 | 1200 | 100 |
| 10TH | 95 | - | - | - |
| 11TH | 85 | - | - | - |
| 12TH | 95 | - | - | - |

**Waves of Odd and Peculiar Shape**

You may wonder if we would get the same result—a straight line—if the wave had some other shape. As long as the repetition is perfectly regular, the shape of the wave makes no difference whatever. This fact is illustrated in the table that follows:

TABLE 12.
A 5-YEAR MOVING AVERAGE OF AN IRREGULAR 5-YEAR WAVE

| YEAR | A<br>DATA EVIDENCING AN IRREGULAR SHAPED 5-YEAR REPETITIVE PATTERN | B<br>5-YEAR MOV. AVERAGE OF THE DATA |
|---|---|---|
| 1ST | 100 | - |
| 2ND | 125 | - |
| 3RD | 85 | 100 |
| 4TH | 105 | 100 |
| 5TH | 85 | 100 |
| 6TH | 100 | 100 |
| 7TH | 125 | 100 |
| 8TH | 85 | 100 |
| 9TH | 105 | - |
| 10TH | 85 | - |

The reason we get a straight line is because the value we add is always the same as the value we drop.

**b. When the Length of the Moving Average is An Integral Multiple of the Length of the Wave**

A moving average that is two or three (or any other integral multiple) times the length of the wave will also completely eliminate any regular wave.

Thus, if we have a perfectly regular 9-year wave, an 18-year moving average will completely eliminate it, and so will a 27-year moving average, or a 36-year moving average.

If we have a perfectly regular 6-year wave, a 12-year, 18-year, 24-year, or 30-year moving average would give the same result.

You should also note that an 18-year moving average would completely eliminate **both** the 9-year and 6-year waves, because 18 is a multiple of both 9 years and 6 years.

**c. When the Moving Average is of a Length That is Different From the Length of the Wave, or from some Integral Multiple of It.**

You may wonder what a 3-year moving average of a 9-year wave might look like, or a 5-year moving average, or a 7-year moving average, or an 11-year moving average, or a 13-year moving average.

At this point the shape of the wave begins to make a difference. Let us therefore consider first the effect upon a rectilinear (saw tooth) wave. Such a wave is given in Table 13 on the page following, together with moving averages of various lengths. The various values are plotted in Fig. 8 on page 313.

You will note that as the moving averages get longer they become flatter until, when the length of the moving average equals the length of the wave, the moving average becomes a straight line. When the moving average is longer than the length of the wave, the wave reappears in inverse (upside down) phase. That is, for the 11-year and 13-year moving averages, the 9-year wave reappears with troughs where there were crests in the original data, and with crests in the moving average where we originally had troughs.

The reason for this is very easy to see. The 13-year moving average, for example, centering on a trough, groups together two highs and one low and is therefore obviously **above** the average of one 9-year wave, at time of trough. As we progress in time to a position centering on a crest, the 13-year moving average includes two lows and one high and is therefore obviously **below** the average of one 9-year wave at time of crest.

TABLE 13.
VARIOUS MOVING AVERAGES OF A 9-YEAR WAVE

| YEAR | A DATA EVIDENCING A REGULAR RECTILINEAR 9-YEAR WAVE | B 3-YEAR MOVING AVERAGE OF DATA | C 5-YEAR MOVING AVERAGE OF DATA | D 7-YEAR MOVING AVERAGE OF DATA | E 9-YEAR MOVING AVERAGE OF DATA | F 11-YEAR MOVING AVERAGE OF DATA | G 13-YEAR MOVING AVERAGE OF DATA |
|---|---|---|---|---|---|---|---|
| 1ST | 105 | | • | • | • | • | • |
| 2ND | 115 | 115.0 | • | • | • | • | • |
| 3RD | 125 | 118.3 | 113 | • | • | • | • |
| 4TH | 115 | 115.0 | 111 | 106.4 | • | • | • |
| 5TH | 105 | 105.0 | 105 | 103.6 | 102.8 | • | • |
| 6TH | 95 | 95.0 | 97 | 100.7 | 102.8 | 104.1 | • |
| 7TH | 85 | 88.3 | 93 | 97.8 | 102.8 | 105.9 | 106.5 |
| 8TH | 85 | 88.3 | 93 | 97.8 | 102.8 | 105.9 | 106.5 |
| 9TH | 95 | 95.0 | 97 | 100.7 | 102.8 | 104.1 | 105.0 |
| 10TH | 105 | 105.0 | 105 | 103.6 | 102.8 | 102.3 | 102.7 |
| 11TH | 115 | 115.0 | 111 | 106.4 | 102.8 | 100.5 | 100.4 |
| 12TH | 125 | 118.3 | 113 | 107.8 | 102.8 | 99.5 | 99.6 |
| 13TH | 115 | 115.0 | 111 | 106.4 | 102.8 | 100.5 | 100.4 |
| 14TH | 105 | 105.0 | 105 | 103.6 | 102.8 | 102.3 | 102.7 |
| 15TH | 95 | 95.0 | 97 | 100.7 | 102.8 | 104.1 | 105.0 |
| 16TH | 85 | 88.3 | 93 | 97.8 | 102.8 | 105.9 | 106.5 |
| 17TH | 85 | 88.3 | 93 | 97.8 | 102.8 | 105.9 | 106.5 |
| 18TH | 95 | 95.0 | 97 | 100.7 | 102.8 | 104.1 | 105.0 |
| 19TH | 105 | 105.0 | 105 | 103.6 | 102.8 | 102.3 | 102.7 |
| 20TH | 115 | 115.0 | 111 | 106.4 | 102.8 | 100.5 | 100.4 |
| 21ST | 125 | 118.3 | 113 | 107.8 | 102.8 | 99.5 | 99.6 |
| 22ND | 115 | 115.0 | 111 | 106.4 | 102.8 | 100.5 | 100.4 |
| 23RD | 105 | 105.0 | 105 | 103.6 | 102.8 | 102.3 | 102.7 |
| 24TH | 95 | 95.0 | 97 | 100.7 | 102.8 | 104.1 | 105.0 |
| 25TH | 85 | 88.3 | 93 | 97.8 | 102.8 | 105.9 | • |
| 26TH | 85 | 88.3 | 93 | 97.8 | 102.8 | • | • |
| 27TH | 95 | 95.0 | 105 | 100.7 | 102.8 | • | • |
| 28TH | 105 | 105.0 | 111 | • | • | • | • |
| 29TH | 115 | 115.0 | • | • | • | • | • |
| 30TH | 125 | • | • | • | • | • | • |

(HIGHS OF EACH CYCLE UNDERLINED)

**d. Generalization for Rectilinear Waves**

Fig. 9 was worked out by Benjamin Foote and James A. Mitchell of the Hartford Electric Light Company to generalize these facts for rectilinear waves. The chart was drawn for you by Mr. Mitchell. It gives you the percentage of the original amplitude remaining in the moving average for all simple arithmetic moving averages up to four times the length of the wave. (Fig. 9 will be found later in this bulletin on page 314.) This chart is a most useful one for all cycle analysts. I use mine constantly. Let us work out an example or two.

**Two Examples**

Suppose we have taken a 22-year moving average of a series of figures that contains a 17-year rectilinear (zigzag) wave. How much of the 17-year wave would remain in the 22-year moving average? Twenty-two is approximately 129.4% of 17. Find 129 on the horizontal scale at the bottom of Fig. 9. Construct a perpendicular at this point. This perpendicular will cut the curved line at about minus 16 (read from the scale at position 50. The scale at the extreme left is for values on the horizontal scale from 0 to 50). There will therefore be minus 16% of the 17-year wave remaining in the moving average; that is, the wave in the moving averages will be in reverse phase or upside down from the wave in the original data.

Suppose we had taken a 38-year moving average of the same series of figures. How much of the original 17-year wave would be present in this 38-year moving average? Thirty-eight is 223.5% of 17. Therefore, we find 224 on our horizontal scale, construct a perpendicular. This perpendicular intersects the curve at plus 8 (read from the scale at position 50). Therefore, we know that 8% of the original amplitude of the 17-year rectilinear wave is still present in the 38-year moving average of these figures. If the amplitude of the moving average should prove to be 4, let us say, we could easily calculate that in the original figures it was 50 because 4 is 8% of 50.

**Use of Tables**

You may prefer to use a table instead of the chart. If so, you can refer to Table A below.

Let us work an example: Suppose we have a 23-year moving average of a regular zigzag shaped 54-year rhythm. How much of the rhythm remains in the moving average? Twenty three divided by 54 is 42.6%. Look up 42.6% in the first column in Table A—the column headed "The length of the moving average expressed as a percentage of the length of the wave." We find no value for 42.6 but we do find values for 40 and for 45. The percentage of the original amplitude remaining in the moving average for 40 is 60%, for 45 is 55%. By interpolation it is easy to compute that the correct percentage for 42.6% is 57.4%, the required answer.

TABLE A

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE, WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND RECTILINEAR OR SAW-TOOTH IN SHAPE, FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE.

| THE LENGTH OF THE MOVING AVERAGE EXPRESSED AS A PERCENTAGE OF THE LENGTH OF THE WAVE | PERCENTAGE OF ORIGINAL AMPLITUDE REMAINING IN THE MOVING AVERAGE | A CONT'D. | B CONT'D. | A CONT'D. | B CONT'D. | A CONT'D. | B CONT'D. |
|---|---|---|---|---|---|---|---|
| 0 | 100. | 100 | .0 | 200 | 0 | 300 | .0 |
| 5 | 95. | 105 | -4.5 | 205 | 2.3 | 305 | -1.6 |
| 10 | 90. | 110 | -8.2 | 210 | 4.3 | 310 | -2.9 |
| 15 | 85. | 115 | -11.1 | 215 | 5.9 | 315 | -4.0 |
| 20 | 80. | 120 | -13.3 | 220 | 7.3 | 320 | -5.0 |
| 25 | 75. | 125 | -15.0 | 225 | 8.3 | 325 | -5.8 |
| 30 | 70. | 130 | -16.2 | 230 | 9.1 | 330 | -6.4 |
| 35 | 65. | 135 | -16.8 | 235 | 9.7 | 335 | -6.8 |
| 40 | 60. | 140 | -17.1 | 240 | 10.0 | 340 | -7.1 |
| 45 | 55. | 145 | -17.1 | 245 | 10.1 | 345 | -7.2 |
| 50 | 50. | 150 | -16.7 | 250 | 10.0 | 350 | -7.1 |
| 55 | 45. | 155 | -16.0 | 255 | 9.7 | 355 | -7.0 |
| 60 | 40. | 160 | -15.0 | 260 | 9.2 | 360 | -6.7 |
| 65 | 35. | 165 | -13.8 | 265 | 8.6 | 365 | -6.2 |
| 70 | 30. | 170 | -12.4 | 270 | 7.8 | 370 | -5.7 |
| 75 | 25. | 175 | -10.7 | 275 | 6.8 | 375 | -5.0 |
| 80 | 20. | 180 | -8.9 | 280 | 5.7 | 380 | -4.2 |
| 85 | 15. | 185 | -6.9 | 285 | 4.5 | 385 | -3.1 |
| 90 | 10. | 190 | -4.7 | 290 | 3.1 | 390 | -2.3 |
| 95 | 5. | 195 | -2.4 | 295 | 1.6 | 395 | -1.2 |
| 100 | 0. | 200 | 0.0 | 300 | 0.0 | 400 | 0.0 |

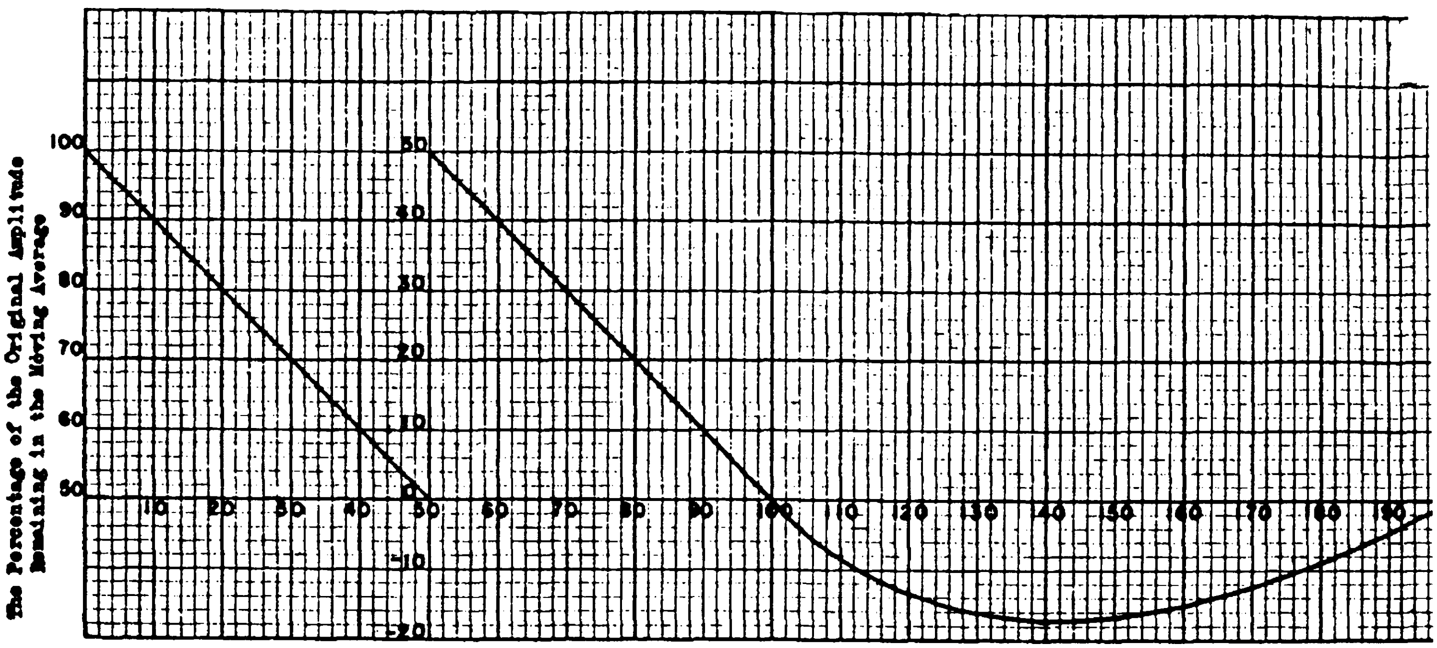



FIG. 9. FOR RECTILINEAR (SAW-TOOTH) WAVES — The Length of the

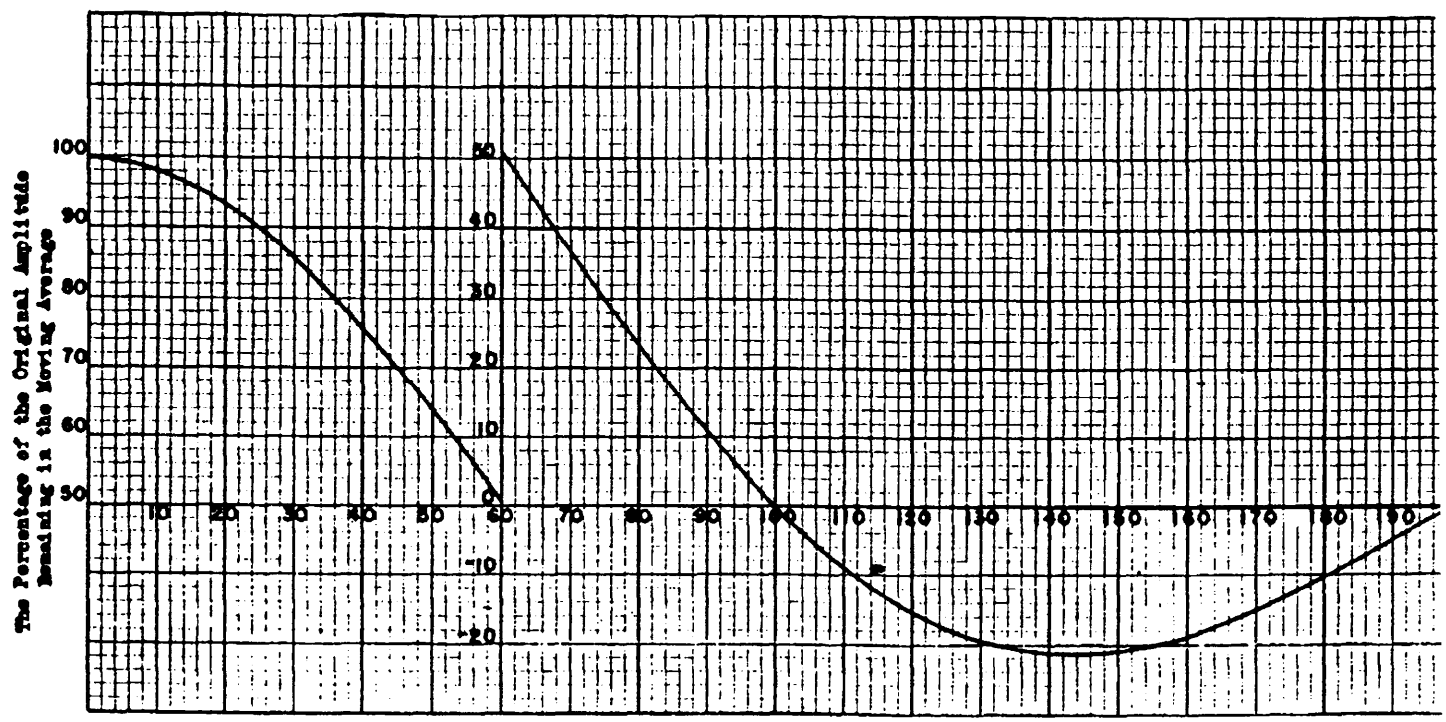
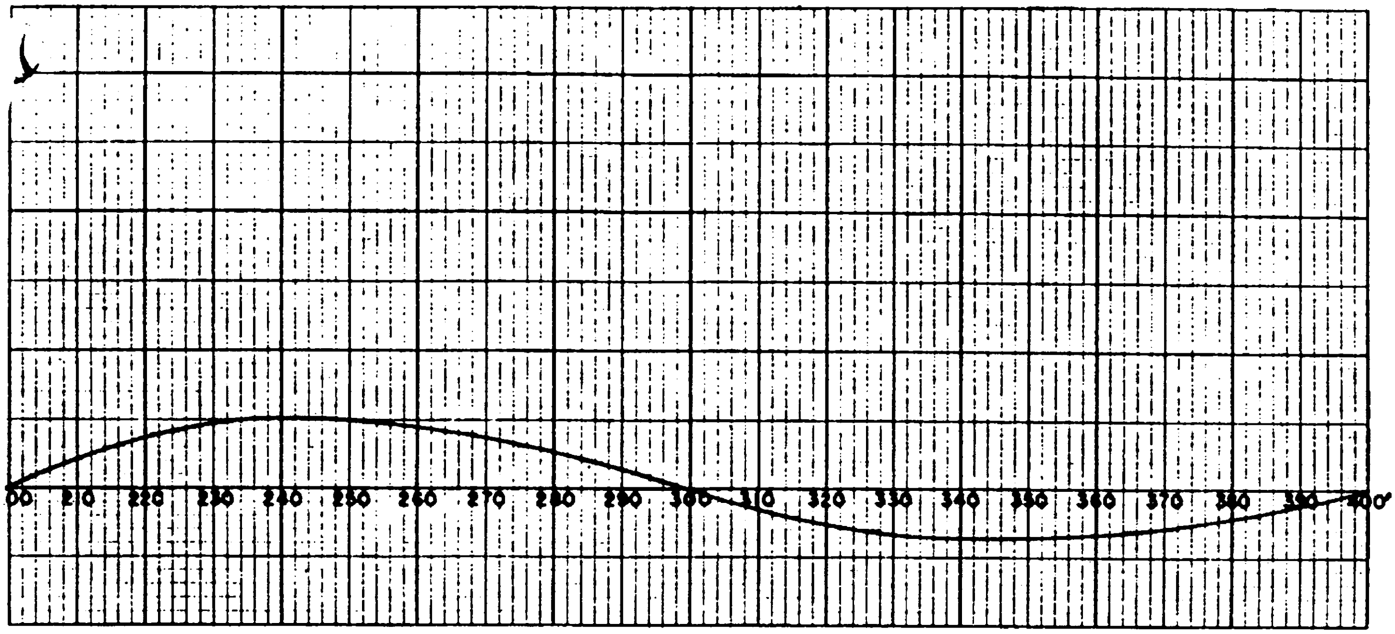
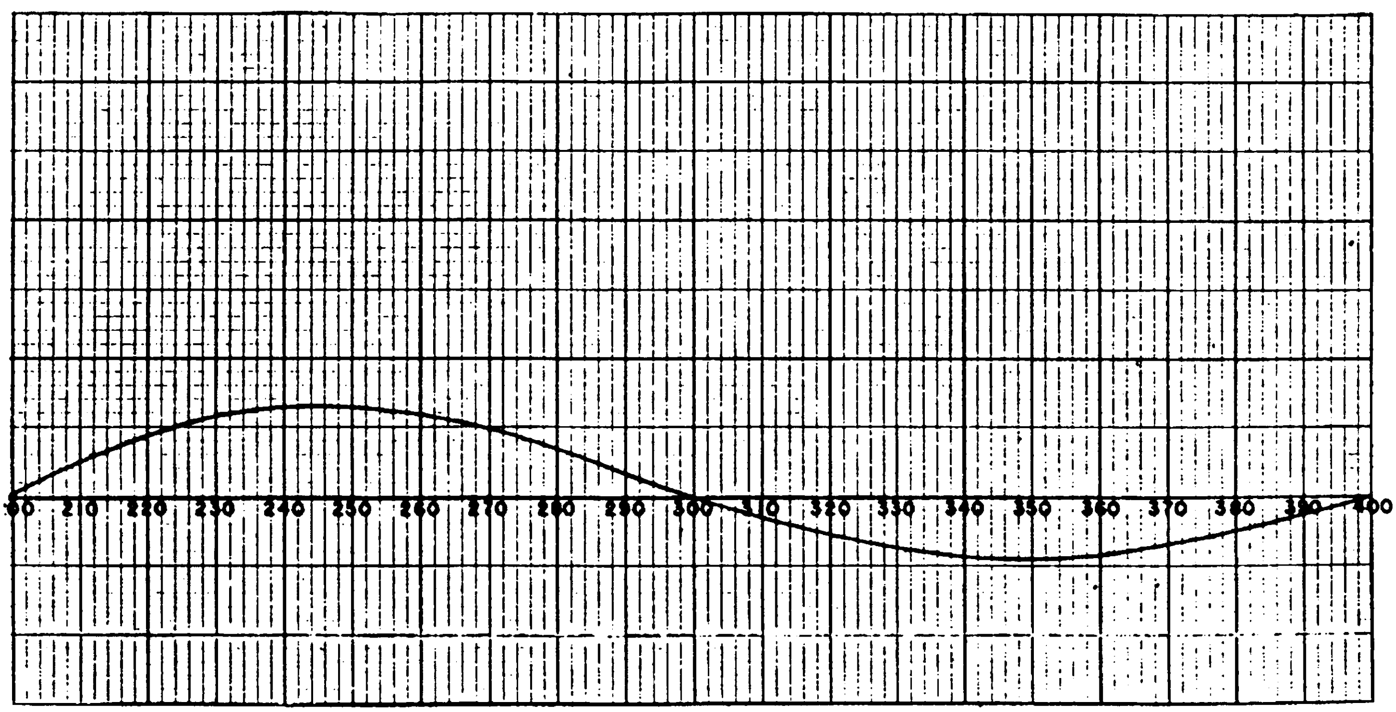



FIG. 10. FOR SINE WAVES — The Length of the Movin




Moving Average Expressed as a Percentage of the Length of the Rhythm




Average Expressed as a Percentage of the Length of the Rhythm

**e. Generalization for Sine Waves**

At this point you may ask, does this chart hold true for sine waves and for other waves that are not rectilinear or zigzag shape? No, it does not. Each shape of wave requires a separate diagram. For sine waves Mr. Foote and Mr. Mitchell constructed Fig. 10, inserted previously in this bulletin on page 314, to show the percentage of the original amplitude remaining in the moving average of a sine wave for all given lengths of moving average up to four times the length of the wave.

If you prefer to use a table, refer to Table B below.

TABLE B

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE,
WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND SINE SHAPED
FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE

| A<br>The Length of the Moving Average Expressed as a Percentage of the Length of the Wave | B<br>Percentage of Original Amplitude Remaining in the Moving Average | A<br>Cont'd. | B<br>Cont'd. | A<br>Cont'd. | B<br>Cont'd. | A<br>Cont'd. | B<br>Cont'd. |
|---|---|---|---|---|---|---|---|
| 0 | .0 | 100 | .0 | 200 | .0 | 300 | .0 |
| 5 | 99.4 | 105 | -4.7 | 205 | 2.4 | 305 | -1.6 |
| 10 | 98.4 | 110 | -8.9 | 210 | 4.7 | 310 | -3.2 |
| 15 | 96.4 | 115 | -12.6 | 215 | 6.7 | 315 | -4.6 |
| 20 | 93.6 | 120 | -15.6 | 220 | 8.5 | 320 | -5.8 |
| 25 | 90.1 | 125 | -18.0 | 225 | 10.0 | 325 | -6.9 |
| 30 | 85.9 | 130 | -19.8 | 230 | 11.2 | 330 | -7.8 |
| 35 | 81.0 | 135 | -21.0 | 235 | 12.1 | 335 | -8.5 |
| 40 | 75.7 | 140 | -21.6 | 240 | 12.6 | 340 | -8.9 |
| 45 | 69.9 | 145 | -21.7 | 245 | 12.8 | 345 | -9.1 |
| 50 | 63.7 | 150 | -21.2 | 250 | 12.7 | 350 | -9.1 |
| 55 | 57.2 | 155 | -20.3 | 255 | 12.3 | 355 | -8.9 |
| 60 | 50.5 | 160 | -18.9 | 260 | 11.6 | 360 | -8.4 |
| 65 | 43.6 | 165 | -17.2 | 265 | 10.7 | 365 | -7.8 |
| 70 | 36.8 | 170 | -15.1 | 270 | 9.5 | 370 | -7.0 |
| 75 | 30.0 | 175 | -12.9 | 275 | 8.2 | 375 | -6.0 |
| 80 | 23.4 | 180 | -10.4 | 280 | 6.7 | 380 | -4.9 |
| 85 | 17.0 | 185 | -7.8 | 285 | 5.1 | 385 | -3.7 |
| 90 | 10.9 | 190 | -5.2 | 290 | 3.4 | 390 | -2.5 |
| 95 | 5.2 | 195 | -2.5 | 295 | 1.7 | 395 | -1.2 |
| 100 | 0. | 200 | 0. | 300 | 0. | 400 | 0. |

**2. Compound Waves**

**Moving Averages of Time Series Influenced by Two or More Concurrent Cycles**

Let us now add together the two series of figures containing the 6-year wave and the 9-year wave respectively that we dealt with above and which were charted in Figs. 6 and 7. This addition is performed in the table on the following page and the result is plotted in Fig. 11 on page 319.

This sum is the kind of a pattern one might expect in the total sales of a company that were equally divided between two products, the sales of one of which fluctuated with a 9-year rhythm, and a second product, the sales of which fluctuated with a 6-year rhythm.

TABLE 14.
6-YEAR, 9-YEAR, AND 18-YEAR MOVING AVERAGES OF A COMPOUND WAVE

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| YEAR | DATA EVIDENCING THE 9-YEAR CYCLE | DATA EVIDENCING THE 6-YEAR CYCLE | SUM OF COL. A AND COL. B | 6-YEAR MOVING AVERAGE OF COL. C, CENTERED | 9-YEAR MOVING AVERAGE OF COL. C | 18-YEAR MOVING AVERAGE OF COL. C, CENTERED |
| 1ST | 105 | 105 | 210 | • | • | • |
| 2ND | 115 | 115 | 230 | • | • | • |
| 3RD | 125 | 105 | 230 | • | • | • |
| 4TH | 115 | 95 | 210 | 208.3 | • | • |
| 5TH | 105 | 85 | 190 | 204.2 | 205.6 | • |
| 6TH | 95 | 95 | 190 | 199.2 | 204.4 | • |
| 7TH | 85 | 105 | 190 | 195.8 | 201.1 | • |
| 8TH | 85 | 115 | 200 | 195.8 | 200.0 | • |
| 9TH | 95 | 105 | 200 | 199.2 | 201.1 | • |
| 10TH | 105 | 95 | 200 | 204.2 | 204.4 | 202.8 |
| 11TH | 115 | 85 | 200 | 208.3 | 205.6 | 202.8 |
| 12TH | 125 | 95 | 220 | 210.0 | 204.4 | 202.8 |
| 13TH | 115 | 105 | 220 | 208.3 | 201.1 | 202.8 |
| 14TH | 105 | 115 | 220 | 204.2 | 200.0 | 202.8 |
| 15TH | 95 | 105 | 200 | 199.2 | 201.1 | 202.8 |
| 16TH | 85 | 95 | 180 | 195.8 | 204.4 | 202.8 |
| 17TH | 85 | 85 | 170 | 195.8 | 205.6 | 202.8 |
| 18TH | 95 | 95 | 190 | 199.2 | 204.4 | 202.8 |
| 19TH | 105 | 105 | 210 | 204.2 | 201.1 | 202.8 |
| 20TH | 115 | 115 | 230 | 208.3 | 200.0 | 202.8 |
| 21ST | 125 | 105 | 230 | 210.0 | 201.1 | 202.8 |
| 22ND | 115 | 95 | 210 | 208.3 | 204.4 | 202.8 |
| 23RD | 105 | 85 | 190 | 204.2 | 205.6 | 202.8 |
| 24TH | 95 | 95 | 190 | 199.2 | 204.4 | 202.8 |
| 25TH | 85 | 105 | 190 | 195.8 | 201.1 | 202.8 |
| 26TH | 85 | 115 | 200 | 195.8 | 200.0 | 202.8 |
| 27TH | 95 | 105 | 200 | 199.2 | 201.1 | 202.8 |
| 28TH | 105 | 95 | 200 | 204.2 | 204.4 | • |
| 29TH | 115 | 85 | 200 | 208.3 | 205.6 | • |
| 30TH | 125 | 95 | 220 | 210.0 | 204.4 | • |
| 31ST | 115 | 105 | 220 | 208.3 | 201.1 | • |
| 32ND | 105 | 115 | 220 | 204.2 | 201.1 | • |
| 33RD | 95 | 105 | 200 | 199.2 | • | • |
| 34TH | 85 | 95 | 180 | • | • | • |
| 35TH | 85 | 85 | 170 | • | • | • |
| 36TH | 95 | 95 | 190 | • | • | • |

(HIGHS OF EACH CYCLE UNDERLINED)

Let us now take these figures that evidence this composite wave and compute first a 9-year moving average, second, a 6-year moving average and third, an 18-year moving average as in the table on the preceding page. The various moving averages are plotted in Fig. 11 on page 319. The 9-year moving average has the effect of completely eliminating the 9-year component of the series and shows the 6-year wave in reverse phase, that is to say, with tops where bottoms used to be and vice versa, but with reduced amplitude, all as we would expect from the foregoing discussion.

The 6-year moving average has the effect of completely eliminating the 6-year wave and leaving the 9-year wave in proper phase position (that is, with tops of the moving average where there were tops in the data, and bottoms in the moving average where there were bottoms in the data), but with greatly reduced amplitude.

The 18-year moving average of course eliminates both 6-year and 9-year waves, and would also have eliminated any 4½-year wave (¼ of 18 years), any 3.6-year wave (1/5 of 18 years), any 3-year wave (1/6 of 18 years) and so on if there had been such in the original data. By the same token, it would have revealed any wave longer than 18 years, or shorter waves that were not integral fractions of the length of 18 years, or both, if these had also been present in the data.

**Discussion**

It should be clear from the foregoing demonstrations that every time you take a moving average of a series of figures, you are performing an operation that has an effect upon the amplitude, and sometimes reverses the phase, of all the regularly recurring waves that may be present in the original figures. It is this fact that prompts the Celticism that one is really not able to start a rhythmic analysis until after one has finished it.

In other words, until one knows the length of all the waves that are present in a series, one is not fully in a position to choose the lengths of the moving averages to use to emphasize some and subordinate others.

## Comparison of the Raw Data With with the Moving Average

In the section which began on page 309, you had demonstrated for you the fact that when the length of the moving average is the same as the length of the wave, the effect is the complete elimination of the wave.

Where the original data consist of nothing but a wave (and a horizontal trend line) as in Tables 10 and 11, it is obvious that if we compare the original data with the moving average (which is a horizontal straight line) the result will merely reconstitute the wave in its entirety. This fact is illustrated in Curve EE of Fig. 12 on page 321 and in Col. EE of Table 15 on page 320.

When the moving average is of a length which differs from the length of the wave in the original data, or some multiple of it, some part of the original wave will remain in the moving average. This residue of the original wave remaining in the moving average will be either in phase with the original wave or in reverse phase (upside down). All of this was demonstrated in Table 13 and illustrated in Fig. 8.

When the moving average retains some of the wave **in phase** with the original wave, and when the original data are compared with such a moving average, it should be clear that the difference between the two will show the original wave with **reduced** amplitude. For example, if we have a 9-year wave with an amplitude of 10, and the moving average also contains a 9-year wave coming at the same time with an amplitude of 2, the series of figures evidencing the difference between the two waves will show an amplitude of 8.

When the moving average shows the wave **in reverse phase**, or upside down, and when the original data are compared with it, the difference between the two will show the original wave with **increased** amplitude. For example, if the wave just discussed with an amplitude of 10 were being compared with a moving average that was of such a length that it evidenced a 9-year wave with an amplitude of -1, the wave in the series of figures evidencing the difference would show an amplitude of 11. These facts are all illustrated for the moving averages given in Table 13, in Fig. 12 and in Table 15 on page 320.

It should be noted that the comparisons above have been made by subtraction for the sake of si plicity. In actual practice one ordinarily makes the comparison by division and determines the **percentages** that the original data are of their moving average.

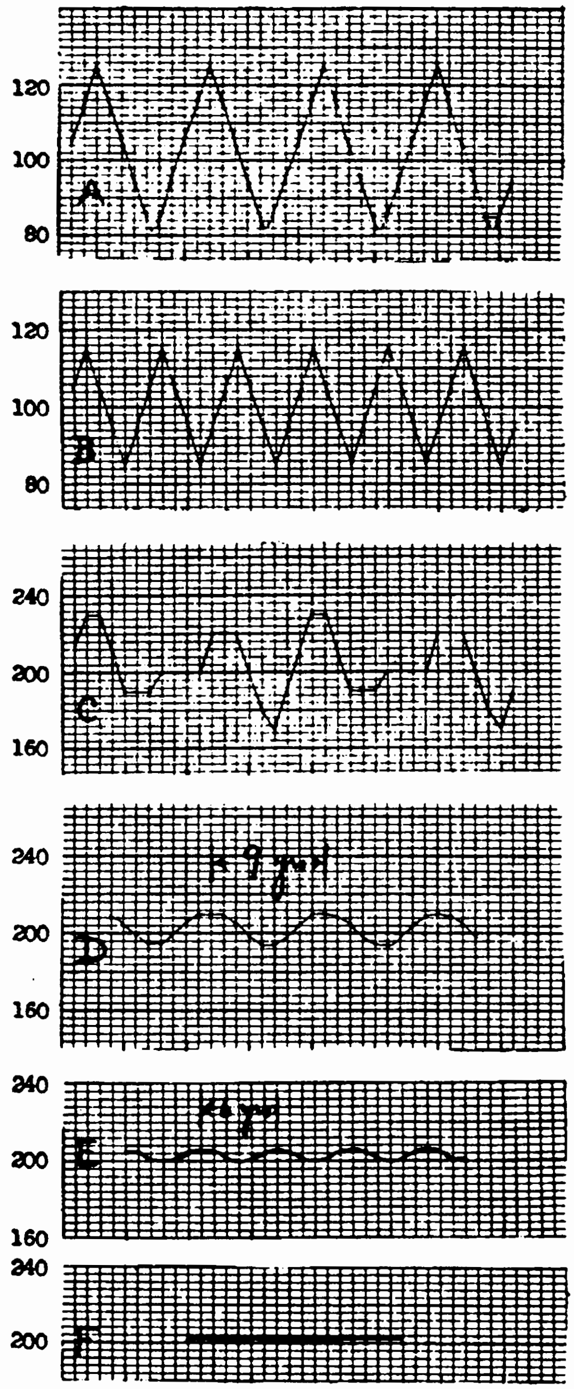
There are several reasons for making the comparisons on a percentage basis. One has already been mentioned—the fact that mostly the waves seem to be the same **percentage** above and below the axis. When one uses percentages on real waves therefore, one tends to get waves that are symmetrical with respect to the axis.

A second reason is that in actual practice, most waves with which one deals are superimposed upon trend lines. That is, the phenomenon with which we deal, let us say the abundance of lynx or the thickness of tree rings or the size of a business, has a long term increase or decrease over a period of time. Experience indicates that the waves that are associated with these various phenomena are usually of approximately constant **percentage** amplitude.

A third reason for making percentage comparisons, even if the trend should be horizontal, is that if there should be other waves it will usually be found that the various waves combine by multiplication and not by addition. They therefore must be unscrambled by division instead of by subtraction.

A fourth reason for making comparison on a percentage basis is that in making projections for most series, one must talk of the waves in terms of percentages. "If the wave continues, the sales of the company at such

Fig. 11




A. A series of 9-year waves

B. A series of 6-year waves

C. Their summation (A + B)

D. A 6-year moving average of the summation ( reveals a 9-year wave in phase with the original 9-year wave

E. A 9-year moving average of the summation ( reveals a 6-year wave in <u>reverse</u> phase from the original 6-year wave (upside down)).

F. An 18-year moving average of the summation (completely eliminates both waves) and such a time will be 10% above its then trend line." Where the trend line will be at the time must be computed separately.

The only exceptions to the above rule that occurs to me at the moment are (a) the case where the waves in the original data are expressed in plus and minus values, and (b) the case where some of the values of the raw data are zero. In these instances, comparisons between the moving average and the data should usually be made by subtraction.

TABLE 15.

COMPARISON OF ORIGINAL CONTROLLED DATA WITH VARIOUS MOVING AVERAGES

| | A | B | BB | C | CC | D | DD | E | EE | F | FF | G | GG |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | Data Evidencing a Regular Saw-tooth 9-Year Wave | 3-Year Moving Average of the Data | Data Divided by Their 3-Year Moving Average (%) | 5-Year Moving Average of the Data | Data Divided by Their 5-Year Moving Average (%) | 7-Year Moving Average of the Data | Data Divided by Their 7-Year Moving Average (%) | 9-Year Moving Average of the Data | Data Divided by Their 9-Year Moving Average (%) | 11-Year Moving Average of the Data | Data Divided by Their 11-Year Moving Average (%) | 13-Year Moving Average of the Data | Data Divided by Their 13-Year Moving Average (%) |
| 1st | 105 | . | . | . | . | . | . | . | . | . | . | . | . |
| 2nd | 115 | 115.0 | 100.0 | . | . | . | . | . | . | . | . | . | . |
| 3rd | 125 | 118.3 | 105.7 | 113.0 | 110.6 | . | . | . | . | . | . | . | . |
| 4th | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | . | . | . | . | . | . |
| 5th | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | . | . | . | . |
| 6th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | . | . |
| 7th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 8th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 9th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | 105.0 | 90.5 |
| 10th | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | 102.3 | 102.6 | 102.7 | 102.2 |
| 11th | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | 102.8 | 111.9 | 100.5 | 114.4 | 100.4 | 114.5 |
| 12th | 125 | 118.3 | 105.7 | 113.0 | 110.6 | 107.8 | 116.0 | 102.8 | 121.6 | 99.5 | 125.6 | 99.6 | 125.5 |
| 13th | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | 102.8 | 111.9 | 100.5 | 114.4 | 100.4 | 114.5 |
| 14th | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | 102.3 | 102.6 | 102.7 | 102.2 |
| 15th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | 105.0 | 90.5 |
| 16th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 17th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 18th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | 105.0 | 90.5 |
| 19th | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | 102.3 | 102.6 | 102.7 | 102.2 |
| 20th | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | 102.8 | 111.9 | 100.5 | 114.4 | 100.4 | 114.5 |
| 21st | 125 | 118.3 | 105.7 | 113.0 | 110.6 | 107.8 | 116.0 | 102.8 | 121.6 | 99.5 | 125.6 | 99.6 | 125.5 |
| 22nd | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | 102.8 | 111.9 | 100.5 | 114.4 | 100.4 | 114.5 |
| 23rd | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | 102.3 | 102.6 | 102.7 | 102.2 |
| 24th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | 105.0 | 90.5 |
| 25th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 26th | 85 | 88.3 | 96.3 | 93.0 | 91.4 | 97.8 | 86.9 | 102.8 | 82.7 | 105.9 | 80.3 | 106.5 | 79.8 |
| 27th | 95 | 95.0 | 100.0 | 97.0 | 97.9 | 100.7 | 94.3 | 102.8 | 92.4 | 104.1 | 91.3 | . | . |
| 28th | 105 | 105.0 | 100.0 | 105.0 | 100.0 | 103.6 | 101.4 | 102.8 | 102.1 | . | . | . | . |
| 29th | 115 | 115.0 | 100.0 | 111.0 | 103.6 | 106.4 | 108.1 | . | . | . | . | . | . |
| 30th | 125 | 118.3 | 105.7 | 113.0 | 110.6 | . | . | . | . | . | . | . | . |
| 31st | 115 | 115.0 | 100.0 | . | . | . | . | . | . | . | . | . | . |
| 32nd | 105 | . | . | . | . | . | . | . | . | . | . | . | . |

(ALL CRESTS UNDERLINED)

It will be noted both in the chart and in the table that when we compare the original data with their various moving averages that the rhythm that was present in the original data continues present in the comparison. That is, if the wave in the original data is 9 years long, and we run a 7-year moving average through the series (Col. D above) and compare the original data with the 7-year moving average (Col. DD), we get the 9-year rhythm with

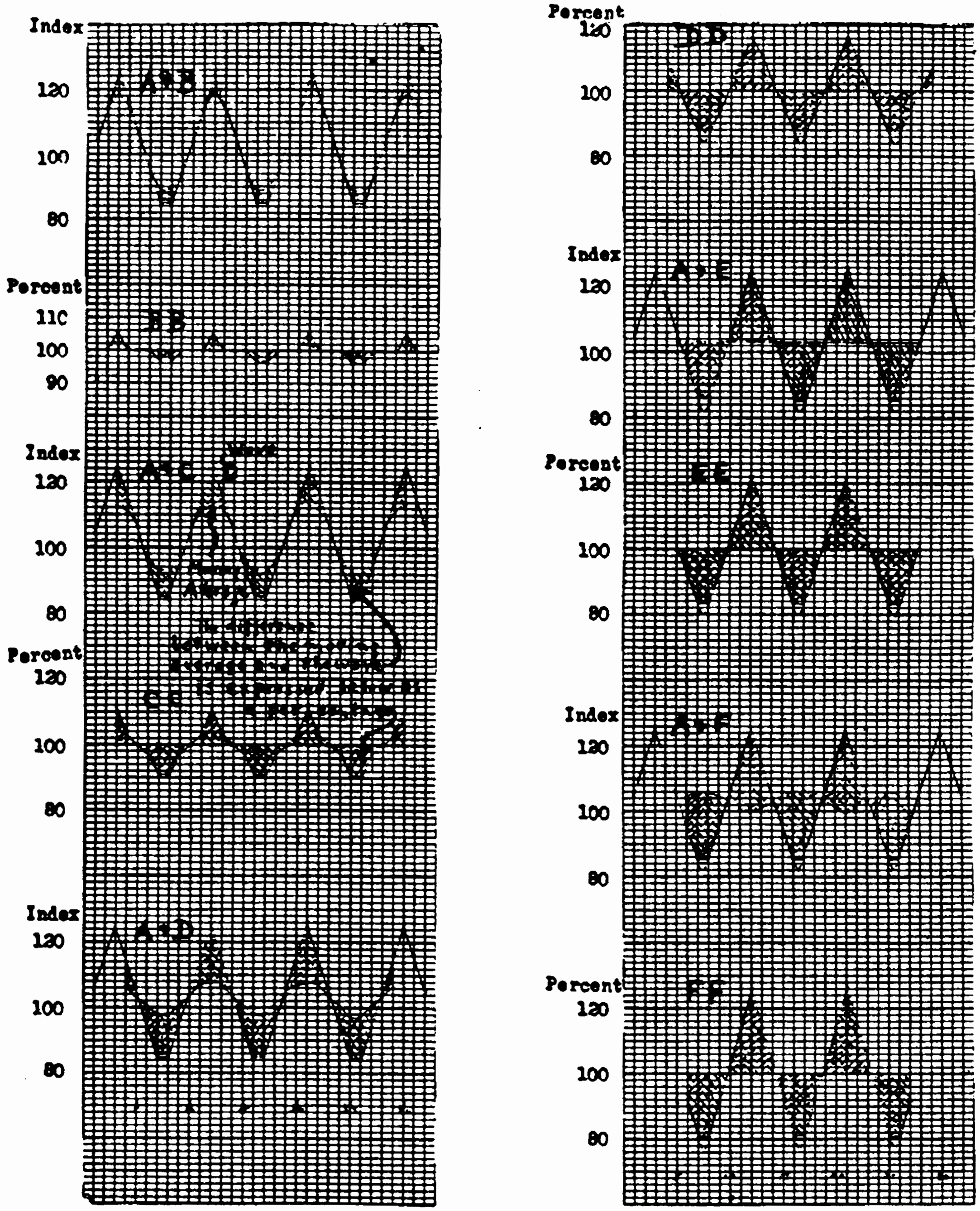



Fig. 12

A & B. 9-Year Rectilinear Wave Together With Its 3-Year Moving Average

BB. Data Divided by Their 3-Year Moving Average

A & C. 9-Year Rectilinear Wave Together With Its 5-Year Moving Average

CC. Data Divided by Their 5-Year Moving Average

A & D. 9-Year Rectilinear Wave Together With Its 7-Year Moving Average

DD. Data Divided by Their 7-Year Moving Average

A & E. 9-Year Rectilinear Wave Together With Its 9-Year Moving Average

EE. Data Divided by Their 9-Year Moving Average

A & F. 9-Year Rectilinear Wave Together With Its 11-Year Moving Average

FF. Data Divided by Their 11-Year Moving Average

Note: Note that as the number of terms in the moving average increase and the moving average gets flatter, the original wave reappears more and more in the percentages. When the moving average equals the length of the wave, the wave reappears fully. As the number of items in the moving average increase further, the amplitude of the original wave is magnified.

Note also that the percentages that the data are of their moving averages always evidence waves of the length of the wave in the original data and not the length of the moving average.

which we started, albeit with greatly reduced amplitude.

In other words, the 7-year moving average of the data does **not** in any sense of the word **introduce** a 7-year rhythm into the comparisons.

The converse of this statement is that if we have a 9-year rhythm in the original data and take a 9-year moving average of the series, compare the original data with this 9-year moving average and find a 9-year wave, the 9-year wave we find can in no sense be construed as a **result** of a 9-year moving average, either. This seems to be the hardest thing about moving averages for people to realize.

It is suggested that you prove these statements to yourself by computing the percentages that actual figures which evidence a rhythm are of moving averages of various lengths.

## Comparison of One Moving Average with Another

The comparison of one moving average with another is merely an extension of the principles that have been explained fully in the foregoing pages. One moving average can be used to eliminate one or more of the minor waves and minimize random fluctuations, another can be used to approximate the trend line. The comparison of the two, if the lengths have been properly chosen, will often reveal or emphasize waves of intermediate length.

## Summary

The moving average is a useful tool for cycle analysts.

By smoothing out random fluctuations and shorter cycles, it aids the eye to see more clearly the waves of intermediate and longer length.

When the length is suitably chosen, the moving average provides an approximation of the underlying growth trend with, however, the disadvantage that the moving average will lie above or below the true trend, unless the trend is increasing by a constant amount.

By the proper choice of length, the moving average can often effect a complete separation of two interacting wave systems present concurrently in the same time series.

By means of the technique of first computing the moving average of a series and then computing the percentages that the original data are of the moving average, it is possible to obtain a curve in which the distorting effect of trend and of longer cycles are minimized.

The use of moving averages can be compared to the use of color filters on a camera. By the proper choice of a filter, you can reveal characteristics of the article being photographed, such as grain in a piece of wood, that might be completely lost in an ordinary photograph and might be overlooked even by the naked eye. The moving average can be used in the same way.

Used with intelligence, and with a full knowledge of its limitations, the moving average is a very valuable tool for the cycle analyst.