Detecting chaos in financial time series

Rossitsa Yalamova; Liu Qi; Lin Wang; Yudong Chen; Spencer Mathews and Guannan Gong
Univ. of Lethbridge; Peking Univ.; Indiana Univ., Tsinghua Univ., UC Davis and Jilin Univ.

Abstract

We use the Tisean C++ package to quantify chaos around Black Monday (crash of 1987). We calculate Lyapunov exponents following the algorithm proposed by Rosenstein et al (1993). A positive Lyapunov exponent correlates with chaos. The negative exponent during the time period centered around Black Monday suggests a lack of chaos surrounding the financial crisis. . However, further tests must be done to confirm this interpretation.

Acknowledgement 1 *This work was partially supported by the US National Science Foundation, via award OISE 0623953.*

1 Introduction

Although it sounds like an oxymoron, the concept of deterministic chaos has been gaining popularity in many fields including economics and finance. Deterministic chaos is defined as the appearance of irregular chaotic motion in purely deterministic dynamical systems. This phenomenon is considered one of the most fundamental discoveries in the theory of dynamical systems in the second part of the last century.

Lovejoy and Schetzer (1999) argue that nonlinear scale invariant dynamics lead to stochastic chaos (universal multifractals). One of the differences between deterministic and stochastic chaos is in the degrees of freedom; from few in the former to an infinite number in the latter. Stochastic chaos allows for the presence of richer scale invariance, e.g. multifractals produced in cascade processes.

In finance, Corcos *et al.* (2002) suggest that exponentially growing bubbles cross over to a non-linear power-law growth rate leading to a finite time singularity. Their model is based on imitative and contrarian behaviors in a stock market with only two agents states - bullish or bearish. They assert that mimicry and contradictory behavior can lead to chaotic prices.

Guanersdorfer (2000a) study the adaptive rational equilibrium dynamics in a simple asset pricing model, analyzing complicated dynamics and bifurcation routes to chaos. Chen *et al* (2001) assert that market dynamics should be tested for the presence of chaos and non-linearity. Kyrtsou and Terraza (2002) test for stochastic chaos in the evolution of price series.

As for a stochastic process, it may not be a chaos as a whole, however, there may exist chaos in subset of the data. Here, we use the largest Lyapunov exponent of a small data set to describe the dynamics of the Dow Jones Index daily A positive largest Lyapunov exponent indicates the existence of chaos. We want to detect chaos in Dow Jones Stock return and find some pattern around the October 19, 1987 market crash.

The multifractal nature of the time series was demonstrated in our previous paper and the generalized dimensions revealed the topology of the series. We would like to examine the dynamics of the system in this paper.

2 Dynamic Stability (level of chaos)

To quantify the stability of orbits around an attractor, it is necessary to calculate the Lyapunov exponents, i. e. the rates at which those orbits converge or diverge. Dechert and Gençay(1992) propose an algorithm for estimation of the Lyapunov exponents when the equations generating the chaos are unknown. A positive largest Lyapunov exponent indicates chaos. Bask and Gençay (1998) propose a test statistic for the presence of chaotic dynamics using the Lyapunov exponents.

To calculate the Lyapunov exponents from time series, it is first necessary to reconstruct the state space from the experimental data record:

2.1 Reconstruction of the state space

The original time series data and its time-delayed copies determine the topological structure of a dynamical system according to the Embedding Theorem of Takens (1981).

$$Y(n) = [X(t), X(t+T), ..., X(t+(d-1)T)] \tag{1}$$

where $Y(n)$ is the reconstructed $d$dimensional state vector, $X(t)$ is the observed variable, T is a time lag, and d is the embedding dimension.

$\rightarrow T$ is calculated from the first minimum of the Average Mutual Information (AMI) function - Fraser and Swinney (1986), Abarbanel (1996).

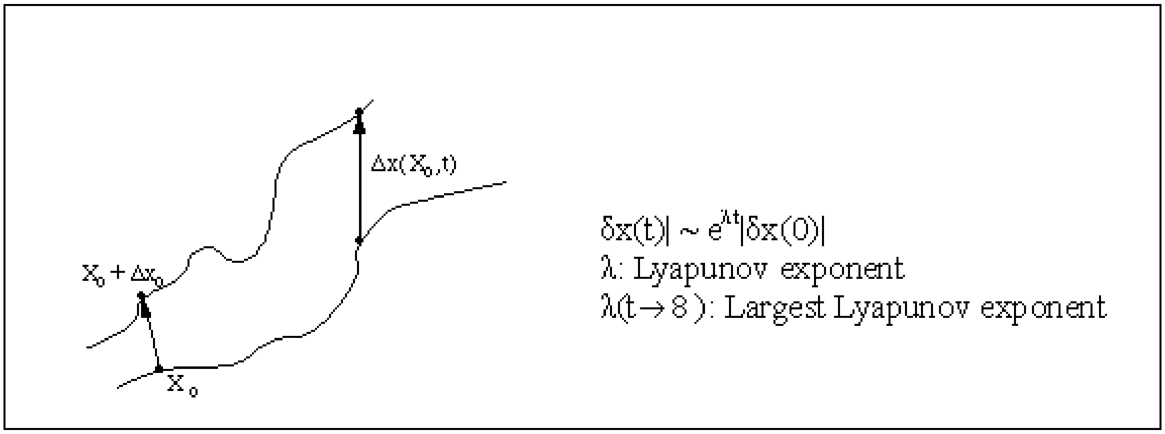

Figure 1: Definition of Lyapunov exponent

$\rightarrow d$ is computed with the Global False Nearest Neighbors (GFNN) method described in Abarbanel (1996).

The dynamical properties of the system are the same in both the original and reconstructed state spaces, providing multidimensional dynamic information from a unidimensional time series.

2.1.1 Methodology

The correlation dimension gives an estimate of a systems complexity, while the Lyapunov characteristic exponent estimates the level of chaos in the dynamical system.

There are different methods for calculation of the Lyapunov exponents in time series data. Here, we followed the algorithm proposed by Rosenstein et al (1993). The algorithm is fast, easy to implement, and robust to changes in the embedding dimension, size of data set, reconstruction delay, and noise level. The authors suggest that the existing methods suffer from at least one of the following:

a) unreliable for small data sets

b) computationally intensive

c) relatively difficult to implement

Therefore, they propose an algorithm estimating the separation between pairs of neighboring points in state space.

1. Reconstruction of the phase space is implemented:

I. Find the lag d as the first minimum of delayed mutual information (MI).

II. Estimate the minimum sufficient embedding dimension m using false nearest neighbor (FNN) method.

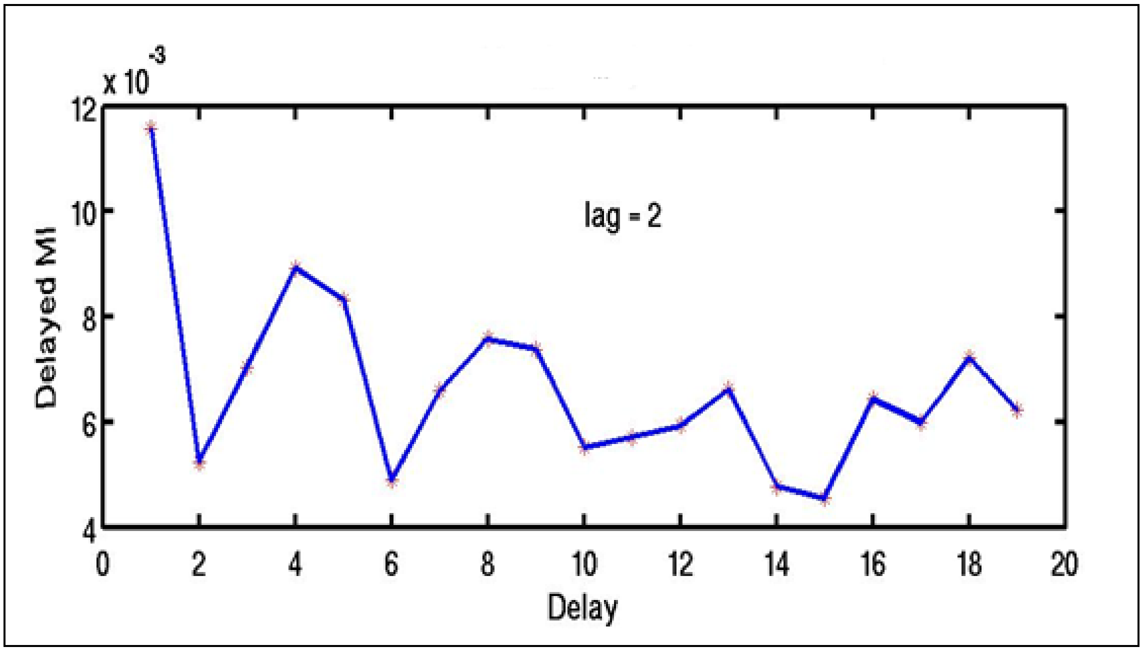

Figure 2: Delayed mutual information of data subset [10798 ~14804], this plot indicates a lag of 2.

III. Phase space representation: $(x(i), x(i+d), ...x(i+md))$

2. Estimation of the Lyapunov exponent of a small data set:

We employ Hegger and Kantz's Tisean software package to calculate Lyapunov exponents on a subset of the Dow Jones Industrial Average Index return series from December 9, 1971 until October 19, 1987.

3 Results

The delayed mutual information is calculated as:

$$s = -\sum_{ij} p_{ij}(\tau) \ln \frac{p_{ij}(\tau)}{p_i p_j} \tag{2}$$

where p_i is the probability of finding value in the $i-$th interval; while p_{ij} is the joint probability for points with value in the $i-$th interval and the observation time j.

Figure 1 gives graphical representation of the results

The false nearest neighbor method is applied to determine the embedding dimension. The best estimation is the value when the percent of false nearest neighbor falls down to zero. However, in the stochastic data set, the percent of false nearest neighbor does not decreases to zero.

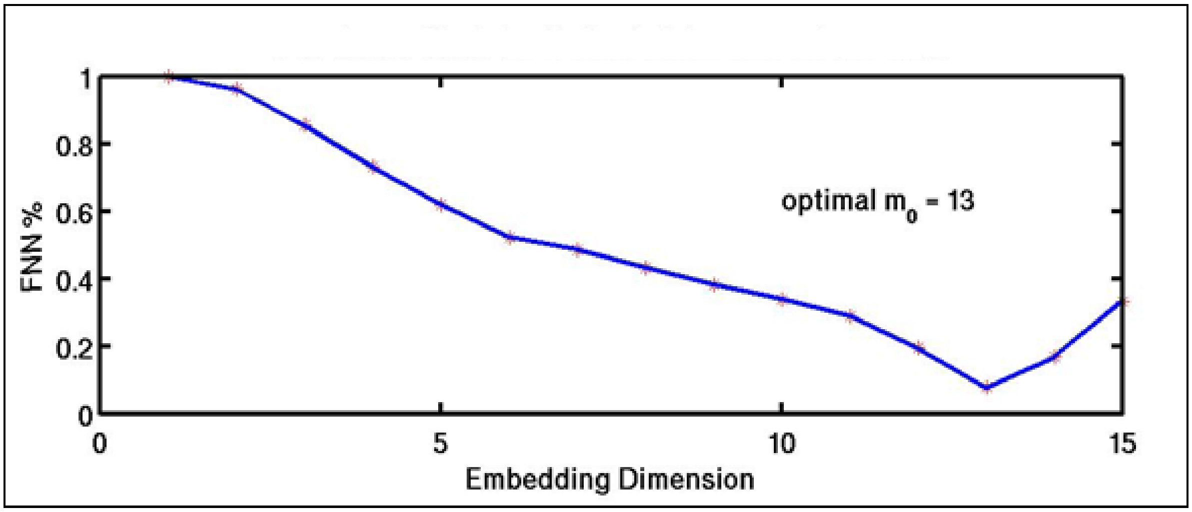

Figure 3: The percentage of FNN generally does not go to zero for non-deterministic data. Here, we choose the global optimum, 13, as the embedding dimension.

Figure 2 illustrates the FNN procedure results:

Following the algorithm of Rosenstein, we estimated the largest Lyapunov exponent of the data subset. In Figure 3, the $\log(\delta x(t))$ is plotted versus t. The slope of the line is the largest Lyapunov exponent (after normalization of the unit of x axis). Here, we use the slope to represent Lyapunov exponent, leaving out normalization.

In order to detect some pattern around the period of the market crash, we apply 5000 days moving window and steps of 300. We calculate and report the Largest Lyapunov exponents on these subsets of Dow Jones Industrial Average index return. The largest Lyapunov exponent is plotted against time.

Before the crisis, the market is unstable as indicated by the consistently positive largest Lyapunov exponents. The largest Lyapunov exponent specifies the maximum average rate of divergence, or instability. The presence of a positive exponent is necessary for diagnosing chaos and represents local instability in a particular direction. For the existence of an attractor, the overall dynamics must be dissipative, i.e. the sum across the entire Lyapunov spectrum is negative.

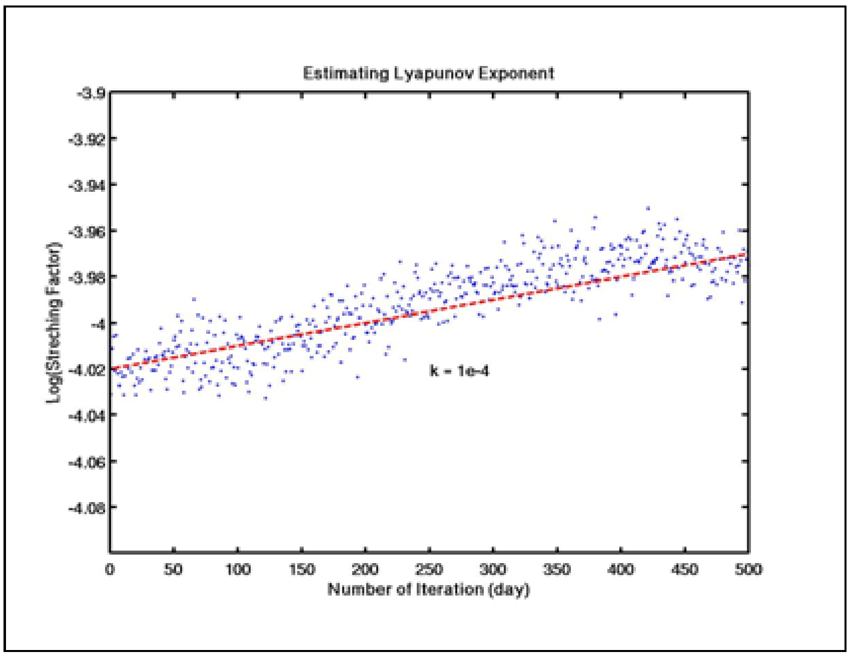

Figure 4: By plotting the log of the divergence versus time,t he exponent is estimated by a least squares fit to the linear region. The slope of the line is the largest Lyapunov exponent. Blue: data points. Red: best linear fit. Here the largest Lyapunov exponent of Data I is 1e-4 (before normalization), which indicates a possible existence of chaos. For comparison, I calculated the largest Lyapunov exponent following the same methodology. Lorenz attractor: 0.95, which is aggreable with established value. Random time series uniformly distributed between 0 and 1: -6e-6.

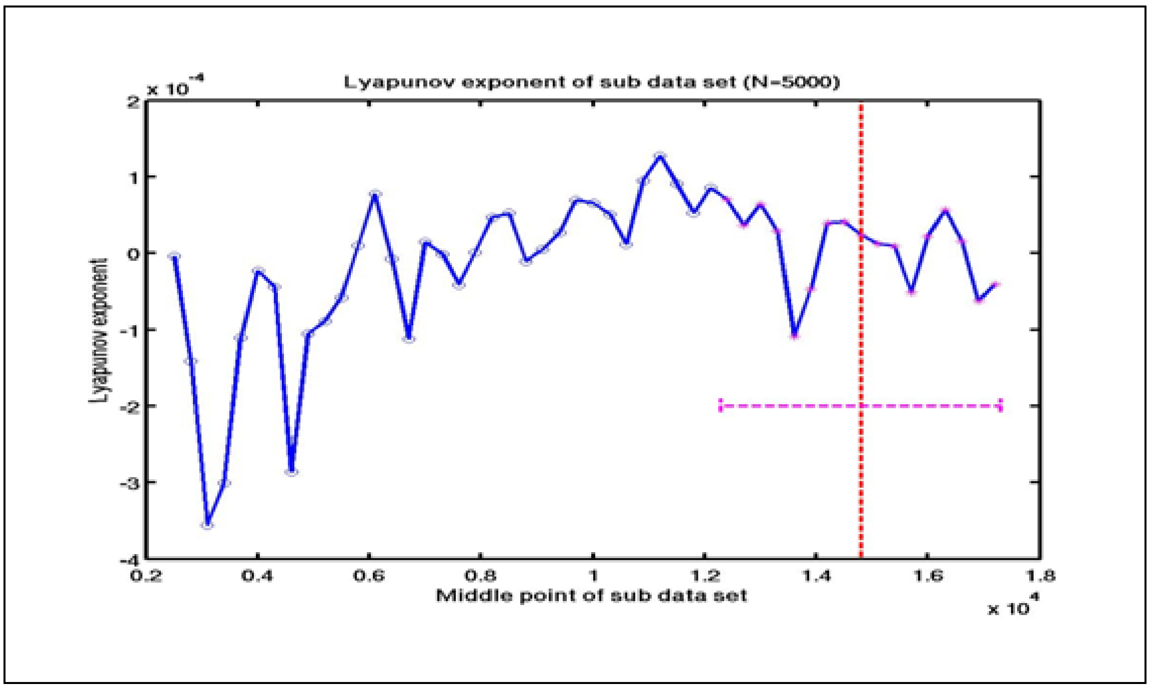

Figure 5: The largest Lyapunov exponent of subset of Dow Jones Stock return as a function of time. Red line: point of financial crisis (Pc); Blue circle: LE of data set (no Pc); Magenta *: LE of data set (including PC); Magenta line: indicating the window size.

4 Conclusions

The fluctuation of the maximal Lyapunov exponents around the October 1987 indicates the existence of chaos before and after the crash, while no such indication can be seen during the crash. Given the limited time and data resources for this project we can not bring any conclusive evidence of chaos but we would like to suggest few directions for further research:

1. Using time series patterns detected around 1987 crash as a starting point, can stock market crashes be predicted?

2. What are the basic behaviors of individual investors leading to fat-tail distributions in financial time series?

3. We need to understand the underlying economic principles for a full understanding of the dynamics surrounding stock market crashes.

4. What is the relationship between multi-fractals and chaos?

This project involves methods with only limited current popularity among finance researchers. Mathematicians and physicists have advanced chaos theory research in many fields, including economics and finance. But empirical research on non-linear dynamics in finance is falling behind because of missing linkages in research methods. Our experience navigating research of different areas shows that sometimes it is hard to seize an analogous approach because of terminology and interpretation differences. Loss of synergy and research benefits can be enormous under such circumstances. Therefore, one of the goals of this research is to bring a better understanding of chaos theory and research methods to the finance area.

Deductive discovery comes when meaningful empirical evidence builds toward a solid theory. There is no extant theory supporting the chaotic dynamics of financial markets, therefore collecting empirical evidence about the emergence of chaotic patterns is the first logical choice.

5 References

Abarbanel H. D. I. (1996). Analysis of Observed Chaotic Data. Springer-Verlag.

Bask, M and R Gençay (1998), Testing chaotic dynamics via Lyapunov exponents, *Physica D 114*, pp.1-2..

Bellazzini, J.(2001) Hölder regularity and chaotic attractors, arXiv:nlin.CD/0104013v2 pp.3.

Boccaletti, S., C. Grebogi, Y.-C. Lai, H. Mancini and D. Maza (2000), The control of chaos: theory and applications, Phys. Rep. 329, 103.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic

Dynamics and Control, 22, (8-9), 1235-1274.

Chen, S.-H., T. Lux, and M. Marchesi (2001), "Testing for Nonlinear Structure in an Artificial Financial Market," Journal of Economic Behavior and Organization 46, 327-342

Corcos, A., J-P Eckmann, A Malaspinas, Y Malevergne and D Sornette, (2002), Imitation and contrarian behaviour: hyperbolic bubbles, crashes and chaos, *Quantitative Finance* 2, pp. 264-281.

Dechert, W. and R. Gençay (1992), Lyapunov exponents as a nonparametric diagnostic for stability analysis, Journal of Applied Econometrics, 7, S41-S60.

Fraser A. M. and Swinney H. L. (1986). Independent Coordinates for Strange Attractors from Mutual Information. *Physical Review A*, 33, 1134-1140.

Gaunersdorfer, A. (2000). Adaptive beliefs and the volatility of asset prices. Technical report, SFB Adaptive Information Systems and Modeling in Economics and Management Science.

Lovejoy, S., D. Schertzer (1999) Stochastic chaos, symmetry and scale invariance, ECO-TEC: Architecture of the In-between, edited by Amerigo Marras, Storefront Book series, copublished with Princeton Architectural Press, 80-99.

Rosenstein, M.T., J.J. Collins, and C.J. De Luca (1993), A practical method for calculating largest Lyapunov exponents from small data sets, Physica D 65:117-134.

Stoop, R. and J. Parisi (1991)., Calculation of Lyapunov exponents avoiding spurious elements, Physica D 50, 89.

Takens F. (1981) Springer Lecture Notes in Mathematics #898: 266-281.