Dynamic Strategies: A Correlation Study

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Abstract

This paper investigates similarities and differences between dynamic strategies applied to varied financial markets from an investor point of view. The similarity in returns is the most important to traders and portfolio managers seeking to reduce risk by diversifying across strategies and markets. This is measured in this study by using the correlation coefficient.

Firstly, expected correlations between returns are established using stochastic modelling. Results apply to any strategies and markets under the only assumption of a binormal random walk without drift. It is shown that the correlation between dynamic strategies applied to different markets is lower in absolute value than the correlation between underlying markets. The process of trading reduces correlation below the one of a buy and hold strategy. For diversification purposes, the application of dynamic strategies might therefore be especially relevant for highly correlated underlying markets.

Secondly, analytical formulae are extended to take into account that positions can be triggered at different times of the day or for different holding periods. Although the time of the day effect can be non negligible, time aggregation is likely to affect the most returns generated by dynamic strategies.

Finally, previous findings are checked against empirical data for varied financial markets and trend-following strategies. Analytical formulae seem to hold for markets as different as spot foreign exchange, futures stock indices, bonds and commodities. The success of theoretical correlations in forecasting observed correlations between dynamic strategies should not be surprising. This is a similar idea to Elton and Gruber (1973), and Elton, Gruber and Urich (1978) who have exhibited the usefulness of averaging (smoothing) some of the data in the historical correlation matrix as a forecast of the future. Theoretical correlations have many potential applications. They might help to build efficient portfolios and also allow to build new tests of random walk from the joint profitability of dynamic strategies.

Keywords:
Dynamic strategies, Rule returns, Gaussian process, Technical analysis.

In recent years, and particularly for banking institutions, involved with unstable financial markets, the need for worthwhile forecasts has been generally recognised by treasurers and academics alike. The choice of which dynamic strategy to follow depends on the expectations one has about the stochastic process which drives prices. Many forecasting strategies can be used to predict future price moves from fundamental to technical indicators without ignoring more advanced techniques such as neural networks and genetic algorithms. Unfortunately, the study of financial forecasts used for trading is relatively new. Many of the previous studies have used historical returns to exhibit the profitability of technical forecasts (Dacorogna et al, 1994; Dunis, 1989; LeBaron, 1991, 1992; Levich and Thomas, 1993; Silber, 1994; Surujaras and Sweeney, 1992; Taylor, 1986, 1990a, 1990b, 1992). Only recent work by Neftci (1991), Taylor (1994), Brock et al (1992), Levich and Thomas (1993) and LeBaron (1991, 1992) have stressed the statistical properties of technical trading rules and the insight they might give us about the underlying process. Although Neftci (1991) has examined the Markovian properties of trading strategies, there is only a scattered literature on their analytic properties.

This study is concerned with the forecasting of correlation between the rates of returns generated by dynamic strategies. In order to calculate the risk of a portfolio, a passive investor applying buy-and-hold strategies needs to know how related are the two underlying markets. This information is necessary for an active investor but not sufficient alone. The active investor also needs to know how the two forecasting strategies she/he follows can differ. Correlation coefficients are often used, particularly in formulating hedging strategies. However, correlations are notoriously unstable, as many hedgers have found. In addition, most financial institutions follow between 150 and 250 assets and as many trading rules. It seems unlikely that analysts will be able to directly estimate correlation structure. Their ability to do so is severely limited by the vast number of correlation coefficients that must be estimated. Recognition of this has motivated the search for development of formulae to describe and predict the correlation structure between assets and strategies. The rationale of this study is that accurate measurement of correlation can be better achieved using stochastic modelling.

Establishing theoretical correlations between trading rules has been considered as an extremely difficult task (Brock et al, 1992). However we show that exact analytical results can be obtained under the assumption that the forecast and underlying process of price returns follow a bivariate normal random walk without drift. Examples are provided for technical indicators although theoretical results are more general and only require that underling asset and forecast follow a bivariate normal law. Analytical formulae can therefore be applied to any forecasters (fundamental, technical, neural network...) under those assumptions.

There are three reasons for investigating correlations between trading rules. Firstly, rules correlations would provide a measure of similarity between trading systems. With the exception of Lukac, Brorsen and Irwin (1988), rules have been merely listed rather than classified on the basis of their properties. Many forecasting systems have been proposed and are often considered to be different when they are extremely similar if not completely identical. Secondly, rules correlations would permit the construction of an efficient portfolio of rules. Until now such portfolios have been build empirically for given financial time series, (Brorsen and Lukac, 1990) but have never been established theoretically for given stochastic processes. Thirdly, and perhaps more important, it will allow the joint profitability of a set of trading rules to be tested. Brock, Lakonishok and LeBaron (1992), Surujaras and Sweeney (1992), Prado (1992) have emphasised that such a test might have power, specially against non-linear alternatives. The resulting tests of non-zero profitability could then be more powerful than any single test.

Section 1 establishes the necessary preliminaries, trading rules correlations under the random walk assumption. Then Section 2 studies the effect of times of the day and time aggregation. Finally, Section 3 tests the adequacy of analytical formulae with empirical results observed for markets as different as spot foreign exchange, futures stock indices, bonds and commodities. The last section summarises and concludes our results.

1 Trading rules correlations under the random walk without drift assumption

Section 1.1 defines our forecasting strategies and the rate of return they generate. Examples are given for some popular technical trading rules, although results can easily be extended to other forecasting strategies. Section 1.2 establishes the correlations between two trading rules applied to a bivariate random walk without drift.

1.1 Forecasting strategies and rule returns

Suppose that at each day t, a decision rule is applied with the intention of achieving profitable trades. It is the price trend which is based on market expectations that determines whether the asset is bought or sold. When the asset is bought, the position initiated in the market is said to be "long". When the asset is sold, the position initiated in the market is said to be "short". A forecasting technique is assessed as useful and will subsequently be used if it has economic value. In short, the forecast is seen as useful if in dealer terms, it can "make money". For achieving this purpose, market participants use price-based forecasts. Therefore the predictor F_t is completely characterised by a mathematical function f of past prices $F_t = f\{P_t, P_{t-1}, P_{t-2} \ldots, P_0\}$.

The only crucial feature which is required from the forecasting technique is its ability to accurately predict the direction of the trend in order to generate profitable buy and sell signals. Trading signals, buy (+1) and sell (-1), can then be formalised by the binary stochastic process B_t:

$$\text{'Sell'} \quad : \quad B_t = -1 \quad \text{iff} \quad F_t = f\{P_t, P_{t-1}, P_{t-2}, \ldots, P_0\} < 0$$

It must be remarked that the signal of a trading rule is completely defined by the inequality giving a sell order, because if the position is not short, it is long. A particular class of forecasters are linear rules which can be expressed by a linear combination of logarithmic returns $X_t = \mathrm{Ln}(P_t / P_{t-1})$:

$$F_t = \delta + \sum_{j=0}^{t} d_j X_{t-j} \qquad (1.1)$$

with δ and the d_j being constants.

Only in the trivial case of a Buy and Hold strategy, the signal B_t is deterministic and is +1 irrespective of the underlying process. Otherwise, trading signals B_t are stochastic variables. By nature, the signal is a highly non-linear function of the observed price series P_t (Neftci and Policano, 1984; Neftci, 1991), and therefore it can be highly dependent through time. B_t remains constant for a certain random period, then jumps to a new level as P_t behaves in a certain way. Trading in the asset occurs throughout the investment horizon at times that depend upon a fixed set of rules and future price changes. The most well known example of technical linear rule is provided by the simple moving average method of order 'm' [1] for which:

$$F_t = \sum_{j=1}^{m-1}(m-j)X_{t-j+1}$$

Returns at time 't' made by applying such a decision rule are called *rule returns* and denoted R_t. Their value can be expressed as:

$$R_t = B_{t-1}\,X_t \quad \Leftrightarrow \quad \begin{Bmatrix} R_t = -X_t \;\; if \;\; B_{t-1} = -1 \\ R_t = +X_t \;\; if \;\; B_{t-1} = +1 \end{Bmatrix} \qquad (1.2)$$

where B_{t-1} is the signal triggered by the trading rule at time t-1.

Two important remarks should be made. Rule returns are the product of a binary stochastic signal and a continuous returns random variable. Except in the trivial case of a Buy and Hold strategy, the signal B_t is a stochastic variable and so rule returns are conditional on the position taken in the market. That is the main feature of rule returns. In addition, our rule return definition clearly corresponds to an unrealised return. By unrealised we mean that rule returns are recorded every day even if the position is neither closed nor reversed, but simply carries on.

1.2 Linear rule returns correlation under the random walk assumption

We are now assuming that two financial series, with underlying returns $X_{1,t}$ and $X_{2,t}$, follow a centred bivariate normal law with variances σ_1^2 and σ_2^2 and correlation coefficient ρ_x. Then two unbiased linear trading rules (similar or different) $F_{1,t}$ and $F_{2,t}$ are respectively applied to the two processes $\{X_{1,t}\}$ and $\{X_{2,t}\}$.

$$F_{1,t} = \sum_{i=0}^{m_1-2} d_{1,i}X_{1,t-i} \qquad (1.3) \qquad\qquad F_{2,t} = \sum_{i=0}^{m_2-2} d_{2,i}X_{2,t-i} \qquad (1.4)$$

m_1 and m_2 are called the orders or lengths of the trading rules.

The linear rule $F_{i,t-1}$ generates signal $B_{i,t-1}$ and return $R_{i,t}$ from the underlying process $\{X_{i,t}\}$, given by, $R_{i,t}$=$B_{i,t-1}X_{i,t}$. Precise theoretical correlations are now being established for any linear rules without constant and highlighted, for the sake of clarity, throughout the popular technical linear rule which is the simple moving average method.

[1] See Acar(1993) for a detailled discussion on the stochastic properties of the simple moving average method.

Proposition 1[2]

Assuming that two underlying time series, $X_{1,t}$ and $X_{2,t}$, follow a centred bivariate normal law with underlying correlation ρ_x, linear rule returns, $R_{1,t}$ and $R_{2,t}$, exhibit linear correlation coefficient ρ_R, given by:

$$\rho_R = \rho(R_{1,t}, R_{2,t}) = \frac{2}{\pi}\rho_x \operatorname{Arc\,sin}(\rho_x \rho_F) \tag{1.5}$$

where ρ_F is the correlation between the two different forecasters which would have been applied to the same underlying process. We call it systems correlations. It is given by:

$$\rho_F = \sum_{i=0}^{\mathrm{Min}(m_1,m_2)-2} d_{1,i}\, d_{2,i} \Big/ \left(\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}\right) \tag{1.6}$$

In addition, $\rho(R_{1,t}, R_{2,t+h}) = \rho(R_{1,t+h}, R_{2,t}) = 0$ for $h>0$. (1.7)

Expression (1.5) suggests a few comments:

(a) rule returns correlation, ρ_R, is an even function of the underlying correlation ρ_x and an odd function of the systems correlation ρ_F. That means that rule returns will be negatively (positively) correlated if, and only if, the systems correlation is negative (positive).

(b) rule returns correlation is always lower in absolute values than the underlying correlation.

If one wants to minimise the risk of an investment, it turns out that diversifying trend-following systems between positively correlated assets can be beneficial beyond diversification of passive strategies, because the correlation between trading systems will be lower (property a). However, this will be disadvantageous if the underlying assets are negatively correlated, because trading systems will be positively correlated (property b).

For the remainder of this section we shall primarily focus our interest on returns rather than signals correlation since it has more implications vis-à-vis a portfolio context. We shall detail and interpret previous results by considering three cases from the simplest to the most general: different rules applied to the same underlying process, the same rule applied to different underlying processes and different rules applied to different underlying processes.

[2] Proofs of propositions are given in Appendix.

Different rules applied to the same underlying process

When two different unbiased linear trading systems are applied to the same underlying process, $X_{1,t}=X_{2,t}=X_t$ and $\rho_x=1$. In this case correlations between rule returns, equation (1.5) becomes equal to:

$$\rho_R = \frac{2}{\pi}\mathrm{Arc}\sin(\rho_F) \tag{1.8}$$

Table 1 shows correlations between various systems. For instance ρ[S(5),S(10)] means the rule returns correlation between the simple moving average of order 5 and the simple moving average of order 10. It is equal to 0.666. Figure 1 illustrates correlations between rule returns as a function of correlations between forecasts.

Table 1: Rules correlations

ρ	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.705	0.521	0.358	0.25	0.175	0.124
S(3)		1	0.71	0.484	0.336	0.236	0.166
S(5)			1	0.666	0.460	0.322	0.226
S(10)				1	0.680	0.472	0.331
S(20)					1	0.685	0.478
S(40)						1	0.688
S(80)							1

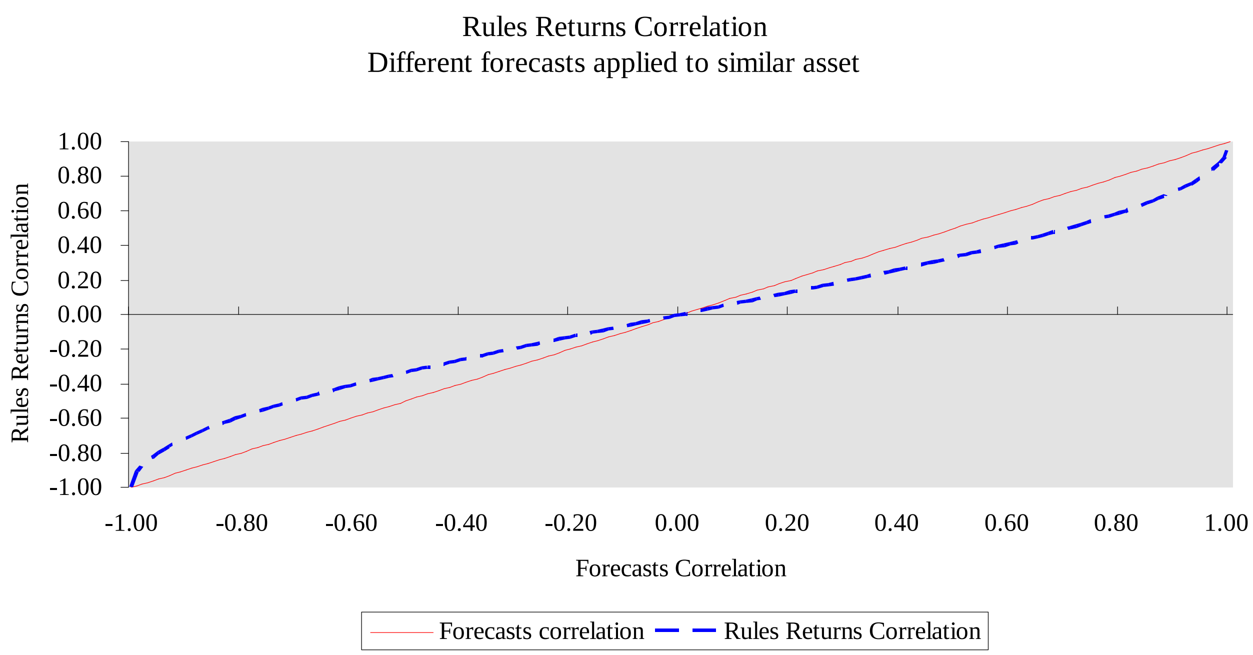

Figure 1: Rules returns correlation as a function of the forecast correlation

Rather than listing differences between systems and orders which could happen to be endless due to the infinite number of linear rules, it is worth emphasising two points. Firstly, trend-following systems are positively correlated. Zero or negative correlation obviously requires the combination of trading rules of different nature such as trend-following and contrarian strategies. Secondly, buy and sell signals and then returns of technical systems are not independent over time under the random walk assumption. Related findings are attributable to Working (1960). This established that if in a time series constructed from independent increments, the individuals items are replaced let say by monthly averages, spurious correlation is introduced between successive first differences of the averages. Correlation between trading signals would contradict, however, the hypothesis of Lukac, Brorsen and Irwin (1988) who considered, as an approximation, that buy and sell signals of systems are independent over time. They then concluded that all the systems are on the same side of the market significantly more than might randomly be expected and that monthly returns are positively correlated. Our results show that it is not absolutely certain that the similarities between systems Lukac, Brorsen and Irwin (1988) found are nothing more than would randomly be expected.

Same rule applied to different underlying processes

When the same linear rule (non-deterministic, and so excluding Buy and Hold, or Sell and Hold strategies) is applied simultaneously to two assets, $\rho_F = 1$ and equations (1.5) becomes:

$$\rho_R = \frac{2}{\pi}\rho_x \mathrm{Arc}\sin(\rho_x) \qquad (1.9)$$

We can see two additional properties, when the same rule is applied to two different assets. Firstly, rule returns correlations become independent of the rule itself and the sole function of the underlying correlation. Secondly, rule returns correlations are now an even function of the underlying correlation and thus are always positive. Figure 2 highlights formulae (1.9) for some values of correlations of the underlying process.

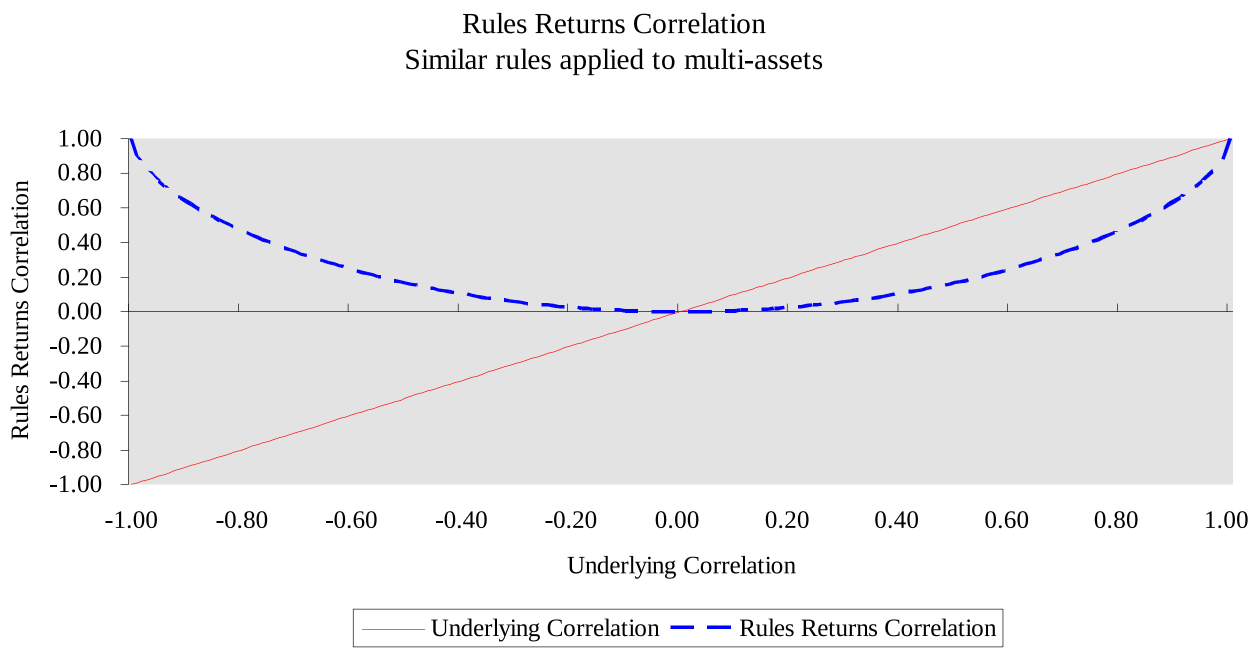

Figure 2: Rules return correlation as a function of the underlying correlation

Different rules applied to different underlying processes

Let us now examine the most general case where different rules are applied to different underlying processes. We use for this purpose different orders of simple moving averages. Having just proved that correlations between rule returns (when the same rule is applied to two different processes) do not depend on the rule itself, Table 2 exhibits constant diagonals.

Table 2: Rule returns correlations ρ_R for underlying correlations $\rho_x = 0.85$

Underlying correlation: $\rho_x = 0.85$					
ρ	S(2)	S(5)	S(10)	S(20)	S(40)
S(2)	0.55	0.362	0.254	0.179	0.126
S(5)		0.55	0.447	0.323	0.229
S(10)			0.55	0.455	0.331
S(20)				0.55	0.457
S(40)					0.55

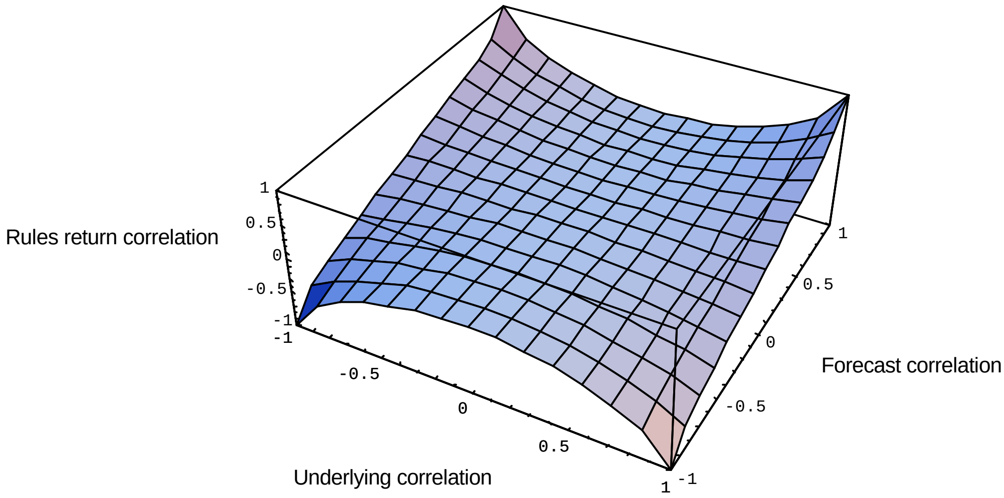

Figure 3: Rules return correlation as a function of the underlying correlation and forecasts correlation

Our results are consistent with Praetz (1979) but disagree with Sweeney (1986) and Surujuras and Sweeney (1992).

On the one hand, Praetz (1979) noted that the results from both different securities and trading rules are likely to be positively correlated due to the presence of the market factor among security returns and due to the presence of many common rates in the returns from short selling of similar trading rules.

On the other hand, Sweeney (1986: 177) concluded that "even if [exchange] rates are correlated, excess rates of return on trading strategies should be virtually uncorrelated because the signals are only randomly synchronised across currencies". Surujuras and Sweeney (1992) then expressed the assumption that on the one hand, under efficiency rules signals would be completely out of synchronism and, on the other hand, inefficiencies would create positive cross correlations. This section comes to a different conclusion, i.e. even when underlying processes are correlated white noises, rules correlations although lower in absolute value cannot be zero. The presence of inefficiencies, more specifically positive autocorrelations, would even increase rules correlations. Our results clearly indicate that correlations between trading rules are strongly dependent on underlying correlations. That could explain why the correlations between trading rules can be low for equities (Sweeney, 1988) and high for currencies (Surujaras and Sweeney, 1992). Accordingly 't'-statistics can be highly sensitive to whether the covariance terms are included or not.

Overall, these results suggest that correlations between the same system applied to various assets can be much lower than correlations between various trend-following systems applied to the same asset. It seems that these results might hold empirically since diversification between assets has been found more beneficial than diversification between systems (Taylor 1990b; Brorsen and Boyd, 1990).

2 Times effects on trading rule returns

2.1 Time of the day

In some cases, investors choose to take positions in the financial market at different times of the day. This is frequently the case with the foreign exchange market which is a twenty-four hour market. Most investors initiate their positions during their time zone. Geographical components related to the business hours of the different trading centers must be taken into account. Realistic trading models should be configured for traders located in particular centers. The European time zone is often perceived as the most important reason for the dominance of Europe as a currency centre. Most European trading desks come in towards the close of Tokyo trading and exit towards the middle of New-York trading day. Dacarogna et al (1994) find that trading models work best for certain currencies at their most active times which are shifted to accommodate their main markets (Japan for the JPY, London for the GBP). Their systematic variation of the business hours of the trading models reveals the geographical structure of the FX market and its daily seasonality by concentrating its most profitable trading times where the market is the most liquid.

In other cases, investors may be forced to take positions at different times of the day. This most often occurs when financial markets do not exhibit overlapping times. For instances, the Tokyo Stock Exchange approximately opens at 0.00 GMT to close at 6.00 GMT whereas the New York Stock Exchange opens at 14.30 GMT to close at 21.00 GMT. This "nonsynchronous trading problem", or "stale quote problem" may induce spurious lagged spillovers (See for instance Lin et al, 1994; Aggarwal and Park, 1994).

This section deals with the "nonsynchronous trading problem" by establishing the correlation coefficient of trading strategies based on daily rates but applied at different times of the day. More specifically, let us assume that two trading rules are applied daily but at different times of the day. The first one is applied at time t, and the second one at time $t^* = t - y$ where y is included between zero and one. A value of y=n/24 means that the second trading rule is applied n hours before the first one. For instance, a value of y equal to 0.25 means that the lag between the two trading rules is equal to 6 (=0.25*24) hours.

Proposition 2

Assuming that two underlying time series, $X_{1,t}$ and $X_{2,t}$, follow a centred bivariate normal law with underlying correlation ρ_x, linear rule returns, $R_{1,t}$ and $R_{2,t}$, lagged y fraction of the unit time (y=n/24 for daily unit time and hourly lag n) exhibit linear correlation coefficient ρ_R, given by:

$$\rho_R = (1-y)\frac{2}{\pi}\rho_x \operatorname{Arcsin}\left(\rho_x \frac{(1-y)\sum_{i=0}^{\mathrm{Min}(m_1,m_2)-2} d_{1,i}d_{2,i} + y\sum_{i=0}^{\mathrm{Min}(m_1,m_2)-3} d_{1,i}d_{2,i+1}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}}\right) + y\frac{2}{\pi}\rho_x \operatorname{Arcsin}\left(\rho_x \frac{y\sum_{i=0}^{\mathrm{Min}(m_1,m_2)-2} d_{1,i}d_{2,i} + (1-y)\sum_{i=0}^{\mathrm{Min}(m_1,m_2)-3} d_{1,i+1}d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}}\right) \quad (2.1)$$

There are two obvious remarks from previous formulae. Firstly, if y=0 or 1, we find the correlation coefficient of two systems applied at the same time given in proposition 1. Secondly, the correlation coefficient is symmetrical around y=0.5 (n=12 hours). This means that one hour lag will produce the same correlation coefficient than a 23 hours lag.

Table 3 quantifies the correlation coefficient of a same system applied to the same underlying market ($\rho_x = 1$) such that the time of the day effect is isolated. The systems we have considered are the simple moving averages of orders 2, 5, 10, 20, 40 and 80. For instance the correlation coefficient between simple moving average of orders 2 lagged 4 hours is equal to 0.540. The rows indicate not surprisingly that the correlation coefficient will be minimum if y=0.5 (12 hours). The columns exhibit that the longer the system the bigger the correlation coefficient and the less value the time diversification. If we consider a simple moving average of order 40, the smallest coefficient correlation we can get using a different time of the day to trigger the position is 0.876. Therefore there can be some value in time diversification, but this must be for very short order systems (up to 5). As trading rules become increasingly short-term in their focus, the "time of the day" effect becomes more significant (Figure 4).

Table 3: Correlation $\mathrm{Corr}(R_{1,t}, R_{1,t*})$ between a same system applied at lagged times

System\LAG	0	1	2	3	4	5	6	7	8	9	10	11	12
S(2)	1	0.783	0.681	0.603	0.54	0.488	0.445	0.41	0.382	0.36	0.345	0.336	0.333
S(3)	1	0.833	0.756	0.697	0.65	0.61	0.578	0.551	0.53	0.514	0.503	0.496	0.494
S(5)	1	0.876	0.82	0.776	0.741	0.712	0.689	0.669	0.654	0.642	0.634	0.629	0.627
S(10)	1	0.915	0.876	0.847	0.823	0.803	0.787	0.774	0.763	0.755	0.75	0.746	0.745
S(20)	1	0.941	0.914	0.893	0.877	0.863	0.852	0.843	0.835	0.83	0.826	0.824	0.823
S(40)	1	0.958	0.94	0.925	0.914	0.904	0.896	0.89	0.885	0.881	0.878	0.876	0.876
S(80)	1	0.971	0.957	0.947	0.939	0.932	0.927	0.922	0.919	0.916	0.914	0.913	0.912

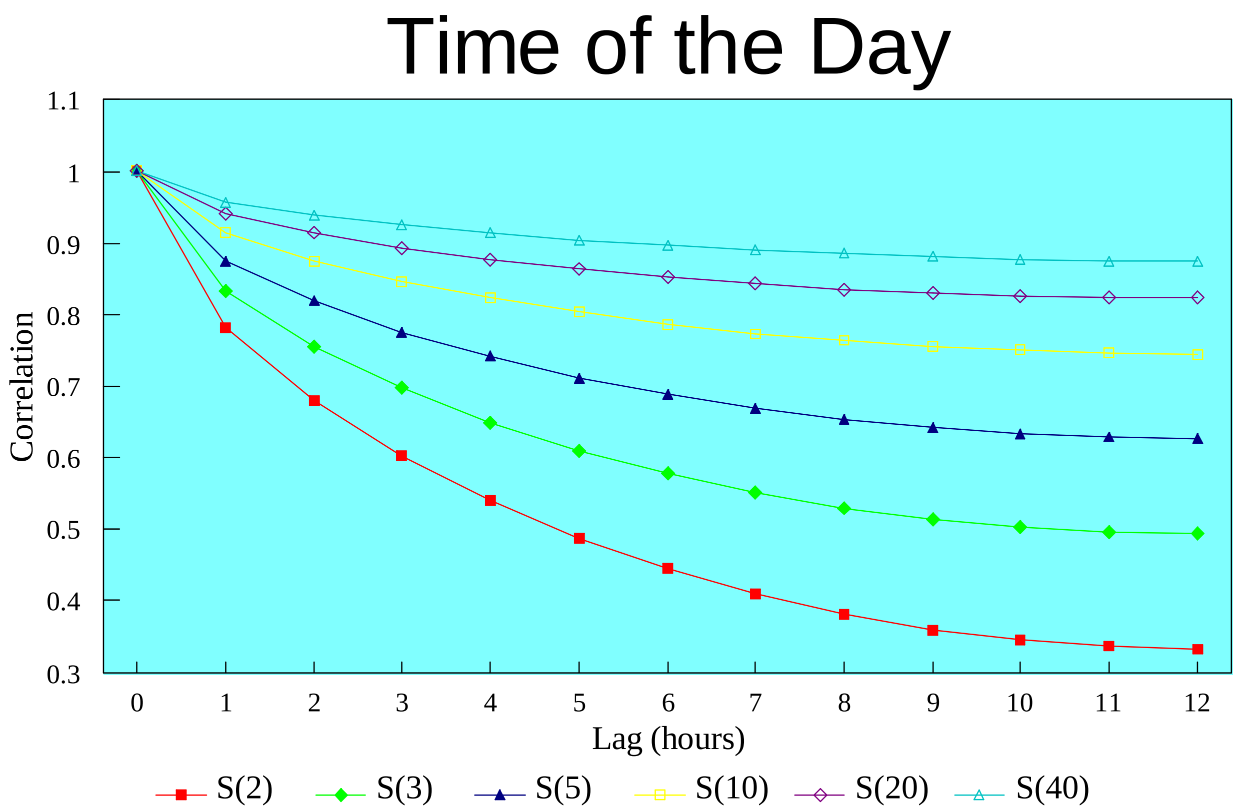

Figure 4: Rules returns correlation as a function of the lag

Formulae (2.1) has many potential applications. Observing only rule return correlation does not provide insights on what contributes the most to the rule return correlation:

(a) the underlying correlation: ρ_x.

(b) the systems correlation: product of linear coefficients $d_{1,i}$ $d_{2,i}$.

(c) the lagged time: y.

On the other hand, the only estimate of underlying correlation ρ_x between financial markets is sufficient to get an estimate of any linear rule returns correlations applied at different times of the day which may not be observable in any cases. Indeed the systems correlation and the lagged time are constant which need not to be estimated.

2.2 Time aggregation

The key characteristic of the markets are the different time scales of the markets participants. Some trade short-term, others have long-term horizons. In the foreign exchange market, market makers are at the short end of the scale and central banks at the long-term end. For instance to take advantage of the lag adjustment between interest rate and exchange rates moves, investors need to tie up their money for months or even years. This is a very long time for a forex trader. Some investors will thus tend to ignore these profits opportunities while other will invest on it. Even if these investors choose similar strategies such as trend-following rules, their rate of returns are likely to be different because of different holding periods. Long term investors such as pension funds might want to consider only low frequency data (daily or weekly) where as spot dealers will look at high frequency data (minute

by minute). We present here another study where we vary the length of the period (daily, weekly) but keeping constant the time a new signal is triggered (close of the market).
For instance, let us assume that a pension fund follows a simple moving average of order two, on weekly rates, when a trader applies the same system to daily rates. The aggregate time unit is here equal to five if we assume that a week includes five working days. Once again, the theoretical correlation between the returns generated by the two strategies can be worked out under the random walk assumption (see Appendix). This is equal to 0.059. This elementary example shows that time aggregation can differentiate as much, if not more, trading rules than different orders of trading rules applied to a same disaggregate series. Table 4 exhibits that simple moving average methods applied every other day exhibit low positive correlations with systems applied daily. This effect is, as expected, accentuated when the aggregation is done over a week (five working days, Table 5).

Table 4: Rule returns correlation under the random walk assumption

Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.250	0.218	0.173	0.123	0.087	0.062	0.044
S(3)	0.500	0.414	0.318	0.223	0.157	0.111	0.078
S(5)	0.583	0.658	0.520	0.365	0.256	0.181	0.128
S(10)	0.468	0.621	0.784	0.599	0.422	0.297	0.210
S(20)	0.342	0.461	0.626	0.858	0.641	0.452	0.318
S(40)	0.245	0.329	0.449	0.658	0.904	0.665	0.467
S(80)	0.174	0.233	0.318	0.467	0.674	0.934	0.677

Table 5: Rule returns correlation under the random walk assumption

Disaggregate\Aggregate(5)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.059	0.052	0.042	0.031	0.022	0.016	0.011
S(3)	0.108	0.095	0.076	0.055	0.039	0.028	0.020
S(5)	0.229	0.199	0.158	0.113	0.080	0.057	0.040
S(10)	0.483	0.470	0.377	0.262	0.187	0.133	0.095
S(20)	0.474	0.606	0.648	0.481	0.336	0.240	0.171
S(40)	0.368	0.492	0.656	0.740	0.542	0.380	0.271
S(80)	0.269	0.3661	0.492	0.712	0.798	0.577	0.407

3 **Trading rules correlations: an empirical study**

Having established expected correlations under the random walk assumption, we can now test if correlations observed in financial markets are more likely to be issued from random walks than inefficient markets. On the one hand, an excessively high correlation between trend-following strategies would mean that the trading rules are on the same side of the market significantly more than might randomly be expected. This would implicitly imply that the market exhibits trends and therefore positive autocorrelations. On the other hand, an excessively low correlation between trend-following strategies would suggest that the trading rules are on the opposite side of the market significantly more than might randomly be expected. This could be explained by the presence of contrarian moves or negative autocorrelations.

3.1 Underlying financial markets

Varied financial markets have been considered in this study. They are:

(a) Spot foreign exchange transactions: Dollar against Deutschmark, (Usd/Dem) and Pound Sterling against Dollar (Gbp/Usd).

(b) Futures contracts: Cac-Matif, Ftse-Liffe, German Government Bonds-Liffe (GGB), Gilts-Liffe, Cocoa-LCE (London Commodity Exchange), Coffee-LCE.

Our simulations roll forward each futures contract as it approaches the settlement date, just as a futures trader would. The futures contracts are the first future contract until the last trading day for Liffe and Matif contracts. For the London Commodity Exchange, contracts rollovers have occurred two weeks before the first trading day of the delivery month. For all futures contracts, an unique time series of logarithmic returns has been build as $X_t = \mathrm{Ln}(P_t / P_{t-1})$. An example of rollovers is given in Table 6 for the Cocoa series. This particular time series is for the period 13/06/90-19/06/90: $\{X_t\}$={-3.05%, 1.41%, -0.62%, -1%}.

The times of the day investigated in this paper are specified in Table 7. Table 8 provides summary statistics including autocorrelations of order k, r[k], for the closing time series. The assets investigated here include a wide range of volatilities, Skewness, Kurtosis and autocorrelations. The daily volatility varies from 0.343% for the German Bunds to 1.641% for the Cocoa. Therefore, this is believed that the sample is representative of various market conditions.

Table 6: Futures Time Series

Date	Delivery Month	Price(July 90)	Price(Sep 90)	Logarithmic returns
13-Jun-90	Jul-90	800		
14-Jun-90	Jul-90	776		-3.05%=Ln(776/800)
15-Jun-90	Jul-90	787	808	1.41%=Ln(787/776)
18-Jun-90	Sep-90		803	-0.62%=Ln(803/808)
19-Jun-90	Sep-90		795	-1.00%=Ln(795/803)

Table 7: London times of price series

Usd/Dem	Gbp/Usd	Cac	Ftse	Cocoa	Gilts	GGB
8.00	8.00	9.00 (open)	8.35 (open)	9.35 (open)	8.32 (open)	7.30 (open)
12.00						
16.00	16.00	16.00 (close)	16.10 (close)	16.45 (close)	16.15 (close)	16.15 (close)

Table 8: Summary Statistics

	Usd/Dem	Gbp/Usd	Cac	Ftse	Cocoa	Gilts	GGB
Sample size	1386	1386	1371	1388	1388	1409	1409
Average	-0.0073%	-0.0122%	-0.0073%	0.0119%	-0.0566%	-0.00219%	-0.00482%
Median	-0.0060%	0.0110%	0.0000%	0.0000%	-0.1250%	0.00000%	0.00000%
Variance	0.0052%	0.0052%	0.0154%	0.0100%	0.0269%	0.00299%	0.00118%
Standard deviation	0.7213%	0.7189%	1.2397%	0.9996%	1.6411%	0.54663%	0.34302%
Minimum	-3.4826%	-3.9481%	-7.6764%	-6.5567%	-6.6335%	-2.19125%	-1.94871%
Maximum	3.3418%	3.2609%	8.6266%	4.9636%	9.7672%	3.65441%	2.29646%
Skewness	0.202	-0.263	-0.113	-0.182	0.268	0.40092	0.00754
Standardized skewness	3.074	-3.992	-1.714	-2.776	4.079	6.14384	0.11551
Kurtosis	1.820	2.137	4.292	2.419	2.414	3.68470	5.31640
Standardized kurtosis	13.831	16.236	32.441	18.396	18.357	28.23270	40.73500
r[1]	0.04105	0.0967*	-0.00127	-0.01158	0.05341*	0.02931	0.03129
r[2]	-0.02201	0.0119	-0.01141	-0.02108	-0.05805*	-0.00073	-0.04589
r[3]	-0.0284	-0.03141	-0.02096	-0.01793	-0.01594	-0.01067	0.03289
r[4]	0.04379	0.05231	0.05556*	0.05513*	0.06241*	0.02160	0.05920*
r[5]	0.03944	0.04713	-0.02782	0.01614	0.00842	0.02870	-0.01410
r[6]	-0.01555	-0.00583	-0.03244	-0.03038	-0.04939	-0.02543	-0.02962
r[7]	-0.03556	-0.04909	-0.01328	-0.05958*	-0.00723	-0.04542	0.03844
r[8]	0.03418	0.02692	0.00563	0.05455*	0.0214	0.05087	0.01079
r[9]	0.03075	0.02136	0.02034	0.00899	0.01909	0.01183	-0.06821*
r[10]	-0.00938	0.03993	0.05751*	0.0214	0.05255	0.04389	-0.04472
r[11]	-0.03833	-0.03366	-0.01163	-0.01585	-0.02881	0.02396	0.04210
r[12]	0.00652	0.02144	0.01643	0.0166	-0.06949*	-0.02716	0.02780
r[13]	-0.00143	0.00008	-0.01608	0.02979	-0.0063	0.03072	0.00849
r[14]	0.02758	0.01933	-0.03982	0.02634	0.01728	-0.01409	0.04395
r[15]	-0.00148	0.02189	0.00466	-0.01437	0.02667	-0.04923	0.03625
r[16]	-0.03074	0.01143	-0.01201	0.00178	-0.03237	-0.01963	-0.02450
r[17]	-0.02052	-0.03533	-0.00998	-0.03766	-0.02405	0.02636	0.00353
r[18]	-0.032	-0.03969	-0.00082	0.01577	0.05286	-0.00957	-0.00431
r[19]	0.05988*	0.00061	-0.05745*	-0.00804	0.04603	-0.02314	0.06408*
r[20]	0.03875	0.01422	0.01593	0.00051	0.00719	0.03362	-0.05739*

* significant at the critical level of 5%

3.2 Correlation Intra-markets

The sample correlation between two times series x and y of length n has been estimated through the standard formulae:

$$r(x,y)=\frac{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\overline{y})^2}} \quad \text{where } \overline{x}=\sum_{i=1}^{n}x_i \qquad \overline{y}=\sum_{i=1}^{n}y_i \tag{3.1}$$

To determine the significance level, we use the fact that $z=\frac{1}{2}\mathrm{Log}(\frac{1+r}{1-r})$ is approximately distributed by a normal law with means $\mu_z=\frac{1}{2}\mathrm{Log}(\frac{1+\rho}{1-\rho})$ and variance $\sigma_z^2=\frac{1}{n-3}$ where ρ is the expected correlation coefficient. Application and more details about this test can be found in Johnson and Wichern (1982).

Overall, theoretical correlations between trading systems applied to a same market (Table 1) are close estimate of observed correlations (Table 9). The most significant departures occur by decreasing orders, for the Cocoa, Ftse, Gilts, and Cac. This may mean that the random walk assumption is less appropriate for these contracts and more acceptable for the Usd/Dem, Gbp/Usd and German Government Bond. In any cases, theoretical correlations are good ex-ante estimates since there are almost as many statistically significant overestimations of correlations (29) than underestimations (30).

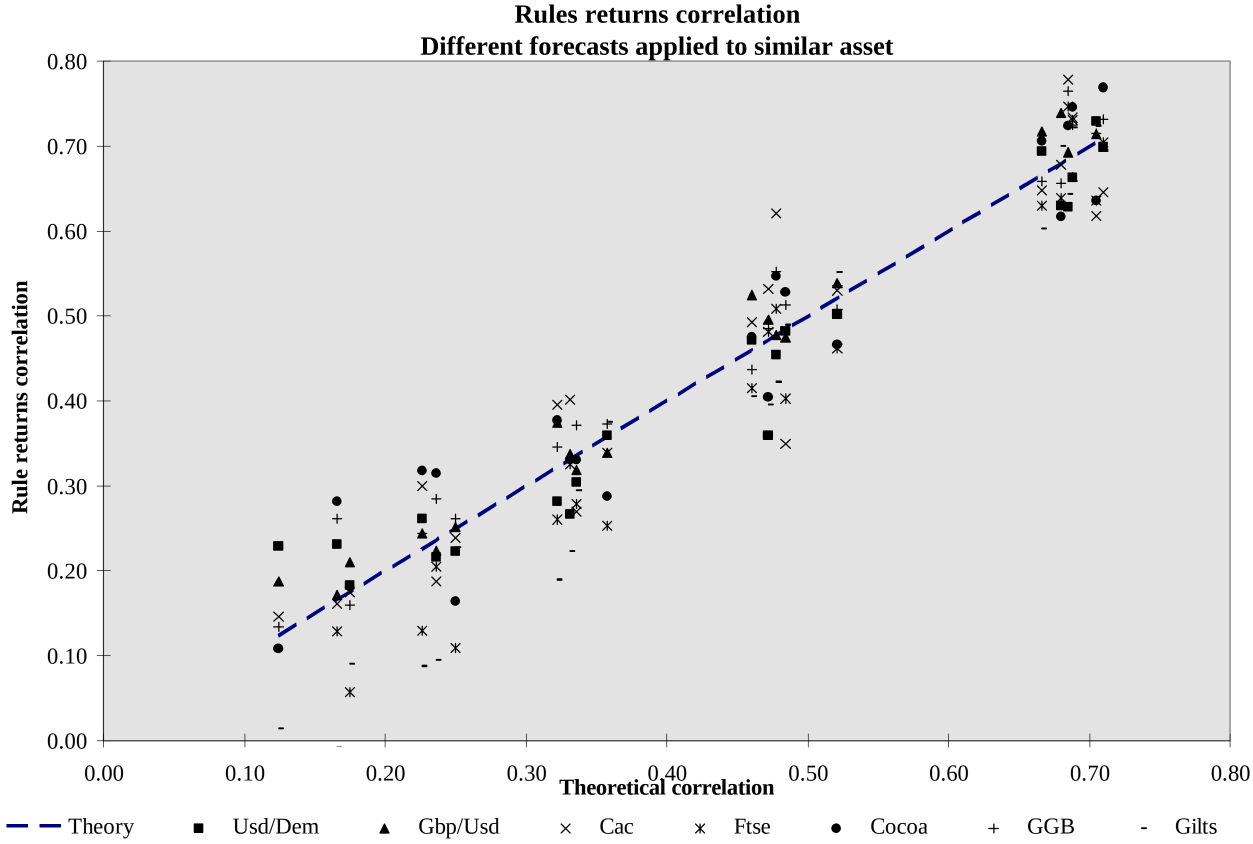

Figure 4: Rules returns correlation: Inter-Markets

Table 9: Empirical correlations from time series on close hour

Usd/Dem							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.729	0.502	0.359	0.223	0.183	0.229*
S(3)		1	0.699	0.482	0.304	0.216	0.231*
S(5)			1	0.694	0.472	0.281	0.261
S(10)				1	0.630*	0.359*	0.266*
S(20)					1	0.628*	0.454
S(40)						1	0.663
S(80)							1
Gbp/Usd							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.714	0.539	0.339	0.251	0.210	0.188*
S(3)		1	0.699	0.474	0.319	0.224	0.172
S(5)			1	0.718*	0.525*	0.374*	0.244
S(10)				1	0.739*	0.496	0.337
S(20)					1	0.693	0.478
S(40)						1	0.664
S(80)							1
Cac							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.618*	0.530	0.339	0.239	0.174	0.146
S(3)		1	0.646*	0.350*	0.270*	0.188	0.161
S(5)			1	0.648	0.493	0.396*	0.300*
S(10)				1	0.678	0.532*	0.402*
S(20)					1	0.779*	0.621*
S(40)						1	0.734*
S(80)							1
Ftse							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.636*	0.462*	0.253*	0.109*	0.057*	-0.048*
S(3)		1	0.704	0.403*	0.279*	0.204	0.128
S(5)			1	0.630*	0.415*	0.260*	0.129*
S(10)				1	0.639*	0.481	0.326
S(20)					1	0.747*	0.509
S(40)						1	0.731*
S(80)							1
Cocoa							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.636*	0.466*	0.288*	0.164*	0.181	0.108
S(3)		1	0.769*	0.528*	0.331	0.315*	0.281*
S(5)			1	0.706*	0.475	0.377*	0.318*
S(10)				1	0.617*	0.404*	0.331
S(20)					1	0.724*	0.547*
S(40)						1	0.746*
S(80)							1

*significantly different from ρ_0 at the critical level of 5%

Table 9 (Continued): Empirical correlations from time series on close hour

German Government Bond							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.715	0.508	0.373	0.261	0.159	0.134
S(3)		1	0.732	0.513	0.372	0.285	0.261*
S(5)			1	0.658	0.437	0.346	0.244
S(10)				1	0.657	0.485	0.333
S(20)					1	0.765*	0.552*
S(40)						1	0.725*
S(80)							1
Gilts							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	1	0.724	0.551	0.375	0.227	0.090*	0.014*
S(3)		1	0.696	0.489	0.295	0.095*	-0.009*
S(5)			1	0.603*	0.405*	0.189*	0.088*
S(10)				1	0.700	0.396*	0.223*
S(20)					1	0.643*	0.422*
S(40)						1	0.722*
S(80)							1

*significantly different from ρ_0 at the critical level of 5%

3.2 Correlation Inter-markets

The correlations between similar trading rules applied to different underlying markets has been studied but only for time series for which the closing times are strictly identical. They are the Usd/Dem, Gbp/Usd and Cac at 16.00, on the one hand, and the Gilts and Government Bonds at 16.15, on the other hand. Underlying market correlations have been estimated here using (3.1). Then expected correlations between similar trading rules applied to different markets have been worked out assuming the underlying correlation to be known through the use of formulae (1.9). Again observed correlations do not significantly deviate from their theoretical expectations (Table 10). There are however significant departures between the Gilts and the GGB, especially for long-term orders trading rules. Gilts and GGB seem to be more correlated over long-term periods than the underlying correlation would let think.

Table 10: Inter-markets correlations

Assets	Usd/Dem, Gbp/Usd	Usd/Dem , Cac	Gbp/Usd , Cac	GGB, Gilts
Underlying Correlation ρ_x	-0.841	-0.101	0.085	0.450
Expected Correlation of identical rule applied to both assets $\rho_0 = \frac{2}{\pi}\rho_x \text{Arcsin}(\rho_x)$	0.535	0.007	0.005	0.134
S(2)	0.554	0.017	-0.003	0.234*
S(3)	0.558	0.057	0.010	0.200*
S(5)	0.557	0.073*	-0.004	0.182
S(10)	0.558	0.029	-0.030	0.174
S(20)	0.479*	0.140*	0.009	0.288*
S(40)	0.603*	0.118*	0.067*	0.308*
S(80)	0.588*	0.098*	0.064*	0.313*

*significantly different from ρ_0 at the critical level of 5%

3.3 Correlations between Close to Close trading rules with Open to Open trading rules

The time of the day effect has been investigated by comparing positions triggered by a similar trading rule at the opening time and positions triggered at closing times of the futures contract. For the spot foreign exchange, the opening and closing times have been respectively fixed at 8.00 and 16.00 London time.

Observed correlations are slightly under their expectations for the Cac, Ftse and above for the Gilts, Cocoa and Gbp/Usd (Table 11 and Figure 5). For instance, the correlation between the simple moving average of order 2 triggered on opening time and on the closing time for the Cac is equal to 0.303 which is under the value 0.410 which would be expected from a seven hour lagged random walk.

In the case of GGB, observed correlations are close to their expectations, confirming it may follow a random walk. For the Usd/Dem, correlations between close and open rules are slightly above their expectations for short order systems and slightly below for long order ones. The time of the day effect has been investigated further for the Usd/Dem by considering as well positions triggered at twelve pm London time (Table 12). This is just before the announcement of major news and might therefore significate a split between morning and afternoon activities. The results are in this case quite spectacular. All the correlations between midday and open rules are largely above their expectations where most of the correlations between close and midday rules are under their expectations. The reason may well be due to the presence of respectively positive/negative autocorrelations.

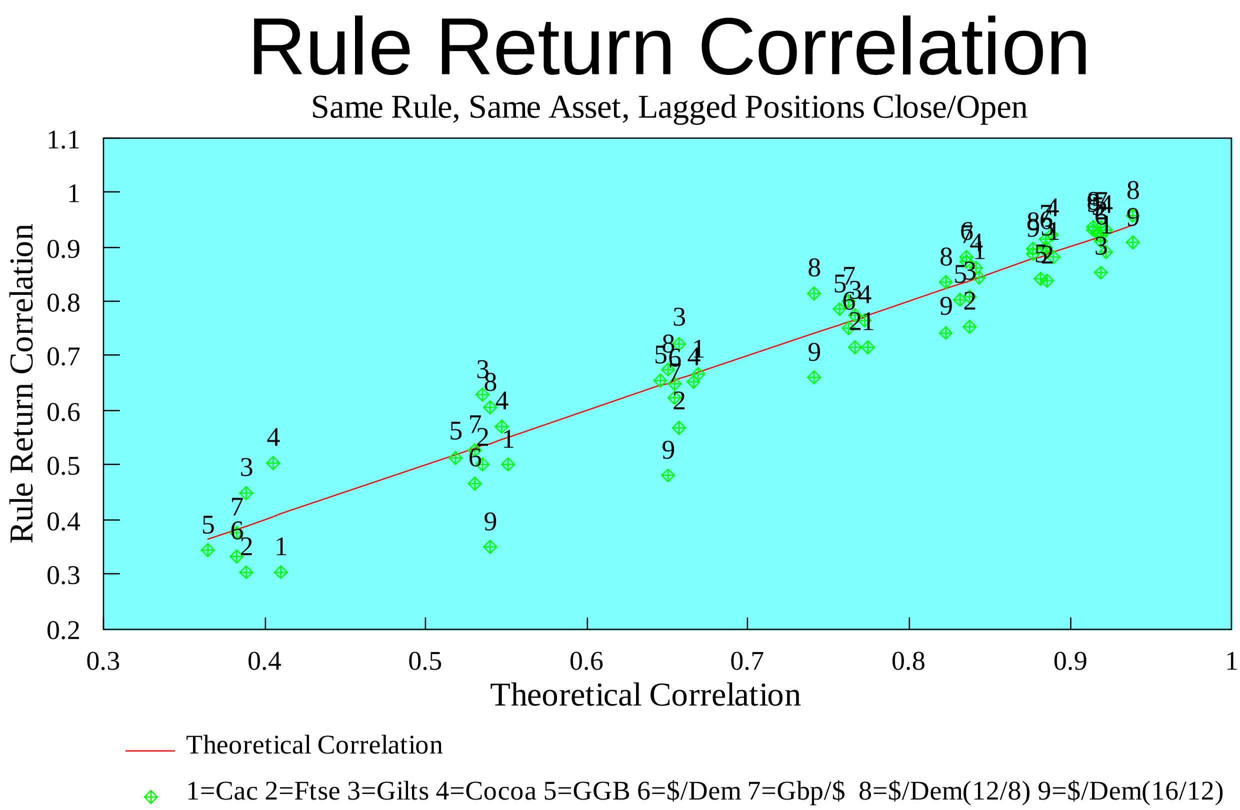

Figure 5: Rules returns correlation: Lagged positions

Table 11: Correlations between identical trading rules triggered on close and on open

	Theory 7h	Cac	Theory 7h45	Ftse	Gilts	Theory 7h10	Cocoa	Theory 8h45	GGB
S(2)	0.410	0.303*	0.388	0.304*	0.448*	0.405	0.503*	0.365	0.345
S(3)	0.551	0.500*	0.535	0.502	0.628*	0.547	0.571	0.518	0.514
S(5)	0.669	0.665	0.657	0.569*	0.723*	0.666	0.651	0.645	0.654
S(10)	0.774	0.716*	0.766	0.715*	0.773	0.772	0.765	0.757	0.786*
S(20)	0.843	0.845	0.837	0.753*	0.809*	0.841	0.860*	0.831	0.803
S(40)	0.89	0.881	0.886	0.837*	0.889	0.889	0.923*	0.882	0.840*
S(80)	0.922	0.891*	0.919	0.918	0.853*	0.922	0.931*	0.917	0.926*

* significantly different from ρ_0 at the critical level of 5%

Table 12: Usd/Dem and Gbp/Usd: Correlations between Close and Open rules

	Theory 8h	Usd/Dem Close/Open	Gbp/Usd Close/Open	Theory 4h	Usd/Dem Midday/Open	Usd/Dem Close/Midday
S(2)	0.382	0.332*	0.376	0.540	0.605*	0.349*
S(3)	0.53	0.466*	0.528	0.650	0.674	0.480*
S(5)	0.654	0.649	0.623	0.741	0.815*	0.660*
S(10)	0.763	0.752	0.800*	0.823	0.835	0.743*
S(20)	0.835	0.880*	0.874*	0.877	0.897*	0.886
S(40)	0.885	0.900*	0.913*	0.914	0.937*	0.932*
S(80)	0.919	0.911	0.936*	0.939	0.957*	0.907*

*significantly different from ρ_0 at the critical level of 5%

3.4 Time aggregation

Theoretical correlations are good ex-ante estimates of observed correlations (Table 13). The GGB exhibits the most significant departures. Overall there are slightly more observations over their expected values (15) than under (9). This would favour the hypothesis of trends over long-term intervals.

Table 13: Correlations between daily and each two days trading rules

Disaggregate\Aggregate(2)	Theory	Usd/Dem	Gbp/Usd	Ftse	Cac	Cocoa	GGB	Gilt
S(2)	0.250	0.253	0.363*	0.191*	0.202	0.331*	0.176*	0.297
S(3)	0.414	0.362*	0.462*	0.408	0.346*	0.379	0.415	0.403
S(5)	0.520	0.559*	0.588*	0.478*	0.529	0.485	0.510	0.475*
S(10)	0.599	0.552*	0.678*	0.570	0.622	0.617	0.633*	0.645*
S(20)	0.641	0.628	0.629	0.697*	0.742*	0.711*	0.748*	0.585*
S(40)	0.665	0.671	0.667	0.699*	0.711*	0.596*	0.752*	0.650

*significantly different from ρ_0 at the critical level of 5%

4 Conclusions

Correlations between trading rules applied to a same asset are non-zero, and even highly positive for trend-following systems. Correlations between a same trading rule applied to multi-assets are positive but lower in absolute value than correlations between underlying markets.

On the one hand, theoretical correlations between trading rules applied to a same underlying market do not depend on the market itself but only on the rules. This is equivalent to the assumption that the past correlation matrix contain information about what the rules correlation will be in the future but no information about the market. This assumes that there is a common mean correlation between rules irrespective of the markets.

On the other hand, theoretical correlations between a same trading rule applied to different underlying markets, do not depend on the rule itself but only on the underlying markets. This assumes that there is a common mean correlation between underlying markets irrespective of the dynamic trading rules.

The time of the day effect is measured by establishing the correlation between identical daily trading rules applied to the same underlying market but lagged a few hours. The correlation is a negative function of the lag up to twelve hours. This increases when the order of the rule increases.

The success of theoretical correlations in forecasting observed trading rule correlations should not be surprising. Indeed, Elton and Gruber (1973), Elton, Gruber and Urich (1978) have exhibited the usefulness of averaging (smoothing) some of the data in the historical correlation matrix as a forecast of the future. The knowledge of theoretical trading rule correlations has many potential applications. Firstly, this might help to build an efficient portfolio. Secondly, this might also allow to build new tests of random walk from the joint profitability of technical trading rules.

APPENDIX

PROOFS OF PROPOSITIONS

Proposition 1

By assumption, $X_{1,t}$ and $X_{2,t}$ are normally distributed with:

$E(X_{1,t}) = 0 \quad E(X_{2,t}) = 0 \quad Var(X_{1,t}) = \sigma_1^2 \quad Var(X_{2,t}) = \sigma_2^2$

That implies that $F_{1,t}$ and $F_{2,t}$ are normally distributed with:

$$E(F_{1,t}) = 0 \quad E(F_{2,t}) = 0 \quad Var(F_{1,t}) = \sigma_1^2 \sum_{i=0}^{m_1-2} d_{1,i}^2 \quad Var(F_{2,t}) = \sigma_2^2 \sum_{i=0}^{m_2-2} d_{2,i}^2$$

$$Cov(F_{1,t},F_{2,t}) = E(F_{1,t}\, F_{2,t}) = \sigma_1 \sigma_2 \rho_x \sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i} d_{2,i}$$

$$\Rightarrow \qquad Corr(F_{1,t},F_{2,t}) = \rho_{F_{1,2}} = \rho_x\ \rho_F \text{ where } \rho_F = \frac{\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i} d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}}$$

$E(B_{1,t}) = Pr(F_{1,t}>0)–Pr(F_{1,t}<0) = 1–2.Pr(F_{1,t}<0) = 0.$

That is due to the fact that the distribution of the linear unbiased forecaster, $F_{1,t}$, is symmetrical around zero, as for the underlying returns X_t. Then, it follows that:

Similarly, $E(B_{2,t}) = 0$.

$E(B_{1,t}^2) = E(B_{2,t}^2) = 1$

$\Rightarrow Var(B_{1,t}) = Var(B_{2,t}) = 1$

$$\begin{aligned}
\Rightarrow \qquad \rho(B_{1,t},B_{2,t}) &= Cov(B_{1,t},B_{2,t}) = E(B_{1,t}.B_{2,t}) \\
&= Pr(F_{1,t}>0,F_{2,t}>0)+Pr(F_{1,t}<0,F_{2,t}<0)–Pr(F_{1,t}>0,F_{2,t}<0)–Pr(F_{1,t}<0,F_{2,t}>0) \\
&= 2\,\{Pr(F_{1,t}>0,F_{2,t}>0)–Pr(F_{1,t}>0,F_{2,t}<0)\} \text{ by symmetry reason.} \\
&= 2\,\{[0,0](\rho_{F_{1,2}})–[0,0](-\rho_{F_{1,2}})\}
\end{aligned}$$

where $\rho_{F_{1,2}}$ has just be defined and [0,0] is the bivariate truncated probability given by Johnson and Kotz (1972):

$$\rho_B = \rho(B_{1,t},B_{2,t}) = \frac{2}{\pi} Arc\sin(\rho_x \rho_F)$$

Acar (1993, Chapter 3) has shown that if the underlying time series X_t are independent identically distributed following a normal law without drift and variance σ^2, linear rule returns R_t are independent identically distributed following a normal law without drift and variance σ^2.

Applying this result, it follows that rule returns $R_{1,t}$ and $R_{2,t}$ are normally distributed with:

$E(R_{1,t}) = 0 \qquad E(R_{2,t}) = 0 \qquad \text{and} \qquad Cov(R_{1,t},R_{1,t+h}) = 0$

$Var(R_{1,t}) = \sigma_1^2 \qquad Var(R_{2,t}) = \sigma_2^2 \quad \text{and} \quad Cov(R_{2,t},R_{2,t+h}) = 0$

Covariances between trading rules are deduced from:

$\mathrm{Cov}(R_{1,t},R_{2,t}) = E(R_{1,t}R_{2,t}) = E(B_{1,t-1}B_{2,t-1}X_{1,t}X_{2,t}) = E(B_{1,t-1}B_{2,t-1})E(X_{1,t}X_{2,t})$

Applying equation [**Error! Bookmark not defined.**]:

$E(B_{1,t-1}B_{2,t-1}) = \rho(B_{1,t-1},B_{2,t-1}) = \frac{2}{\pi}\mathrm{Arc}\sin(\rho_x\rho_F)$

Since by assumption $E(X_{1,t}X_{2,t}) = \sigma_1\sigma_2\rho_x$, it follows that:

$\mathrm{Cov}(R_{1,t},R_{2,t}) = \sigma_1\sigma_2\rho_x\frac{2}{\pi}\mathrm{Arc}\sin(\rho_x\rho_F)$, and then:

$$\rho(R_{1,t},R_{2,t}) = \frac{E(R_{1,t}R_{2,t})}{\sqrt{\mathrm{Var}(R_{1,t})\mathrm{Var}(R_{2,t})}} = \frac{\sigma_1\sigma_2\rho_x\frac{2}{\pi}\mathrm{Arc}\sin(\rho_x\rho_F)}{\sigma_1\sigma_2}$$

$$\Rightarrow \qquad \rho_R = \rho(R_{1,t},R_{2,t}) = \frac{2}{\pi}\rho_x\mathrm{Arc}\sin(\rho_x\rho_F) \qquad (1.5)$$

In addition, $\rho(R_{1,t},R_{2,t+h})=\rho(R_{1,t+h},R_{2,t})=0$ for $h>0$

That can be shown considering that:

$\mathrm{Cov}(R_{1,t},R_{2,t+h}) = E(B_{1,t-1}B_{2,t+h-1}X_{1,t}X_{2,t+h}) = E(B_{1,t-1}B_{2,t+h-1}X_{1,t})E(X_{2,t+h}) = 0$

$\mathrm{Cov}(R_{1,t+h},R_{2,t}) = E(B_{1,t+h-1}B_{2,t-1}X_{1,t+h}X_{2,t}) = E(B_{1,t+h-1}B_{2,t-1}X_{2,t})E(X_{1,t+h}) = 0$

Proposition 2: Times of the day

Let us note:

$P_{i,t}$ is the daily price of asset i {i=1,2}, at close time t. The opening time is noted t*. It is supposed that t*=t-y where y=n/24 and n is the number of hours separating the opening from closing time. Values of y are included between 0 and 1.

$X_{i,t} = \mathrm{Ln}(P_{i,t} / P_{i,t-1})$ the underlying close to close return.

$X_{i,t^*} = \mathrm{Ln}(P_{i,t^*} / P_{i,t^*-1})$ the underlying open to open return.

The first trading rule is applied to asset 1 from close to close and the second one to asset 2 from open to open. The close and open are supposed to be lagged n hours (y=n/24 daily fraction). The rules we apply are linear and can be expressed as follow:

$$F_{1,t} = \sum_{j=0}^{m_1-1} d_{1,j}X_{1,t-j}$$

$$F_{2,t^*} = \sum_{j=0}^{m_2-1} d_{2,j}X_{2,t^*-j}$$

To establish the correlation coefficient, we must therefore evaluate rates of return over the same period, let say from close to close. Then, following previous notations:

$R_{1,t} = B_{1,t-1}\mathrm{Ln}(P_{1,t} / P_{1,t-1}) = B_{1,t-1}\mathrm{Ln}(P_{1,t^*} / P_{1,t-1}) + B_{1,t-1}\mathrm{Ln}(P_{1,t} / P_{1,t^*})$

$R_{2,t} = B_{2,t^*-1}\mathrm{Ln}(P_{2,t^*} / P_{2,t-1}) + B_{2,t^*}\mathrm{Ln}(P_{2,t} / P_{2,t^*})$

We now assume that the underlying assets follow a continuous bivariate normal random walk with:

$E(X_{1,t}) = 0$, $E(X_{2,t}) = 0$, $\mathrm{Var}(X_{1,t}) = \sigma_1^2$, $\mathrm{Var}(X_{2,t}) = \sigma_2^2$ and $E(X_{1,t}X_{2,t}) = \rho_x\sigma_1\sigma_2$

Then:

$$E(R_{1,t}R_{2,t}) = E(B_{1,t-1}B_{2,t^*-1})\,(1-y)\rho_x\sigma_1\sigma_2 + E(B_{1,t-1}B_{2,t^*})\,y\rho_x\sigma_1\sigma_2$$

Using the results of proposition 1 and after some arrangements, it is 4traightforward to show that:

$$E(B_{1,t-1}B_{2,t^*-1}) = \frac{2}{\pi}\,\mathrm{Arcsin}(\rho_x \frac{(1-y)\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i} + y\sum_{i=0}^{Min(m_1,m_2)-3} d_{1,i}d_{2,i+1}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}})$$

$$E(B_{1,t-1}B_{2,t^*}) = \frac{2}{\pi}\,\mathrm{Arcsin}(\rho_x \frac{y\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i} + (1-y)\sum_{i=0}^{Min(m_1,m_2)-3} d_{1,i+1}d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}})$$

Since $Var(R_{1,t}) = \sigma_1^2$, $Var(R_{2,t}) = \sigma_2^2$, we have:

$$\rho_R = (1-y)\frac{2}{\pi}\rho_x\,\mathrm{Arcsin}(\rho_x \frac{(1-y)\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i} + y\sum_{i=0}^{Min(m_1,m_2)-3} d_{1,i}d_{2,i+1}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}}) + y\frac{2}{\pi}\rho_x\,\mathrm{Arcsin}(\rho_x \frac{y\sum_{i=0}^{Min(m_1,m_2)-2} d_{1,i}d_{2,i} + (1-y)\sum_{i=0}^{Min(m_1,m_2)-3} d_{1,i+1}d_{2,i}}{\sqrt{\sum_{i=0}^{m_1-2} d_{1,i}^2}\sqrt{\sum_{i=0}^{m_2-2} d_{2,i}^2}}) \quad (2.1)$$

Proposition 3: Times aggregation

Let us note X_t^* the underlying return over n days:

$$X_t^* = X_t + X_{t-1} + ... + X_{t-n+1}$$

We assume that a first trading rule is applied to daily rates of returns. The forecast is defined as:

$$F_{1,t} = \sum_{i=0}^{m_1-1} \lambda_i X_{t-i}$$

Then a second trading rule is applied to aggregated rates of returns. The forecast is defined as:

$$F_{2,t} = \sum_{i=0}^{m_2-1} \gamma_i X_{t-i}^* = \sum_{i=0}^{m_2-1} \gamma_i \left(\sum_{j=i}^{i+n-1} X_{t-j}\right)$$

The rates of return generated by both strategies are over the period of n days:

$$R_{1,n} = B_{1,t}X_{t+1} + B_{1,t+1}X_{t+2} + ... + B_{1,t+n-1}X_{t+n}$$

$$R_{2,n} = B_{2,t}X_{t+1} + B_{2,t}X_{t+2} + ... + B_{2,t}X_{t+n}$$

Then we can work out what is the correlation coefficient between the two returns.

$$E(R_{1,n}) = E(R_{2,n}) = 0$$

$$V(R_{1,n}) = V(R_{2,n}) = n\sigma^2$$

$$E(R_{1,n}R_{2,n}) = \{E(B_{1,t}B_{2,t}) + E(B_{1,t+1}B_{2,t}) + ... + E(B_{1,t+n-1}B_{2,t})\}\sigma^2$$

where $E(B_{1,t}B_{2,t+k}) = \frac{2}{\pi}\mathrm{Arcsin}(\rho_{t+k})$ where $\rho_{t+k} = \frac{E(F_{1,t}F_{2,t+k})}{\sqrt{Var(F_{1,t})Var(F_{2,t})}}$ k=0, n-1 (See proposition 1).

Then

$$Corr(R_{1,n}R_{2,n}) = \frac{E(B_{1,t}B_{2,t}) + E(B_{1,t+1}B_{2,t}) + ... + E(B_{1,t+n-1}B_{2,t})}{n}$$

REFERENCES

Acar, E. (1993), "Economic evaluation of financial forecasting", Ph.D Thesis, City University London

Aggarwal, R., and Park, Y.S. (1994), "The Relationships Between Daily U.S. and Japanese Equity Prices: Evidence from Spot Versus Futures Markets", *Journal of Banking and Finance* 18, pp 757-773

Brock, W., Lakonishok, J. and LeBaron, B. (1992), "Simple Technical Rules and the Stochastic Properties of Stock Returns", *Journal of Finance*, 47, pp 1731-1764

Brorsen, B.W. and Boyd, M.S. (1990), "Reducing profit variability from technical trading systems", *Technical Analysis of Stocks and Commodities*, March, pp 23-32

Brorsen, B.W. and Lukac, L.P. (1990), "Optimal portfolios for commodity futures funds", *Journal of Futures Markets*, 10(3), pp 247-258

Dacorogna, M.M., Müller, U.A., Jost, C., Pictet O.V., Olsen R.B., and J.R. Ward (1994), "Heterogeneous Real-Time Trading Strategies in the Foreign Exchange Market", *Presentation at the International Conference on Forecasting Financial Markets: New Advances for Exchange Rates and Stock Prices, 2-4 February 1994, in London*

Dunis, C. (1989), *"Computerised technical systems and exchange rate movements"*, in C.Dunis and M.Feeny (eds), *Exchange Rates Forecasting*, Woodhead-Faulkner, pp 165-205

Elton, E.J. and Gruber, M.J. (1973), "Estimating the dependence structure of share prices-Implications for portfolio selection", *Journal of Finance*, 8(5), pp 1203-1232

Elton, E.J., Gruber, M.J. and Urich, T.(1978), "Are betas best ?", *Journal of Finance*, 23(5), pp 1375-1384

Johnson, N.L. and Kotz, S. (1972), "*Distributions in Statistics: Continuous Multivariate Distributions*", New York: Wiley

Johnson, R.A and Wichern, D.W. (1982), *"Applied Multivariate Statistical Analysis"*, New Jersey: Prenctice Hall

Lukac, L.P., Brorsen, B.W. and Irwin, S.H. (1988), "Similarity of computer guided technical trading systems", *Journal of Futures Markets*, 8(1), pp 1-13

LeBaron, B. (1991), "Technical Trading Rules and Regime Shifts in Foreign Exchange", University of Wisconsin, Social Science Research, Working Paper 9118

LeBaron, B. (1992), "Do Moving Average Trading Rule Results Imply Nonlinearities in Foreign Exchange Markets", University of Wisconsin, Social Science Research, Working Paper 9222

Levich, R.M. and Thomas, L.R. (1993), "The significance of technical trading-rule profits in the foreign exchange market: a bootstrap approach", *Journal of International Money and Finance*, 12, pp 451-474

Lin, W.L., Engle, R.F., and Ito, T. (1994), "Do Bull and Bears Move Across Borders ? International Transmission of Stock Returns and Volatility", *The Review of Financial Studies*, Fall 7(3), pp 507-538

Neftci, S.N. (1991), "Naive trading rules in financial markets and Wiener-Kolmogorov prediction theory: a study of "technical analysis"", *Journal of Business*, 64, pp 549-571

Neftci, S.N. and Policano, A.J. (1984), "Can chartists outperform the market? Market efficiency tests for "Technical Analysis"", *Journal of Futures Markets*, 4(4), pp 465-478

Prado, R. (1992), *"Design, Testing, and Optimization of Trading Systems"*, John Wiley & Sons

Praetz, P.D. (1979), "A general test of a filter effect", *Journal of Financial and Quantitative Analysis*, 14, pp 385-394

Silber, W. (1994), "Technical trading: when it works and when it doesn't", *The Journal of Derivatives*, Spring, pp 39-44

Surujaras, P. and Sweeney, R.J. (1992), *"Profit-making speculation in foreign exchange markets"*, The political economy of global interdependence, Westview Press

Sweeney, R.J. (1986), "Beating the Foreign Exchange Market", *Journal of Finance*, 41, pp 163-182

Sweeney, R.J. (1988), "Some new filter rule tests: method and results", *Journal of Financial and Quantitative Analysis*, 23, pp 285-300

Taylor, S.J. (1986), *"Modelling Financial time series"*, Chichester, England: John Wiley and Sons

Taylor, S.J. (1990a), "Reward available to currency futures speculators: Compensation for risk or evidence of inefficient pricing ? ", *Economic Record (supplément)*, 68, pp 105-116

Taylor, S.J. (1990b), *"Profitable currency futures trading: a comparison of technical and time-series trading rules"*, in L.R Thomas (eds), *The Currency hedging debate*, IFR Publishing Ltd., London, pp 203-239

Taylor, S.J. (1992), *"Efficiency of the Yen futures market at the Chicago Mercantile Exchange"*, in Barry A Goss (eds), *"Rational expectations and efficient in Future Markets"*, pp 109-128

Taylor, S.J. (1994), "Trading Futures Using the Channel Rule: A Study of the Predictive Power of Technical Analysis with Currency Examples", *Journal of Futures Markets*, 14(2), pp 215-235

Working, H. (1960), "Note on the correlation of first differences averages in a random chain", *Econometrica*, 28, pp 916-918

3.3 *Correlation Close/Close with Open/Open*

4 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.54	0.526	0.424	0.302	0.213	0.15	0.106
S(3)		0.65	0.593	0.427	0.302	0.213	0.15
S(5)			0.741	0.604	0.428	0.302	0.213
S(10)				0.823	0.645	0.455	0.321
S(20)					0.877	0.667	0.469
S(40)						0.914	0.678
S(80)							0.939
7 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.41	0.441	0.375	0.273	0.195	0.138	0.098
S(3)		0.551	0.535	0.399	0.284	0.202	0.143
S(5)			0.669	0.572	0.412	0.292	0.207
S(10)				0.774	0.628	0.447	0.316
S(20)					0.843	0.658	0.465
S(40)						0.89	0.674
S(80)							0.922
7.5 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.395	0.431	0.369	0.27	0.193	0.137	0.097
S(3)		0.54	0.527	0.395	0.282	0.2	0.142
S(5)			0.661	0.568	0.41	0.291	0.206
S(10)				0.768	0.626	0.446	0.315
S(20)					0.839	0.657	0.464
S(40)						0.887	0.673
S(80)							0.92
7.75 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.388	0.426	0.366	0.268	0.192	0.136	0.096
S(3)		0.535	0.524	0.393	0.281	0.199	0.141
S(5)			0.657	0.566	0.409	0.29	0.205
S(10)				0.766	0.625	0.445	0.315
S(20)					0.837	0.656	0.464
S(40)						0.886	0.673
S(80)							0.919
8 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.382	0.421	0.363	0.267	0.191	0.135	0.096
S(3)		0.53	0.521	0.392	0.28	0.199	0.141
S(5)			0.654	0.565	0.408	0.29	0.205
S(10)				0.763	0.624	0.445	0.315
S(20)					0.835	0.656	0.464
S(40)						0.885	0.672
S(80)							0.919
8.75 H	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.365	0.409	0.356	0.263	0.188	0.133	0.095
S(3)		0.518	0.512	0.388	0.278	0.197	0.14
S(5)			0.645	0.56	0.406	0.288	0.204
S(10)				0.757	0.621	0.444	0.314
S(20)					0.831	0.654	0.463
S(40)						0.882	0.672
S(80)							0.917

Usd/Dem Close\Open							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.332*	0.358*	0.297*	0.222	0.178	0.118	0.174*
S(3)		0.466*	0.488	0.360	0.276	0.147	0.181
S(5)			0.649	0.550	0.444	0.263	0.240
S(10)				0.752	0.609	0.372*	0.246*
S(20)					0.880*	0.669	0.434
S(40)						0.900*	0.691
S(80)							0.911
Usd/Dem Close/Midday							
Clo/Mid	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.349*	0.321*	0.287*	0.212*	0.198	0.161	0.157
S(3)		0.480*	0.447*	0.360	0.294	0.185	0.169
S(5)			0.660	0.548	0.484*	0.246	0.206
S(10)				0.743*	0.629	0.342*	0.239*
S(20)					0.886*	0.643	0.415*
S(40)						0.932*	0.693
S(80)							0.907*
Usd/Dem Midday/Open							
Mid\Open	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.605*	0.529	0.429	0.291	0.183	0.076*	0.097
S(3)		0.674	0.607	0.437	0.304	0.179	0.136
S(5)			0.815*	0.629	0.386	0.191*	0.174
S(10)				0.835	0.633	0.383*	0.265*
S(20)					0.897	0.652	0.442
S(40)						0.937*	0.726*
S(80)							0.957*
Gbp/Usd							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.376	0.400	0.324	0.245	0.185	0.150	0.153*
S(3)		0.528	0.502	0.380	0.244	0.173	0.133
S(5)			0.623	0.594	0.451	0.350*	0.215
S(10)				0.800*	0.667*	0.477	0.325
S(20)					0.874*	0.665	0.478
S(40)						0.913*	0.654
S(80)							0.936*
Cac							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.303*	0.409	0.364	0.179*	0.148	0.140	0.095
S(3)		0.500*	0.510	0.277*	0.217*	0.143*	0.106
S(5)			0.665	0.523*	0.377	0.316	0.220
S(10)				0.716*	0.576*	0.432	0.340
S(20)					0.845	0.734*	0.571*
S(40)						0.881	0.683
S(80)							0.891*

*significantly different from ρ_0 at the critical level of 5%

Ftse							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.304*	0.319*	0.267*	0.136*	0.069*	-0.027*	-0.109*
S(3)		0.502	0.497	0.290*	0.229	0.113*	0.065*
S(5)			0.569*	0.454*	0.284*	0.164*	0.085*
S(10)				0.715*	0.513*	0.378*	0.285
S(20)					0.753*	0.650	0.485
S(40)						0.837*	0.695
S(80)							0.918
Cocoa							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.503*	0.553*	0.428*	0.317*	0.210	0.132	-0.038*
S(3)		0.571	0.483*	0.379	0.333*	0.283*	0.074*
S(5)			0.651	0.593	0.414	0.330	0.137
S(10)				0.765	0.635	0.455	0.209*
S(20)					0.860*	0.761*	0.406*
S(40)						0.923*	0.612*
S(80)							0.931*
German Government Bond							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.345	0.385	0.362	0.328*	0.270*	0.198*	0.094
S(3)		0.514	0.543	0.443*	0.384*	0.289*	0.210*
S(5)			0.654	0.546	0.433	0.343*	0.244
S(10)				0.786*	0.617	0.497*	0.332
S(20)					0.803	0.671	0.544*
S(40)						0.840*	0.718*
S(80)							0.926*
Gilts							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.448*	0.471*	0.417*	0.293	0.108*	0.087	-0.054*
S(3)		0.628*	0.545	0.396	0.197*	0.110*	-0.053*
S(5)			0.723*	0.563	0.311*	0.214*	0.057*
S(10)				0.773	0.568*	0.420	0.174*
S(20)					0.809*	0.721*	0.425
S(40)						0.889	0.659
S(80)							0.853*

*significantly different from ρ_0 at the critical level of 5%

3.4 *Time aggregation*

Rule returns correlation under the random walk assumption

Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.250	0.218	0.173	0.123	0.087	0.062	0.044
S(3)	0.500	0.414	0.318	0.223	0.157	0.111	0.078
S(5)	0.583	0.658	0.520	0.365	0.256	0.181	0.128
S(10)	0.468	0.621	0.784	0.599	0.422	0.297	0.210
S(20)	0.342	0.461	0.626	0.858	0.641	0.452	0.318
S(40)	0.245	0.329	0.449	0.658	0.904	0.665	0.467
S(80)	0.174	0.233	0.318	0.467	0.674	0.934	0.677

	Usd/Dem					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.253	0.177	0.182	0.084	0.070	0.180*
S(3)	0.475	0.362*	0.318	0.178	0.119	0.180*
S(5)	0.608	0.630	0.559*	0.365	0.235	0.216
S(10)	0.474	0.658*	0.776	0.552*	0.335*	0.232*
S(20)	0.270*	0.476	0.593	0.854	0.628	0.462
S(40)	0.136*	0.206*	0.314*	0.612*	0.887*	0.671
S(80)	0.168	0.189	0.241*	0.454	0.651	0.948*
	Gbp/Usd					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.363*	0.282*	0.218	0.170	0.126	0.101
S(3)	0.628*	0.462*	0.363	0.252	0.147	0.122
S(5)	0.634*	0.729*	0.588*	0.442*	0.307*	0.178
S(10)	0.479	0.706*	0.847*	0.678*	0.450	0.280
S(20)	0.377	0.556*	0.706*	0.917*	0.629	0.401*
S(40)	0.255	0.427*	0.445	0.653	0.924*	0.667
S(80)	0.198	0.307*	0.294	0.449	0.705*	0.958*
	Ftse					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.191*	0.147*	0.078*	0.018*	-0.030*	-0.110*
S(3)	0.513	0.408	0.250*	0.192	0.122	0.079
S(5)	0.554	0.637	0.478*	0.323	0.193*	0.083*
S(10)	0.405*	0.534*	0.770	0.570	0.413	0.287*
S(20)	0.374	0.395*	0.610	0.889*	0.697*	0.480
S(40)	0.233	0.241*	0.467	0.724*	0.903	0.699*
S(80)	0.180	0.169*	0.334	0.487	0.725*	0.936
	Cac					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.202	0.243	0.196	0.144	0.129	0.097
S(3)	0.469	0.346*	0.213*	0.181	0.145	0.119
S(5)	0.515*	0.690*	0.529	0.424*	0.358*	0.274*
S(10)	0.403	0.607	0.810	0.622	0.516*	0.378*
S(20)	0.311	0.475	0.675*	0.900*	0.742*	0.578*
S(40)	0.255	0.391*	0.552*	0.761*	0.926*	0.711*
S(80)	0.173	0.299*	0.437*	0.600*	0.751*	0.948*
	Cocoa					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.331*	0.279*	0.203	0.140	0.084	-0.052*
S(3)	0.497	0.379	0.300	0.311*	0.217*	0.070
S(5)	0.566	0.636	0.485	0.365	0.275	0.107*
S(10)	0.482	0.644	0.798	0.617	0.392	0.189*
S(20)	0.297	0.432	0.630	0.915*	0.711*	0.401*
S(40)	0.205	0.301	0.411	0.701*	0.964*	0.596*
S(80)	0.045*	0.136*	0.192*	0.423*	0.646	0.939
	German Bunds					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.176*	0.172	0.220	0.183*	0.123*	0.068
S(3)	0.450*	0.415	0.345	0.296*	0.230*	0.217*
S(5)	0.558	0.638	0.510	0.397	0.306*	0.228
S(10)	0.506	0.622	0.822*	0.633*	0.467*	0.341
S(20)	0.448*	0.493	0.717*	0.919*	0.748*	0.573*
S(40)	0.440*	0.427*	0.548*	0.757*	0.955*	0.752*
S(80)	0.393*	0.396*	0.415*	0.591*	0.753*	0.964*

	Gilt					
Disaggregate\Aggregate(2)	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)
S(2)	0.297	0.215	0.135	0.157	0.002*	-0.042*
S(3)	0.520	0.403	0.284	0.232	0.023*	-0.069*
S(5)	0.614	0.629	0.475*	0.354	0.107*	0.042*
S(10)	0.535*	0.690*	0.755*	0.645*	0.359*	0.181*
S(20)	0.350	0.502*	0.656	0.912*	0.585*	0.374*
S(40)	0.150*	0.260*	0.402*	0.662	0.892*	0.650
S(80)	0.055*	0.088*	0.213*	0.431	0.705	0.963*

*significantly different from ρ_0 at the critical level of 5%

Table 12: Usd/Dem and Gbp/Usd: Correlations between Close and Open rules

	Theory 8h	Usd/Dem Close/Open	Gbp/Usd Close/Open	Theory 5h	Usd/Dem One/Open	Theory 3 h	Usd/Dem Close/One
S(2)	0.382	0.332*	0.376	0.488	0.605*	0.603	0.349*
S(3)	0.53	0.466*	0.528	0.610	0.674*	0.697	0.480*
S(5)	0.654	0.649	0.623	0.712	0.815*	0.776	0.660*
S(10)	0.763	0.752	0.800*	0.803	0.835*	0.847	0.743*
S(20)	0.835	0.880*	0.874*	0.863	0.897*	0.893	0.886
S(40)	0.885	0.900*	0.913*	0.904	0.937*	0.925	0.932
S(80)	0.919	0.911	0.936*	0.932	0.957*	0.947	0.907*

*significantly different from ρ_0 at the critical level of 5%

Correlation between Different Markets

Theoretical correlations

Correlation: (Usd/Dem, Gbp/Usd = -0.841)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.535	0.456	0.354	0.249	0.175	0.124	0.087
S(3)		0.535	0.458	0.331	0.234	0.166	0.117
S(5)			0.535	0.436	0.316	0.224	0.159
S(10)				0.535	0.443	0.324	0.231
S(20)					0.535	0.446	0.327
S(40)						0.535	0.448
S(80)							0.535

Correlation: (Usd/Dem , Cac = -0.101)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.007	0.006	0.005	0.003	0.002	0.002	0.001
S(3)		0.007	0.006	0.004	0.003	0.002	0.002
S(5)			0.007	0.006	0.004	0.003	0.002
S(10)				0.007	0.006	0.004	0.003
S(20)					0.007	0.006	0.004
S(40)						0.007	0.006
S(80)							0.007

Correlation: (Gbp/Usd , Cac = 0.085)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.005	0.004	0.003	0.002	0.002	0.001	0.001
S(3)		0.005	0.004	0.003	0.002	0.002	0.001
S(5)			0.005	0.004	0.003	0.002	0.002
S(10)				0.005	0.004	0.003	0.002
S(20)					0.005	0.004	0.003
S(40)						0.005	0.004
S(80)							0.005

Correlation: (German Government Bonds, Gilts Underlying = 0.450)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.134	0.119	0.096	0.069	0.05	0.035	0.025
S(3)		0.134	0.119	0.09	0.066	0.047	0.033
S(5)			0.134	0.115	0.087	0.063	0.045
S(10)				0.134	0.116	0.089	0.065
S(20)					0.134	0.117	0.089
S(40)						0.134	0.117
S(80)							0.134

Again observed correlations do not significantly deviate from their theoretical expectations. The most significant departures occur between the Gilts and the GGB and long-term orders trading rules.

Empirical correlations

Correlation: (Usd/Dem, Gbp/Usd = -0.841)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.554	0.434	0.329	0.213	0.134	0.084	0.038
S(3)		0.558	0.442	0.305	0.194	0.110*	0.056*
S(5)			0.557	0.460	0.365*	0.244	0.118
S(10)				0.558	0.444	0.275*	0.153*
S(20)					0.479*	0.458	0.299
S(40)						0.603*	0.524*
S(80)							0.588*
Correlation: (Usd/Dem , Cac = -0.101)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.017	0.021	0.032	0.005	-0.024	-0.071*	-0.057*
S(3)		0.057	0.079*	0.034	0.009	-0.025	-0.038
S(5)			0.073*	0.052	0.078*	0.036	-0.001
S(10)				0.029	0.073*	0.063*	0.023
S(20)					0.140*	0.128*	0.098*
S(40)						0.118*	0.113*
S(80)							0.098*
Correlation: (Gbp/Usd , Cac = 0.085)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	-0.003	-0.009	0.005	0.007	-0.010	-0.038	-0.057*
S(3)		0.010	0.033	0.007	-0.024	-0.015	-0.038
S(5)			-0.004	0.014	-0.013	-0.014	-0.016
S(10)				-0.030	-0.003	-0.003	-0.017
S(20)					0.009	0.028	0.009
S(40)						0.067*	0.049
S(80)							0.064*
Correlation: (German Government Bonds, Gilts Underlying = 0.450)							
	S(2)	S(3)	S(5)	S(10)	S(20)	S(40)	S(80)
S(2)	0.234*	0.182*	0.110	0.104	0.057	0.043	0.038
S(3)		0.200*	0.156	0.136*	0.094	0.073	0.035
S(5)			0.182	0.161	0.109	0.125*	0.069
S(10)				0.174	0.196*	0.171*	0.152*
S(20)					0.288*	0.277*	0.240*
S(40)						0.308*	0.306*
S(80)							0.313*

*significantly different from ρ_0 at the critical level of 5%